Failure Probability of a FinFET-based SRAM cell utilizing the Most Probable Failure Point

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Abstract

Application requirements along with the unceasing demand for ever-higher scale of device integration, has driven technology towards an aggressive down-scaling of transistor dimensions. This development is confronted with variability challenges, mainly the growing susceptibility to time-zero and time-dependent variations. To model such threats and estimate their impact on a system's operation, the reliability community has focused largely on Monte Carlo-based simulations and methodologies. When assessing yield and failure probability metrics, an essential part of the process is to accurately capture the lower tail of a distribution. Nevertheless, the incapability of widely-used Monte Carlo techniques to achieve such a task has been identified and recently, state-of-the-art methodologies focusing on a Most Probable Failure Point (MPFP) approach have been presented. However, to strictly prove the correctness of such approaches and utilize them on large scale, an examination of the concavity of the space under study is essential. To this end, we develop an MPFP methodology to estimate the failure probability of a FinFET-based SRAM cell, studying the concavity of the Static Noise Margin (SNM) while comparing the results against a Monte Carlo methodology.

Keywords: Most Probable Failure Point (MPFP), Reliability, Monte Carlo, SRAM

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1. Introduction

Shrinking of device dimensions has brought to surface reliability issues such as aging and time-zero variability. Actually, process variation phenomena such as the random discrete dopant fluctuation (RDF) [1] and time-dependent ones (like Bias Temperature Instability (BTI) and Hot Carrier Injection (HCI) [2]) deteriorate the device's performance since they significantly impact the threshold voltage among others. Aside from performance degradation and reliability threats, these phenomena cause a non-deterministic behavior of the device and therefore of the system. This stochastic nature of performance can be captured through the estimation of distributions, rather than fixed values, to describe performance metrics and device characteristics. In fact, this shift in probabilistic behavior also explains the utilization of Statistical Static Timing Analysis tools (SSTA) instead of Static Timing Analysis (STA) ones [3]. SSTA tools, first model delay from device characteristics and then propagate these distributions. Nevertheless, traditional SSTA methodology still suffers from certain inaccuracies like for example the incapability to include the spatial correlations of process variation and the delay correlation of different circuit paths [4].

To overcome these disadvantages, the reliability community has also largely focused on Monte Carlo experiments while combinations of Monte Carlo algorithms with SSTA methodology have also been discussed [5], [6]. The standard Monte Carlo approach randomly selects samples from the variation space therefore to accurately capture yield and probabilities of rare events either requires a large number of samples and prohibitive simulation times or focuses on fitting data of a reasonable number of samples to known distributions. This fitting process of the latter methodology inevitably introduces errors hence, the objective to capture failure probability values with a higher accuracy still remains. Although approaches like the Quasi-Monte Carlo technique have also been proposed, their implementation is still of questionable accuracy, especially when focusing on aggressively downscaled devices [7]. Then, the Importance Sampling technique can be used for such estimations, however, the accuracy of this approach heavily depends on the alternate distribution which ideally should be close to the final one. However, the final distribution is scarcely known [8].

The drawbacks of these conventional methods have long been identified and another approach, utilizing the Most Probable Failure Point (MPFP) has been introduced [9]. The main advantage of this technique is that it is

distribution-independent in a sense that the metric under study (i.e. delay of a gate, Static Noise Margin of an SRAM cell e.t.c.) is not fitted to any distribution. As expected, the MPFP methodology has been since then used in many works [8], [10]. Recently, a new MPFP approach was discussed that utilizes the chi-square distribution to improve probability mass coverage and achieves even higher accuracy compared to the standard MPFP estimations [11]. Although all these works produce promising results, a fundamental aspect of this methodology has been swept under the carpet: the *concavity* of the metric space under study. In the case that this space is not concave, the MPFP methodology is rendered inapplicable.

Our work focuses on an implementation based on recent MPFP approaches while comparing the failure probability estimations against the typical Monte Carlo methodology. For our case-study, we focus on a FinFET-based 6T SRAM cell suffering from both time-zero and time-dependent variability while examining hold stability. The novelty of our work lies on a thorough study of the *concavity* of the SNM space, differentiating the current paper from prior art.

The rest of this paper is organized as follows: Section 2, presents an overview of the typical MPFP methodology and discusses the recently proposed MPFP approach, focusing on its ability to produce results of higher accuracy. In Section 3 we study the *concavity* of SNM space. In Section 4 we compare our failure probability estimations against a typical Monte Carlo approach. After studying the SNM space, we optimize the discussed MPFP approach on Section 5. Finally, Section 6 concludes this work.

2. Prior Art on MPFP

Although methodologies on finding a most probable point of system failure had been utilized by mechanical and aerospace engineers, the scheme was eventually adopted by reliability engineers in electronic design and architecture [9]. Its core concept is based on the following assumptions:

- Due to process variation and aging effects, device characteristics of the circuit (V_{th} in our case), are described by a vector $\mathbf{x}$. The space of $\mathbf{x}$ is depending on the size of the netlist, specifically the number of involved devices. For instance, for a 6T SRAM cell, $\mathbf{x} \in R^6$ ¹.

¹For a memory cell, the wires in the BL/WL can easily contribute to the variability,

- 71 • For every combination of V_{th} values of the involved devices, namely
72 for every $\mathbf{x}$, corresponds a value y of a metric, which in our case is
73 the SNM. This is captured through the formulation $y(\mathbf{x})$. Of course, a
74 mathematical formulation of the function $y(\mathbf{x})$ is extremely difficult, if
75 not impossible, to find. Therefore, in most cases through experiments
76 or simulation results a y_A value is matched to a certain $\mathbf{x}_A$ vector.
- 77 • We assume a certain margin Y for the metric value we examine. De-
78 pending on the metric under study, the failure criterion can be either
79 $y < Y$ or $y > Y$. In our case, a sample is considered failed when $y < Y$
80 where Y is a margin value for SNM metric. Nevertheless, assuming the
81 metric under study is the delay of a path, it is self-evident that the
82 failure criterion switches to $y > Y$, considering maximum delay Y .
- 83 • After sampling the $\mathbf{x}$ space - either exhaustively or by following an
84 optimization method - we focus on a) identifying the most probable
85 failure point $\mathbf{x}_{MPFP}$ (that is the point that leads to failure and is most
86 likely to appear) and b) isolating F space, that contains all the failed
87 points and of course $\mathbf{x}_{MPFP} \in F$. Actually, locating the most probable
88 failure point is crucial in estimating the F space. Finally, the failure
89 probability is simply $P(\mathbf{x} \in F)$.

90 Certain flavors of MPFP implementation can be found in the literature.
91 Regarding the sampling of the space, as stated in [9], we can find tools that
92 exhaustively sample the space. This provides the complete picture of F and
93 therefore the most accurate results however, leads to prohibitive simulation
94 times. Considering that the complexity of an exhaustive search increases
95 dramatically with the number of devices, it is crucial to examine trade-offs
96 between a reasonable step of the sampling space. Another technique fol-
97 lows the assumption that the function of some devices is more significant
98 to system's operation compared to others. Therefore, instead of sampling a
99 space $\subset R^6$, one can move on a lower-dimensional space neglecting variance
100 of other devices. For example, the work of [12] assumes a 1-D approximation
101 while the work of [10] realizes that WL devices for an SRAM hold operation
102 are not crucial and minimizes search space of an SRAM cell to 4-D. Other
103 reasonably efficient sampling algorithms are also mentioned [8].

especially for large SRAM blocks. Thus $\mathbf{x}$ space can be even more complex. However,
such a study is outside the scope of the current work

Then, it is important to locate the most probable failure point that assists in estimating the failure probability. Looking into a bulk of prior art ([9], [10] and [13]) we can see that the most probable failure point is estimated from Equation 1, which represents an optimization problem. Since the MPFP point (ideally) lies on the failure boundary, the probability of this point is equal to the failure probability of the circuit under test.

$$\mathbf{max} \left\{ P_{\text{fail}} = \prod_{i=0}^{N-1} P(|\Delta V_{th,i}| \geq x_i) \right\} \quad (1)$$

so that $y(\mathbf{x}) < Y$.

Nevertheless, we have already mentioned that failure probability is $P(\mathbf{x} \in F)$ and regardless how accurate our estimations on the MPFP location are, it is important to have a solid indication of the F space. This problem is discussed in [11] where the authors differentiate their implementation from previous approaches by adopting x^2 distribution and drawing hypersphere-like shapes to include all pass (or failed) samples. This methodology achieves a more accurate estimation of the F space and therefore of the failure probability. Interestingly, a hypersphere sampling method is also described in [8] however, this methodology only focuses on the identification of the MPFP point and does not proceed in transforming the metric space under study.

All previous work is based on the assumption that regarding variation, failure is monotonic [9]. Specifically, it is mentioned that “as variation increases the cell will move towards failure, and when failure is reached the cell will keep failing” focusing on SRAM cells. This assumption on the *concavity* of the SNM space however, to the best of our knowledge, has not been proven.

The SNM of the hold operation of an SRAM cell describes how easily a cell flips under hold conditions. In principle, the SNM is quantified graphically through the butterfly curve. Specifically, it is estimated as the side length of the maximum square that can be fitted inside the lobes of this curve as in Figure 1[14]. Nevertheless, stability of a cell

Figure 1: Example of a butterfly curve. SNM is extracted graphically

can be related to other figures of merit as well. For instance, relevant work [15] develops a methodology based on a prediction tool for the estimation of read SNM, that indicates the robustness of the cell during its read operation. The effect of variability on a cell's read operation is also studied in another paper [16]. Here, we will study the *concavity* of the SNM hold space and then, by following the methodology described in [11], we will proceed in accurately the estimating failure probability. In general, the FinFET devices have major advantages compared to the planar CMOS technology regarding their scalability and independent gate control[17]. In our work, FinFET-based 6T SRAM cell is implemented in 10 nm technology nodes using Multi-Gate Predictive Technology Modelcards [18].

3. Utilizing the MPFP – SNM Focus

3.1. Modeling Variability

A typical 6T SRAM cell is shown in Figure 2. For our estimations, we choose to model variations on all 6 devices. This way, we achieve greater accuracy at the cost of elevated simulation times. The approach we follow is similar to the one thoroughly presented in previous work [11]. A major differentiator between the two methodologies is that we model both time-zero and time-dependent variability. Specifically, we focus on a FinFET-based, high performance SRAM cell after 3 years of continuous operation, which is a typical life-time assumption for electronics. To model variability we consider the threshold voltage of the devices distributed rather than fixed, with a Gaussian being a sufficient first-order approximation ($\Delta V_{th}(t) \sim \text{Norm}(\langle \Delta V_{th,TD}(t) \rangle, \sigma_{\Delta V_{th,TOT}}^2(t))$). Specifically, we assume the BTI effect to be the dominant factor of time-dependent variability and describe the mean threshold voltage shift through the formulation [19]:

Figure 2: The typical 6T SRAM cell

$$\langle \Delta V_{th,TD}(t) \rangle \cong At^a E_{ox}^\gamma \quad (2)$$

Regarding the $\sigma_{V_{th,TOT}}$, it is depending both on time-zero and time-dependent variability ($\sigma_{\Delta V_{th,TOT}} = \sigma_{\Delta V_{th,0}} + \sigma_{\Delta V_{th,TD}}$) [20]. Actually, when

time-zero variation phenomena are more aggressive, time-dependent variability is deteriorating. Specifically, for the time-dependent part, it is claimed that [20]

$$\sigma_{\Delta V_{th,TD}}(t) = \sigma_{\Delta V_{th,0}} \sqrt{\frac{\langle \Delta V_{th,TD}(t) \rangle}{100mV}} \quad (3)$$

where $\sigma_{\Delta V_{th,0}}$ represents the threshold voltage shift variability at time-zero due to process variation phenomena. Its value is estimated from Equation 4 where A_{VT} is the Pellgrom’s mismatch parameter and W_{eff} is the effective width of the FinFET’s channel [19], [21]

$$\sigma_{V_{th,0}} = \frac{A_{VT}}{\sqrt{2W_{eff}L}} \quad (4)$$

and finally

$$\sigma_{V_{th,TOT}}(t) = \left(\sqrt{\frac{\langle V_{th,TD}(t) \rangle}{100mV}} + 1 \right) \sigma_{V_{th,0}} \quad (5)$$

3.2. Concavity of the SNM space

Following the methodology of previous work [11], to locate the MPFP and therefore estimate the failure probability, we first need to explore the SNM space and identify the point $y(\mathbf{x})$ with the highest SNM value. To this end, using the Cadence Spectre simulator[22], we measure the SNM at each $\mathbf{x}$ point. It should be noted that for each device we focus on “realistic” values of threshold voltage shifts, therefore $|\Delta V_{th_i}| \leq 0.2V$, while the step size is $50mV$. Aggressively decreasing the step size would of course lead to a more accurate view of the space, however, it would also require prohibitive simulation times. Hence, a trade-off exists between the accuracy of the analysis (in terms of space mapping) and simulation time. In fact, when studying netlists with numerous devices, inhibitive simulation times would render the exhaustive sampling of the space impractical.

After the sampling procedure, one important observation is the symmetry of the SNM space. From a closer look at Figure 2 and at basic design principles of the typical 6T SRAM cell [23], we easily realize that the cell is symmetric. This symmetry is confirmed in our analysis of the SNM space as well. Specifically, we have verified that the impact of threshold voltage shifts on any of the M1, M2 and M5 devices to the SNM value is identical

197 to the same amount of shifts on the M3, M4 and M6 respectively as shown
 198 in Figures 3a–3c. Obviously, this is not the case for other combinations of
 199 devices, like for example M1 and M6 as we can verify from Figure 3d. Since
 200 a thorough representation of the whole SNM space is practically infeasible,
 201 we present certain instances where the 4 ΔV_{th} of the devices remain fixed
 202 while for the other two transistors, we sweep the threshold voltage shift and
 203 estimate the SNM.

Figure 3: Instances of the SNM space

204 Next, we move forward to the development of a tool that locates any
 205 maxima of the SNM space. To this end, we are based on the coordinate ascent
 206 algorithm as presented in previous work [11]. Actually, from our simulations
 207 we have observed that the SNM space contains a global maximum, while
 208 local maxima also exist. Interestingly, we have observed that this global
 209 maximum “lies in” the symmetry axis (the pairs of M1-M3, M2-M4 and M5-
 210 M6 have the same threshold voltage shift). Our tool starts by initializing the
 211 threshold voltage shifts near zero while the results of the coordinate ascent

Figure 4: Results for the Coordinate Ascent algorithm

212 tool are shown in Figure 4. It is clear that as the algorithm proceeds to next
 213 steps, the SNM value increases until it reaches the maximum at the final
 214 step. We later verified through exhaustive search that this maximum is in
 215 fact the global maximum by comparing the aforementioned point with all
 216 the other sampled points of the space in terms of SNM value. It should be
 217 stressed that when utilizing a coordinate ascent algorithm and have reached
 218 a maximum, we cannot guarantee if this is the global maximum or a local
 219 one. To assess whether the first or the second case stands, a study on the
 220 *concavity* of the SNM space near this point is required. Should the maximum
 221 be local, we will observe points in a distance r to have a SNM value higher
 222 than the aforementioned point.

223 A strict mathematical proof of the *concavity* of the space is infeasible since
 224 the exact formulation of the equation $f(\mathbf{x}) = y$ is unknown. Nevertheless, we
 225 can focus on the space near the maximum and examine whether all points
 226 have an SNM value lower than the one estimated on the maximum. In
 227 addition, we expect that as distance r from the maximum increases, the SNM

228 value decreases. This observation would be rendered as substantial evidence
 229 on the *concavity* of the SNM space and show whether failure is monotonic.
 230 Figure 5 shows a representation of the SNM space near our maximum. The
 231 curves depict SNM as we are moving away from the maximum at multiple
 232 directions and each curve depicts the movement away from the maximum at
 233 a different direction. Specifically, the directions were calculated as stated in
 234 the following equation, where $\mathbf{x}_{max}$ is the identified maximum point and $\mathbf{x}$
 235 specifies the direction that we move parallel to each time.

$$\mathbf{y} = t * \mathbf{x} + \mathbf{x}_{max} \quad (6)$$

Figure 5: Concavity of the SNM space near $\mathbf{x}_{max}$

236 In Figure 5 the parameter r is the distance ($r = \text{sgn}(t) \times \|\mathbf{y} - \mathbf{x}_{max}\|$) of
 237 the threshold voltage shifts from the maximum. All the curves confirm that
 238 the point we identified is indeed the global maximum since the SNM value
 239 is lessening in each specified direction while we are moving away from it.
 240 Therefore, as variation increases (having as a starting point our maximum)
 241 failure is monotonic. Now, we can safely proceed to the estimation of the
 242 failure probability.

243 3.3. Estimation of the Failure Probability

244 In order to estimate the failure probability, we continue by locating the
 245 ΔV_{th} combination, namely $\mathbf{x}_Y$, with the minimum distance from the max-
 246 imum r_Y , that leads to the failure of the cell. To achieve it we use the
 247 gradient descent algorithm[11]. According to it, we initialize the V_{th} shifts
 248 to the values of the maximum and at every step, we move towards the most

249 descending direction of SNM until we reach a local minimum, as shown in
 250 Figure 6. It has to be noted that this algorithm follows a different concept
 251 than the coordinate ascent. Instead of searching one coordinate at a time, it
 252 examines numerous coordinates and then moves to the point with the lower
 253 SNM value hence, this methodology requires more computations per itera-
 254 tion. In the previous Subsection we chose the coordinate ascent algorithm,
 255 since the path that led to the maximum is not of interest.

Figure 6: Results for the Gradient Descent algorithm

256 After the identification of the most descending direction, we utilize the
 257 non central chi-square distribution (χ^2) to draw hypersphere-like shapes
 258 around the maximum with a radius equal to r_Y each time. This way, we
 259 are able to estimate the failure probability at all points of the previous steps.
 260 Specifically, we will compute the probability $P(\mathbf{x} \in F)$, where the F space
 261 represents the set in the ΔV_{th} variation space that leads to samples failing the
 262 SNM criterion. Next, we use Equation 7 to calculate the random variable
 263 which is distributed according to the non central chi-squared distribution

264 where N is the number of devices and σ_i represent the standard deviations
 265 of the threshold shifts of devices. To compute the non centrality parameter
 266 λ we use μ to represent the mean ΔV_{th} value of each device at the global
 267 maximum.

$$z^2 = \sum_{i=1}^N \frac{x_i^2}{\sigma_i^2} \quad \lambda = \sum_{i=1}^N \frac{\mu_i^2}{\sigma_i^2} \quad (7)$$

268 Finally, the failure probability is estimated using the following metho-
 269 dology: For each point $\mathbf{x}_Y$, with distance from the maximum r_Y that the
 270 gradient descent algorithm has indicated, we calculate the failure probability
 271 regarding as Y the respective SNM and using the χ^2 distribution. Specifi-
 272 cally, the failure probability for each design margin Y satisfies Equation 8,
 273 where CDF and PDF are the cumulative and probability density functions
 274 of the chi-square distribution respectively.

$$P_{fail} = 1 - CDF(r < r_Y) = 1 - \int_{-\infty}^{r_Y} PDF_r dr \quad (8)$$

Figure 7: Failure Probability for various values of the SNM margins Y

275 The results of the failure probability versus the SNM margin Y are demon-
 276 strated in Figure 7. We can see that the decrement of Y leads to a lower
 277 failure probability and vice versa. This is expected, considering that failure
 278 is directly depending on the criterion and the specified SNM design margin.
 279 Hence, the decisive role of the Y is obvious: when high levels of SNM margin
 280 are demanded, the failure probability of our cell rapidly drops.

281 4. Comparing MPFP to Monte Carlo

282 Since the Monte Carlo (MC) technique has been largely employed to
 283 evaluate the reliability of CMOS designs, in this Section we will estimate the
 284 failure probability according to this methodology and compare it against the
 285 results of the MPFP technique we study. The simulation framework for our
 286 MC experiments is shown in Figure 8. First, we add time-zero variability
 287 to a median population size (namely 10^3 samples) of “fresh” SRAM netlists
 288 of the same technology node, according to the mathematical formulation
 289 4. Subsequently, we inject time-dependent variability based on Equations 2
 290 and 5 to produce the aged netlists that suffer from both time-zero and time-
 291 dependent variations. Next, through simulations, we estimate the SNM value
 292 of each sample.

Figure 8: The Flow of our Simulation Framework

293 The histogram of the estimated SNMs after approximately three years of
 294 operation is presented in Figure 9a along with the design margin Y . Next,
 295 we present the respective failure probability estimated by the MPFP tool
 296 (Figure 9b) for the same Y and we compare the two results. We can observe
 297 that for a high Y value, the estimated failure probabilities by the two different
 298 methodologies are approximately equal.

299 For failure probabilities at significantly lower orders of magnitude (i.e. in
 300 the scale of 10^{-6}) the MC methodology requires a vast population number
 301 and long simulation times. When low CPU time is a constraint, computation
 302 accuracy is poor. This is confirmed in our case, since the number of failed
 303 samples is zero when we choose a more realistic margin as shown in Figure
 304 10. A quantitative comparison of failure probabilities for certain values of
 305 Y between MC and MPFP methodologies is given in Table 1. It should be
 306 stressed that in order to alleviate prohibitive computation times, engineers
 307 work with a small population of MC samples and fit the output estimations

Figure 9: Comparison between MC and MPFP for a high value of Y

Figure 10: Comparison of Monte Carlo with MPFP for a realistic value of Y

308 to a known distribution (usually the Gaussian). However, the SNM metric
 309 is observed to have a rather prolonged tail towards lower values that the
 310 aforementioned fitting fails to capture [24]. Hence, we can conclude that
 311 while both MC and MPFP methodology are accurate for estimating failure
 312 probabilities that have high values, when focusing on probabilities of lower
 313 scales, the typical MC methodology suffers in terms of accuracy. To overcome
 314 this issue, one can select a vast number of samples however, this would render
 315 the technique inefficient.

316 **5. An Efficient MPFP Approach**

317 So far, we have shown how the MPFP approach can accurately esti-
 318 mate failure probability nevertheless, we have exhaustively sampled the SNM
 319 space. This required elevated simulation times. Since we have shown the *con-*
 320 *cavity* of the SNM space along with its symmetry, we now focus on developing
 321 new tools based on the coordinate ascent and gradient descent algorithms.
 322 This time, however, we do not sample the whole space but use Spectre “in-

Table 1: A quantitative comparison for failure probabilities estimated by the two methodologies for certain Y values.

Y (V)	Failure Probability MC	Failure Probability MPFP
0.132	0.9382	0.9429
0.113	0.4683	0.6042
0.095	0.0656	0.107
0.076	0.0055	0.0066
0.046	0	$1.34e - 7$
0.020	0	$8.06e - 10$

(a) Opt. CA for $\Delta V_{th,M1}$

(b) Opt. CA for $\Delta V_{th,M2}$

(c) Opt. CA for $\Delta V_{th,M3}$

(d) Opt. CA for $\Delta V_{th,M4}$

(e) Opt. CA for $\Delta V_{th,M5}$

(f) Opt. CA for $\Delta V_{th,M6}$

(g) Opt. CA for SNM

Figure 11: Results for the optimized Coordinate Ascent of the Efficient MPFP approach

323 *the-loop*", while the two algorithms are executed. In addition, we have im-
324 proved the coordinate ascent tool exploiting the symmetry of the SNM space.
325 We have shown and verified that the global maximum lies on the symmetry
326 axis, therefore, our second version of the coordinate ascent tool utilizes the

Figure 12: Results for the Gradient Descent algorithm of the Efficient MPFP approach

coordinates of the symmetry axis. An example of this implementation of the coordinate ascent algorithm is shown in Figure 11. Since simulation time is not a bottleneck, we have used a step size of $25mV$. Interestingly, on a typical 6T SRAM cell, the symmetry of the SNM space between devices M1-M3, M2-M4 and M5-M6 will exist regardless the technology node. Hence, when studying the failure probability of a 6T SRAM cell, the current efficient MPFP approach can be utilized without recurring computations on the *concavity* or symmetry of the SNM space.

Within the same spirit, we move forward with the gradient descent tool starting from the maximum. Because of the decreased step size, the steps to reach the minimum are nearly double. In addition, since our simulator is running “*in-the-loop*” with the gradient descent, the total computation time

339 is negligible. In fact, we can decrease step size to achieve even higher accuracy
 340 in our estimations. The results of the optimized version of the gradient
 341 descent tool are shown in Figure 12. This time, according to Equation 8 we
 342 estimate the cumulative distribution of Y and eventually the distribution of
 343 P_{fail} in Figure 13. We outline the difference in computation time on a Intel
 344 CPU i3 8100 processor for our previous Monte Carlo approach versus the
 345 current efficient MPFP scheme in Figure 14.

Figure 13: CDF_Y and, eventually, the P_{fail} for our SRAM cell in terms of SNM design margin

Figure 14: Difference in Computation time for the two methodologies

346 6. Conclusion

347 In this paper we have presented a complete framework for estimating
 348 the failure probability of a 10nm FinFET-based SRAM cell suffering from
 349 process and time-dependent variations. Specifically, we focused on the hold
 350 stability and evaluated the SNM metric. We have assumed the dominant
 351 source of time-dependent variability is the BTI effect and have estimated
 352 the degradation of devices under 3 years of operation. The first step to the
 353 discussed approach was to show the *concavity* of the SNM space and validate
 354 the assumptions concerning the failure monotonicity of a typical 6T SRAM

cell. We also verified the symmetry of the SNM space, a consistency of the cell’s design principles. To estimate the failure probability, we exhaustively sampled the SNM space, while the core tools of our framework consist of implementing a coordinate ascent and a gradient descent algorithm. In addition, we briefly showed the advantages of the MPFP methodology compared against a simple Monte Carlo-based technique in terms of accuracy. Lastly, we moved forward by optimizing our tools to avoid exhaustive sampling of the space, since *concavity* has already been shown. In this new version, we exploited the symmetry of the SNM space while we “adjusted” our simulator to work on top of the coordinate ascent and gradient descent tools. Now, our final tool presents significant advantages over a typical Monte Carlo approach both in terms of accuracy and efficiency, saving up to 97% in terms of computation time.

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