An automatic constructive matheuristic for the shift minimization personnel task scheduling problem

Reshma Chirayil Chandrasekharan ·
Pieter Smet · Tony Wauters

Received: date / Accepted: date

Abstract The shift minimization personnel task scheduling problem is an NP-complete optimization problem that concerns the assignment of tasks to multi-skilled employees with a view to minimize the total number of assigned employees. Recent literature indicates that hybrid methods which combine exact and heuristic techniques such as matheuristics are efficient as regards to generating high quality solutions. The present work employs a constructive matheuristic (CMH): a decomposition-based method where sub-problems are solved to optimality using exact techniques. The optimal solutions of sub-problems are subsequently utilized to construct a feasible solution for the entire problem. Based on the study, a time-based CMH has been developed which, for the first time, solves all the difficult instances introduced by Smet et al. (2014) to optimality. In addition, an automated CMH algorithm that utilizes instance-specific problem features has also been developed that produces high quality solutions over all current benchmark instances.

Keywords Task Scheduling · decomposition · SMPTSP · matheuristic

Reshma Chirayil Chandrasekharan
KU Leuven, Department of Computer Science, CODES, Gebroeders De Smetstraat 1, 9000
Gent, Belgium
E-mail: ReshmaCC@kuleuven.be

Pieter Smet
KU Leuven, Department of Computer Science, CODES, Gebroeders De Smetstraat 1, 9000
Gent, Belgium
E-mail: pieter.smet@kuleuven.be

Tony Wauters
KU Leuven, Department of Computer Science, CODES, Gebroeders De Smetstraat 1, 9000
Gent, Belgium
E-mail: tony.wauters@kuleuven.be

1 Introduction

Personnel scheduling concerns assigning tasks to a given workforce such that a certain cost is minimized. Due to its widespread applications and connection to other optimization problems, personnel scheduling continues to constitute one of the most active research areas within operations research (den Bergh et al., 2013). The influence and applicability of other research streams such as behavioural sciences and human rights also serves to maintain the particularly interesting status of personnel scheduling.

The problem of determining the size and composition of a minimal workforce required to perform a specific operation is encountered often in personnel scheduling. It becomes even more difficult when there is a multi-skilled workforce of which only some are skilled to perform certain tasks. This problem was studied by Dowling et al. (1997) to develop a day-to-day planning tool and to estimate a minimal staff set capable of operating as the ground staff of an international airport. The same problem can also be viewed as a machine scheduling problem if shifts are mapped to machines and tasks to jobs (Kolen et al., 2007). Another way of looking at the problem is in the form of a colouring problem on the interval graph defined by the time intervals associated with tasks. Motivated by its applicability in personnel rostering, Krishnamoorthy et al. (2012) formally defined the problem and introduced it as a variant of the personnel task scheduling problem. A benchmark dataset of 137 instance was proposed and two heuristics were utilized to generate 135 feasible solutions of which 70 were optimal. To enable this dataset to also be applicable for machine scheduling, it is assumed that shifts/employees are available throughout the planning horizon (24 hrs). In personnel scheduling, these long shifts can be subdivided and performed by one or more personnel with similar skills. The problem is therefore equivalent to determining the minimal set of shifts required and is called the shift minimization personnel task scheduling problem (SMPTSP). For the ease of discussion, the SMPTSP will be presented as a problem concerning assignment of employees to tasks similar to how it has been treated in every other paper in the literature.

This problem is proven NP-Complete and therefore, with the exception of very small instances involving only a few tasks and shifts, exact approaches turn out to be inefficient at generating high quality solutions. As a consequence, most of the solution approaches found in the literature are heuristic in nature. Lin and Ying (2014) introduced a three-phase heuristic approach which was able to generate 135 feasible solutions, of which 101 were optimal. Their approach involved a constructive heuristic and an iterated greedy heuristic, with the generated solutions utilized by the general purpose solver Gurobi. Employing a multi-stage approach, Smet et al. (2014) was successful in solving all 137 instances to optimality for the first time. In addition to their promising results, a major contribution of the paper was the proposal of a constructive matheuristic (CMH) approach. The authors also conducted a detailed study of the benchmark instances and a new benchmark dataset of ten difficult instances was proposed, with their approach being capable of solving

five of these to optimality. Since then, few methods have been proposed in an attempt to solve the remaining unsolved instances to optimality. Baatar et al. (2015) introduced a novel branching scheme based on the mathematical structure of the problem and applied two column generation methods and a heuristic. Their approach was able to solve 6 out of these 10 instances to optimality. An optimality-preserving decomposition method was discussed in Niraj Ramesh et al. (2018) which was capable of solving 5 of them and 83 of the 137 original benchmark instances to optimality. Four of these ten difficult instances therefore, still remain unsolved. In addition, a new benchmark dataset with different structure was introduced by Fages and Lapgue (2013) and their approach solved 46 of them to optimality. Later, 84 of them were solved by Hojati (2018).

The aim of this study is twofold. Existing methods in the literature are not efficient at producing optimal solutions for all benchmark instance sets. The first aim of this study therefore is to develop a solution approach that takes instance structure into account to produce high quality solutions for all sets of instances. Second, since previous research such as Smet et al. (2014) has motivated the implementation of a CMH approach so that large real world scenarios can also be solved effectively, is to investigate the applicability of CMH techniques for the SMPTSP and to test additional CMH-design parameters defined in Chandrasekharan et al. (2019).

To begin with, the first part of the study focuses on selecting an appropriate decomposition for the SMPTSP that is suitable to be employed in a CMH framework. This has led to the development of a CMH approach that assumes fixed values for the design parameters predefined by the user. In the second part, interaction of the design parameters with the instance structure is studied and as a result, the CMH approach has been extended to be able to arrive at suitable values for certain design parameters based on the instance structure. This automated heuristic approach has been able to generate high quality solutions for a large number of benchmark instances over the three different instance sets. The following section formally introduces the problem and its mathematical formulation. Section 3 introduces the overall strategy utilized in this paper and provides a comparative study concerning two possible decomposition approaches. Computational experiments are detailed in Section 5 and Section 5.2 describes the improved CMH strategy. Section 6 presents the final results and compares these against the state of the art. Finally, Section 7 summarizes the conclusions.

2 The shift minimization personnel task scheduling problem

The SMPTSP involves a set of tasks and employees. A task is spread over some time interval and has specification concerning the skill set required to perform it. An employee is characterised by its start and end times and has a set of tasks associated which it is skilled to perform. Tasks must be performed

one at a time by a given employee. The aim of the SMPTSP is to determine a minimal set of employees capable of performing all tasks.

Let the set of tasks and employees be denoted by $J = \{1, 2, \dots, n\}$ and $W = \{1, 2, \dots, m\}$, respectively. Task j can be performed by employee w if w has the skills required to perform task j and is available during the time interval associated with task j . Let the set of employees that can perform task j be denoted by $W_j \subseteq W$. Similarly, for every employee w , let $J_w \subseteq J$ be the set of tasks which w can perform. Note that two tasks which overlap in time cannot be assigned to the same employee.

Consider graph G defined with respect to the set of tasks J . Two tasks in G are connected if they overlap in time. Note that any two tasks in a maximal clique of G overlap in time and therefore cannot be assigned to the same employee. For each employee w , consider the induced subgraph G_w on the vertices from set J_w . Let $C^w = \{K_1, K_2, \dots, K_t\}$ be the set of maximal cliques in G . Note that no two tasks in $K_l \in C^w$ can be assigned to the same w .

The objective of the SMPTSP is to find a minimal subset of W that is capable of performing all the tasks in J . In this paper, the Integer Programming (IP) formulation introduced by Krishnamoorthy et al. (2012) is utilized, which makes use of the following two sets of decision variables:

$$x_{jw} = \begin{cases} 1 & \text{if task } j \in J \text{ is assigned to employee } w \in W \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$y_w = \begin{cases} 1 & \text{if employee } w \in W \text{ is active} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

The model can then be formulated as:

$$\text{minimize: } \sum_{w \in W} y_w \quad (3)$$

$$\text{subject to: } \sum_{w \in W_j} x_{jw} = 1 \quad \forall j \in J \quad (4)$$

$$\sum_{j \in K_l^w} x_{jw} \leq y_w \quad \forall w \in W, \forall K_l \in C^w \quad (5)$$

$$0 \leq y_w \leq 1 \quad \forall w \in W \quad (6)$$

$$x_{jw} \in \{0, 1\} \quad \forall j \in J, \forall w \in W \quad (7)$$

The objective of the SMPTSP is provided by way of Equation (3) and seeks to minimize the number of employees utilized. Constraints (4) ensure that a task is assigned to exactly one employee. Constraints (5) stipulate that for any employee w , no two tasks in a maximal clique of G_w are assigned to it and that tasks are assigned only to active employees. Constraints (6) and (7) specify bounds of the decision variables.

2.1 Benchmark data set

Krishnamoorthy et al. (2012) proposed the dataset comprising of 137 instances, denoted by Data_137. The parameters associated with each instance are its number of employees, task lengths, multi-skilling level of employees and the tightness ($T\%$), defined as the total length of all tasks as a percentage of the total duration of employees. Time interval for each instance was 24 hours (1440 mins). Therefore, starting time of the instance t_{min} and its end time t_{max} are fixed as 0 and 1440 respectively. Smet et al. (2014) proved that the size of largest maximal clique in graph G is a valid lower bound for the SMPTSP, referring to it as the clique lower bound (CLB). The same properties also hold for the 10 difficult instances proposed, denoted by Data_10, as well. However, for the 100 instances proposed by Fages and Lapgue (2013), denoted by Data_100, CLBs not necessarily correspond to optimal solutions. Following this work, Solyali (2016) proposed a general lower bounding procedure which established efficient lower bounds for these instances.

3 Constructive matheuristic

Maniezzo et al. (2009) define matheuristics as algorithmic hybrids of mathematical programming and heuristics. Lately, various heuristics have been designed which incorporate Mixed Integer Programming (MIP) solvers, given recent advances in the area. These methods are referred to as MIP heuristics and, by definition, belong to the class of matheuristic algorithms. The local branching technique presented in Fischetti and Lodi (2003), which iteratively solves subsets of variables mathematically and fix them, is an example of one such heuristic. Other successful examples of MIP heuristics include heuristics utilized in Santos et al. (2016) and Della Croce et al. (2014).

Smet et al. (2014) introduced a simple MIP heuristic which sequentially solves subproblems using a commercial MIP solver, thereby arriving at a feasible solution for the entire problem. The generality of the CMH strategy presented in Smet et al. (2014) makes it easily adaptable and therefore is utilized by the present paper. Two different decompositions are employed in this framework and additional design parameters defined in Chandrasekharan et al. (2019) are tested to arrive at conclusions regarding the interaction of problem structure and the two decompositions. Much like any other decomposition-based method, the challenge is to identify an appropriate decomposition strategy which results in high quality solutions. Once a decomposition approach has been determined, an order for solving these subproblems is determined and they are then solved one by one. It is often the case that not all hard constraints can be evaluated locally within individual subproblems. Such constraints are therefore converted into appropriate soft constraints and employed in the subproblems such that their feasibility is maintained in the final solution produced by the CMH. The decomposition strategy, the sub problems'

formulation and order of execution of these subproblems, all constitute key factors that define a CMH strategy.

3.1 The general CMH framework for the SMPTSP

A given problem is decomposed into subproblems referred to as blocks. An IP formulation for each block is solved using a generic MIP solver as it is encountered. The values of a block's decision variables are then fixed. The CMH continues solving blocks until either the final block is solved or an infeasibility is encountered. The following design parameters are utilized.

1. Block size (η): Subproblem size is central in determining the solution quality and CMH runtimes. While larger subproblems are typically associated with better solutions, they also result in longer run times. The size of blocks must therefore be carefully chosen in order to define an effective decomposition strategy.
2. Overlap (θ): A challenge when defining subproblems is that the fixed values of previously-solved blocks affect the feasibility and solution quality of the blocks yet to be solved. The CMH permits overlap between subproblems so that the feasibility of the subproblems yet to be solved is taken into some degree of consideration while solving the current one. This design parameter defines the extent of overlap between subproblems.
3. Relaxed future (ρ): While larger subproblems often lead to better solutions, increasing subproblem size is not always feasible due to runtime restrictions. This parameter extends a subproblem by including a part of the next subproblem in a relaxed manner. This relaxed portion of the next subproblem is named relaxed future and the parameter ρ determines its size. In problems where there are hard constraints spanning across different blocks and for which applying a relax and fix method (Wolsey, 1998) might not be feasible due to long root relaxation solve times, incorporating a large relaxed future can be helpful. Note that ρ must be set such that the relaxed future of the current block is re-solved in the following blocks with the original formulation in order to maintain feasibility.

The CMH parameter configuration is thus given by a tuple of the format (η, θ, ρ) . An overlap θ means that the current block is solved with a $\theta\%$ overlap with the next block. Similarly, relaxed future ρ indicates the percentage of the future block relaxed and included in the current one.

Two decomposition strategies are presented in this section and they define subproblems or blocks as a subset of employees and tasks. The primary difference between the two strategies can be attributed to the very nature of decompositions employed: employee-based and time-based.

3.2 Employee-based decomposition

This is the decomposition strategy employed in the CMH introduced by Smet et al. (2014) for the SMPTSP. In this decomposition, a block is defined by a subset of employees $W' \subseteq W$ and the set of unassigned tasks J which can be performed by W' . An IP formulation of the problem is solved and the block's assignments are fixed. Since Constraints (4) cannot be evaluated on a subset of the tasks, the IP formulation for the full problem cannot be evaluated for a block. Instead, a modified version where the objective is to maximize total assigned task duration is utilized. This in turn reduces the number of employees required. The following is the modified objective function utilized by blocks.

$$\text{maximize: } \sum_{w \in W'} \sum_{j \in J_w} d_j \quad (8)$$

where d_j is the duration of task j . The last block is re-optimized with Objective (3) to minimize the number of employees. Note that in this CMH there can be unassigned tasks after the last block is solved. In order to remove remaining infeasibilities, Smet et al. (2014) applied a local branching-based improvement heuristic on the solutions of the CMH.

3.3 Time-based decomposition

The problem can also be decomposed by way of separating the problem's timeline into intervals, each of which corresponds to a block. Associated with each block is the following: a time interval, the tasks which begin in this time interval and the set of employees qualified to perform these tasks. The original IP formulation is utilized here with the only modification being that Constraints (4) only considers tasks belonging to the current block. When a task is assigned to an employee, the time interval associated with the task is deleted from the employee's timeline in order to prevent overlapping task assignments in future blocks. A block may become infeasible if there are no employees with the required skill set left to execute tasks which are part of it. Note, however, that in this decomposition, either a block becomes infeasible or all tasks are assigned when CMH completes execution.

Figure 1 presents a snapshot in time of the two different decompositions. Employees and tasks are presented as time intervals and rectangular windows represents blocks. Tasks are numbered and the list of numbers written on an employee are the tasks which it is skilled to perform. As evident from the figure, block in the employee based decomposition are defined by subsets of employees whereas blocks in the time based decomposition are time intervals.

4 Automatic time-based CMH

For a given instance, all employees span the entire time horizon while tasks are distributed over it. Figure 2 plots the number of tasks active at each moment

Fig. 1: A snapshot of the two CMH decompositions. The solid window is the current block, the gray window represents the previous block and the dashed window represents the relaxed future of the current block.

in time for the instances 33 and 135 of Data_100 and Data_137, respectively. The zeroes of such a graph correspond to time where no tasks are simultaneously active. Tasks active between pairs of consecutive zeroes can therefore be allotted to separate sets which can then be independently assigned to employees without compromising the optimality of the overall solution. The two graphs presented correspond to instances with contrasting properties. Instance 135 has a near-equal number of active-tasks throughout the time horizon, while the graph concerning instance 33 is characterized by values quite close to zero. Such points in time are good candidates for decomposition with respect to

time. It has been observed that a large number of the Data_100 instances exhibit this potential for decomposability while most of the Data_137 and Data_10 instances do not.

Fig. 2: The active-task distribution graph of instance 33 of Data_100 and instance 135 of Data_137

Based on the observations concerning instance structure, a strategy has been arrived at which constructs blocks based on the distribution of tasks over the time horizon. Given an instance, its active task distribution graph over time is plotted and small fluctuations are ignored. If t_{min} and t_{max} denote the start and end time values of the time horizon, the graph's set of minima in the interval (t_{min}, t_{max}) , denoted by set $M = \{m_1, m_2, \dots, m_l\}$, is identified. Based on M , the time horizon is decomposed as $[t_{min}, m_1], [m_1, m_2], \dots, [m_l, t_{max}]$. Set of tasks starting in each time interval and employees skilled to perform them constitute blocks. Clearly, blocks here are of various sizes and therefore, this strategy is referred as the *variable blocking strategy*.

While it is possible to identify an efficient variable blocking strategy for some instances, not all can be effectively decomposed using this strategy. For instances which have a near-equal number of tasks distributed throughout the time horizon, this decomposition strategy fails at finding an effective minima for the graph (such as in instance 135 in Figure 2). It may also happen that the minima are located too close to each other such that the resulting blocks are too small to lead to high quality final solutions. An opposite scenario is when the minima are located too far from each other and, as a consequence, certain blocks are too large to be efficiently solved as a subproblem.

To be able to utilize the advantages of the variable blocking strategy while ensuring an effective decomposition, an automatic time-based CMH approach has been designed that selects between the fixed and variable blocking strategies based on instance structure. Given an instance, this approach generates

a variable blocking strategy. If the variable blocking identified is such that all minima of the active task distribution graph are at least $r\%$ less than the maxima, the variable blocking strategy is accepted. In addition, the distance between any two consecutive minima are not allowed to be less than d_{min} in order to prevent blocks which are too small. If the approach is unable to identify a feasible variable blocking strategy, an alternative blocking strategy is applied: *fixed blocking strategy*. This strategy applies a simple CMH which employs the time-based decomposition. Depending on the value of η , the time horizon is decomposed into fixed equal intervals, each of which corresponds to a block.

5 Computational study

The CMH algorithms were coded in Java and the commercial mathematical solver Gurobi 7.0.2 was utilized to solve all subproblems. Experiments were performed on four threads of an Intel(R) Xeon(R) CPU E5-2650 v2 @ 2.60GHz computer running Ubuntu 16.04.2 LTS. The benchmark time limit is considered to be 1800 seconds.¹ For the ease of discussion, the Data_137 instances are subdivided into three groups, such that instances 1-50, 51-100 and 101-137, and henceforth are referred to as small, medium and large instances, respectively.

5.1 Comparing employee-based and time-based decompositions

This section presents a comparative study of the employee-based and time-based decompositions when applied to the SMPTSP. The aim of the study is to test the effect of the CMH's design parameters upon final solution quality when coupled with the decompositions. Smet et al. (2014) has already tested the employee-based CMH on the Data_137 instances. This section repeats some of those experiments and extends them to the whole set of benchmark instances. Trends in design parameters are analyzed separately for Data_137 and Data_100 in order to present the variation with respect to the dataset utilized. Additional CMH design parameters are also discussed.

For a given value of η , this decomposition divides the employees into groups of size η with each of these groups constituting a block. The order in which employees are assigned may have an impact on solution quality. Smet et al. (2014) tested the employee-based the decomposition when they were arranged in three different orders: (i) when employees which are least skilled were assigned first, (ii) when employees which are the most skilled were assigned first and (iii) when employees were randomly assigned. It was observed that these

¹ The benchmark data sets Data_137, Data_10 and Data_100 are available at <http://people.brunel.ac.uk/~mastjjb/jeb/info.html>, <https://people.cs.kuleuven.be/~pieter.smet/smptsp.html> and <https://sites.google.com/site/ptsplib/smptsp/instances> respectively.

orders did not have a significant impact in the solution quality when the employees have a random skill structure. Therefore in the following experiments, employees are selected in a random order to construct blocks.

Figure 3 compares the solution quality and the algorithm runtime of the employee-based decomposition for block sizes $\eta = \{10, 15, 20\}$. This decomposition solves almost all Data_137 instances while most of the Data_100 instances remain unsolved. It is evident from the graph that larger block sizes in general lead to improved performance, but at the expense of longer run times. The employee-based decomposition executes block sizes $\eta = \{10, 15, 20\}$ in ascending order until an optimal solution is found or the runtime limit is reached. As expected, the results of Data_137 results coincide with those of the same experiments presented in Smet et al. (2014). Note that the block objective function (8), which results naturally from the employee-based decomposition, does not guarantee that all tasks are assigned when the algorithm finishes execution. In other words, a feasible solution for the algorithm need not be a feasible solution for the actual problem. Smet et al. (2014) utilized a local-branching based improvement heuristic to remove infeasibilities and to improve the solution. The solutions obtained by the employee-based decomposition and the results after employing this improvement heuristic, which constitutes the current best known solutions for the SMPTSP, are summarized in Table 1 for comparison purposes. The employee-based decomposition which is capable of generating optimal solutions for most Data_137 instances is, however, not efficient in solving most of the Data_100 instances to optimality. This can be attributed to the less uniform skill structure of the Data_100 instances. While improvements might be possible by allowing multiple iterations of the algorithm with varying random employee-ordering, to accommodate instances with a non-uniform skill structure the employee-based decomposition requires determining a suitable employee-order to be utilized by the CMH.

Fig. 3: Performance and algorithm runtime of the employee-based decomposition for different block sizes

Utilizing design parameters such as overlap and relaxed future has proven useful in similar CMH approaches such as Chandrasekharan et al. (2019). In an attempt to produce optimal solutions with the employee-based decomposition without the improvement heuristic and to possibly generate optimal solutions

for the unsolved difficult Data_10 instances, overlap and relaxed future parameters were tested with respect to the employee-based decomposition. Figure 4 presents the trend of employee-based decomposition's outcome with respect to increasing overlap and relaxed future. Here, relaxed future is achieved by relaxing the integrality constraints. It is evident from the graph that neither overlap nor relaxed future brings any significant difference to solution quality. Before concluding that the overlap and relaxed future features are not beneficial for this problem, the progress of the employee-based CMH was observed in detail when overlap is implemented. Different employee-orders were tested to arrive at a decomposition that enables employees with tasks corresponding to overlapping time-windows to be assigned tasks simultaneously. This idea motivates the time-based decomposition in which sets of employees with the maximum number of overlapping tasks form blocks.

Fig. 4: Trend in the solution quality of the employee-based decomposition with respect to increasing overlap and relaxed future

In the time-based decomposition, tasks are ordered based on their start times which range from T_{min} to T_{max} . Block size η is defined as $1440/\beta$, where β assumes values in $\{2, 3, 4, 5, 6\}$. Once a value of β is chosen, the timeline of the instance is subdivided into β equal parts, each of which corresponds to a block. The block-objective is to optimally assign all the tasks within it to the set of available employees such that the total number of employees utilized is minimized. In contrast with the employee-based decomposition, a solution generated by the task-based decomposition is a feasible solution for the problem. However, it may still happen that employees are not available to perform all the tasks in a block resulting in an infeasibility. The algorithm, therefore, may fail to construct a solution. Figure 5 compares the time-based decomposition's outcome and runtime for different block sizes on data_100 and Data_137 instances when implemented with no overlap or relaxed future. It is evident from the graphs that the solutions improve with increasing block size, and so do the runtimes. In contrast with the employee-based decomposition, the time-based decomposition exhibits similar trend in both the datasets. It is also worth noting that for many large instances, the run time exceeds benchmark time limits for the largest block size corresponding to $\beta = 2$. Such data points are outliers and therefore are not visible in the graphs.

Fig. 5: Performance and algorithm runtime of time-based decomposition for different block sizes

Similar overlap and relaxed future experiments were also conducted for the time-based CMH, with Figure 6 comparing the time-based CMH's solution quality when overlap and relaxed future are added. Although the trends are not monotonic, this plot clearly shows how overlap and relaxed future parameters can be utilized to bring significant solution quality improvements.

Fig. 6: Trend in the solution quality of time-based decomposition with respect to increasing overlap and relaxed future

A time-based decomposition approach was developed, which executes the configurations $(\beta, 80, 0)$, $(\beta, 90, 0)$, $(\beta, 80, 50)$ and $(\beta, 90, 50)$ one after the other. The values of θ and ρ are selected based on the analysis from Figure 6. While larger values of ρ may improve the solution quality, runtime restrictions do not support this. For each of the configurations, all values of β (2, 3, 4, 5 and 6) are tested in the descending order. The algorithm terminates when the runtime limit is reached, when an optimal solution is found or when all configurations have been executed. Table 1 and 2 summarize the performance details of this algorithm on the benchmark dataset. For the first time, all Data_10 instances have been solved to optimality. Moreover, all Data_137 instances have also been solved to optimality. The Data_100 instances, however, have not all been solved to optimality using this time-based decomposition. A closer look at the results of the Data_100 instances reveal that most of the unsolved instances are those which include very large numbers of staff ($m > 500$). Concerning the

Data_100 instances, there certainly exists general trend upwards in solution quality with respect to increasing block size, overlap and relaxed future parameters, but one which fluctuates with some irregularity. In many Data_100 instances, larger block sizes produced optimal solutions much faster than when utilizing a smaller block size. In order to investigate this, the active task distribution in the instance-timeline was studied, which led to the development of an automatic time-based CMH.

Table 1: Results of time-based decomposition (TBD), employee-based decomposition (EBD) and the current best known results of Smet et al. (2014)(SWMV).

	EBD			SWMV			TBD		
Dataset	137	100	10	137	100	10	137	100	10
No of optimal solutions	133	28	0	137	98	5	137	77	10
Average calculation time(s)	44.62	6.4	19.17	55	88.46	1,294.15	48.25	746.97	162.59

Table 2: Comparison of results of time-based decomposition (TBD), employee-based decomposition (EBD) and Smet et al. (2014)(SWMV) on Data_10 instances. LB, Obj. and T(s) denote lower bound, objective-value and algorithm runtime (seconds) respectively.

	EBD			SWMV		TBD	
Instances	LB	Obj.	T(s)	Obj.	T(s)	Obj.	T(s)
1_50_258_20_90_100_200_300	40	45	0.44	40	641.58	40	196.15
2_44_510_20_90_0_100_200	40	44	5.47	41	683.31	40	15.97
3_102_525_30_90_100_200_300	77	78	7.86	77	938.62	77	127.78
4_113_647_20_90_100_200_300	98	101	2.32	98	163.24	98	34.77
5_77_777_30_90_0_100_200	59	60	96.48	59	1,615.46	59	12.98
6_135_777_20_90_100_200_300	116	121	3.26	116	1,699.28	116	663.7
7_70_781_20_90_0_100_200	59	66	9.27	61	1,800	59	370.46
8_88_1022_20_90_0_100_200	79	83	15.19	80	1,800	79	23.69
9_125_1308_20_90_0_100_200	98	103	29.24	99	1,800	98	60.6
10_153_1577_20_90_0_100_200	116	122	22.25	118	1,800	116	119.86

5.2 Automatic time-based CMH

Depending on instance structure, the automatic time-based algorithm selects either a fixed or blocking strategy. A brief analysis shows that the mean (μ) percentage difference between the minima and maxima of the active-task distribution graph over all the instances is 48.19 with standard deviation,

$\sigma=29.82$. Therefore, the value $\mu - \sigma = 18.37$ is used to filter instances that do not have an effective minima and therefore, $r = 18.37\%$ is selected. d_{min} was set as 240, which corresponds to the smallest block size given by $\beta = 6$ of the time-based decomposition. The variable strategy utilizes four configurations, $(\eta, 0, 0)$, $(\eta, 25, 0)$, $(\eta, 0, \eta')$ and $(\eta, 25, \eta')$, where η' corresponds to size of the following block. In this strategy, relaxed future adds the variables corresponding to an entire next block to the current block. This is to preserve the selected decomposition. The algorithm ends when an optimal solution is found or when all configurations have been tested. The fixed blocking strategy executes the configurations $(\beta, 80, 0)$, $(\beta, 90, 0)$, $(\beta, 80, 50)$ and $(\beta, 90, 50)$ in this order for all the block sizes (the same configuration utilized by the time-based decomposition). Tables 3, 4 and 5 summarize the results of the automatic CMH algorithm on Data_100 and Data_137 instances.

The automatic time-based CMH solves 136 Data_137 instances and 96 Data_100 instances to optimality. Out of the four unsolved Data_100 instances, optimal solutions for two have not yet been established. The column f indicates whether the fixed (0) or variable (1) blocking strategy has been implemented for the specific instance. Results demonstrate how a variable blocking strategy was utilized for all Data_100 instances, whereas a fixed strategy was adopted for almost all large Data_137 instances. The best results obtained by this CMH algorithm within the benchmark time limit of 1800 s are presented in aggregated Tables 6 and 7. The best results obtained by other methods from the literature are also presented for comparison purposes. Table 8 provides a summary of the results obtained for the Data_10 instances along with the results obtained by other methods from the literature. KEB denotes Krishnamoorthy et al. (2012), SV denotes Smet and Vanden Berghe (2012), SWMV denotes Smet et al. (2014), Lin-Ying denotes Lin and Ying (2014), FL denotes Fages and Lapgue (2013), DMA denotes Baatar et al. (2015), NMA denotes Niraj Ramesh et al. (2018) and MH denotes Hojati (2018).

6 Discussion of results

Smet et al. (2014) and the automatic time based decomposition perform significantly better on the original Data_137 instances compared to other algorithms from the literature. For the Data_100 instances, Smet et al. (2014) produces best results followed by the automatic time-based CMH. When it comes to the Data_10 instances, the time-based decomposition is, for the first time, capable of solving all Data_10 instances within the benchmark time limit of 1800 seconds. The automatic time-based CMH, when utilizing the same parameter configuration for the whole set of benchmark instances also generates 10 optimal solutions, utilizing similar calculation times. In summary, the time-based decomposition and automatic time-based CMH has been able to produce optimal solutions for all the unsolved Data_10 instances. While there are competent techniques in the literature which are capable of producing equally good results over Data_100 or Data_137, the automatic time-based CMH proves

Table 3: Results of automatic time-based CMH on the small and medium Data_137 instances. f , LB, Obj. and T(s) denote Fixed, lower bound, objective-value and algorithm runtime (seconds) respectively.

Instance	f	Obj.	T(s)	LB	Instance	f	Obj.	T(s)	LB
1_23_40_66	1	20	0.02	20	51_196_480_33	1	160	12.28	160
2_24_40_33	1	20	0.02	20	52_205_480_66	1	160	6.01	160
3_25_40_66	1	20	0.02	20	53_127_487_66	0	100	9.81	100
4_23_59_33	1	20	0.17	20	54_175_492_66	0	140	24.45	140
5_25_60_33	1	20	0.13	20	55_85_493_66	1	70	12.13	70
6_48_80_66	1	40	0.07	40	56_163_500_66	0	140	11.13	140
7_51_80_66	1	40	0.08	40	57_88_508_66	1	70	16.2	70
8_48_85_33	1	40	0.22	40	58_158_517_66	1	140	6.92	140
9_49_104_33	1	41	0.6	40	59_70_525_33	1	59	78.96	59
10_51_111_66	1	40	0.11	40	60_181_549_66	0	139	15.54	139
11_24_119_33	1	20	0.27	20	61_121_557_66	0	100	10.38	100
12_49_119_33	1	40	0.33	40	62_101_571_33	1	80	60.93	80
13_25_120_33	1	20	0.86	20	63_97_577_66	1	80	5.12	80
14_75_124_33	1	60	0.43	60	64_176_595_66	1	160	9.69	160
15_72_126_33	1	60	0.42	60	65_179_596_66	0	159	16.34	159
16_75_131_66	1	60	0.25	60	66_348_600_33	1	300	6.06	300
17_23_139_66	1	20	0.66	20	67_371_600_66	1	300	13.42	300
18_48_160_66	1	40	0.16	40	68_359_613_66	1	300	15.7	300
19_97_160_33	1	80	0.17	80	69_148_614_33	0	120	13.03	120
20_99_163_33	1	80	0.64	80	70_192_623_66	1	160	32.2	160
21_93_175_33	1	80	1.49	80	71_197_624_33	0	158	38.85	158
22_47_180_66	1	40	0.77	40	72_205_624_66	1	160	14.9	160
23_74_180_66	1	60	0.28	60	73_155_661_66	0	120	42.16	120
24_110_200_33	1	100	0.31	100	74_209_664_33	0	180	8.56	180
25_120_200_33	1	100	0.24	100	75_72_665_33	1	60	128.88	60
26_116_203_66	1	100	0.55	100	76_162_683_66	0	140	22.45	140
27_49_204_66	1	40	0.6	40	77_180_688_33	1	160	43.6	160
28_75_208_66	1	60	0.99	60	78_199_688_66	1	160	11.52	160
29_22_219_66	1	20	2.48	20	79_94_689_33	0	80	6.96	80
30_25_219_66	1	20	1.35	20	80_112_691_33	0	99	16.25	99
31_90_230_66	1	80	0.63	80	81_97_692_66	1	80	38.15	80
32_70_236_66	1	60	2.32	60	82_89_697_66	1	80	24.2	80
33_76_240_66	1	60	0.43	60	83_222_700_66	0	180	31.3	180
34_152_240_33	1	120	0.44	120	84_136_718_66	1	120	479.85	120
35_171_280_33	1	140	0.66	140	85_217_720_66	1	180	11.39	180
36_175_280_33	1	140	0.69	140	86_178_721_33	0	140	20.26	140
37_145_321_33	0	120	12.26	120	87_203_735_33	1	170	341.31	170
38_147_347_66	0	120	5.22	120	88_137_777_66	0	120	22.55	120
39_45_351_66	1	40	2.49	40	89_88_788_33	1	70	47.53	70
40_138_360_33	1	120	0.71	120	90_157_791_66	0	139	27.06	139
41_144_360_66	1	120	1.76	120	91_147_851_66	0	118	85.27	118
42_101_380_66	1	80	2.68	80	92_126_856_66	0	98	35.52	98
43_156_387_66	0	140	6.1	140	93_141_856_66	0	119	30.53	119
44_121_400_33	1	100	0.7	100	94_93_881_33	1	80	102.76	80
45_67_420_33	0	60	2.04	60	95_204_882_33	0	170	15.54	170
46_147_423_33	1	120	4.05	120	96_98_886_66	1	80	50.75	80
47_150_430_33	1	120	4.91	120	97_383_895_33	1	300	38.3	300
48_120_434_66	1	100	20.37	100	98_91_896_33	0	80	14.37	80
49_211_446_66	1	180	278.72	180	99_176_956_66	1	160	61.24	160
50_187_447_66	0	160	10.18	160	100_194_956_66	1	160	1,068.23	160

to be an efficient strategy which is capable of generating high quality solutions over all datasets. More specifically, this strategy performs well even on very large instances and therefore can be adapted for solving real-world scenarios.

The merit of the automatic-CMH lies in the fact that it is a simple constructive heuristic that efficiently generates very high quality solutions (optimal solutions for 242 out of 247 instances) in a single pass of the algorithm.

Table 4: Results of automatic time-based CMH on the large Data_137 instances. f , LB, Obj. and T(s) denote Fixed, lower bound, objective-value and algorithm runtime (seconds) respectively.

Instance	f	Obj.	T(s)	LB	Instance	f	Obj.	T(s)	LB
101_166_997_66	0	140	51.92	140	120_228_1341_33	0	187	47.67	187
102_179_997_66	0	138	57.65	138	121_147_1345_33	0	120	50.54	120
103_348_1024_33	0	300	347.05	300	122_422_1358_66	0	348	235.6	348
104_181_1057_33	0	146	25.02	146	123_187_1376_33	0	159	45.29	159
105_173_1075_66	0	150	77.31	150	124_198_1383_33	0	158	55.13	158
106_121_1096_33	0	100	25.78	100	125_157_1448_33	1	130	208.91	130
107_114_1112_33	1	100	69.79	100	126_193_1462_33	0	167	60.44	167
108_162_1115_33	0	128	29.36	128	127_192_1472_66	0	170	172.72	170
109_205_1115_33	0	157	29.95	157	128_207_1542_66	0	178	181.07	178
110_183_1143_66	0	155	82.42	155	129_233_1546_33	0	178	79.51	178
111_155_1211_33	0	139	33.29	139	130_176_1562_66	1	140	433.69	140
112_200_1213_33	0	169	32.02	169	131_415_1610_33	0	348	95.08	348
113_141_1221_66	0	110	83.43	110	132_216_1645_33	0	186	266.06	186
114_157_1227_33	0	138	36.49	138	133_211_1647_33	0	185	294.14	185
115_228_1257_33	0	177	43.97	177	134_184_1776_66	0	160	239.77	160
116_205_1262_66	0	176	105.18	176	135_213_1988_33	0	179	153.74	179
117_192_1285_33	0	149	47.04	149	136_216_2000_66	0	179	431.04	179
118_180_1302_33	0	147	46.04	147	137_245_2105_33	0	190	206.14	190
119_236_1335_33	0	188	47.9	188					

The key feature of the automatic-CMH which makes it efficient at solving large and difficult instances is its underlying decomposition strategy. The employee-based decomposition applied by Smet et al. (2014) requires employing larger block sizes in order to improve solution quality, which in turn affects the algorithm run time. The advantage of the time-based decomposition lies in its ability to utilize much smaller blocks and yet still generate high quality solutions when combined with sufficient overlap and relaxed future, the presence of which impacts runtime far less than increasing block size. Experiments show, however, that implementing overlap in a employee-based decomposition does not significantly improve the solution quality. In order to be able to implement overlap or relaxed future, it requires identifying an appropriate order of solving subproblems. This is to ensure that the compromise in local optimality of subproblems when employing overlap or relaxed future reflects in the final solution quality improvement for the entire problem. In this problem, time-based decomposition imposes a natural ordering for the subproblems which enables overlap and relaxed future features to be effectively implemented.

7 Conclusions and future work

The present paper compared two different decomposition strategies applied within a constructive matheuristic framework when applied to the shift minimization personnel task scheduling problem (SMPTSP). The paper also studied the features of the decomposition strategy which makes it suitable for implementing key CMH design parameters such as overlap and relaxed future. Not only do these features improve solution quality, but they also play a pivotal role in improving the scalability of the algorithm, which makes it suitable for

Table 5: Results of automatic time-based CMH on the Data_100 instances. f , LB, Obj. and T(s) denote Fixed, lower bound, objective-value and algorithm runtime (seconds) respectively.

Instance	f	Obj.	T(s)	LB	Instance	f	Obj.	T(s)	LB
01_0062_0071_025	1	29	0.2	29	51_0514_0832_025	1	197	123.3	197
02_0080_0105_025	1	33	0.14	33	52_0448_0767_025	1	171	70.43	171
03_0082_0093_025	1	31	0.12	31	53_0486_0746_025	1	186	56.31	186
04_0078_0086_025	1	29	0.1	29	54_0498_0850_025	1	190	71.66	190
05_0081_0110_025	1	31	0.8	30	55_0524_0920_025	1	201	182.43	201
06_0084_0094_025	1	31	0.13	31	56_0538_0911_025	1	206	151.7	206
07_0076_0094_025	1	29	0.1	29	57_0440_0737_025	1	169	43.15	169
08_0083_0097_025	1	33	0.26	33	58_0348_0562_025	1	132	28.84	132
09_0082_0117_025	1	32	0.15	32	59_0460_0689_025	1	176	63.72	176
10_0072_95_025	1	28	0.33	28	60_0443_0783_025	1	173	67.91	173
11_0164_0228_025	1	63	1.39	63	61_0551_0891_025	1	222	152.63	222
12_0186_0273_025	1	73	3.34	73	62_0610_1096_025	1	262	254.33	262
13_0138_0212_025	1	52	0.64	52	63_0524_0905_025	1	203	117.44	203
14_0182_0232_025	1	69	1.13	69	64_0366_0570_025	1	140	25.67	140
15_0152_0204_025	1	58	3.4	58	65_0456_0764_025	1	179	81.26	179
16_0161_0235_025	1	62	0.94	62	66_0492_0775_025	1	189	99.23	189
17_0164_0217_025	1	62	1.32	62	67_0597_0949_025	1	230	254.18	230
18_0147_0204_025	1	57	0.83	57	68_0561_0958_025	1	219	79.54	219
19_0150_0196_025	1	55	3.56	55	69_0550_0891_025	1	211	114.42	211
20_0152_0235_025	1	64	0.51	64	70_0550_0990_025	1	211	302.18	211
21_0197_0292_025	1	79	9.01	79	71_0528_0895_025	1	202	193.27	202
22_0288_0456_025	1	110	65.38	110	72_0604_0997_025	1	230	543.07	230
23_0287_0414_025	1	110	8.57	110	73_0604_0999_025	1	239	262.39	239
24_0236_0362_025	1	94	9.83	94	74_0563_0941_025	1	217	157.56	217
25_0192_0311_025	1	75	2.81	75	75_0518_0870_025	1	204	217.5	204
26_0212_0376_025	1	93	3.13	93	76_0642_1107_025	1	246	343.58	246
27_0282_0437_025	1	107	8.67	107	77_0648_1123_025	1	249	1,800	248
28_0262_0402_025	1	105	13.35	105	78_0548_0941_025	1	210	100.11	210
29_0248_0355_025	1	95	8.6	95	79_0638_1052_025	1	246	172.93	246
30_0236_0375_025	1	93	5.58	93	80_0578_0885_025	1	222	165.53	222
31_0290_0488_025	1	116	11.28	116	81_0647_1181_025	1	265	154.37	265
32_0332_0527_025	1	127	30.57	127	82_0734_1265_025	1	289	148.21	289
33_0338_0534_025	1	132	19.66	132	83_0551_0941_025	1	206	110.42	206
34_0300_0468_025	1	114	17.18	114	84_0644_1121_025	1	247	457.87	247
35_0308_0469_025	1	118	10.87	118	85_0688_1111_025	1	263	117.84	263
36_0320_0535_025	1	129	13.89	129	86_0628_1136_025	1	241	251.61	241
37_0296_0437_025	1	115	12.43	115	87_0765_1323_025	1	288	237.64	288
38_0340_0525_025	1	129	19.14	129	88_0808_1346_025	1	310	1,396.74	310
39_0284_0446_025	1	108	14.25	108	89_0790_1371_025	1	320	1,677.57	319
40_0384_0576_025	1	147	23.98	147	90_0810_1443_025	1	312	651.57	312
41_0358_0556_025	1	137	28.63	137	91_0754_1287_025	1	289	330.8	289
42_0360_0601_025	1	141	29.83	141	92_0948_1583_025	1	364	722.84	364
43_0422_0674_025	1	166	39.19	166	93_0820_1478_025	1	340	247.61	340
44_0364_0572_025	1	145	11.47	145	94_0812_1394_025	1	318	1,802	313
45_0376_0586_025	1	144	33.57	144	95_0696_1162_025	1	266	190.09	266
46_0409_0635_025	1	157	35.98	157	96_0710_1250_025	1	272	601.47	272
47_0373_0600_025	1	142	38.76	142	97_0886_1605_025	1	340	691.2	340
48_0390_0614_025	1	152	96.16	152	98_0750_1334_025	1	289	295.72	289
49_0354_0549_025	1	140	78.25	140	99_0564_0955_025	1	221	311.81	221
50_0318_0536_025	1	123	18.59	123	100_0806_1374_025	1	310	491.27	310

application in large real-world scenarios. The results of an extensive computational study conducted shows that the resulting automated CMH strategy is capable of producing optimal solutions for a large number of benchmark instances. More specifically, this strategy has been able to solve all Data_10 instances to optimality for the first time. Another major contribution of the

Table 6: Results of automatic time-based CMH on the Data_137 instances.

	KEB	SV	SWMV	Lin-Ying	FL	MH	auto-CMH
Number of optimal solutions	67	81	137	105	137	129	136
Average solution	127	123.04	122.24	122.72	122.24	122.48	122.25
Average calculation time(s)	544.3	958.91	55.52	637.5	–	15.3	59.54
Maximum calculation time(s)	1800	1800	604.87	1800	–	1690.9	1068.23

Table 7: Results of automatic time-based CMH on the Data_100 instances

	FL	MH	SWMV	auto-CMH
Number of optimal solutions	46	84	98	96
Average solution	166.2	164.56	164.4	164.46
Average calculation time(s)	–	154.5	88.46	179.21
Maximum calculation time(s)	1800	1800	683.07	1,800

Table 8: Results of automatic time-based CMH on the Data_10 instances. f , Obj. and T(s) denote Fixed, objective-value and algorithm runtime (seconds) respectively.

Instance	auto-CMH			SWMV		DMA		NMA	
	f	Obj.	T(s)	Obj.	T(s)	Obj.	T(s)	Obj.	T(s)
1_50_258_20_90_100_200_300	1	40	9.42	40	641.58	41	1,800	40	33.8
2_44_510_20_90_0_100_200	1	40	176.52	41	683.31	40	61	40	3007
3_102_525_30_90_100_200_300	0	77	128.75	77	938.62	77	198	77	591.3
4_113_647_20_90_100_200_300	0	98	33.41	98	163.24	98	324	98	299.7
5_77_777_30_90_0_100_200	0	59	12.76	59	1,615.46	59	495	–	–
6_135_777_20_90_100_200_300	0	116	664.78	116	1,699.28	121	1,801	116	3691.8
7_70_781_20_90_0_100_200	0	59	371.83	61	1,800	59	147	–	–
8_88_1022_20_90_0_100_200	0	79	23.54	80	1,800	79	1,286	–	–
9_125_1308_20_90_0_100_200	0	98	60.99	99	1,800	106	1,801	–	–
10_153_1577_20_90_0_100_200	0	116	119.22	118	1,800	128	1,806	–	–
Number of optimal solutions			10		6		6		5
Average solution			78.2		78.9		80.8		–
Average calculation time(s)			160.12		1,294.15		971.9		–
Maximum calculation time(s)			664.78		1,800		1,800		5000

paper is the development of a CMH strategy which automatically generates a suitable time-based decomposition based on instance structure.

For the SMPTSP, the time-based decomposition proves to be beneficial. Future research can be aimed at identifying problem features on which decomposition can be based such that the resulting order of solving subproblems is suitable for implementing an effective CMH strategy and the design parameters discussed. It is also worth identifying problem classes or problem features which makes a time or time-like feature based decomposition suitable.

Acknowledgements We would like to thank Luke Conolly (KU Leuven) for providing editorial consultation.

References

- D. Baatar, M. Krishnamoorthy, and A. T. Ernst. A triplet-based exact method for the shift minimisation personnel task scheduling problem. In N. Bansal and I. Finocchi, editors, *Algorithms - ESA 2015*, pages 59–70. Springer Berlin Heidelberg, 2015.
- R. C. Chandrasekharan, T. A. M. Toffolo, and T. Wauters. Analysis of a constructive matheuristic for the traveling umpire problem. *Journal of Quantitative Analysis in Sports*, 15(1):41–57, 2019.
- F. Della Croce, A. Grosso, and F. Salassa. A matheuristic approach for the two-machine total completion time flow shop problem. *Annals of Operations Research*, 213:67–78, 2014.
- J. V. den Bergh, J. Beliën, P. D. Bruecker, E. Demeulemeester, and L. D. Boeck. Personnel scheduling: A literature review. *European Journal of Operational Research*, 226(3):367 – 385, 2013.
- D. Dowling, M. Krishnamoorthy, H. Mackenzie, and D. Sier. Staff rostering at a large international airport. *Annals of Operations Research*, 72:125–147, 1997.
- J. Fages and T. Lapègue. Filtering at most n value with difference constraints: Application to the shift minimisation personnel task scheduling problem. In C. Schulte, editor, *Principles and Practice of Constraint Programming*, pages 63–79. Springer Berlin Heidelberg, 2013.
- M. Fischetti and A. Lodi. Local branching. *Mathematical Programming*, 98: 23–47, 2003.
- M. Hojati. A greedy heuristic for shift minimization personnel task scheduling problem. *Computers and Operations Research*, 100:66–76, 2018.
- A. W. Kolen, J. K. Lenstra, C. H. Papadimitriou, and F. C. Spieksma. Interval scheduling: A survey. *Naval Research Logistics (NRL)*, 54(5):530–543, 2007.
- M. Krishnamoorthy, A. Ernst, and D. Baatar. Algorithms for large scale shift minimisation personnel task scheduling problems. *European Journal of Operational Research*, 219:34–48, 2012.
- S. Lin and K. Ying. Minimizing shifts for personnel task scheduling problems: A three-phase algorithm. *European Journal of Operational Research*, 237: 323–334, 2014.
- V. Maniezzo, T. Stützle, and S. Voß. *Matheuristics: Hybridizing Metaheuristics and Mathematical Programming*. Springer Publishing Company, Incorporated, 1st edition, 2009. ISBN 144191305X, 9781441913050.
- D. Niraj Ramesh, M. Krishnamoorthy, and A. T. Ernst. Efficient models, formulations and algorithms for some variants of fixed interval scheduling problems. In R. Sarker, H. A. Abbass, S. Dunstall, P. Kilby, R. Davis, and L. Young, editors, *Data and Decision Sciences in Action*, pages 43–69, Cham, 2018. Springer International Publishing.
- H. G. Santos, T. A. M. Toffolo, R. A. M. Gomes, and S. Ribas. Integer programming techniques for the nurse rostering problem. *Annals of Operations Research*, 239:225–251, 2016.

- P. Smet and G. Vanden Berghe. A matheuristic approach to the shift minimisation personnel task scheduling problem. *Proceedings of the 9th international conference on the practice and theory of automated timetabling, PATAT, 2012*, pages 145–151, 2012.
- P. Smet, T. Wauters, M. Mihaylov, and G. V. Berghe. The shift minimisation personnel task scheduling problem: A new hybrid approach and computational insights. *Omega*, 46:64 – 73, 2014.
- O. Solyali. The shift minimization personnel task scheduling problem: An effective lower bounding procedure. *Hacettepe Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi*, 34(2), 2016.
- L. A. Wolsey. *Integer programming*. Wiley-interscience series in discrete mathematics and optimization. Wiley, New York (N.Y.), 1998. ISBN 0471283665.