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NUMERICAL STUDY ON THE INFLUENCE OF ARTIFICIAL INTERNAL STRESS RELIEF GROOVE ON FRETTING FATIGUE IN A SHRINK-FITTED ASSEMBLY

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ABSTRACT

Fretting fatigue failure occurs in shrink-fitted assemblies due to the combination of high stresses and relative displacements near the contact edge. Due to these high stresses, fatigue crack initiates followed by crack propagation until final rupture. Additive manufacturing (AM) is a game changing technology, which enables new component capabilities that cannot be manufactured with conventional techniques. This research work analyses numerically the influence of an artificial internal stress relief toroidal groove inside a shrink-fitted shaft, which could be manufactured using AM technology. Due to the toroidal void, the stress/strain fields are redistributed improving the fretting fatigue crack initiation and propagation lifetimes. To do so, 2D finite element models are created in Abaqus software with and without the internal groove. To estimate the fretting fatigue initiation and propagation lifetime and crack propagation direction, critical plane methods are used. In terms of the crack propagation, eXtended Finite Element Method (XFEM) is used to simulate mixed mode crack advancing in a single mesh structure. Finally, the obtained results with and without void were compared concluding with significant improvements in terms of total fatigue lifetime.

1. INTRODUCTION

The failure of shrink-fit assemblies is commonly produced by fretting fatigue phenomena [1-2]. The contact pressure produced by the interference fitting and relative displacements found at the shaft/hub edge, produced by external loads, lead to the failure of the shaft due to fretting fatigue. In general, the combination of contact stresses and relative displacements between bodies lead to three well-differentiated damage types: wear, oxidation and crack nucleation [3]. Fretting phenomena is observed in

several mechanical components, such as metallic cables [4-7], turbine blade dovetails [8], shrink fitted shafts assemblies [9–11] and bolted joints [12] among others.

Shrink-fitted assemblies are commonly used at gear shafts assemblies and railway wheelsets among others. Therefore, they are commonly subjected to rotary bending and torsional cyclic loads. In the present work only rotary bending boundary conditions are analysed. Several papers studied the type of failure of these assemblies, noticing that the failure is observed in the vicinity of the contact edge at the shaft side, in which the effect of fretting wear is significant, governing the crack initiation point [9-10,13]. To delay the failure, different palliatives are found in practice, such as the application of deep-rolling to the contact surfaces [11], and the use of shaft overhangs and slots in the hub side [14].

Several authors have analysed different palliatives in order to reduce the negative effect of fretting fatigue applied to different fretting problems. These palliatives can be divided in three well differentiated groups. Those that modify some geometrical aspect [14], those that modify the material properties [15-16] and those that induce compressive residual stresses below the surfaces in contact [17-19]. All these palliatives, in one way or another, achieve significant life enhancements mitigating the influence of fretting. The proposed palliative could be considered in the first group.

The palliative analysed in the current work is directly linked with AM technology, which is the only way to reproduce the suggested internal geometrical feature in an actual case. This field is still under development so there are still many unknowns. From the mechanical point of view, the porosity of the new material, the surfaces roughness and the apparent anisotropy of the material due to the layer-by-layer deposition play an important role in the mechanical behaviour of components manufactured by AM [20-21]. All these variables are not considered in the present work, assuming that the behaviour of the new material is the same as the traditional one. Although AM is still under development, there is no doubt that it is gaining a strong foothold in the manufacturing field so it is interesting to investigate about the benefits that could offer in the near future.

The main objective of current research work is moving forward in the analysis of voids as fretting fatigue stress relievers for the specific case of shrink-fitted shaft assemblies as proposed in former works of the authors [22-23]. In [22] it is studied the influence of a toroidal artificial void, with the shape of a groove, inside the shaft of a shrink-fitted assembly subjected to rotary bending. Concluding that, with the adequate void position and size, a decrease in terms of a multiaxial fatigue parameter was achievable if compared with a traditional case without voids. In basis of the aforementioned idea, the objective of the present work is applying this palliative to an actual assembly and numerically study its influence at the initiation and propagation crack stages. Besides life enhancements produced by the void, crack orientation is also analysed at both stages. In this sense, a SWT based crack propagation orientation method is applied by means of XFEM.

XFEM approach is an extension of the conventional Finite Element Method (FEM) and is based on the concept of partition of unity [24]. Based on a finite element model, local enrichment functions are introduced in the model, and therefore new degrees of freedom (DOFs). These new DOFs are linked with the nodes of the elements that are penetrated by the crack. This feature allows modelling the discontinuity produced by a crack and its propagation without modifying the discretization. Therefore, it is possible to propagate a crack in a single mesh. For the purpose of fracture analysis, these functions consist of the near-tip asymptotic functions that capture the singularity around the crack tip. The approximation for a displacement vector function u with the partition of unity enrichment is shown in Eq. (1).

$$u = \sum_{I=1}^N N_I(x) \left[u_I + H(x)a_I + \sum_{\alpha=1}^4 F_{\alpha}(x)b_I^{\alpha} \right] \quad (1)$$

where $N_I(x)$ are the traditional shape functions and u_I is the nodal displacement vector of the finite element solution. The second addend is the product of the nodal enriched degree of freedom vector, a_I , and the associated discontinuous jump function $H(x)$ across the crack surfaces. The jump function can only take the values $H(x)=\pm 1$, depending on the relative position of the enriched node with respect to the crack face. The third term is the product of the nodal enriched degree of freedom vector, b_I^{α} , and the associated elastic asymptotic crack-tip functions. The first term on the right-hand side is applicable to all the nodes in the model; the second term is valid for nodes whose

shape function support is cut by the crack interior; and the third term is used only for nodes whose shape function support is cut by the crack tip.

The asymptotic crack tip functions in an isotropic elastic material, $F_\alpha(x)$, are shown in Eq. (2) and obtained from [24].

$$F_\alpha(x) = \left[\sqrt{r} \sin \frac{\theta}{2}, \sqrt{r} \cos \frac{\theta}{2}, \sqrt{r} \sin \theta \sin \frac{\theta}{2}, \sqrt{r} \sin \theta \cos \frac{\theta}{2} \right] \quad (2)$$

where r and θ are polar coordinates with its origin at the crack tip and $\theta = 0$ is tangent to the crack at the tip.

Therefore, with the XFEM formulations it is possible to propagate a predefined initial crack in a single mesh. In the literature there are several criteria to estimate the crack propagation direction specifically designed for proportional loading conditions. Some of them are briefly developed in the following section. Some authors have observed that when applying these propagation methods to fretting fatigue conditions, erroneous crack paths are predicted [25-26]. The main reason of the failure of these approaches is the non-proportional loading conditions that appear in fretting fatigue conditions. To tackle this problem, a crack propagation direction method based on the SWT parameter is applied with satisfactory results. The method was previously developed by the author's and applied to a FEM model [27]. Besides, the traditional proportional methods to estimate the crack propagation direction and some non-proportional ones are studied in the next section and compared later in the results section.

2. Background

2.1. Crack initiation

The value of the SWT multiaxial fatigue parameter is used as a crack initiation criterion using the procedure developed by Vázquez et al. [28] and applied to fretting fatigue problems in subsequent works [23]. The equation defining the traditional SWT parameter for a non-proportional loading state is shown in Eq. (3) [29].

$$SWT = \left(\sigma^n \frac{\Delta \epsilon}{2} \right)_{\max.} \quad (3)$$

Where σ'' is the normal stress of the material plane and $\Delta\epsilon$ is the range of the normal strain along the loading cycle, both for a specific orientation. The value of SWT is the product of these parameters at the orientation where it is maximum.

Fig. 1 shows a schematic representation of the method proposed based on the traditional SWT parameter with some modifications. At first, it is necessary to seek for the most unfavourable contact surface point, considering it as the one with the maximum value of the SWT parameter according to Eq. (3). This critical point is considered to be the origin of different material lines distributed homogeneously between $\theta = 0^\circ$ and $\theta = 180^\circ$ with radius R , defining the size of the influence area (see Fig. 1). The SWT parameter is calculated along each of these lines (defined by θ) at different points, but with the novelty that, for each of these points, the orientation of the material plane considered to evaluate the parameter is not the critical one, i.e., according to Eq. (3), but the one that coincides with the orientation imposed by θ . For example, for the 90 degrees' plane (vertical) the stresses and strains used to obtain the SWT parameter are σ_{xx} and ϵ_{xx} (perpendicular to the material line), which could produce a crack in vertical direction. With this procedure a SWT distribution is obtained along each material line. Next, the mean value of the SWT distribution along each line is calculated. Finally, the orientation having the highest mean value of the parameter is considered to be the most likely to initiate a crack as shown in Fig. 1.

The mean SWT values depends on the R parameter. To estimate the initiation lifetimes, the R parameter should be calibrated for each problem by means of the fatigue curve of the material. The procedure is developed at the results section. It is important to emphasise that this method is only applicable for comparison purposes. In the present work this methodology is used to compare the results obtained for two case studies of the same material and subjected to the same boundary conditions but with different geometries. From now on references to SWT parameter are referred to the mean value, at the critical orientation defined by the angle θ_0 obtained with the defined method and not according to Eq. (3).

2.2. Crack propagation

The crack propagation phase is studied via XFEM implemented in Abaqus software. This numerical technique allows the simulation of cracks, which are introduced after the

meshing process. Due to this feature, it is not necessary to develop different models with different crack sizes and fine meshes at the crack tip [24]. To estimate the crack propagation lifetime, stress intensity factors K_I and K_{II} are computed via the interaction integral method implemented in Abaqus for each loading step. The interaction integral is a modification of the J-integral in which the line integral is converted to an area integral. In the case of Abaqus software it is possible to define the crack tip and several contours to evaluate the integral and obtain the Stress Intensity Factor (SIFs) and crack propagation direction at each contour. Each contour is defined by one ring of elements surrounding the crack tip as shown in Fig.2.

Once the SIFs are known it is possible to estimate the crack propagation lifetime by integration of the well know Paris' law shown in Eq. (4) [30].

$$\frac{da}{dN} = C(\Delta K_{eq})^m \quad (4)$$

Where da/dN is the crack growth rate, C and m are material constant and ΔK_{eq} is obtained from Eq. (5) [31].

$$\Delta K_{eq} = \sqrt{\Delta K_I^2 + \Delta K_{II}^2} \quad (5)$$

Where ΔK_I and ΔK_{II} is the range of the SIF along the loading cycle for the modes I and II respectively. It will be considered that the crack is closed for negative values of K_I , therefore any negative value of K_I will be considered as zero.

Crack propagation direction could be estimated if stress and strain fields are known at the crack tip. Three different methods are commonly used for proportional loading states: Maximum Tangential Stress criterion (MTS) [32], Maximum Energy Release rate criterion (MERR) [33] and $K_{II} = 0$ criterion (KII0) [35]. MTS criterion consider the next crack propagation direction to be orthogonal to the orientation with maximum tangential stress at crack tip. In plane strain behaviour the stress field close to the crack tip is defined by Eq. (6) and Eq. (7) for an isotropic linear elastic material.

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right] \quad (6)$$

$$\tau_{r\theta} = \frac{1}{2\sqrt{2\pi r}} \cos \frac{\theta}{2} [K_I \sin \theta - K_{II} (3 \cos \theta - 1)] \quad (7)$$

Where r and θ are polar coordinates and the origin is at the crack tip in a plane perpendicular to the crack face. The crack propagation direction is obtained with the

condition $\tau_{r\theta} = 0$. Therefore, the crack propagation orientation θ_p , measured with respect to the former crack direction is obtained with Eq. (8).

$$\theta_p = \arccos \left[\frac{3K_{II}^2 + \sqrt{K_I^4 + 8K_I^2 K_{II}^2}}{K_I^2 + 9K_{II}^2} \right] \quad (8)$$

A negative value of θ_p should be considered if $K_{II} \geq 0$ and a positive value for $K_{II} \leq 0$.

MERR criterion considers the crack propagation in the direction in which the energy release rate G_k , defined by Eq. (9), is maximum.

$$G_k = \frac{1}{E} \left[(K_I^k(\theta))^2 + (K_{II}^k(\theta))^2 \right] \quad (9)$$

Where K_I^k and K_{II}^k are defined by:

$$K_I^k(\theta) = C_{11}(\theta)K_I + C_{12}(\theta)K_{II} \quad (10)$$

$$K_{II}^k(\theta) = C_{21}(\theta)K_I + C_{22}(\theta)K_{II} \quad (11)$$

And the constants C_{ij} are given in [33].

Finally, the KII0 criterion postulates that the propagation is defined by the direction in which K_{II} is zero or minimum in some loading conditions where a zero value is not possible.

However, it is well known that under fretting loading conditions these criteria do not predict the correct direction due to high stress gradient zones and the existence of mixed mode non-proportional loading [25-26]. Besides, due to the formulation of these methods, they do not take into consideration the complete range of the loading cycle. In the present work, the crack orientation directions obtained according to the aforementioned criteria are obtained for the loading step in which the crack is open.

With the aim of solving these difficulties, assuming that the stress range in fatigue field is vital, different crack propagation criteria are found in practice for the cases of non-

proportional loading conditions. Dubourg et al.[36] proposed the crack propagation in the direction for which the range of the circumferential stress is maximum without taking into account compressive stresses, $\max(\Delta\sigma_{eff}(\theta))$. Ribeaucourt et al. [37] compared the results obtained using two different approaches. The first one considers that the crack propagation direction is defined by the maximum value of the crack growth rate, $\max(\frac{da}{dN}(\theta))$. The second one is based on the combination of mode I and II stress intensity factors, this criterion looks for the direction with the maximum value of the range of Eq. (10), $\max(\Delta K_I^k(\theta))$. Giner et al.[26] developed a non-proportional criterion defining the crack orientation in the direction with the minimum value of the shear stress range, $\min(\Delta\tau(\theta))$. Finally, the method applied in the current work is based in the procedure developed at the crack initiation stage and was proposed and validated by Bohórquez et al [27]. Therefore, a more detailed explanation of the method is developed next. A schematic view of the proposed methodology is depicted in Fig.3.

The first step is defining an initial crack length and orientation in the numerical model. The initial crack orientation is obtained using the method elaborated in section 2.1 and the initial crack length is defined by the user. Once the problem is numerically solved and the stress/strain fields are known, specifically at the crack tip. The SWT procedure described earlier in section 2.1 is applied at the postprocessor stage moving the critical point to the crack tip, O_1 in Fig. 3. At this stage, instead of evaluating the SWT parameter along the material planes defined by θ , the parameter is evaluated at different points ahead of the crack tip at a constant distance r^* (black dots in Fig.3). With this modification the effect of the singularity at the crack tip is avoided. We assume that the crack will not go backwards. The orientation with the highest value of the parameter is considered the more prone to propagate a crack (θ_1 in Fig. 3). With the new orientation and defining the crack propagation increment (Δa) the next crack path and tip (O_2) are determined, being the next critical point of the method at O_2 . The procedure should be repeated up to the desired crack length or up to final crack length which results in fracture of specimen.

The value of the parameter r^* does not play an important role at the propagation stage. The variation of r^* will give different magnitudes of the SWT parameter; however, the maximum one is always at the same orientation. This is due to the fact that the stress

and strain fields ahead of the crack tip for a fixed orientation are proportional to $1/\sqrt{r^*}$ as demonstrated in [27] for plane strain conditions. Therefore, this procedure is also applicable for the minimum shear stress range criterion which is also based on a critical plane method that depends on the stress field ahead the crack tip. From a practical point of view, r^* should be much smaller than the crack length and larger than the mesh size at the crack tip, to ensure that the stress/strain fields evaluated are far enough from the crack tip. In the current work $r^* = 50 \mu\text{m}$, although several lengths have been numerically analysed confirming the independence of the r^* parameter.

3. NUMERICAL MODEL

In order to use actual geometries and boundary conditions, the values used here are obtained from the experimental results reported in [13]. The geometry and boundary conditions of the test are shown in Fig. 4. The interference pressure, p , between the shaft and the hub is 141 MPa, produced by an interference fit of $21 \mu\text{m}$. The material used for both components were manufactured from an alloy steel (35CrMo). Rotary bending fatigue test is done with a pair of vertical loads Q . These loads produce a nominal bending stress at both contact edges of 220 MPa. The total life reported is 1.31×10^6 cycles. The authors analysed the fretted surfaces at 10% and 30% of the total life. Noticing that no micro cracks were found at 10% and micro cracks between 30 and 50 microns were found at a 30% of total life. It was also observed that cracks initiate inside the contact zone, between 50 and 200 microns far from the contact edge due to the effect of fretting wear. According to these observations, the initiation stage is considered between 10% and 30% of the total lifetime, as reported in [13].

3.1. Numerical geometry and boundary conditions

The aforementioned test is numerically reproduced in a 2D simplified model. Due to symmetry boundary condition, only half of the test configuration is modelled. The global dimensions are shown in Fig. 5a. In the case in which a void is introduced, it is aligned with the contact edge and three parameters are necessary to define it, as depicted in Fig. 5b. The groove diameter $D_g = 0.1D$, the groove length $L_g = 2D_g$ and the vertical distance between the horizontal axis of the groove and the contact edge $L_v = 0.3D$, being the parameter $D = 13 \text{ mm}$. The size and position of the groove are obtained according to the optimum position obtained from the authors' previous published work in [22].

The loading conditions of a rotary bending fatigue test are reproduced as shown in Fig. 6. The contact edges (upper and lower) are two critical points according to the geometry considered in this study. However, both edges are subjected to the same load history, so only the strain/stress states at point A in Fig. 5 are considered.

The rotary bending load cycle could be defined numerically using two static loading steps. The first loading step is the one considering load $Q(t)$ upwards, at this moment, point A is subjected to tensile (σ_A^{max}), defined in Eq. (12). The second loading step applies load $Q(t)$ downwards and therefore the stresses at the lower edge are now under compressions ($\sigma_A^{min} = -\sigma_A^{max}$), as shown in Fig 6.

$$\sigma_A^{max} = \frac{Q L D}{I} \frac{D}{2} \quad (12)$$

Where L is the distance between the point A and the end of the shaft ($L = 22$ mm) and I is the cross sectional moment of inertia. The analytical nominal bending stress considered at point A is $\sigma_A^{max} = 220$ MPa. In the 2D model and considering plane strain conditions the section could be considered rectangular ($D \times 1$ mm) therefore the absolute value of the load Q for the 2D model, according to Eq. (12), is 269 N. The bending load is applied by means of a master node coupled with the nodes of the right face of the shaft (see Fig. 7).

The interference pressure is applied as a remote pressure to the lower line of the hub as shown in Fig. 7. The displacement of the left side of the hub and the shaft are horizontally restricted to reproduce symmetry boundary conditions. The upper part of the hub is restricted to move in vertical direction to avoid solid rigid movements. The same boundary conditions are applied to both models, with and without void.

The model is elastic and the steel properties (35CrMo) considered are shown in Table 1 and obtained from [13].

Where E is the Young modulus', ν the Poisson's coefficient, σ_y the yield stress and μ the friction coefficient.

The element used to mesh the geometry is quadrilateral with a bilinear formulation and plane strain features (CPE4R). The mesh size around the lower contact edge is $10 \mu\text{m} \times$

10 μm . The size of the remaining elements is $0.5 \text{ mm} \times 0.5 \text{ mm}$. The contact between the shaft and the hub is defined using the master-slave algorithm in Abaqus for contact between two surfaces. The shaft is defined as the master and the hub as the slave. Lagrange multiplier formulation is considered in the contact pair to define the frictional behaviour of the parts assembled. Contact of the crack faces in the XFEM model are considered as frictionless. The number of elements for the reference case is 135648 and for the case with the void 110902.

The loading sequence is applied in three steps. First, the contact pressure is applied and is kept constant for the remaining loading steps. Next, the load Q is applied upwards and eventually the same load is applied downwards, thus reproducing the actual loading conditions of experimental configuration, as reported in [13].

In this work fatigue lifetimes have been obtained by means of a simplified 2D model. Authors' previous work [39], [40], show that fretting fatigue life predictions can be done with success using this type of geometrical simplification in extruded geometries. The case that concerns us is more complex, the shaft-hub assembly is a revolute geometry in nature, therefore the most direct simplification is an axisymmetric model. Nevertheless, rotary boundary conditions could not be reproduced by means of an axisymmetric model. Therefore, in order to understand the behaviour of the crack in presence of a void a 2D model is considered as a first approach. The results obtained for the fatigue crack initiation phase with this 2D model are in agreement with those previously obtained by means of a 3D model [22]; the decrease achieved in terms of the SWT parameter, obtained as explained in section 2.1, is similar in both 2D and 3D models. Therefore, the use of a 2D model seems to be a reasonable simplification for the initiation stage.

The propagation stage is governed by the SIFs. The crack emanating from the 2D shaft could be understood as a crack growing into a semi-infinite plane. In these conditions, the well-known geometry correction factor, $F = K_1/\sigma\sqrt{\pi a}$, is about 1.12. Depending on the crack propagation stage, the aspect ratio of the semi-elliptical cracks observed in this type of assemblies ranges between 0.6 and nearly zero [41]. Under these conditions, the value of F could be obtained from Shin et al. [42] where a semi-elliptical crack growing in a circular specimen under tension loads is considered. According to this work, and

for an average crack of 1.6 mm, F oscillates between 0.9 and 1.1. Therefore, the use of a 2D model seems to be a reasonable approach for the crack propagation stage.

4. RESULTS

4.1. Fretting fatigue crack initiation

At the initiation stage, the reference case and the case with the void are compared using the initiation procedure described in section 2.1 (see Fig. 1). The critical point of the shrink-fitted shaft assembly in the void case is the bottom contact edge (Point A of Fig. 5a). The experimental values obtained for the crack initiation lifetime are in the range 131000-393000 cycles as reported in [10]. The R value is calibrated looking for a value of the mean SWT at the critical orientation, so that applying Eq. (13) results in a lifetime initiation value, N_i , in the range of the experimental values. The optimum mean SWT value was found at $R = 0.5$ mm, obtaining approximately 190000 cycles from Eq. (13). Therefore, only for comparison purposes, a value of $R = 0.5$ mm is used for the cases with and without voids. The constants of Eq. (13) are shown in Table 2 and are obtained from reference [10].

$$SWT = \frac{\sigma_f'^2}{E} (2N_i)^{2b} + \sigma_f' \epsilon_f' (2N_i)^{b+c} \quad (13)$$

The initiation procedure is applied now for the case with the void, considering the critical point at the contact edge (Point A of Fig. 5b) and $R = 0.5$. The mean SWT value obtained at the critical orientation (see Fig. 1) is introduced in Eq. (13) and 1.06×10^6 cycles are obtained. The numerical results show that an increase of 5.6 times of the initiation lifetime is achieved due to the artificial groove introduced in the shaft, from 1.9×10^5 cycles up to 1.06×10^6 cycles. This result suggests that the initiation stage could be substantially delayed with the adequate groove position. The critical orientation predicted in both cases is $\theta_0 = 103^\circ$ (critical orientation as defined in Fig.1), pointing inside the contact, which is in good agreement with the 108° orientation observed in the experimental results [13].

Besides the initiation life estimation obtained, the stresses and the sliding amplitude along the shaft contact surface (Path in Fig. 8) for the first loading step (i.e. upwards bending load) are shown in Fig. 8. It can be noticed that the axial stress σ_{xx} , shear stress

σ_{xy} and normal stress σ_{yy} decrease substantially in the case with the void with respect to the reference case close to the critical zone. In special, it is important to note the decrease in the value of σ_{xx} (~25%). That is the main reason of the numerically predicted crack initiation lifetime improvement.

A remarkable featured produced by the void in the assembly here analysed, is the increase in the slip. FEM simulations show that the sliding distance, δ , for the case with the void is larger than the corresponding for the reference case. However, this increment in the slip is not traduced in an increment in the direct stress at the contact edge, which could be the main agent for the crack initiation. According to Archard's law, fretting wear is proportional to the product of the contact normal pressure, σ_{yy} , and the sliding distance [43]. This product is analysed in order to give a qualitatively idea of the fretting wear process in the contact zone. The results are shown in Fig. 8b, which shows that due to the groove, wear is expected to be more pronounced far from the contact edge.

4.2. Fretting fatigue crack propagation

The fretting fatigue crack propagation stage is performed according to the SWT method proposed in section 2.2 and compared with the $\min(\Delta\tau(\theta))$ method. First, an initial crack is defined with a length of 0.1 mm and an initial orientation of 108° at 50 μm inside the contact from the contact edge. These initial crack parameters are obtained according to the experimental results measurements shown in reference [13] and the mesh size used in the vicinity of the crack. Once the stress/strain fields are known for each crack length, at the postprocessing stage, the SIFs are obtained by means of the interaction integral method and the next crack propagation direction are obtained according to the propagation methods. Crack increments, Δa , of 0.1 mm are considered. The propagation process is carried out automatically by means of a Python script. The comparison of the results obtained for the methods already implemented in Abaqus, the proposed one as described in section 2.2 and the non-proportional method $\min(\Delta\tau(\theta))$, are shown together with two actual cracks, in Fig. 9. The evaluation of $\min(\Delta\tau(\theta))$ is carried out in the same manner as shown in Fig. 3 to calculate both methods with the same stress/strain fields. The crack obtained with traditional cases (MTS, MERR and KII0), quickly tends to the vertical direction and convergence problems arose. Nevertheless, the predicted crack propagation path using the SWT proposed method and the $\min(\Delta\tau(\theta))$ are in good agreement with the observed experimental results in Fig. 9.

Therefore, the non-proportional crack propagation methods are used to investigate the fretting fatigue crack propagation response of the shrink-fit assembly. The predicted cracks for the reference case, the case with the groove and the actual cracks of Fig. 9 are plotted in Fig. 10. The maximum crack length analysed is 2.6 mm. The limit is imposed by the height of the refined mesh. The results obtained for both propagation methods are very congruent. The slope of the $\min(\Delta\tau(\theta))$ method is slightly larger than the slope of the SWT method, although the difference is almost negligible. The crack path in the cases with the void, independently of the method, tend more quickly to the vertical direction than the reference case crack. This difference is more visible with the $\min(\Delta\tau(\theta))$ method.

The propagation cycles for each crack increment, Δa , are computed using the Paris' law (see Eq. (4)). The SIFs values are obtained at each crack increment with the contour integral method by defining 9 contours and averaging the results of the last five (see Fig. 1). The material's constants C and m defining Eq. (4) depends in a great extent of the specific type of material. Due do the lack of data two different constant of two types of steel are analysed with comparative purposes [44]. The constant used and the predicted propagation lifetimes (N_p) are shown in Table 3.

The predicted lifetimes values depend on the Paris' law constants. However, life improvements around 1.5 times are observed in the case with the void with respect to the reference case independently of the propagation method used. This increment is substantially smaller than the enhancements achieved at the initiation stage. For a better understanding of the void influence along the crack propagation process, the number of cycles to propagate an initial crack, with a length $a=a_0 = 0.1$ mm, up to a crack length of $a = a_0 + \Delta a$ are shown in Fig. 11. Analysing Fig. 11 it can be observed that the propagation lifetime enhancements shown in Table 3 are mainly due to the improvements obtained at the very beginning of the crack, independently of the propagation criteria. From this figure it can be observed that the difference in the crack propagation cycles are produced during the first steps. Then both crack propagation curves tend to be almost parallel due to the small difference between the propagation cycles; this behaviour is observed approximately from a crack increment around $\Delta a=0.9$

mm. Therefore, the influence of the void is remarkable for the initiation stage and for crack lengths below 1 mm approximately.

The cycles required to propagate the crack from 2.5 mm up to 2.6 mm are lower than the 0.8% of the total propagation lifetime. Therefore, and although the crack could propagate more than the imposed limit of 2.6 mm (up to 3 mm in the void case), this extra number of cycles are negligible. This effect can be observed at Fig.11, where the slope of the curves tends to infinity.

Fig. 12 compares the first principal stress at loading step 1 (i.e. load Q upwards) between the reference case and the case with the artificial void for a crack length of 1 mm and the SWT propagation method. The zone with values higher than the yield stress is represented with grey colour. It can be noticed that, thanks to the void introduced, lower maximum principal stress is observed in the case with the void although the difference is negligible. In general, the stress fields are very similar, therefore for this crack length the void influence is null as was observed from Fig.11.

5. CONCLUSIONS

The conclusions could be divided into two well differentiated points. On the one hand a recent method to estimate the crack propagation orientation has been analysed for the case of mixed mode non-proportional loading. The method is based in a multiaxial fatigue parameter. The results show that the proposed method estimates correctly the crack propagation direction while traditional methods, already implemented in Abaqus, predict wrong crack propagation directions when they are applied to a fretting fatigue problem. Besides, the SWT propagation method has been compared with the $\min(\Delta\tau(\theta))$ method. In general, although some differences are observed discrepancies are minimum. On the other hand, a new fretting fatigue palliative has been analysed for the case of an actual shrink-fitted shaft assembly subjected to rotary bending loading conditions. The proposed palliative consists on the introduction of a toroidal void with a previously defined optimum position and geometry, beneath the contact edge. To analyse the void influence, two strategies have been developed by means of a 2D simplified model of an actual test. The first strategy was analysing the crack initiation lifetime for the cases with and without void, noticing life improvements of more than five times the actual

case lifetime. Besides, the initial crack orientation estimated agrees very well with the one measured from the tests.

The second strategy was analysing the crack propagation orientation and lifetime. The results suggest that improvements in terms of crack propagation lifetime are also observed due to the palliative proposed. These enhancements of lifetime are of the order of one and a half times the one estimated with the reference case. Therefore, the improvements at the initiation stage are more significant than those observed at the propagation stage. In general, it can be drawn that with the adequate void position and size, important improvements in terms of crack initiation and propagation lifetimes could be achieved.

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Table 1. Mechanical properties used for the hub and the shaft (35CrMo).

E	212 GPa
ν	0.28
σ_y	705 MPa
μ	0.6

Table 2. SWT constants for 35CrMo.

σ'_f	1146.78 MPa
ε'_f	0.95
b	-0.075
c	-0.81

Table 3. Propagation lifetime comparison.

Method	Material	C	m	N_p (Reference case)	N_p (Void case)	Improvement
SWT	Steel (Tempered martensite)	1.35e-11	2.8	264500	377700	1.4
	Steel (Pearlite)	7.15e-13	3.4	1436400	2426500	1.7
$\min(\Delta\tau(\theta))$	Steel (Tempered martensite)	1.35e-11	2.8	301200	412400	1.4
	Steel (Pearlite)	7.15e-13	3.4	1663400	2674300	1.6

Figure 1. SWT critical plane initiation procedure.

Figure 2. Contours to evaluate SIFs.

Figure 3. Crack propagation direction SWT method.

Figure 4. Experimental geometry, loading and boundary conditions [13].

Figure 5. Geometry a) Reference case, b) Case with void.

Figure 6. Rotary bending loading conditions simplification.

Figure 7. Mesh and boundary conditions, a) Reference case, b) Case with void.

Figure 8. a) Contact stress and sliding distribution along path, b) $\delta\sigma_{yy}$ parameter.

Figure 9. Crack propagation direction [13].

Figure 10. Predicted and experimental crack paths with SWT propagation method.

Figure 11. Accumulative cycles to propagate the initial crack a_0 up to $a_0 + \Delta a$.

Figure 12. σ_I for 1mm crack, a) Assembly with void, b) Reference case assembly.

HIGHLIGHTS

- Shrink-fitted shaft fretting fatigue palliative
- Evaluation and comparison of crack propagation direction methods
- Analysis of crack propagation direction using XFEM
- Voids could delay crack initiation and propagation stages.

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: