On the Leydesdorff-Wagner-Bornmann proposal for diversity measurement

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In a recent article Leydesdorff, Wagner and Bornmann (2019) propose a new diversity indicator (DIV) which operationalizes “variety,” “balance,” and “disparity” and then combines them. Concretely, they define the DIV-value of a unit c , as follows:

$$DIV(c) = \frac{n(c)}{N} * (1 - G(c)) * \sum_{\substack{i,j=1 \\ i \neq j}}^{n(c)} \frac{d_{ij}}{n(c) * (n(c) - 1)}$$

Here $n(c)$ is the number of non-empty cells in c for a categorization, having by definition N cells; $G(c)$ is the Gini concentration value of c , so that $1-G(c)$ is the Gini evenness value; d_{ij} is the disparity value between cells i and j . Disparity takes values between zero and one.

I have three problems with the formula for DIV.

1) The first factor is a relative value and hence assumes that there is a general framework consisting of N categories or cells. This is not wrong but it is a restriction on the applicability of DIV. If one uses DIV in a context such as that of WoS categories then N exists (it may not be the best categorization or it even may not be the most appropriate, but this is not the issue here). Yet, if one collects a sample of mosquitoes in a tropical forest, then the possible number of different mosquitoes that one may sample is probably unknown. In a scientometric context one may collect journals that publish articles on a given topic, e.g. interdisciplinarity, but the total number of such journals is not known: one has just an observed number.

2) The third factor in DIV, taking disparity into account is just a relative (normalized) sum. The fact that the d -value between category i and category j takes the value d_{ij} is not taken into account, only the total sum of all disparity values is. For example, assuming three cells and an exact

disparity matrix $D = \begin{pmatrix} 0 & 0.4 & 0.6 \\ 0.4 & 0 & 1 \\ 0.6 & 1 & 0 \end{pmatrix}$, then replacing D by $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ does not influence the value of D .

3) DIV is normalized between the values zero (zero not included) and one. Hence it cannot be a 'true' diversity measure in the sense of Jost (2009). Such measures must lead to the value N if one studies a community of N equally abundant, totally dissimilar items. This objection implies that there is no way to discuss DIV -diversity in terms of percentage growth or decline.

Can one do better in the context described by Leydesdorff et al. (2019)? I think that the answer is yes: objection 3 can easily be solved, simply by removing the divisor N in the variety factor. We refer to this new measure by $DIV^* = N \cdot DIV$. In this case a community of N equally abundant, totally dissimilar items, leads to a value N as $DIV^* = N \cdot 1.1 = N$. This shows that DIV^* meets the "effective number" property of Leinster and Cobbold (2012).

Objection 1 is related to the "effective number" property, but also to the "absent species" property of Leinster and Cobbold (2012). This property states that "diversity is unchanged by adding a new cell of abundance zero". Whether N is known or not (in which case only $n(c)$ is), in both cases DIV and DIV^* do not meet this requirement as adding an empty cell increases $G(c)$, and hence decreases Gini evenness $= 1 - G(c)$. Yet, we think that this requirement is not important: if N is known one might say that it is not allowed to add a cell, as this changes the framework (and we assume a fixed framework); and if one allows to expand the framework, then the situation has changed and there is no a priori reason to obtain the same diversity value. Finally, if N is not known, then $n(c)$ is the observed number of non-empty cells: in these circumstances one cannot observe an absent cell. In our opinion the "absent species" property is not essential.

In this way, only objection 2 remains, which makes DIV^* an acceptable diversity measure, in the sense that if a disparity value increases, then the diversity measure increases, be it clearly not a refined one. We recall that the well-known Rao-Stirling measure takes these exact values into account.

Leydesdorff et al. (2019) proposed DIV , among other reasons, because the well-known Rao-Stirling measure or its modified version (Leinster &

Cobbold, 2012; Zhang et al., 2016), do not meet the 'monotonicity of balance' requirement (Rousseau, 2018) and DIV does.

In conclusion, I refer to Stirling (2007) who stated that one may propose a large number of acceptable properties a diversity measure should meet, but that in reality it might be impossible that one measure meets all requirements. Yet, I still urge colleagues to find a better framework for diversity measurement. One should at least be fully aware of possible requirements which are not met in the framework used by the investigator.

References

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