On the assembly of Archimedean spiral cavities for sound absorption applications: design, optimization and experimental validation

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Abstract

This work presents the design, optimization and combination of fully parameterized Archimedean spiral cavities (ASCs) as acoustic resonators, in order to achieve perfect absorption at frequencies below 1000 Hz while using a completely modular design. The assembly of two ASCs in the same system is also presented. An optimization is performed and results are verified through Finite Element Analysis (FEA) and validated experimentally for almost perfect absorption peaks at 700 Hz and 800 Hz. A good agreement is achieved between theory, simulation and experiments.

Keywords: Archimedean spiral cavities, Metasurfaces, Acoustic absorption

1. Introduction

Acoustic treatment of rooms is required to diminish noise coming from machinery and other sources [1]. Acoustic treatments are also applied to make the listener's experience more pleasant [2] in case the room is devoted to entertainment, such as listening to music. Sound absorbing materials and structures are often applied to obtain an optimal reverberation time in a pre-determined range of frequencies. Low frequency absorption proves to be difficult as the thickness of the absorbing material needs to be larger than $\lambda/4$, λ being the wavelength of the sound wave at a particular frequency, in

order to provide high absorption coefficients [3]. If foams or fibrous materials are to be applied, this leads to bulky and impractical solutions.

Micro-perforated panels (MPPs) backed by an air cavity are widely applied to obtain high sound absorption coefficients at low frequencies [4, 5, 6, 7]. The small holes provide high viscous losses, but, as for foams and wools, the depth of the backing cavity becomes large to be effective at lower frequencies.

Helmholtz resonators (HRs) have also been used in literature [8, 9] as acoustic absorbers, but only provide a good absorption coefficient in a small frequency band, thus limiting their application for broadband absorption.

Coiled channels and spirals are structures that can provide a high absorption coefficient at low frequencies while keeping the thickness of the structure in the deep-subwavelength scale. These show great potential for practical applications in room acoustics and noise control [10]. This approach for sound absorption first appeared in literature in the work of Cai et al. [10], whose structure has a thickness of $\lambda/102$. The frequency at which the absorption peaks occur is mainly determined by the length of the channel. Spiralled channels are also used in the work of Li et al. [11], who combine an MPP and a coiled space in order to achieve a thickness of $\lambda/223$. Their theory is verified by numerical simulations; no experiments are performed. Further improvements are made by Huang et al. [12], who combined an embedded aperture with a coiled channel to achieve more tunability in their system, which resulted in a thickness of $\lambda/100$ at very low frequencies. All of these strategies provide a good ratio of thickness to λ , but the absorption peak is very narrow and perfect absorption only occurs at a single frequency due to the natural resonance of the coiled channel. Chen et al. [13] designed a spiralled channel with different cross section areas along its segment, achieving two peaks of absorption, at 700 Hz and 800 Hz, respectively, within a thickness of $\lambda/38.5$. Between those frequencies, the absorption coefficient remains higher than 0.8. Multiple coiled spaces based on the theory of HRs also have been studied [14, 15], with the system being optimized accounting for more than one HR at once. Combination of MPPs and HRs [16, 17], arrays of HRs with different dimensions [18, 19] and honeycomb cells that mimic the behaviour of HRs [20] have also been tested to target a broader frequency range.

Structures with more than one coiled channel, such as Fabry-Perot acoustic cavities, are present in literature [21]. The channels are optimized following the causality constraint, which links frequency-dependent absorption coeffi-

cient to sample thickness. The authors demonstrate the theory with a design with a thickness of only 10.56 cm, which consists of 16 channels, and gives a high sound absorption coefficient starting from 400Hz onwards. The thickness of the sample is thus $\approx \lambda/10$. Combinations of HRs specifically tuned for a broadband absorption also have been applied in the so-called rainbow trapping absorbers [22]. The HRs are loaded in an array of varying cross section waveguides, and the low cut-off frequency is determined by the resonance frequency of the largest HR. By adding different HRs to the waveguide, a cascade effect is obtained and quasi-perfect absorption is reported from 300 Hz to 1000 Hz. The structure has a thickness of ≈ 11 cm, thus being 10 times smaller than the wavelength at 300 Hz.

Inspired by the aforementioned works, in this paper a strategy is presented to combine different resonators, namely Archimedean spiral cavities (ASCs), that are embedded in the same geometry in order to provide sound absorption peaks in more than one frequency. Each resonator is optimized for a targeted frequency and manufactured in a way that makes it fully independent from the others, while making the system modular at the same time. Special attention is paid to their design, allowing for a straightforward production by 3D-printing process. The strategy is verified numerically through Finite Element Analysis (FEA) and validated with experiments in an impedance tube.

This work is divided as follows: Section 2 presents the theoretical models used for the geometry; Section 3 presents the design and numerical simulation processes; Section 4 discusses the results and Section 5 draws the main conclusions.

2. Models and methods

This section presents (i) the parameterization of the Archimedean spiral cavities, (ii) the analytical model to account for sound absorption in these channels and (iii) the combination of more than one channel in the same geometry.

2.1. Parameterization of the Archimedean spiral cavities

Archimedean spiral cavities provide a thin geometry for sound absorption in low frequencies due to space coiling. In addition, the full parameterization ensures that the geometry can be straightforwardly optimized to obtain a

high absorption coefficient at targeted frequencies. A schematic representation of such a spiral is found in Figure 1.

Figure 1: Archimedean spiral (a) and parameters in polar coordinates (b).

Equation (1) gives the spiral radius r as a function of the initial radius a_i and the angle θ :

$$r = a_i + b\theta. \quad (1)$$

For the final angle $\theta = \theta_f$, the final radius is $r = a_f$. The x and y components of r , as a function of polar coordinates, are

$$x = r \cos(\theta) \quad (2)$$

and

$$y = r \sin(\theta). \quad (3)$$

The parameter b is the spiral growth ratio, which is given by

$$b = \frac{a_f - a_i}{2\pi k}, \quad (4)$$

where a_f is the final radius and k is the number of turns of the spiral.

The distance d between successive turns is then

$$d = 2\pi b. \quad (5)$$

The spiral shown in Figure 1 is a 1-dimensional (1D) curve, which can be expanded into a 2D geometry as seen in Figure 2, thus becoming a spiralled

channel whose 2D cross section has a thickness t . The distance between turns d is dependent on the thickness t and on the thickness of the wall d_{wall} : $d = d_{wall} + t$. Finally, it is possible to turn the 2D channel into a 3D cavity by assigning a height h , in case the cross section is rectangular, or a radius $r_t = t/2$ in case the cross section is circular.

Figure 2: Spiral with expanded sides with thickness t and distance between turns d_{wall} .

Finally, the final angle of the spiral can be written as

$$\theta_f = \frac{a_f - a_i}{b}. \quad (6)$$

The total length of the spiral is then determined by [23]:

$$L_{ASC} = \int_{\theta_i=0}^{\theta_f} \sqrt{a_f^2 + (da_f/d\theta)^2} d\theta. \quad (7)$$

2.2. Acoustic impedance calculation

An ASC (Figure 3) whose front panel has area A , has an impedance of

$$Z_{ASC} = -jZ_c(A/s_0)\cot(k_{eff}L_{ASC}), \quad (8)$$

where s_0 is the cross section area of the ASC and $Z_c = (\rho_{eff}K_{eff})^{1/2}$ is its characteristic impedance [24]. ρ_{eff} and K_{eff} are the effective density and effective compressibility, respectively [24, 2], and the effective wavenumber is given by $k_{eff} = \omega\sqrt{\rho_{eff}/K_{eff}}$.

Figure 3: Unit cell with one ASC and a rectangular cross section.

The characteristic impedance Z_c of a channel, whether it is a spiral or any other geometry, is dependent on its cross sectional shape, since the effective density ρ_{eff} and compressibility K_{eff} are cross section dependent. Circular cross sections are shown to perform better, i.e. thinner sample thickness and high absorption, than rectangular ones [24]. For a channel with a circular cross section of radius r_t , the effective density and compressibility are given by [25]:

$$\rho(\omega)_{eff} = \rho_0 \left[1 - \frac{2}{r\sqrt{-i\omega\nu}} G(\xi) \right]^{-1} \quad (9)$$

and

$$K(\omega)_{eff} = (\gamma P_0)^{-1} \left[1 + \frac{2(\gamma - 1)}{r\sqrt{-i\omega\gamma\nu'}} G(\xi') \right], \quad (10)$$

where $G(\varepsilon) = J_1(\varepsilon)/J_0(\varepsilon)$, J_i is the Bessel function of the i -th order, ρ_0 and P_0 are the air density and pressure, respectively, $\gamma = 1.4$ is the specific heat ratio and ν and ν' are the dynamic and thermal viscosity of air, respectively; $\xi = r\sqrt{-i\omega/\nu}$ and $\xi' = r\sqrt{-i\omega\gamma/\nu'}$.

2.3. Sound absorption coefficient

In a system with more than one ASC, the resonators must be combined in such a way that they do not interact with each other, i.e. each spiral is isolated from the other by solid walls. An example of assembly is given in Figure 4, considering the combination of 2 ASCs.

Figure 4: System containing 2 ASCs. The wave enters both ASC 1 and ASC 2 channels at the same time through a front panel with a circular cross section of total area A .

Since the systems are being designed targeting frequencies whose wavelength is much higher than the distance between the entrance of the channels, and accounting for the fact that each resonator works in a deep subwavelength frequency, it can be considered that the absorptive effect of one resonator will not be affected by the presence of the other [21]. For parallel assemblies of N different systems, each one with impedance Z_N , the total impedance is then given by [24]

$$\frac{1}{Z} = \sum_{k=1}^N \frac{1}{Z_k}. \quad (11)$$

From Equation (11), it is then possible to calculate the total sound absorption coefficient α as

$$\alpha_{ASC} = 1 - \left| \frac{Z - \rho_0 c_0}{Z + \rho_0 c_0} \right|^2. \quad (12)$$

3. Design and numerical analysis

This section presents the optimization procedure to obtain dimensions that lead to a maximum absorption peak at a chosen frequency. The numerical modelling of the system is presented, as well as the manufacturing process. For the latter, 3D-printing is chosen, although other techniques, such as laser cutting, could also be used.

3.1. Optimization

Being simply a coiled channel, the ASC will have a main resonance frequency and its harmonics [2]. In order to find the optimum parameters to obtain unity absorption at a chosen frequency, an optimization procedure is performed with the solver *fmincon()* from MATLAB. The constraint equation $c(x)$ in the solver is set as

$$c(x)_{circ} = a_f + r_t - (D/2 - 0.1) \leq 0, \quad (13)$$

thus stating that the radius $D/2$ of the sample must be bigger than the outer spiral radius a_f plus the cross section radius r_t of the channel, in case of a circular cross section, leaving 0.1 mm of gap between the outer radius of the spiral and the edge of the sample. This gap is a tolerance implemented into the solver foreseeing experimental testing, i.e. if the tube has a diameter of 40 mm, it is necessary for the physical sample to have a maximum diameter of 39.8 mm, as a warranty that it will fit into the tube for the experiment to be done. The function to be minimized, since the target is a single frequency and not a range of frequencies, is $f(x) = -\alpha_{ASC}$. Other functions, such as $f(x) = 1/\alpha_{ASC}$, could be used, but, in the case of a system with many resonators, the division operation makes the optimization algorithm more computationally expensive.

3.2. Numerical simulation

The numerical simulation of the optimized UC is performed in COMSOL Multiphysics® by means of a Finite Element Analysis (FEA), using the Pressure Acoustics Module with a *Frequency Domain* study. The spiralled shapes are modelled using Solid Edge software and imported as an .igs file. For each ASC, a *Narrow Acoustic Region* is assigned as the domain and the type of tube is set as *Circular Tube*. To excite the channels, a cylindrical air cavity with a diameter of 40 mm is created and excited on the boundary opposite to the ASC, with a unit pressure to generate a plane wave. For the tube domain, the fluid model is set as *Thermally Conducting and Viscous*. The whole system is meshed with *Free Tetrahedral* elements. The element length is verified to be smaller than one sixth of the smallest wavelength. The mesh consists of 68420 degrees of freedom. A representation of the meshed system is shown in Figure 5.

Figure 5: Implementation of the 2ASC system in COMSOL software for a FEA analysis.

3.3. 3D-Printing and experiments

To provide a modular design, the geometry is 3D-printed in different parts, as shown in Figure 6. This allows not only for the removal of the supporting material left by the printing process, but also for an easy assembly of the parts. The channels are printed with a square cross section with sides of size $2r_t$, thus keeping the hydraulic radius the same. As long as the hydraulic radius is maintained, the change from circular to square cross sections can be made without compromising the results for sound absorption [24].

Figure 6: 3D-printed layers of the 2ASC system.

The sample is manufactured using a Objet30 Prime printer, using approximately 35 ml of the material Vero White RGD835. The layers are as-

sembled together with superglue. The experimental test follows ISO 10534, in an impedance tube with diameter $D = 40$ mm. The test setup and its schematic representation are represented in Figure 7.

Figure 7: Impedance tube used for experiments: (a) setup used and (b) schematic representation.

The acoustic cavity is excited via a sound source at the left end of the tube. The pressure variation in the medium is captured by microphones $M1$ and $M2$, spaced $s = 35$ mm from each other. Microphone $M2$ is $x = 290$ mm away from the sample, which is rigidly backed. The test rig is connected to a SCADAS mobile acquisition system. The data is processed with LMS Test-Lab software, using the “Measuring Sound Absorption with an Impedance Tube module”. The frequency response is calculated up to 6400 Hz with a frequency resolution of 1.25 Hz and using 50 averages per run.

4. Results and discussion

This section presents and discusses the results of the optimization, numerical implementation and experimental validation.

4.1. Design and optimization results

A system containing 2 ASCs, namely ASC_1 and ASC_2 , is optimized for absorption peaks at 700 Hz and 800 Hz, respectively. Each channel is optimized separately as it is considered that there is no interaction effect between them. The frequencies are chosen considering the diameter of the tube available for testing, which is 40 mm. The lower boundary (LB), upper boundary

(UB) and initial guesses (IG), as well as the optimization results, are shown in Table 1.

Table 1: Optimization parameters and boundaries for the 2ASC system.

ASC	Parameter	LB	UB	IG	Result	L_{ASC}
ASC_1	a_f	10 mm	49 mm	10 mm	17.5 mm	120.1 mm
	r_t	3 mm	25 mm	6 mm	3.4 mm	
ASC_2	a_f	10 mm	49 mm	10 mm	15.33 mm	105.2 mm
	r_t	3 mm	25 mm	6 mm	3.3 mm

Figure 8: Theoretical results for 2ASC system.

The optimization delivers a maximum sound absorption coefficient at 700 Hz and 799.4 Hz for ASC_1 and ASC_2 , respectively, as seen in Figure 8. When the two resonators are placed in parallel by using Equation (11), there is a small deviation of the peaks: from 700 Hz and 799.4 Hz to 702.2 Hz and 797.5 Hz, respectively. The absorption coefficient at 700 Hz and 800 Hz, which are the frequencies targeted by this system, are 0.9934 and 0.9928, respectively, with almost zero reflection, thus showing that, even with the small deviation at the peaks due to the assembly, which affects the initial hypothesis of no interaction between the resonators, the 2ASC geometry performs almost

full absorption at the frequencies for which it was designed. In addition, an impedance analysis, shown in Figure 9, reveals that the structure perfectly matches the impedance of the air at the optimized frequencies, since $Re(Z/Z_{air}) \approx 1$ and $Im(Z/Z_{air}) \approx 0$.

Figure 9: Impedance analysis of the 2ASC system.

4.2. Numerical and experimental validation

The analytical and FEA results, as well as the experimental ones, are shown in Figure 10. For the FEA analysis, two runs are performed: one considering the coiled channel or pore (CP) and the other considering a straight pore (SP) with the same length, in order to assess any effects due to the coiling, which are reported in literature [26, 27] as a decrease in the effective propagation speed of the wave. From the simulations, it can be seen that this effect is small: in the lowest and highest peaks there is a shift from 702.5 Hz to 705 Hz, and from 795 Hz to 792.5 Hz, respectively. In terms of amount of absorption, at 700 Hz (lowest targeted frequency) it is 0.978 (SP) and 0.988 (CP), therefore a change of only $\approx 1\%$. As for the peak at 800 Hz (highest targeted frequency), the absorption values are 0.938 (SP) and 0.965 (CP), a change of $\approx 2.7\%$.

The position of the peaks in both theoretical and numerical analysis match well, and the difference between these curves has a maximum of $\approx 4\%$ at ≈ 747 Hz. This might be due to some interaction effects, although small, as discussed previously, captured by the simulation.

Figure 10: Experimental and numerical results for the 2ASC system.

The position of the experimental peaks have a fair match with the theoretical and numerical ones. Experimental peaks occur at ≈ 706 Hz and ≈ 794 Hz, obtaining a total absorption of 98.73% and 99.33%, respectively. The deviation of the experimental results from the predicted peaks (theory and FEA) might have been caused by geometrical inaccuracies of the 3D-printing machine. The total absorption at 700 Hz and 800 Hz, the frequencies at which the system is tuned, are above 97% and 98%, respectively. An overall 20% additional absorption is spotted between the two peaks as compared to the simulations and theory, which may indicate that effects of roughness are taking place, as well as the possibility of an existing air gap between the walls of the sample and the walls of the tube. Some papers argue that roughness of the sample might play a role [28, 29], however then both peaks would be expected to shift in the same direction.

The thickness of 18.4 mm of the sample represents $1/26.4$ of the wavelength at 706 Hz (experimental), thus making the unit cell a subwavelength

absorptive structure. The length of the channel sets the lowest frequency that can be achieved with this kind of system. From Table 1, the length of each ASC is close to $\lambda/4$ at their respective frequencies. If such length is to be increased, thus targeting a lower frequency, the diameter of the sample would need to be higher than 40 mm, making experiments not possible since the tube available for testing has a diameter of 40 mm. As such, with optimized values for r_t and other parameters it is then feasible to obtain absorption for $f \ll 1000$ Hz, as seen in literature [10, 12], and have the system validated experimentally, provided a tube with feasible dimensions is available.

5. Conclusion

This paper presents a fully parameterized Archimedean spiral cavity that works as an acoustic resonator and that is 3D-printed in a fully modular way. It is also shown how to combine two of these ASCs in the same structure, with the aim of obtaining a compact geometry that provides unity sound absorption at more than one frequency for frequencies below 1000 Hz. The system is validated through finite element analysis simulations as well as experimental results. A good agreement is found between experiments and theory, thus confirming the validity of the method.

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