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TAX REFORM RESULTS WITH DIFFERENT
DEMAND SYSTEMS

by

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ABSTRACT

We calculate the marginal welfare costs for a twelve commodity-classification of the Belgian indirect tax system. Price elasticities are estimated with Rotterdam, AIDS and CBS. The ranking of the marginal welfare costs is about the same when we use the unrestricted estimates of these systems. After the symmetry condition of the Slutsky matrix is imposed, the general picture of the results still remains similar, but there appear some important differences for individual commodities. These differences diminish with increasing inequality aversion.

1. INTRODUCTION

A major obstacle to the practical application of *optimal taxation* theory seems to be the dependency of the results on the specification of the demand system, used to estimate the reactions of the consumers. The problem is nicely summarized by Deaton (1981), who states that "it is likely that empirically calculated tax rates, based on econometric estimates of parameters, will be determined in structure, not by the measurements actually made, but by arbitrary, untested (and even unconscious) hypotheses chosen by the econometrician for practical convenience". In that same article, he shows theoretically what are the consequences of different separability and Engel curve assumptions. A first empirical illustration of the problem has been presented by Ray (1986).

There are many reasons, however, to hypothesize that the problem might be much less severe in the context of *marginal tax reform*. This is emphasized by Ahmad and Stern (1984), who argue that for the tax reform problem we need only information about aggregate demand derivatives for the point at which we find ourselves. It is not necessary to have information about individual reactions, nor to use fitted values for configurations away from the starting point. This is more a (reasonable) intuition than a theoretical proof, however. In fact, to the best of our knowledge, there are no unambiguous theoretical results about the dependency of tax reform calculations on the specification of the demand system. It seems therefore interesting to present some empirical evidence on the subject.

In this paper we follow closely the approach proposed by Ahmad and Stern (1984). This approach is summarized in section 2, where we also give an overview of our data. These refer to the Belgian indirect tax system, and we work with twelve aggregate commodities. In section 3 we present the three estimated demand systems : Rotterdam, AIDS and CBS. For each of the systems we have estimated different variants, including complete preference independence for the Rotterdam-version. In section 4 we compare the tax reform results with these different systems. Section 5 concludes.

As the main objective of this paper is methodological, we will not spend too much attention to the policy conclusions of our calculations. These are discussed in more detail in a companion paper (Decoster and Schokkaert (1987)).

2. TAX REFORM FORMULAE

Because of the limitations of the data, we have concentrated on indirect taxes only. We assume factor income to be fixed and untaxed. Producer prices $p_1, \dots, p_N$ of the N commodities also are fixed and there are no pure profits. Indirect taxes $t_1, \dots, t_N$ therefore increase the consumer prices $q_1, \dots, q_N$:

$$q_i = p_i + t_i \quad i = 1, \dots, N \quad (1)$$

There are H households in the economy. Total consumption of commodity i is given by

$$X_i = \sum_h x_i^h \quad i = 1, \dots, N \quad (2)$$

where x_i^h is the amount purchased by household h , and function of the consumer prices. The government collects a certain amount of revenue R through indirect taxes

$$R = \sum_i t_i X_i \quad (3)$$

and it cares about social welfare, represented by the function $W[v^1(q), \dots, v^H(q)]$, where $v^h(q)$, $h = 1, \dots, H$, refers to the indirect utility function of household h .

This is the model, proposed by Ahmad and Stern (1984), and they define MC_i , the marginal cost in terms of social welfare of an extra franc raised via the i^{th} good, as

$$MC_i = - \frac{\partial W / \partial t_i}{\partial R / \partial t_i} \quad (4)$$

If e.g. $MC_i < MC_j$ then social welfare will be increased by lowering the tax rate t_j and by an offsetting increase in the tax t_i , so as to keep global tax revenue constant.

It is immediately obvious that within our simplified model of the economy, (4) can be specified as

$$MC_i = \frac{\sum_h \beta^h x_i^h}{X_i + \sum_k t_k \left(\frac{\partial X_k}{\partial t_i} \right)} \quad (5)$$

where $\beta^h = \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial m^h}$, m^h being the lump sum income of household h . The parameter β^h therefore gives the marginal social valuation of one unit of income accruing to household h .

Multiplying numerator and denominator in (5) by q_i , leads to an expression which can be operationalized easily :

$$MC_i = \frac{\sum_h \beta^h (q_i x_i^h)}{q_i X_i + \sum_k \epsilon_{ki} t_k^* (q_k X_k)} \quad (6)$$

where ϵ_{ki} refers to the uncompensated price elasticity and $t_k^* = t_k/q_k$, the tax rate as a fraction of consumer price.

To apply (6) we have to combine information from different sources. We constructed 12 aggregate commodities in such a way that it was possible to match time series information from the Belgian National Accounts and cross section-material from the Belgian consumer expenditure survey 1978/1979. On

the basis of the latter, we constructed the consumption data $(q_i x_i^h)$ for ten representative consumers, each corresponding with one decile of the total expenditure distribution in the survey. The indirect tax rates t_i^* were computed from the more detailed study of Paraire-Laguesse et al. (1986). The commodity classification and the tax rates are shown in table 1. Information from the National Accounts was used to estimate the complete matrix of cross price-elasticities. These elasticities were evaluated at the average budget shares, taken from the Consumer Expenditure Survey.

TABLE 1

As a sensitivity analysis with respect to these price elasticities is the main topic of this paper, we will return to their estimation more extensively in the following section. The last piece of information needed to calculate (6) are the welfare weights β^h . Following Ahmad and Stern, we define them starting from the well known iso-elastic specification

$$W = \sum_h k \frac{(m^h)^{1-e}}{1-e} \quad e \neq 1 \quad e \geq 0 \quad (7a)$$

$$W = \sum_h k \log m^h \quad e = 1 \quad (7b)$$

which, with the normalisation $\beta^1 = 1$, leads to

$$\beta^h = \left(\frac{m^h}{m} \right)^{-e} \quad (8)$$

Equation (8) immediately shows that with an increase in the parameter of inequality aversion e , higher income households get a relatively smaller welfare weight. For the application of (8) we defined m^h to be the total expenditures per equivalent adult¹.

3. DEMAND SYSTEMS : SPECIFICATION AND ESTIMATION

People's reactions towards changes in indirect taxes appear in (6) through the price elasticities ϵ_{ki} . One could use different sources of information to get an idea about the order of magnitude of these elasticities, but the most sophisticated approach undoubtedly is the estimation of a demand system. For our exercise, we started from the well-known Rotterdam model (see e.g. Theil (1975/1976), Barten and Geyskens (1975)).

$$\bar{w}_{it} \Delta \ln X_{it} = b_i \sum_j \bar{w}_{jt} \Delta \ln X_{jt} + \sum_j s_{ij} \Delta \ln q_{jt} + v_{it} \quad (9)$$

where w_{it} is the budget share of commodity i in period t and

$\bar{w}_{it} = (w_{i,t} + w_{i,t-1})/2$. The vector b and the matrix S are fixed coefficients to be estimated and interpreted as marginal budget shares and Slutsky

coefficients respectively. Expenditure elasticities and price elasticities therefore are not fixed, but dependent on the budget shares. The relevant expression for the price elasticities is

$$\epsilon_{ij} = \frac{s_{ij}}{w_i} - \frac{b_i w_j}{w_i} \quad (10)$$

The restrictions of demand theory can be formulated immediately in terms of the coefficients in (9) :

$$\begin{array}{lll} \iota' b = 1 & \text{and} & \iota' S = 0 & \text{(adding up)} \\ S \iota = 0 & & & \text{(homogeneity)} \\ S = S' & & & \text{(symmetry)} \\ S \text{ negative semi-definite} & & & \text{(negativity)} \end{array}$$

We will compare the tax reform results for these different variants.

The Rotterdam model is a flexible specification for complete demand systems². For our purposes, an interesting additional feature is the possibility of imposing additivity of the utility function within the more general model, requiring no specification of such function. Additivity (or "complete preference independence" in the Rotterdam-jargon) can be imposed through the restrictions

$$s_{ii} = \varphi b_i (1 - b_i) \quad (11)$$

$$s_{ij} = -\varphi b_i b_j \quad (i \neq j)$$

where φ is a constant to be estimated and interpreted as the reciprocal of the elasticity of the marginal utility of income with respect to income.

Starting from (9), two other systems can be derived immediately through a simple change in the dependent variable, while leaving the right-hand side unchanged. The first one is Deaton and Muellbauer (1980)'s "almost ideal demand system". This is specified as follows :

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} \ln q_{jt} + \beta_i \ln \{m_t/P_t\} + u_{it} \quad (12)$$

where $\ln P_t$ is a price index, defined as :

$$\ln P_t = \alpha_0 + \sum_k \alpha_k \ln q_{kt} + \frac{1}{2} \sum_j \sum_k \gamma_{kj} \ln q_{jt} \ln q_{kt} \quad (13)$$

This expression makes (12) highly nonlinear and estimation becomes much easier when we approximate it by

$$\ln P_t^* = \sum_k w_{kt} \ln q_{kt} \quad (14)$$

This approximation does not lead to important changes in the results, when the prices are collinear (see Deaton and Muellbauer (1980)). Using (14) in (12), taking differences and replacing w_{kt} by $\bar{w}_{kt}$, we get

$$\Delta w_{it} = \beta_i (\Delta \ln m_t - \sum_k \bar{w}_{kt} \Delta \ln q_{kt}) + \sum_j \gamma_{ij} \ln q_{jt} + u'_{it} \quad (15)$$

Since we know that the term in brackets at the right hand side of (15) is equal to $\sum_k \bar{w}_{kt} \Delta \ln X_{jt}$, we see that the differential form of AIDS leads to the same right hand side as the Rotterdam model (9).

Of course, the interpretation of the coefficients in (9) and (15) is not the same. Indeed, it is easy to show (see e.g. Keller and Van Driel (1985)) that

$$\beta_i = b_i - w_i \quad (16)$$

In the Rotterdam model, the sign of b_i indicates whether a commodity is inferior or superior, in AIDS the sign of β_i shows whether the commodity is a necessity or a luxury. The latter specification implies Engel-curves of the PIGLOG-type, which give an excellent fit in cross-sections and guarantee exact aggregation over consumers. It also can be shown that

$$G = [\gamma_{ij}] = S + \hat{w} - ww' \quad (17)$$

where $\hat{w}$ is a diagonal matrix with the budget shares on the diagonal. Expression (17) implies that concavity of the cost function cannot be imposed in AIDS. The other restrictions can be written as :

$$\begin{aligned} \iota' \beta &= 0 & \text{and} & & \iota' G &= 0 & & \text{(adding up)} \\ G \iota &= 0 & & & & & & \text{(homogeneity)} \\ G &= G' & & & & & & \text{(symmetry)} \end{aligned}$$

In an attempt to combine the nice features of both models, Keller and Van Driel (1985) proposed the so-called CBS-model. This can again be obtained by a change in the dependent variable of (9). Starting from (9), subtracting $\bar{w}_{it} \sum_j \bar{w}_{jt} \Delta \ln X_{jt}$ and using (16), one gets

$$\bar{w}_{it} (\Delta \ln X_{it} - \sum_j \bar{w}_{jt} \Delta \ln X_{jt}) = \beta_i \sum_j \bar{w}_{jt} \Delta \ln X_{jt} + \sum_j s_{ij} \Delta \ln q_{jt} + v_{it} \quad (18)$$

This specification combines the attractive total expenditure coefficients of AIDS with the matrix of price coefficients of the Rotterdam model.

The feature that the Rotterdam model (9), the AIDS specification (15) and the CBS-system (18) all have the same right hand side is exploited in the computer program DEMMOD, constructed by A.P. Barten. This program allows to estimate all variants of the three systems with the maximum likelihood procedure, described in Barten and Geyskens (1975). To impose symmetry and (especially) negativity this procedure uses the Choleski-decomposition of

the Slutsky-matrix. We used this program to estimate the three systems for the twelve commodities-classification, described in table 1. In all three cases we added an intercept, to be interpreted as a trend shift in demand. The hypothesis that this intercept could be omitted was strongly rejected.

TABLE 2

It is perhaps interesting first to have a look at the results for the statistical testing of the different theoretical restrictions. These results are shown in table 2. For this exercise, we have to take into account that the usual test statistics (such as the likelihood ratio) are strongly biased towards rejection of the restrictions for small samples³. For homogeneity we computed the exact small sample test, proposed by Laitinen (1978) : with this exact test, the restriction is rejected for none of the three systems. An exact small sample test for symmetry has not yet been derived. The traditional likelihood ratio tests indicate rejection of the restriction for the three systems. Recently, Italianer (1985) has proposed a small-sample correction for this test : when applying this correction factor, symmetry passes for the three systems. None of the three symmetric systems satisfies the negativity condition, however. This condition cannot be imposed in AIDS. For Rotterdam and CBS, this inequality restriction can be imposed by requiring the Cholesky values to be negative. Table 2 shows that this condition is rejected for both systems, even after the small sample correction⁴. One may wonder about the importance of this result. However, violation of

the concavity of the cost function is a rather disturbing feature for applied welfare analysis and there are many statistical and practical reasons which might explain our empirical results. We therefore feel that in this case theory is more important than statistical test results and we will work further with the Rotterdam and CBS estimates under negativity. We return to the problem in the following section.

While the previous conditions directly follow from theory, additivity of the utility function is an *empirical* restriction. It is well known to have immediate consequences for the optimal taxation results. We therefore estimated an additive system as a standard of comparison for the other systems. This can only be done for the Rotterdam system. When we start from the system under negativity as the maintained hypothesis, additivity is accepted after the small sample correction. This may seem surprising, but one should remember that negativity already was strongly rejected by our data.

Given the different interpretation of the coefficients, it seems advisable to compare the results for the different systems in elasticity terms. As noted in the previous section, these elasticities were evaluated at the average budget shares, taken from the Consumer Expenditure Survey. These shares are very similar to the ones computed with National Accounts data for the same years 1978/1979. In table 3 we show the total expenditure elasticities for ROTT4 (negativity), AIDS3 (symmetry), CBS4 (negativity) and ROTT9

TABLES 3 AND 4

(additivity). Table 4 summarizes all price elasticities for the former three systems. The problems with AIDS immediately show up with the positive own price coefficients for some non-inferior goods. For the rest, the numbers speak for themselves. There are some striking differences between the systems : as could be expected, total expenditure elasticities are similar between CBS and AIDS, while for the price elasticities the resemblance is between CBS and Rotterdam.

4. TAX REFORM RESULTS WITH THE DIFFERENT DEMAND SYSTEMS

Table 4 shows that there are important differences between the price elasticity matrices of the three systems. The question remains, however, what are the consequences of these differences for the tax reform problem, as presented in section 2. We will proceed in three steps. First we analyse the results for the case where the inequality aversion parameter e is zero. In that case equation (6) reduces to

$$MC_i = \frac{1}{1 + \sum_k \epsilon_{ki} t_k^* \left(\frac{q_k X_k}{q_i X_i} \right)} \quad (19)$$

and the price effects play the dominant role in the explanation of differences between the MC_i 's. In this first step, we will compare the results for the "best" variants of the systems, i.e. ROTT4, AIDS3 and CBS4. As standards of comparison we also introduce ROTT9 (Rotterdam under complete preference independence) and the "easy" practical solution where the estimation of demand systems is made unnecessary by the assumption

$$\begin{aligned} \epsilon_{ii} &= -1 \\ \epsilon_{ik} &= 0 \quad \forall i \neq k \end{aligned} \tag{20}$$

We will call this the "diagonal" assumption. In a second step we stick to equation (19), but we will compare the results for the different variants of the systems. We will there return to the discussion in the previous section about imposing the negativity condition. In the last step we consider what are the consequences of increasing e .

What matters for the tax reform problem is the *ranking* of the different marginal welfare costs. In fact, the absolute numbers of these costs are dependent on the normalization, chosen for the β^h 's. We will therefore summarize our comparisons with Spearman rank correlation coefficients.

A. Comparison of results for the case $e = 0$

The results for the case $e = 0$ are shown in table 5.

TABLE 5

For some commodities the estimates seem quite different between the systems, striking examples being "beverages" and, especially, "services". The outlier is AIDS, which is not surprising given the positive own price elasticities for these commodities in that system. Remember that for both CBS and Rotterdam, negativity has been imposed. On the other hand, it is easily possible to draw some generally valid policy conclusions from table 5 : one could say that the marginal welfare cost for increasing taxes on tobacco, heating and transportation (where Belgium has now already important excise taxes) is high for all sets of estimates (including DIAGONAL and ROTT9 !).

The information is summarized in table 6 where we give rank correlation coefficients. The AIDS-system obviously is an outlier. The affinity of Rotterdam and CBS is clear. It is striking also that the "simple" solutions

TABLE 6

(DIAGONAL and ROTT9) are not completely divergent from the more complex systems. It is probably fair to conclude that the differences between the results are not as large as could be expected when considering table 4 and

that there is some truth in Ahmad and Stern (1984, p. 293)'s suggestion that this might be explained by the fact that all the demand effects are summed to compute (19). On the other hand, our results also suggest that the practitioner who would use information from one system only (e.g. AIDS under symmetry), cannot be confident that his results are not influenced heavily by this (more or less arbitrary) choice of system. We will now see whether this result is due to the fact that we compare AIDS under symmetry with the other systems under negativity.

B. Results for the different variants ($e = 0$)

Table 7 shows the rankings of welfare costs for the different variants of the systems and the corresponding rank correlation coefficients. The suf-

TABLE 7

fixes 1 to 4 refer to the variants "free", "under homogeneity", "under homogeneity and symmetry", "under homogeneity, symmetry and negativity" respectively. We see that the divergent behaviour of AIDS is also present when we compare AIDS3 with CBS3 and ROTT3. In fact, table 7 suggests some further interesting conclusions.

Free estimates of the different systems lead to almost identical rankings of the welfare costs. Imposing homogeneity does not change this picture drastically. In fact, these two variants could be seen as exercises of curve fitting, while homogeneity only imposes one restriction per equation. Symmetry, however, leads to pronounced changes in the rankings, even for the same system : compare ROTT2 and ROTT3, AIDS2 and AIDS3, CBS2 and CBS3. At the same time, after imposing symmetry, differences appear also between the different systems. This suggests that Ahmad and Stern (1984)'s suggestion, referred to earlier, gives only part of the truth. When we transform our system of equations into a demand system by imposing integrability, summing the demand effects is no longer sufficient to remove differences. There still is a close connection between CBS3 and ROTT3, however. Further use of theoretical restrictions, by imposing negativity (i.e. concavity of the cost function) leads to a widening of the differences between the respective rankings.

This may be the place to return to a question, raised earlier : should one impose the theoretical restrictions ? In our case, at least three possible positions can be taken. The first one could be called the *theoretical position*. The theoretician will argue that welfare economic applications run into deep trouble when estimated demand parameters are not consistent with a concave cost function. He will point out that there are so many data limitations and remaining theoretical problems (e.g. aggregation, treatment of durables) that one should not attach too much attention to the statistical rejection of the negativity condition. Data only speak within a theoretical

framework, so we have to impose the framework. In our case, the theoretician has to face the problem that results will be dependent on the choice of specification to a certain extent. Either, he will have to compare different systems in order to draw conclusions which hold for all (as is possible with our results), or he will have to argue on theoretical grounds that one system is superior. The second position could be called the *statistical* one. Someone holding this position will not want to impose theoretical restrictions if they are rejected by the data. In our case, he will stick to the symmetry versions, and he will have to face the same choice problem as the theoretician. The third position is the *pragmatic* one. The pragmatist will not like to be faced with a choice between different systems. As the theoretician he will point to the many problems (both theoretical and practical) arising when estimating a full-fledged complete demand system. He will argue that we do not need accurate estimates of all cross price-effects and that to apply (19) only the sum of the different effects is necessary. Why bother about a precise estimate of cross-price effects if we can only estimate them, conditional on the "arbitrary" choice of system? Why impose such a straitjacket on the data? After all one can have doubts about the validity of the theory. The pragmatist probably will be happy with the finding that a free curve-fitting exercise (even after imposing the no money-illusion condition of homogeneity) yields results which are insensitive to the exact specification used and could prefer these non-symmetric estimates.

In our opinion, the statistical position undoubtedly is the weakest of the three⁵. The choice between the other two positions is partly a matter of taste, however, and while endorsing the theoretical position ourselves, we feel that more experience with demand systems will be needed if one wants to convert the pragmatist.

C. Increasing the inequality aversion

Table 8 shows the effects of increasing the inequality aversion parameter e . In this table we use CBS4 as the main standard of comparison and add a variant "ZERO" which simply assumes that $\epsilon_{ki} = 0 \quad \forall k, i$ and, hence, neglects all changes in demand. Table 9 contains the rankings for $e = 1,5$, which seems to be a "reasonable" value (see Stern (1977)).

TABLES 8 AND 9

For $e = 1,5$, there are still important differences between the results with the different systems, while as before it is possible at the same time to draw some stable policy conclusions. Table 8 clearly shows that the differences become smaller when e increases. This result is opposite to the one found by Ray (1986) in his computation of "optimal" taxes. The explanation is obvious, however. When e increases, considerations of distributive justice get a relatively larger weight in (5) and (6). In fact, these considerations become irrelevant when $e = 0$ (see (19)). In the framework of tax re-

form (in contrast with optimal taxation), justice considerations are captured by actual observations of consumption patterns (the numerator in (6)) and, hence, completely independent of the elasticity estimates. For an analysis of tax reform, increasing e therefore amounts to increasing the relative weight attached to actual consumption. A striking illustration of this phenomenon is the high correlation between the results for CBS4 and the results for DIAGONAL and (even !) ZERO.

5. CONCLUSION

A comparison of the results with the different systems shows that it is possible to draw relevant policy conclusions, which hold for all of them. Even the additive version of Rotterdam does not give completely different results. It would be wrong to ascribe this result to the similarity between the specifications of our systems. Indeed, it was shown that the matrices of expenditure and price elasticities show huge differences. From that point of view, it is possible to corroborate the suggestion that the sensitivity to the specification of the demand system is less severe for a tax reform exercise, than it seems to be for the calculation of optimal tax rates. This does not mean, however, that the exclusive use of one system always leads to reliable results. Some differences remain very large and a sensitivity analysis seems to be advisable. Increasing the inequality aversion implies that a relatively larger weight is given to observed consumption patterns. The choice of demand system therefore becomes less crucial.

It is remarkable how strongly the rankings of the marginal welfare costs are affected by imposing the restriction of symmetry of the Slutsky-matrix. Without this integrability condition, the different systems yield very similar results. This could be interpreted by a pragmatist as implying that the estimates of the cross price effects are heavily influenced by the mathematical straitjacket of the chosen specification, and that the symmetrical estimate of the Slutsky-matrix is not really informative about reality. Then

why not use the estimates under homogeneity ? Surely, more work about specification and estimation of complete demand systems will be necessary, if one wants to formulate a completely convincing answer to this pragmatical critique.

NOTES

- 1 The equivalence scales used by the Belgian National Institute of Statistics are those of the League of Nations. These are far from perfect of course. The welfare weights are not very sensitive to the choice of scale, however. For instance, use of total expenditure per capita does not lead to a very different pattern of welfare weights.
- 2 See in this respect the interesting discussion by Byron (1984) and Barnett (1984).
- 3 An overview of this problem can be found in Bewley (1986).
- 4 It is not obvious how to determine the number of degrees of freedom for this test, because the negativity condition is an inequality restriction. Following Barten and Geyskens (1985), we chose the number of positive Cholesky values under symmetry.
- 5 Note that Wiebaut (1987) in his empirical application for Belgium uses estimates from a Rotterdam system with symmetry imposed, but with a positive diagonal element in the Slutsky-matrix. He does not explain why he uses that variant.

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Table 1 : Commodity classification and indirect tax rates

Commodity	Expenditures average consumer (1)	Value added tax (2)	Excise tax (3)	Indirect tax rate (in %) [(2)+(3)]/(1)
FOOD	111.531	6.695	191	6,1741
BEVERAGES	16.519	2.874	2.803	34,3665
TOBACCO	6.918	392	4.339	68,3868
CLOTHING	49.145	7.436	-	15,1307
RENT	115.303	1.146	-	0,9939
HEATING	51.114	6.941	-	13,5794
DURABLES	43.381	7.391	-	15,2767
HOUSING	12.340	885	-	7,1718
PERSONAL CARE	27.144	2.077	-	7,6518
TRANSPORTATION	72.001	12.742	10.329	32,0426
LEISURE GOODS	45.648	5.319	-	11,6522
SERVICES	9.368	135	-	1,4411

Table 2 : Testing the restrictions from demand theory

TESTING HOMOGENEITY						
Demand system	Laitinen test-statistic	F(11,7) critical value		Conclusion		
		0.05	0.01			
ROTTERDAM	2,58589	3,6018	6,5382	homogeneity not rejected		
AIDS	2,92910	3,6018	6,5382	homogeneity not rejected		
CBS	2,76468	3,6018	6,5382	homogeneity not rejected		
TESTING SYMMETRY						
Demand system	2LL (a)	χ^2		Conclu- sion	2LL cor- rected(b)	Conclusion
		0.05	0.01			
ROTTERDAM	118,737712	65,09	73,57	rejected	60,805188	not rejected
AIDS	110,814600	65,09	73,57	rejected	56,747799	not rejected
CBS	115,491486	65,09	73,57	rejected	59,142823	not rejected
TESTING NEGATIVITY						
Demand system	2LL (a)	χ^2		Conclusion	2LL cor- rected (b)	Conclusion
		0.05	0.01			
ROTTERDAM	47,476290	9,49	13,28	rejected	27,822127	rejected
CBS	64,307626	9,49	13,28	rejected	37,685652	rejected

(a) Likelihood ratio test statistic

(b) Likelihood ratio test statistic after small-sample correction suggested by Italianer (1985).

Table 3 : Total expenditure elasticities

	ROTTERDAM	AIDS	CBS	Complete preference independence (Rott.)
FOOD	0,663201 ***	0,597821 *	0,501285 *	0,996022 ***
BEVE	1,966410 ***	0,957932 -	1,103555 -	1,570287 **
TOBA	0,521132 -	-0,205197 *	-0,177631 **	1,521279 ***
CLOT	1,534623 ***	1,493855 *	1,542549 **	1,887129 ***
RENT	0,007779 -	0,496411 ***	0,489058 ***	0,000231 -
HEAT	1,184268 *	1,567252 -	1,477835 -	0,797202 **
DURA	3,049535 ***	2,766847 ***	2,752273 ***	3,542886 ***
HOUS	1,347476 **	0,533417 -	0,683356 -	0,969288 **
PERS	1,363547 ***	0,709665 -	0,884087 -	0,253217 -
TRAN	0,666115 **	0,746789 -	0,806191 -	0,659270 ***
LEIS	0,693243 ***	0,990877 -	0,929644 -	0,600705 ***
SERV	6,881274 ***	3,905420 -	4,388789 *	4,150932 **

Table 4 : Price elasticities

ROTTERDAM

	FOOD	BEVE	TOBA	CLOT	RENT	HEAT	DURA	HOUS	PERS	TRAN	LEIS	SERV
FOOD	-0.707485	-0.013697	0.007473	0.021498	-0.140073	-0.034100	0.018998	0.047331	0.003920	-0.053773	-0.006487	0.133199
BEVE	-0.360432	-0.138549	-0.015697	0.238425	-0.347680	-0.245866	-0.295650	-0.097335	-0.001619	-0.369031	-0.260006	-0.012934
TOBA	0.155243	0.002990	-0.063646	-0.270716	-0.250759	0.022496	0.682101	0.180925	0.637960	0.119948	-0.727023	-0.110735
CLOT	-0.125941	0.084384	-0.047488	-0.801933	-0.326742	0.056353	0.103478	-0.012820	-0.192114	-0.069743	-0.140171	-0.061886
RENT	-0.012931	0.004657	-0.009120	-0.013699	-0.033381	0.018142	-0.035197	0.034654	0.006011	0.080707	-0.020197	-0.027424
HEAT	-0.207270	-0.080339	-0.003781	0.107823	-0.175267	-0.395868	0.023422	-0.022352	-0.094597	-0.327527	-0.001257	-0.007254
DURA	-0.429342	-0.129145	0.069464	-0.020993	-0.676271	-0.103814	-0.748470	0.021151	-0.231504	-0.209405	-0.450524	-0.140681
HOUS	0.228377	-0.085579	0.072441	-0.027561	-0.000168	-0.067656	0.203631	-0.096102	-0.136542	-0.318656	-0.120540	-0.130489
PERS	0.087439	-0.013474	0.123647	-0.290727	-0.242858	-0.122834	-0.197009	-0.063364	-0.288631	-0.209947	-0.052803	-0.092984
TRAN	-0.086705	-0.045255	0.009610	0.027618	-0.002071	-0.139055	0.056817	-0.048556	-0.055440	-0.528818	-0.010521	0.156263
LEIS	-0.017373	-0.027568	-0.077123	-0.034932	-0.167838	0.031627	-0.129599	-0.011352	0.011579	-0.014903	-0.201236	-0.054524
SERV	1.323266	-0.167875	-0.200297	-1.012089	-1.851306	-0.421498	-1.406625	-0.469642	-0.813387	1.103797	-1.362490	-1.663724

AIDS

	FOOD	BEVE	TOBA	CLOT	RENT	HEAT	DURA	HOUS	PERS	TRAN	LEIS	SERV
FOOD	-0.460596	-0.046792	-0.007439	-0.008125	-0.037468	0.063715	-0.041064	0.011853	-0.043651	-0.118415	0.027310	0.062845
BEVE	-0.414672	0.192930	0.070755	0.247618	-0.562051	-0.199896	-0.356308	-0.038813	-0.124246	0.057001	0.221735	-0.052021
TOBA	0.033944	0.195680	-0.195416	-0.442356	-0.149582	-0.000223	0.665174	0.512593	0.748499	0.331999	-1.217646	-0.277468
CLOT	-0.196954	0.060822	-0.078101	-0.647543	-0.309462	0.085899	0.108051	-0.048618	-0.252613	-0.004638	-0.110262	-0.100449
RENT	-0.018209	-0.066153	-0.017286	-0.053100	-0.395764	0.000040	-0.070355	0.050194	0.021755	0.138524	-0.028789	-0.057272
HEAT	-0.000495	-0.099240	-0.020872	0.110027	-0.208149	-0.409102	0.032290	-0.088363	-0.288113	-0.455730	0.069032	-0.168553
DURA	-0.534265	-0.169360	0.061789	0.005703	-0.610854	-0.052722	0.131991	0.023325	-0.264310	-0.166421	-0.717887	-0.473823
HOUS	0.104244	-0.029285	0.224075	-0.082092	0.369952	-0.156023	0.253369	-0.016974	-0.126640	-0.137850	-0.522404	-0.413028
PERS	-0.177481	-0.053521	0.145935	-0.332264	0.033882	-0.281623	-0.214236	-0.062945	0.197429	-0.130229	-0.010692	0.176042
TRAN	-0.219385	0.018213	0.020126	0.063487	0.167514	-0.176353	0.055329	-0.034158	-0.060763	-0.690182	-0.067235	0.176635
LEIS	-0.030540	0.052191	-0.139839	-0.041687	-0.145378	0.043210	-0.366446	-0.130683	-0.021065	-0.104049	-0.152511	0.045911
SERV	0.549721	-0.216992	-0.362889	-1.082282	-1.737461	-1.225417	-3.786465	-1.118531	0.773997	1.729537	0.172276	2.399087

CBS

	FOOD	BEVE	TOBA	CLOT	RENT	HEAT	DURA	HOUS	PERS	TRAN	LEIS	SERV
FOOD	-0.267430	-0.082237	0.028732	0.084157	-0.140135	-0.042612	-0.108274	0.034155	0.005251	0.001593	-0.023068	0.008578
BEVE	-0.722671	-0.243780	0.138233	0.220887	-0.331251	-0.091878	-0.281004	0.099255	0.004814	0.064240	0.010517	0.029086
TOBA	0.623372	0.355295	-0.931866	-0.030654	-0.240015	0.030423	0.127263	0.467379	0.380594	0.114169	-0.534869	-0.123400
CLOT	-0.020017	0.055321	-0.024246	-0.787782	-0.329201	0.035638	0.018085	0.005255	-0.221797	-0.048447	-0.147301	-0.078069
RENT	-0.141359	-0.029658	-0.022340	-0.057165	-0.142553	-0.002393	-0.065939	0.036851	-0.009956	0.014093	-0.060010	-0.008633
HEAT	-0.323952	-0.048180	-0.014031	0.054159	-0.199369	-0.352687	-0.021402	-0.025808	-0.066796	-0.395841	-0.068031	-0.015897
DURA	-0.716566	-0.139617	-0.015931	-0.088183	-0.598824	-0.101423	-0.491883	-0.017733	-0.017736	-0.069334	-0.428133	-0.066909
HOUS	0.227012	0.115908	0.174861	0.095003	0.239102	-0.013348	0.111856	-0.896534	-0.209516	-0.306640	-0.101937	-0.119084
PERS	-0.057730	0.008314	0.067193	-0.294421	-0.111316	-0.039269	0.125473	-0.101786	-0.222189	-0.328009	0.015322	0.054313
TRAN	-0.058300	0.022145	-0.000800	0.031085	-0.039683	-0.165066	0.112328	-0.066876	-0.143420	-0.579497	-0.040632	0.122509
LEIS	-0.125952	0.007256	-0.068265	-0.060970	-0.188266	-0.003277	-0.156828	-0.029565	0.005003	-0.059882	-0.223072	-0.025827
SERV	-0.610710	-0.013064	-0.193574	-0.928074	-0.920414	-0.292856	-0.653974	-0.393241	0.097378	1.026121	-0.677037	-0.829537

Table 6 : Rank correlation between demand models ($e = 0$)

	DIAGONAL	ROTT4	AIDS3	CBS4	ROTT9
DIAGONAL	1,0000				
ROTT4	0,6224	1,0000			
AIDS3	-0,0629	0,2098	1,0000		
CBS4	0,4266	0,7622	0,4545	1,0000	
ROTT9	0,7902	0,7413	-0,1189	0,5734	1,0000

Table 7 : Rankings for all variants (e = 0)

	ROTT1	ROTT2	ROTT3	ROTT4	AIDS1	AIDS2	AIDS3	CBS1	CBS2	CBS3	CBS4
	TOBA	TOBA	TOBA	TOBA	SERV	SERV	SERV	TOBA	TOBA	TOBA	TOBA
	SERV	SERV	TRAN	TRAN	TOBA	TOBA	TRAN	SERV	SERV	TRAN	TRAN
	BEVE	BEVE	LEIS	HEAT	BEVE	BEVE	HEAT	BEVE	BEVE	LEIS	HEAT
	HEAT	HEAT	HEAT	BEVE	HEAT	HEAT	LEIS	HEAT	HEAT	HEAT	LEIS
	LEIS	LEIS	BEVE	LEIS	LEIS	TRAN	TOBA	LEIS	LEIS	BEVE	RENT
	RENT	RENT	RENT	RENT	TRAN	LEIS	PERS	RENT	TRAN	RENT	PERS
	TRAN	TRAN	SERV	FOOD	CLOT	CLOT	FOOD	TRAN	RENT	PERS	FOOD
	CLOT	CLOT	PERS	HOUS	RENT	RENT	RENT	CLOT	CLOT	FOOD	CLOT
	DURA	FOOD	FOOD	CLOT	DURA	FOOD	CLOT	FOOD	FOOD	CLOT	DURA
	FOOD	DURA	CLOT	PERS	FOOD	DURA	HOUS	DURA	PERS	SERV	BEVE
	PERS	PERS	HOUS	DURA	PERS	PERS	DURA	PERS	DURA	HOUS	HOUS
	HOUS	HOUS	DURA	SERV	HOUS	HOUS	BEVE	HOUS	HOUS	DURA	SERV
RANK CORRELATION COEFFICIENTS FOR DEMAND SYSTEMS											
	ROTT1	ROTT2	ROTT3	ROTT4	AIDS1	AIDS2	AIDS3	CBS1	CBS2	CBS3	CBS4
ROTT1	1.0000										
ROTT2	0.9930	1.0000									
ROTT3	0.7133	0.7343	1.0000								
ROTT4	0.4476	0.4755	0.8252	1.0000							
AIDS1	0.9720	0.9650	0.6713	0.3776	1.0000						
AIDS2	0.9510	0.9580	0.6993	0.4266	0.9860	1.0000					
AIDS3	0.3986	0.4266	0.5874	0.2098	0.4615	0.5035	1.0000				
CBS1	0.9930	1.0000	0.7343	0.4755	0.9650	0.9580	0.4266	1.0000			
CBS2	0.9720	0.9860	0.7902	0.5105	0.9650	0.9720	0.5035	0.9860	1.0000		
CBS3	0.5524	0.5804	0.9580	0.8951	0.4965	0.5315	0.4545	0.5804	0.6434	1.0000	
CBS4	0.2587	0.2727	0.7413	0.7622	0.1818	0.2098	0.4545	0.2727	0.3147	0.8462	1.0000

Table 8 : Rank correlations between demand systems

Correlation between	VALUE OF e							
	0,0	0,1	0,5	1,0	1,5	2,0	5,0	10,0
CBS4 - ZERO	-	0,2168	0,3077	0,3217	0,3427	0,4406	0,6573	0,9161
CBS4 - DIAGONAL	0,4266	0,4266	0,4615	0,4825	0,4895	0,4895	0,6853	0,9231
CBS4 - ROTT4	0,7622	0,7622	0,8042	0,8252	0,8322	0,8252	0,8531	0,9091
CBS4 - AIDS3	0,4545	0,4545	0,4336	0,4476	0,4545	0,4336	0,5385	0,7413
CBS4 - ROTT9	0,5734	0,5734	0,6154	0,5804	0,6154	0,7273	0,7273	0,8741
ROTT4 - AIDS3	0,2098	0,2098	0,2587	0,2937	0,3497	0,2797	0,3147	0,4895

Table 9 : Rankings for $e = 1,5$

	ZERO	DIAGONAL	ROTT4	AIDS3	CBS4	ROTT9
1	HEAT	TOBA	TOBA	SERV	TOBA	TOBA
2	FOOD	BEVE	HEAT	HEAT	HEAT	BEVE
3	TOBA	HEAT	TRAN	TRAN	TRAN	HEAT
4	BEVE	TRAN	BEVE	TOBA	FOOD	RENT
5	PERS	CLOT	LEIS	LEIS	LEIS	PERS
6	SERV	DURA	FOOD	PERS	RENT	FOOD
7	RENT	LEIS	RENT	FOOD	PERS	TRAN
8	CLOT	FOOD	PERS	RENT	CLOT	CLOT
9	LEIS	PERS	CLOT	CLOT	DURA	LEIS
10	HOUS	HOUS	HOUS	HOUS	BEVE	DURA
11	DURA	SERV	DURA	BEVE	HOUS	HOUS
12	TRAN	RENT	SERV	DURA	SERV	SERV