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THEORY AND TESTS

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Abstract

The present paper demonstrates that, when some agents who use money have imperfect information regarding the seignorage rates of alternative money suppliers, the equilibrium supply of money involves mixed-strategies. As a consequence, the provision of money is stochastic for strategic reasons. It follows that inflation and seignorage rates are intrinsically stochastic, but that their averages are below the rate of a monopoly central bank. Empirical tests reveal that these predictions are consistent with the free banking era of the United States.

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1. The Free Banking Era

While the provision of base money is now a government operated business, historically there have been several periods of free banking. Over the past decade there has been renewed interest in free banking, and the past performance of the free banking era has been examined from several different perspectives.¹ Three major themes are present in the extant literature: (i) sustainability, (ii) uncertainty and information, and (iii) inflation and seignorage. In many respects, the debate about the effects of deregulation of the money supply is an argument about sustainability; that is, whether a single supplier of money can remain the sole supplier of currency when competitors are free to enter the money supply process. In the Scottish case, for example, White [1984, p. 25-26] argues that when Bank of Scotland lost its monopoly rights it was unable to thwart entry into the money supply process. In addition, many have argued that the era of free banking in the United States led to more uncertainty.² There is also evidence that during the free banking era many members of the nonbank public had imperfect information regarding the alternative currencies and their

¹One focus has been largely normative in spirit, and coincides with the current interest in deregulation; see the provocative essay by Hayek [1978], and the volume edited by Salin [1984]. Another line of research has focused on positive implications of free banking; see the early perceptive analysis by Klein [1974], and the review article by King [1983]. In addition, several interesting empirical studies have been conducted. For example, Rockoff [1974] and Rolnick and Weber [1983, 1984] have investigated the free banking era of the United States, while White [1984] has conducted a detailed study of the Scottish experience.

²See, for example, Rockoff [1974, p. 143] and Friedman [1959].

relative values.³ Lastly, some have argued that the private provision of money may reduce inflation, since competition would provide a check against overissuing; others have argued the converse, see Klein [1974].

While the existing literature offers alternative models to addresses each of these issues, a single coherent framework that integrates all the issues is notably absent; see Fischer's [1986] review essay and King [1983, p. 156]. The link between theory and empiricism has been especially weak; only the hypothesis of wildcat banking has been tested empirically. In Rockoff's [1974] study of free banking in the U.S., he notes a sizeable number of free bank failures and relates these failures to the possibility of wildcat banking. However, Rolnick and Weber [1983, 1984] raise the alternative falling asset price explanation, and refute the wildcat banking hypothesis in favor of the falling asset price hypothesis. While the falling asset hypothesis explains why 15 percent of the banks closed down, it does not explain the behavior of the remaining banks that remained in operation.

The absence of theoretical models of the free banking era not only obscures discussions about the merits of privatization of the money supply, but also hampers the empirical analysis as it is unclear what the hypotheses are that are tested. The aim of the present paper is to develop an analytical framework that captures the above elements, and

³ During the free banking era in the U.S., banks had the obligation to accept notes from other banks a par, while discounts prevailed in the market place. Apparently, complete arbitrage did not prevail, see also Grossman and Stiglitz [1976] who analyze this issue in general and Rolnick and Weber [1986] who provide a particularly relevant example.

to test the predictions of the model based on data from the free banking era in the United States.

We begin by noting that the key difference between a government run central banking system and the free banking system is the difference between a monopoly and an oligopoly model. Hence, we face the difficult task of selecting an appropriate oligopoly model. Toma [1985] has modeled the rivalry between the U.S. Federal Reserve and Treasury in the 1930's and 1940's as an oligopoly game. In Toma's model, each institution could reap part of the total seignorage, by determining their share in the supply of high powered money. The analysis is cast in a Cournot-Nash duopoly setting where the two institutions compete for their share in the supply of the U.S. monetary base. Given the institutional background of that time, the respective quantities of base money, i.e. the shares, are the appropriate strategic variables.

In contrast, during the free banking era different base monies coexisted. As a consequence, 'price competition' is the appropriate strategic variable rather than 'quantity competition'. In particular, when alternative currencies coexist, informed agents are not indifferent between the currencies if the inflation rates differ. Therefore, we focus on an oligopoly in minting under 'price competition,' or more specifically, where debasement rates are the strategic variables.

Most of the free banking laws required banks to accept dollar notes from other banks a par, rendering the different⁴ dollar bills

⁴The notes were distinguishable by the imprinted brand names.

perfectly substitutable and homogenous. If the non bank public were perfectly informed, this would trigger 'price wars' to capture the whole market, see Friedman [1959, p.1] and Klein [1974, p. 430]. However, it is unlikely that all of the nonbank public was perfectly informed about the rates of debasement of alternative suppliers of money. Hence, some consumers demand a currency even if a bank debase above the lowest rate.⁵ This implies, that if the minimum debasement rate is very low, a provider of money may have an incentive to debase rapidly. This occurs if the lost demand due to agents switching to the currencies of another supplier is more than offset by the inflation tax. Thus, there are two opposite forces at work: the incentive to debase slowly in order to attract the fraction of money holders who are informed about the debasement rates, and the incentive to debase rapidly to exploit the uninformed. We show that these two forces render any pure strategy debasement rate non viable. However, it is shown below that a mixed-strategy provides an appropriate response to the two forces.

The upshot of our analysis is that the equilibrium strategies are intrinsically stochastic, thus implying that the individual currency inflation rates are random over time. However, this source of uncertainty is more of a virtue than a vice. While Rolnick and Weber have refuted the wildcat banking hypothesis, the general belief persists that the era of free banking exhibited more uncertainty. This

⁵ Rockoff [1974, p. 145] provides another explanation for the non zero demand. Because of the transactions motive, people used notes of local banks for transactions in their neighborhoods, but switched to more widely accepted notes when travelling.

is precisely what is implied by the mixed-strategy equilibrium, as it implies uncertainty as an equilibrium phenomenon.

2. A Model of Free Banking

We begin by specifying the demand for money, and describing the technology available for minting currency. The money demand functions are based on the quantity equation $MV = PT$. Let $k = k(y, r)$ be the money multiplier that is a function of real income y and the real interest rate r . The relationship between base money B (notes) and money demand M (notes and deposits) is $M = kB$. We employ Cagan's [1956] specification for velocity, so that $V = \exp(\gamma i)$, where the nominal interest rate, i , equals the real interest rate r plus the inflation rate π . Furthermore, we assume a fixed relationship between real income and the transactions: $T = T(y)$. It follows that the demand for real base money, b , is given by

$$b = \frac{B}{P} = \frac{T(y)}{k(y, r)} e^{-\gamma(r+\pi)}.$$

Following Toma [1985], we focus on situations where the real income y and the real interest rate r are exogenous to the model. In this case, the demand for real base money may be written as

$$(1) \quad b = \phi e^{-\gamma\pi},$$

where $\phi = \phi(y, r) = T(y)/k(y, r)e^{\gamma r}$ depends only on y and r , which are assumed to be fixed.

Equation (1) is a reasonable specification of the demand for real base money if money is supplied by one source. During the free banking era, however, several banks issued distinct dollar bills, and one must

modify the demand for the currencies to incorporate interdependencies in the demand for the currencies. One way in which the different currencies are interrelated is through the structure of the demand side. In particular, we will argue that a separation in information causes the demand for monies to become interdependent.

A partition in information, such that some agents are better informed than others, seems natural given the cost involved in gathering information about money supply rates. So many agents are involved in collecting information and arbitrage that the marginal benefits from arbitrage just offset the costs of acquiring the information. For the period of free banking, the nonbank public can be divided into two classes: those who were informed of inflation rates of alternative money suppliers and those who were uninformed about the alternative inflation rates. The former class is comprised of brokers of bank notes and the users of bank note reports. For the latter class (comprised of agents primarily involved in local trade), the costs of gathering information exceed the benefits. It is assumed that the informed wish to hold the currency of the bank that inflates at the lowest rate, while the uninformed simply hold the "local" currency or, alternatively, choose a currency at random. Hence, each bank enjoys some positive demand for its notes from its share of the uninformed. In addition, extra demand materializes if it debases at the lowest rate. In this manner, the demands for district dollar bills were interdependent.

The impact of informed and uninformed agents on the demand for real base money is modelled by means of two multiplicative scale

factors α , β such that $\alpha > \beta$. Define the following generic scale factor τ_i :

$$(2) \quad \tau_i = \begin{cases} \alpha & \text{if } \pi_i < \pi_j \quad \forall i \neq j \\ \beta & \text{if } \pi_i > \pi_j \text{ for some } j \neq i. \end{cases}$$

If m firms simultaneously have the lowest inflation rate, it is assumed that τ_i is a weighted average of α and β . Given this definition of τ_i , the demand for mint i 's base money is given by

$$(3) \quad b_i = \tau_i \phi e^{-\gamma \pi_i}, \quad i = 1, \dots, n.$$

Intuitively, if bank i has the lowest inflation rate, it gets all of the informed and its share of the uninformed, and $\tau_i = \alpha$. If the bank does not have the lowest inflation rate, it loses the informed to the bank with the lowest inflation rate, and τ_i takes on the smaller value of β . Thus the difference $(\alpha - \beta)$ is the factor by which demand increases when a bank sets the lowest inflation rate.

The supply of money derives from seignorage maximization by each individual free bank. For simplicity we detach the provision of money business from the other banking activities provided by the banks, i.e. we assume that the profit function is separable in the two services. Furthermore, we assume that the cost of maintaining the stock of money exhibits decreasing average costs due to increasing returns to scale over the relevant range.⁶ Several scale factors may be identified: (i) some fixed costs are incurred in maintaining the stock of notes; (ii) with sufficient cross sectional independence in individual's stochastic

⁶ The issuance cost like the printing plates are essentially sunk costs, but the maintenance costs are recurrent.

demand for redemption, the reserve ratio can decrease with the size of the bank due to Arrow and Lind's diversification argument, see King [1983, p. 133]; and (iii), as argued by Friedman [1959, p.7] the value of the currency must be above marginal cost to "prevent" the return to a commodity standard. For simplicity, these costs K_i , not necessarily equal for each i , are assumed to be constant.

Following Friedman [1953], Bailey [1956] and Toma [1985], the revenue from the provision of money equals the inflation rate π_i times the real base demanded b_i , i.e. the 'inflation tax'. Hence, the seignorage function of the i -the 'mint' is given by

$$(4) \quad S_i = \pi_i b_i - K_i.$$

In combination with equation (3), the revenues are positive for all inflation rates, and negative in the case of deflation. For K_i sufficiently small, seignorage is positive over some range as well. Note that, for a given r_i , seignorage is maximal at $\pi_i = 1/\gamma$.

Evidently, if money demand was supplied by a central bank acting as a monopolist, it would maximize its seignorage by inflating at the rate $1/\gamma$. Is inflating at $1/\gamma$ a viable strategy as well in case of competitive banking? Consider the case of n free banks participating in the money supply process.⁷ Due to the presence of informed agents the public favors the 'good' money over 'bad' money, i.e. prefers low inflation rates, and the low inflation currency experiences a demand advantage, see equation (2). Hence, it pays for a particular bank to

⁷While entry was free in principle, the free banking acts were such that the number of banks in the industry may be considered as fixed in the short run.

'undercut' the others if the others are inflating at rate $1/\gamma$. By inflating at a slightly lower rate the bank loses some profits, but this loss is more than compensated for by the gain in demand through α . In fact, this type of "deflationary war" destroys pure strategies for any $\pi_i \in (s, 1/\gamma]$, where $s < 1/\gamma$ is the inflation rate such that seignorage is zero when a bank gets all of the informed. More precisely, s is defined implicitly by

$$S_i(\pi_i=s | \tau = \alpha) = S_i(\pi_i=1/\gamma | \tau = \beta).$$

If, however, all competitors choose $\pi = s$, a bank is better off by inflating at $\pi = 1/\gamma$. The reason is that, if multiple banks charge the lowest inflation rate, they must share the scale factor, α . The gain from the switch in π more than outweighs the loss in demand, c.f. equations (2) and (3). It follows that no $\pi_i \in [s, 1/\gamma]$ is a pure strategy Nash equilibrium. The intuition behind this result is as follows. Throughout the interval $(s, 1/\gamma]$, competitive forces exert deflationary pressure on π_i . But at s , the incentive to exploit monopolistic power over the uninformed becomes dominant. This renders any pure strategy non viable. What is needed is a strategy such that the two forces--competitive and monopolistic--exactly offset one another for all $\pi_i \in [s, 1/\gamma]$. Such a strategy is the mixed strategy, where the π_i 's are chosen at random from a particular distribution function. This procedure mixes the two opposite forces in such a way that they balance and generate constant expected profits for any $\pi_i \in [s, 1/\gamma]$. Moreover, by randomizing the inflation rates, a bank precludes its rivals from being able systematically to undercut its own inflation rate.

The mixed-strategy Nash equilibrium for the above game may be derived as follows. Define the profits for the i -the bank, which depend on whether it experiences the scale benefits, as:

$$(5) \quad S_i = \begin{cases} W(\pi) & \text{if } \tau_i = \alpha \\ L(\pi) & \text{if } \tau_i = \beta \end{cases}$$

Recall that $\tau_i = \alpha$ if bank i has the lowest inflation rate, while $\tau_i = \beta$ if some bank other than i has the lowest inflation rate. Suppose all other $n-1$ banks employ a mixed strategy setting π according to a distribution function $F(\pi)$. Hence, with probability:

$A(\pi) = (1 - F(\pi))^{n-1}$ the i -the firm inflates at the lowest rate and captures the scale benefits α . With probability: $1 - A(\pi)$ it does not inflate at the lowest rate and experiences a scale effect on the demand side of $\beta < \alpha$.⁸ Assuming that the i -the firm employs a mixed strategy with distribution function $F(\pi)$ over the domain $[s, m]$, its expected seignorage is

$$(6) \quad ES = \int_s^m \{A(\pi)W(\pi) + (1-A(\pi))L(\pi)\} dF(\pi).$$

The objective of the i -the bank is to maximize ES by choosing $dF(\pi)$ appropriately. We will show that the distribution function (mixed-strategy)

$$F(\pi) = 1 - A(\pi)^{1/(n-1)} \text{ with support } [s, m],$$

such that

$$(7) \quad A(\pi)W(\pi) + (1-A(\pi))L(\pi) = C_1,$$

⁸ Note that with an absolutely continuous distribution $F(\pi)$ the case $\pi_i = \pi_j$ of equation (2) has probability zero. Therefore, this happenstance is not defined in equation (5), as it will play no role in equation (6).

C_1 is a constant such that $C_1 = L(m)$, $m = 1/\gamma$, and $W(s) = C_1$ implicitly defines s , comprises a Nash equilibrium to the above game.⁹

In order to verify that this indeed constitutes a Nash equilibrium, we must show that if all banks use this strategy, no individual bank has an incentive to deviate from the strategy. Moreover, as the equilibrium involves randomized strategies, we must verify F defined above is a well-defined distribution function. We begin by verifying the latter point, and then demonstrate that no firm has an incentive to use an alternative strategy.

In order to show that $F(\pi)$ is well defined, derive an explicit expression for $A(\pi)$ from equation (7) while using (5), (4) and (3):

$$(8) \quad A(\pi) = \frac{\beta}{\alpha - \beta} \left[\frac{m}{\pi} e^{\gamma(\pi - m)} - 1 \right].$$

Note that as $A(\pi)$ is continuous in π and that knowledge of $A(\pi)$ is equivalent to knowledge of $F(\pi)$, since $H(\pi) = 1 - A(\pi)^{1/(n-1)}$. For $F(\pi)$ to be a well-defined distribution function, $A(\pi)$ must exhibit the following properties: (i) $A(r) = 1$ and $A(m) = 0$, (ii) $A(\pi)$ is a monotone and non-increasing function, and (iii) $A(\pi)$ is right continuous. Evidently, for $m = 1/\gamma$, $A(m) = 0$ from equation (8). As $A(\pi) = [L(m) - L(\pi)]/[W(\pi) - L(\pi)]$ from (7), the implicit definition of s in the equation $W(s) = L(m)$ implies that $A(s) = 1$. To show that s is well defined, set $A(s) = 1$ in equation (8). Taking logarithms, this equation may be rewritten as

$$(9) \quad 1 - \gamma s = \log(\beta/\alpha\gamma) - \log s.$$

⁹ A similar equilibrium may be found in Varian [1980]. Note, however, that here m will be derived endogenously rather than being imposed exogenously.

It is clear that this equation has a well defined and nondegenerate solution $0 < s < 1/\gamma$, due to the fact that $\beta/\alpha < 1$. Hence property (i) is satisfied. Next, differentiate $A(\pi)$ defined in equation (8) to obtain

$$(10) \quad \frac{dA}{d\pi} = \frac{\beta}{\alpha-\beta} \left[\gamma - \frac{1}{\pi} \right] \frac{m}{\pi} e^{\gamma(\pi-m)},$$

which is negative for $\pi < 1/\gamma$. Therefore, $A(\pi)$ is monotonically decreasing on its support, and property (ii) is satisfied. Clearly, property (iii) is satisfied as well, as $A(\pi)$ is continuous in (π) . It follows that $F(\pi)$ is a well defined distribution function.

Next, we verify the mutual best response property. Note that for any inflation rate π_i such that $s \leq \pi_i \leq m$, the i -the firm has no incentive to choose a strategy different from the strategies F used by the other firms, as the pay-offs within the curled brackets in (6) are constant; see equation (7). Hence, since the integrand in (6) is constant, varying F does not improve expected seignorage, ES , on the interval $[s, m]$. Can the i -the bank profit by allocating some probability mass outside the interval $[r, m]$? If not, then $F(\pi)$ is a Nash equilibrium strategy for the i -the bank, as a deviation from F does not improve expected seignorage.

We finally show that there is no incentive to allocate probability mass above m or below s . As can be seen from (3) and (4), S_i is decreasing for any $\pi_i > 1/\gamma$. Therefore, no bank will ever inflate at a rate above $1/\gamma$, as it can always make the same seignorage $L(\pi)$ for an appropriately chosen $\pi < 1/\gamma$, and still have a chance to make $W(\pi)$. Alternatively, the chances of earning $W(\pi)$ are zero for $\pi > 1/\gamma$, and hence the firm should never inflate at a rate above the monopoly rate.

Similarly, it does not pay to inflate at a rate below s , as $W(\pi) < W(s)$ for $\pi < r$. In summary, it does not pay to deviate from $F(\pi)$ derived above, as it either generates the same or lower expected seignorage.

3. Implications and Empirical Tests

Prior to the free banking era in the United States, banks enjoyed local monopoly power. For example, Huntington [1964] reports for the case of Ohio that the number of banks were explicitly limited by law for each county. Moreover, the amount of dollar notes each bank could issue was limited by a reserve ratio and the limitations on the capital stock. However, free banking laws, such as the one of Ohio in 1851, challenged the monopoly power of existing banks by allowing (limited) entry. By 1952, 18 percent of all banks in Ohio were free banks. A major factor behind entry was the excess demand for (local) currency, especially in the frontier regions. This meant that an entrant could profitably secure some demand, by competing with the incumbent monopolist.

Rockoff, and Rolnick and Weber note that, while the period of free banking exhibited more uncertainty than before, instabilities were not prevalent. Competitive pressures, as feared by Friedman and Klein, and monopolistic power together destabilize pure strategy equilibria. However, we showed in the previous section that an appropriately chosen mixed strategy balances the two destabilizing forces. Hence, uncertainty in our model is an equilibrium phenomenon, rather than being the sign of destructive competition. The following proposition links this endogenous uncertainty with inflation.

Proposition 1. A free banking oligopoly with imperfect information, such that only some agents are aware of the inflation rates associated with the distinct dollar bills, implies stochastic inflation rates for strategic considerations. However, these inflation rates are below the inflation rate generated by a central bank.

Proof. The first part of the proposition was derived in the previous section. For the second part, note that a central bank exercising its monopoly power has a seignorage function: $S = \pi[\alpha + (n-1)]\alpha \phi e^{-\gamma\pi} - K$, where n is the number of would-be competitors. Hence, the monopolist chooses $\pi = 1/\gamma$, which equals the maximum inflation rate, m , under a free banking oligopoly. Because $F(m) = 1$, $F(m-\epsilon) < 1$ for $\epsilon > 0$, and $f(\pi)$ is continuous, the free banks almost surely inflate at a rate below m . Q.E.D.

From this, it is straightforward to derive the following implication.

Corollary 1. Monopoly seignorage is above the expected free banking industry seignorage. Moreover, the seignorage of free banks is random.

Proof. The monopolist earns seignorage $[\alpha + (n-1)\beta]\alpha \phi / e\gamma - K$, whereas the total expected seignorage of free banks equals $n \beta \phi / e\gamma - nK$; the latter returns are lower and costs are higher. The randomness in the seignorage of free banks derives from the randomness in π_i and τ_i .

The proposition and corollary unite the three themes of free banking. Given multiple suppliers of currency and imperfect

information on the demand side, equilibrium involves mixed-strategies. As a consequence, inflation rates and seignorage are random due to strategic aspects of behavior. This prediction of the model is consistent with the general notion that the free banking era constituted a period with higher financial uncertainty. While Rockoff [1974], Rolnick and Weber [1983, 1984] mention the general notion of increased uncertainty, they also note that in most states "the free banking law proved reasonably successful," Rockoff [1974, p.142]. This, again, is in line with our model of free banking. The uncertainty is an equilibrium feature; if each bank indeed employs the above mixed strategies the banking industry should exhibit stability, in terms of the number of banks that operate, say. Stability is also present in the sense that free banking does not lead to runaway inflations, as feared by Friedman [1959, p.6,7], because the average inflation rate under free banking is below the optimizing inflation rate of a monopolist. Hence, the claim of free banking advocates that deregulation of the money supply process may curb inflation has some validity. The trade-off is an increased volatility in inflation.

The empirical predictions implied by the proposition and corollary offer some interesting test of hypotheses. Fortunately, some data are available for the free banking era. We begin our empirical investigation by confronting the corollary with data on seignorage gains. Corollary 1 suggests that the monopoly rents from the provision of money should diminish under free banking, due to entry. Table 3 in Rockoff [1974], reproduced in the Appendix, lends some support to the

Corollary 1. The average seignorage¹⁰ as fraction of net worth is consistently higher in Philadelphia, which did not allow for free banking, than in the free banking cities of New York and Boston.

While the prediction that seignorage under free banking is lower than under monopoly supports the corollary, virtually any theory of the industry predicts that entry lowers profits. In this respect, the randomness of inflation and seignorage is the more interesting feature of the Proposition and Corollary, as this separates the Bertrand solution from, for example, the Cournot solution. We will focus on the case of Ohio for which reliable data on the money supply are available from Huntington [1964].

From 1846 to 1851 three types of banks existed: Old Banks which were a remnant of past legislation, State Branch Banks which had insurance similar to the current FDIC, and Independent Banks which were bond secured. Prior to 1851, entry to each type of Bank was limited. Each bank provided their own dollar bills, and apart from the central banking aspect, they also conducted the usual commercial banking business. In 1851 a free banking law was passed, allowing for entry by free banks, conditional upon certain solvency and liquidity requirements. This system was in existence until 1863. In the Appendix we present a figure which depicts the ratio of the amount of (base) money each type of bank circulated to the amount of capital. The ratio thus provides a measure of the creation of outside money. From the figure it appears that the ratio becomes much more volatile after 1851,

¹⁰ Seignorage here refers to distributed dividends, which also include profits from the commercial banking activities.

thus corroborating the prediction of the model that free banking leads to increased uncertainty. Moreover, the banks least protected by the state (the independent and free banks) have very low ratios of base money to capital. This indicates that the outside money creation by such banks may have been virtually zero.

In order to provide a test of the proposition, Table 1 gives the mean and variance for each type of bank over both periods. The variances seem to increase after the deregulation, while the mean rates drop. Formal tests of hypothesis are reported in Table 2. Let index 1 refer to the era prior to free banking, and let the index 2 denote the free banking era. Define the sample variances S_i , $i=1,2$, as the sum of squared deviations divided by $n-1$, where n is the number of observations. Let $R = S_1/S_2$ be the likelihood ratio. In all cases $H_0: \sigma_1^2 \leq \sigma_2^2$ cannot be rejected at the 5% level as R is well below the critical values. However, the alternative hypothesis $H_1: \sigma_1^2 \geq \sigma_2^2$ is not rejected in case of State Branch Banks either. Presumably the variance of the circulation to capital ratio in case of State Banks was not affected by the introduction of Free Banking, but the others were affected.

While the capital to circulation ratio provides some evidence in favor of an increased uncertainty, this variable is not directly related to the variables of the proposition and the corollary. The data sources do provide some profit measures, but these profits include net earnings from the commercial banking activities, thereby contaminating this indicator of seignorage. As for the currency specific inflation rates π_i , no reliable data seem to be available,

although one could in principle attempt to exploit data on discounts on bank notes. We opted for a somewhat different procedure whereby the data of Huntington on circulation can be used. From equation (3) we have

$$(11) \quad B_i = P_i \tau_i \phi e^{-\gamma \pi_i}.$$

Taking logarithms and transforming the result into first differences yields approximately

$$(12) \quad \Delta B_i / B_i = \pi_i - \gamma \Delta \pi_i + \Delta \tau_i / \tau_i + \Delta \phi / \phi.$$

Under the null hypothesis that our model of free banking is correct, we have that

$$(13) \quad \begin{aligned} \text{Var} (\Delta B_i / B_i) &= \text{Var} (\pi_i - \gamma \Delta \pi_i + \Delta \tau_i / \tau_i) + \text{Var} (\Delta \phi / \phi) \\ &= \text{Var} (\epsilon_1) + \text{Var} (\epsilon_2) \end{aligned}$$

say. Recall that $\phi = \phi(y, r)$, and hence $\text{Var}(\epsilon_2)$ represents uncertainty due to sources other than strategic reasons, like fluctuations in demand. On the other hand, $\text{Var}(\epsilon_1)$ is entirely due to the mixed strategy feature of the free banking equilibrium whereby banks randomize over π_i . $\text{Cov}(\epsilon_1, \epsilon_2)$ is zero as the two sources of risk are unrelated, i.e. $A(\pi)$ in (8) does not depend on ϕ . Also note that $\text{Var}(\epsilon_1) = 0$ in case of a monopoly, or in case of any pure strategy equilibrium for that matter, as $\Delta \pi_i = 0$ and $\Delta \tau_i = 0$ in such cases.

The idea is now to exploit (13) in order to test Proposition 1. Maintain the hypothesis that the pre free banking period was

essentially a period with local money supply monopolies such that $\text{Var}(\epsilon_1) = 0$. Then, free banking must have increased $\text{Var}(\Delta B_1/B_1)$ by factor $\text{Var}(\epsilon_1)$, assuming that the other source of risk ϕ remained stable. Table 3 provides the necessary information on the means and variances. In line with the Proposition, the means after deregulation are below the means of the pre deregulation period.

Note that taking expectations in (12) yields

$$(14) \quad E(\Delta B_1/B_1) = E(\pi_1) + E(\Delta\phi/\phi).$$

The Proposition implies

$$E_{\text{Pre}}(\pi_1) = 1/\gamma > E_{\text{Post}}(\pi_1),$$

and hence one expects $E(\Delta B_1/B_1)$ to drop. On the other hand the volatility of $\Delta B_1/B_1$ should go up on basis of (13). This is again the case, except for the State Branch Banks.

In Table 4 we test whether the increase in the variances was significant. At the 5% significance level H_0 is accepted in all three cases, and H_1 is rejected for the Independent and Old Banks. However, for the State Branch Banks equality of σ_1 and σ_2 cannot be rejected, as in Table 2. The above evidence suggests that the State Branch Banks were not affected by the free banking laws vis a vis the other banks. This, however, is not the entire story. Given the equality of variances for the State Branch Banks, we may test for the equality of the means as well by using a t-test. The appropriate hypotheses are $H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 > \mu_2$. Computing the test statistic gives a value of 3.18, which is well above the critical value of 1.64 (with 41 degrees of freedom and at the 5% significance level). Hence, H_0 is strongly rejected in favor of H_1 , indicating that the State Banks

inflated at a significantly lower rate after free banking was instituted.

4. Conclusions

The aim of this paper was two fold: first, to provide a much needed framework to analyze the issue of free banking, and second, to account for some of the stylized facts of the free banking era in the U.S. We choose the Bertrand-Nash model as a vehicle of analysis, as it is able to account for the noted increased financial uncertainty for this period. Importantly, this uncertainty is more an indication of stability than of instability; the uncertainty arises for strategic reasons. Given imperfect information on the demand side and the homogeneity of the product, only mixed strategy equilibria are viable Nash equilibria if banks compete in 'price' (the Bertrand feature of the model). The implication is that inflation rates become intrinsically random. On the other hand competition does lower the average inflation and seignorage rates.

When we tested for these implications by means of free banking data for Ohio, both a decline in the mean and an increased volatility were verified for the least regulated type of banks. However, for the State Branch Banks, to which entry was limited and which operated under a common safety fund, only the mean inflation rate seemed to go down significantly. The volatility of the inflation rate of these banks was not affected in an upward direction. This suggests that the State Banks may not have participated in the Bertrand game, as they received sufficient protection from entry.

Table 1
Sample Statistics of the
Circulation to Capital Ratio *)

<u>Type of Bank and period</u>	<u>Number of Observations</u>	<u>Sample Mean</u>	<u>Sample Variance</u>
Independent			
Pre 1852	19	1.57	.0073
Post 1852	26	1.24	.1379
Branch			
Pre 1852	19	1.72	.0124
Post 1852	26	1.72	.0171
Old			
Pre 1852	19	.78	.0340
Post 1852	10	.65	.2918
Free			
Post 1852	26	1.01	.0948
Total			
Pre 1852	57	1.36	.0173
Post 1852	88	1.25	.1020

*) The data are quarterly observations on aggregate measures of base money (circulation to capital that cover the two periods 1846-1851 and 1852-1863, except for those quarters for which non data were available. The data source is Huntington [1964, p. 284-286]. Data are available from the third author upon request.

Table 2
Tests for the Equality of Volatility
in the Circulation to Capital Ratio

<u>Type of Bank</u>	<u>R Ratio</u>	<u>Critical Values (5%)</u>	
		$H_0: \sigma_1^2 \leq \sigma_2^2$	$H_1: \sigma_1^2 > \sigma_2^2$
Independent	.053	F(18,25) - 2.03	F(18,25) - .46
Branch	.724	F(18,25) - 2.03	F(18,25) - .46
Old	.117	F(18,9) - 2.95	F(18,9) - .41
Total	.170	F(56,87) - 1.50	F(56,87) - .66

Table 3
Sample Statistics of $\Delta B_i/B_i$ *)

<u>Type of Bank and Period</u>	<u>Number of Observations</u>	<u>Sample Mean</u>	<u>Sample Variance</u>
Independent			
Pre 1852	18	.05	.0078
Post 1852	25	-.04	.0925
Branch			
Pre 1852	18	.10	.0173
Post 1852	25	-.01	.0096
Old			
Pre 1852	18	-.03	.0423
Post 1852	8	-.77	4.0023
Free			
Post 1852	25	.00	.0532

*) The ratio $\Delta B_i/B_i$ is computed as $\Delta \log B_i$, and where B_i are the notes in circulation as reported in Huntington [1964, p.284-287].

Table 4
Tests for the Equality of Volatility in $\Delta B/B$

<u>Type of Bank</u>	<u>R Ratio</u>	<u>Critical Values (5%)</u>	
		$H_0: \sigma_1^2 \leq \sigma_2^2$	$H_1: \sigma_1^2 > \sigma_2^2$
Independent	.084	$F(17, 24) = 2.07$	$F(17, 24) = .46$
Branch	1.802	$F(17, 24) = 2.07$	$F(17, 24) = .46$
Old	.011	$F(17, 7) = 3.47$	$F(17, 7) = .38$

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Appendix

In this Appendix we reproduce Table 3 from Rockoff [1974, p. 157] and the Figure from Huntington [1964, p. 298].

TABLE 3
BANK PROFITS IN NEW YORK BOSTON, AND PHILADELPHIA,
1849-59*±

Year	DIVIDENDS AS A FRACTION OF THE PAR VALUE OF CAPITAL			DIVIDENDS AS A FRACTION OF NET WORTH		
	New York	Boston	Philadelphia	New York	Boston	Philadelphia
1849	8.79	8.06	9.79	7.63	7.33	8.14
1850	8.70	8.36	10.60	7.33	7.33	9.04
1851	9.38	7.82	10.30	7.51	6.88	8.61
1852	9.03	7.78	10.27	7.69	6.87	8.70
1853	8.85	8.08	11.13	7.84	7.32	9.30
1854	8.87	8.65	11.40	7.86	7.71	9.54
1855	9.08	7.98	11.00	8.08	7.09	9.52
1856	8.61	7.81	10.26	7.72	7.10	9.00
1857	7.73	7.73	7.12	6.91	7.00	6.18
1858	6.91	7.43	8.03	6.20	6.76	6.74
1859	7.56	7.31	8.03	6.78	6.70	6.74

*The first three columns are unweighted averages for all banks in the city. The second columns are the corresponding column to the left multiplied by the aggregate ratio of par capital to par capital plus surplus.

†Sources: 1. *Dividends*. Boston: Joseph G. Martin, *A Century of Finance* (New York: Greenwood Press, 1969) pp. 99-101, 108-11. New York, 1849-51: *Hunt's Merchants' Magazine*, vol. XXIII, p. 89, and vol. XXVII, p. 92; 1852-59: *Banker's Magazine*, vol. XIV, pp. 556-57. Philadelphia, 1849-55: *Banker's Magazine*, vol. X, p. 978; 1856-59: *Communication of the Auditor General of Pennsylvania Relative to Banks and Savings Institutions*, 1856, 1857, 1858, and 1859, passim.
2. *Capital and Surplus*. Boston and New York: *Annual Report of the Secretary of the Treasury on the Condition of the State Banks* (in the House Congressional Documents), 1849-1859. Philadelphia, *annual Report of the Secretary of the Treasury*, 1849-1850, 1859; *Communication of the Auditor General*, 1851-1858.

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