Loose bolt detection in a complex assembly using a vibro-acoustic sensor array

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Abstract

A non-destructive testing strategy is proposed that combines microphone measurements and structural measurements in one array. The aim is to lower the total inspection time by reducing the number of structural sensors. This is possible by making use of the easier and faster installation of microphones as compared to structural sensors. Structural sensors are needed to limit the influence of background noise. The proposed algorithm is based on the theory of the Time-Reversal Multiple Signal Classification (TR-MUSIC) algorithm. In addition to the novelty of using a vibro-acoustic array, this publication presents a strategy on how to select only the most 'valuable' regions of the frequency spectrum for the TR-MUSIC algorithm and an option to use the algorithm with a number of excitations below the theoretically necessary minimum.

The proposed procedure is applied in an experiment that targets the localisation of loose or removed bolts in an assembly of aluminium beams and a honeycomb panel, mounted with rivets, bolts and double-sided tape. Although none of the bolts is in the line of sight of the sensor array, all introduced defects can be found in this experiment. This includes the localisation of multiple defects as well. By adding artificial measurement noise to the microphone measurements, it is demonstrated that a combined vibro-acoustic array can cope with a high ambient noise level, while the process fails if a comparable measurement error is also applied to the structural sensors.

Keywords: Non-destructive testing & evaluation, Time-Reversal Multiple Signal

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1. Introduction

The process of investigating the health status of a structure, which is out of operation without destroying it or adding additional damage is widely referred to as non-destructive testing and evaluation (NDT&E) [1]. Many NDT&E strategies are based on the mechanical vibrations of the test object, either introduced by ambient excitations, such as wind or waves or intentionally excited. Two of the most common and already industrially used methods are ultrasonic arrays and guided waves.

Ultrasonic arrays are arrays of transducers/sensors emitting an ultrasonic wave and recording its reflection. Primarily based on the time of flight of the reflected short-wavelength waves, it is possible to generate a highly detailed image of the area closely around the array. Also the inspection of larger surfaces is possible moving the array across the inspection surface. However, this process is time-consuming and therefore costly for large surfaces. Furthermore, the arrays contain typically a big number of transducers/sensors (the largest example known to the authors contains up to 256 independent channels [2]), which makes it difficult to couple all of them simultaneously to the surface of the test object, as e.g. the efforts made by Chatillon et al. [3] show. An additional restriction caused by the small inspection area nearby the array is, that ultrasonic arrays can only be applied for the investigation of accessible zones [4].

Guided wave methods excite specific wavemodes, which propagate through a test object. Also here, the location of defects can be determined measuring the time of flight of the reflected wavemodes. Though, for guided wave methods, care must be taken to measure the correct scattered wavemode, since the speed of propagation of different wavemodes varies due their dispersive behaviour. Following this procedure, a long range inspection is possible, setting aside the accessibility issues posed for the ultrasonic arrays. However, the application of these methods is restricted to simple test objects [5], since the discrimination of a reflection at a defect and a reflection at a geometrical feature must be possible [6]. Developers of these methods take a big effort to excite only the desired wavemode [7, 8] and as for ultrasonic arrays, also coupling

liquids are needed for some transducer/sensor types [9].

30 As both strategies suffer from the effort that needs to be made to properly couple the test device to the test object, it seems logical to move to other types of sensors. Here microphones come to the fore [10], as the coupling to the test structure is automatically ensured by the surrounding air. In fact, the principle of listening to the sound radiated by a test object is used for centuries for quality control, e.g. checking a porcelain
35 vase for cracks by tipping it and listening to the sound it makes. However, apart from the difficulty of combining this manual process with the high frequency guided waves or ultrasonic arrays (typically at frequencies far above the audible range), there are additional challenges since [11]:

- A manual process is an expensive procedure for mass produced products.
- 40 • The process is not reliable in an environment with severe background noise, which could be caused by workers speaking, welding, cutting or dropping things [12] or other machinery operating nearby the test object.
- No objective data can be documented and thus reproducing the test is difficult.
- A manual process relies on the worker's performance, which can lead to a "great
45 variance in operation of NDT systems", partially due to ambient noise the worker is opposed to [12].

As the examples for noise sources imply, these harsh conditions for acoustic testing are present in nearly every production line. Thus the robustness requirements for all NDT&E processes applied in a mass-production are set by these conditions. Against that
50 background, in view of the ongoing evolution towards Industry 4.0, in order to overcome the shortcomings of manual testing and in order to fulfil normative requirements, automated testing becomes increasingly important [13].

Understanding NDT&E as a 4 step process [14] with:

- 1 Detection: Is there damage in the test object?
- 55 2 Localisation: Where is the damage?
- 3 Assessment: How large is the damage?

4 Consequence: How severe is the influence of the damage on the structural integrity?

this publication addresses step 2, trying to localise a damage in a complex assembly. Therefore, a Time-Reversal Multiple Signal Classification (TR-MUSIC) algorithm in a low-to mid frequency range, as proposed by Becht et al. [15, 16] is employed. This approach differs from most other implementations of this algorithm, where high-frequency waves are used due to their higher sensitivity to small scatterers. The lower frequency range brings two big advantages. Firstly, the wave scattering in complex structures at this frequency range follows a deterministic pattern and not a statistical distribution, which makes it predictable. It is for that reason that the TR-MUSIC algorithm can be applied to test objects with industrial complexity. Secondly, by lowering the frequency range, the acoustic radiation of the test object happens to be in an audible range. Therefore, the acoustic waves propagate further on the one hand and can be picked up using commercial microphones on the other hand. As mentioned above, the microphones are coupled to the test object by the surrounding air, which automatically solves the coupling problem other types of sensors have to cope with.

Nevertheless, the new challenge of severe ambient noise arises by changing to microphone arrays. In order to overcome this, it is proposed to enhance the microphone array with few structural sensors, which are exposed to less noise. These sensors still need to be attached manually, but the expenditure of time decreases as compared to a fully structural array, since the number of structural sensors is smaller. In this context, a novel technique is presented dealing with the combination of different sensor types in one array for TR-MUSIC.

In order to improve the efficiency of the method in terms of total inspection time, the use of microphones is a significant improvement as compared to conventional sensors. The calculation time of the TR-MUSIC algorithm itself takes maximally some few seconds. A step in the test procedure that still provides room for a considerable speed-up of the total inspection time is the excitation. Rather than coupled transducers, in the work presented here, an impact hammer is used to excite the structure. Furthermore, it is explained how the number of necessary impact locations can be lowered, even below

the theoretically given necessary minimum.

Furthermore, it is explained how to select only parts of the frequency spectrum to ensure a good localisation result even under non-perfect conditions.

90 The paper consists of a theoretical part followed by an experimental validation of the developed strategy. The test object consists of multiple aluminium beams and a honeycomb panel, mounted with bolts, rivets and double-sided tape and comes close to industrial complexity. The defects intended to be localised are loose or removed bolts at different connections, which are not visible for the sensor array.

95 2. TR-MUSIC algorithm

The aim of the first subsection in this section is to introduce the reader to the calculation of a damage index based on the TR-MUSIC algorithm. Based on this introduction to the algorithm, the following subsections explain:

- How to use the TR-MUSIC algorithm for undersampled data, meaning that the
100 number of excitation locations and sensors is theoretically insufficient to generate an image of the inspected region.
- Why it is necessary to construct the damage index only based on the potentially valuable parts of the frequency spectrum. Furthermore, a proposal on how to select the 'potentially valuable parts of the frequency spectrum' is made.
- 105 • How to combine different sensor-types in one array.

2.1. TR-MUSIC for damage localisation

In the first step of a TR experiment, the signal transmitted by a source is measured by a sensor array. In a second step, the measured signal at each sensor is reversed in time and re-emitted at the sensors' locations. Due to the constructive interference of
110 the re-emitted signals, temporal and spacial focusing on the primary source location is achieved [17, 18]. This process can be conveniently written in the frequency domain for an array of N degrees of freedom (DOFs):

$$\sum_{n=1}^N H_n(\omega) H_n^*(\omega) F(\omega) = a(\omega) F(\omega), \quad (1)$$

with $H_n(\omega)$ being the transfer function from sensor element n to the source location and $F(\omega)$ being the force input. The superscript $*$ denotes the complex conjugate of a function and $a(\omega)$ is a scalar, real value. From equation (1), it can be seen that the spatial focusing happens independent of the frequency dependency of $a(\omega)$, since each term in equation (1) focuses at each frequency independently on the original source location. In order to obtain a correct reconstruction of the frequency content of $F(\omega)$, the coefficient $a(\omega)$ needs to be frequency-independent.

If the measured signal contains only the waves caused by the scattering of an incident wave-field at a scatterer, the scatterer can be regarded as the source. In consequence, the TR-process would focus on the scatterer's locations. In an experiment, the scattered field $\Psi_s(\omega, x_n, x_m)$ measured at x_n due to an excitation at x_m can be calculated by subtracting the known vibration of the intact structure $\Psi_{init}(\omega, x_n, x_m)$ from the actually measured vibration of the potentially defected structure $\Psi_{meas}(\omega, x_n, x_m)$ [19, 16]:

$$\Psi_s(\omega, x_n, x_m) = \Psi_{meas}(\omega, x_n, x_m) - \Psi_{init}(\omega, x_n, x_m). \quad (2)$$

In order to facilitate readability, unless indicated differently, from equation (3) onwards, dependency on ω is assumed, but not specifically mentioned.

It can be shown [20, 16] that the scattered wave field can be re-written in the form

$$\Psi_s(x_n, x_m) = P_r(x_n)\Theta P_t\rho(x_m), \quad (3)$$

where P_t and P_r are the transmitter and receiver propagators. They describe respectively the propagation of a wave excited in domain D_t to the entire inspection domain D and the propagation from D to the domain D_r , where the field is recorded. $P_r(x_n)$ indicates that only the propagation to the point x_n is evaluated. $\rho(x_m)$ describes the excitation applied at x_m . The scattering itself is represented by Θ .

Applying a singular value decomposition on P_r , it can be decomposed in its singular values λ_n^D and singular functions Φ_n^D :

$$P_r = \sum_{n=1}^{\infty} \lambda_n^D \Phi_n^D, \quad (4)$$

where the superscript D indicates the affiliation of the variables to the entire inspection domain.

140 An approximation of equation (4) is given in equation (5)

$$P_r \simeq \sum_{n=1}^N \lambda_n^D \Phi_n^D, \quad (5)$$

where only the N most relevant (largest) singular values and the corresponding singular vectors are kept.

In a similar manner, the transmitter propagator can be decomposed and truncated to the M most relevant singular values:

$$P_t \simeq \sum_{m=1}^M \lambda_m^D \Psi_m^D. \quad (6)$$

145 The truncation factors N and M must be chosen such that the dynamics in the inspection domain can still be described entirely using only the singular functions Φ_n^D and Ψ_m^D . This condition implies that every scattering event at arbitrary scatterers of extent D_0 within the inspection domain D ($D_0 \subset D$) can be described by

$$P_r^{D_0} \simeq \sum_{n=1}^N \lambda_n^{D_0} \Phi_n^{D_0}, \quad (7)$$

and

$$P_t^{D_0} \simeq \sum_{m=1}^M \lambda_m^{D_0} \Psi_m^{D_0}, \quad (8)$$

150 where the superscript D_0 indicates the affiliation of the singular functions to the actually observed scattering event. It also follows that the space spanned by $\Phi_n^{D_0}$ and

$\Psi_m^{D_0}$ is necessarily a subspace of (maximally the same dimension as) the space spanned by Φ_n^D and Ψ_m^D , respectively.

Introducing equations (7) and (8) into equation (3) leads to the expression for the
 155 scattered wave field

$$\Psi_s(x_n, x_m) = \sum_{n=1}^N \sum_{m=1}^M \Phi_n^{D_0} \Lambda_{n,m}^{D_0} \Psi_m^{D_0} \rho(x_m). \quad (9)$$

The actual scattering event is summarised in $\Lambda_{n,m}^{D_0}$, which is composed of the singular values $\lambda_n^{D_0}$ and $\lambda_m^{D_0}$ and the scattering potential Θ .

Assuming that the number of excitations M and sensors N is sufficiently large and well placed to illuminate the entire inspection region D , a matrix $\mathbf{K}$ with elements
 160 $K_{n,m} = \Psi_s(x_n, x_m)$ can be constructed. It follows from equation (9) that the rank R_0 of this matrix is maximally $\min(M, N)$, but likely to be smaller. This results from the fact that the dimension of the space spanned by $\Phi_n^{D_0}$ and $\Psi_m^{D_0}$, respectively, is necessarily smaller or equal to the space spanned by Φ_n^D and Ψ_m^D , due to $D_0 \subset D$. Therefore, the singular vectors $\Phi_n^{D_0}$ and $\Psi_m^{D_0}$ obtained from a singular value decomposition of $\mathbf{K}$ can
 165 be grouped into two orthogonal spaces. On the one hand side the signal space, S_Ψ and S_Φ , respectively, which is spanned by the singular functions describing the physical wave propagation from the excitation array to the scatterer(s) and from the scatterer(s) to the sensor array, respectively. Those singular vectors correspond to the R_0 largest singular values. On the other hand side the noise space, N_Ψ and N_Φ , which has no
 170 physical interpretation, but the mathematical property of being orthogonal to the signal space:

$$\begin{aligned} S_\Phi \oplus N_\Phi, \\ S_\Psi \oplus N_\Psi. \end{aligned} \quad (10)$$

If a vector $\mathbf{g}(z)$ with each element containing the transfer function from a random point z in the inspection domain to one sensor in the sensor array lies completely in S_Φ , then it follows from the properties in (10) for the multiplication with a noise space
 175 singular vector $\Phi_n \in N_\Phi$:

$$\mathbf{g}(z)^H \mathbf{\Phi}_n = 0, \text{ for } R_0 < n \leq N. \quad (11)$$

If $\mathbf{g}(z)$ also contains parts of N_Φ , then:

$$\mathbf{g}(z)^H \mathbf{\Phi}_n \neq 0, \text{ for } R_0 < n \leq N, \quad (12)$$

From the discussion of the TR-principle, it is known that the best possible focus on the source location is reached by back-propagating the time-reversed version of the measured signal. Since all the information on the physical propagation from the sensors to the defect(s) is contained in S_Φ , it follows that if equation (11) is fulfilled, z is a point in the defect domain and the vector $\mathbf{g}(z)$ can be interpreted as the best possible focus on this part of the scatterer(s) that can be achieved with a given array.

Exploiting the properties of equation (11) and (12), the TR-MUSIC algorithm is defined as [19, 21]:

$$I(z) = \left(\left| \mathbf{g}(z)^H \mathbf{\Phi}_n \right| \right)^{-1}, \text{ for } R_0 < n \leq N, \quad (13)$$

where z is a point in the inspection domain. Therefore, the search for the location of a scatterer can be replaced by finding the location z linked to the vector $\mathbf{g}(z)$ for which equation (13) reaches its maximum. Equation (13) holds for each vector $\mathbf{\Phi}_n \in N_\Phi$ individually. In order to exploit the orthogonality of all $\mathbf{\Phi}_n$'s to N_Φ , it is common practice to sum equation (13) over the entire noise space, increasing the dynamic range and the robustness of the pseudospectrum $I(z)$ [19, 22, 23, 24, 25, 26, 27, 28, 29]:

$$I(z) = \left(\sum_{n=R_0+1}^N \left| \mathbf{g}(z)^H \mathbf{\Phi}_n \right| \right)^{-1}. \quad (14)$$

Since the location of the maxima of $I(z)$ are linked to the location of the scatterers, they appear at the same spot, independent of the frequency at which equation (14) is evaluated. Again in order to increase robustness and dynamic range of the localisation

algorithm, an additional summation over different frequencies was successfully applied,
 195 e.g. by Yavuz and Teixeira [24] (and before using the signal space singular vectors [30]),
 Marengo et al. [20] and Becht et al. [16].

Following the same reasoning, equation (14) can also be derived for the imaging
 from excitation side using Ψ_n with $R_0 < n \leq M$.

In all examples presented in this publication, the vector $\mathbf{g}(z)$ is normalised using
 200 the routine described in Appendix B in order to limit the influence of modal behaviour
 on the detection process.

It should be pointed out that alternative strategies to compute $I(z)$ and to compose
 $\mathbf{K}$ [31, 32] have been published in literature. Especially the computation of the defect-
 indicating function $I(z)$ offers opportunities to adapt the algorithm according to specific
 205 needs. On the one hand, the signal space could be used instead of the noise space,
 which gives a physical meaning to the focusing amplitude and, if performed fully
 experimentally in time-domain, this approach automatically compensates for variances
 in the propagation medium. On the other hand, the resolution of the MUSIC-based
 pseudospectrum is superior and the computational time of the algorithm in equation (14)
 210 significantly smaller than for a full time-domain simulation. In the comparison of the
 robustness against random noise, both computational signal and noise space methods
 perform similarly [22, 33]. Furthermore, Some authors use a square like expression
 $(\mathbf{g}(z)^H \Phi_n \Phi_n^H \mathbf{g}(z))$ instead of $\mathbf{g}(z)^H \Phi_n$ in equation (14), which automatically makes the
 elements in the sum in equation (14) real and positive and hence replaces the operation
 215 $|\bullet|$ [31]. However, in both cases the intrinsic idea is the exploitation of the orthogonality
 between signal and noise space, which holds independent of the use of a linear or a
 squared term.

2.2. *TR-MUSIC with undersampled data*

For a large number of defects or a volume-defect that scatters the incident wave-field
 220 in a very complex manner, the number of necessary sensors N and excitations M to
 completely capture the scattering event is likely to become very large. Therefore, in the
 majority of the application cases found in literature, large arrays (e.g. 48 and 96 DOF
 transmitter/sensor array [27]) are employed. If, due to practical considerations either M

or N are restricted and smaller than R_0 , the rank of $\mathbf{K}$ becomes $R_o^{meas} = \min(M, N) <$
 225 R_0 . In consequence the number of signal space singular vectors is not sufficient to
 capture all possible scattering events in the observed domain. This means that the
 space spanned by the found signal space singular vectors S_Ψ^{meas} or S_Φ^{meas} , respectively,
 is only a part of the theoretical signal space S_Ψ or S_Φ . Therefore, the space spanned
 by the found noise space singular vectors N_Ψ^{meas} or N_Φ^{meas} is no longer guaranteed to
 230 be orthogonal to S_Ψ or S_Φ , respectively, which causes the TR-MUSIC algorithm as
 described in equation (14) to break down.

An example for this case can be found in figure 1, where a 1.5 mm thick steel plate
 (1.608 m x 1.848 m) is excited by $M = 11$ point forces normal to the surface. In order
 to facilitate the illustration, the same locations are used to measure the out-of plane
 235 acceleration ($N = 11$). In total there are $R_0 = 6$ point masses, which act as point
 scatterers in the observation domain (light green). The image constructed based on
 the 5 noise space singular vectors (for point scatterers under the Born approximation
 it is known that the dimension of the signal space is equal to the number of scatterers
 [34, 35, 19]) at 1240 Hz ($\lambda = 12$ cm) is presented in figure 2a. As can be seen, the
 240 locations of all 6 point scatterers are highlighted, although the one in the bottom centre is
 difficult to identify. Although there is a high dynamic range between detected scatterers
 and background noise, in the background of the image still artefacts of the dominant
 modeshape at this frequency are visible.

Since the scatterers' position is independent of the frequency, the location of the
 245 pixels with the largest amplitude will not change as a function of frequency and always
 indicate the scatterers. Equation (11) holds and thus $I(z)$ tends towards infinity (in
 reality a very high, but finite value). At the same time, the pseudospectrum's amplitude
 at pixels z not correlating with a scattering event does depend on the test function, since
 the finite value in equation (12) and in consequence also $I(z)$ changes with frequency.

250 Making use of this knowledge, even under these ideal conditions (no noise, perfect
 model), the dynamic range of the image can be improved by introducing a multiplication
 over different frequencies in equation (14):

Figure 1: FE model of a 1.608 m x 1.848 m plate with 11 excitation/sensor locations (orange) , 6 point scatterers (black) and the observation domain indicated in light green.

$$I(z) = \left(\prod_{f=1}^{N_f} \sum_{n=R_0^{meas}+1}^N |g_f(z)^H \Phi_{n,f}^{meas}| \right)^{-1}, \quad (15)$$

with N_f being the number of frequency evaluations.

This technique was suggested by Solimene et al. [28, 36] and is referred to as
 255 interferometric TR-MUSIC. The effect on the image can be seen in figure 2b.

The case described in the beginning of this section, where $M = 4 = R_0^{meas} < R_0 = 6$ evaluated at a single frequency (equation (14)) can be found in figure 2c. As expected, the algorithm is not able to correctly split S_Φ from N_Φ , which is why the detection fails.

However, the measured signal space is still a subset of the true signal space,
 260 $S_\Phi^{meas} \subset S_\Phi$. Since in this example, the space spanned by the $N = 11$ sensors did not change as compared to the ideal scenario, it follows that the true noise space is a subset of the estimated noise space, $N_\Phi \subset N_\Phi^{meas}$. As a consequence, the measured signal space is still orthogonal to the true noise space, $S_\Phi^{meas} \oplus N_\Phi$. This condition contributes correct information to the pseudospectrum computed according to equation
 265 (14). In addition to this, $I(z)$ is polluted by the contribution of the vectors in N_Φ^{meas} , which are not part of N_Φ . This is the reason for the image in figure 2c to fail.

(a) Damage index based on $M = N = 11$, $R_0 = R_0^{meas} = 6$ at 1240 Hz. Logarithmic color scale.

(b) Damage index based on $M = N = 11$, $R_0 = R_0^{meas} = 6$ for 151 frequency lines between 500 Hz and 1000 Hz. Logarithmic color scale.

(c) Damage index based on $M = 4 = R_0^{meas} < R_0 = 6$, $N = 11$ at 1240 Hz. Logarithmic color scale.

(d) Damage index based on $M = 4 = R_0^{meas} < R_0 = 6$, $N = 11$ for 151 frequency lines between 500 Hz and 1000 Hz. Logarithmic color scale.

Figure 2: Influence of undersampled data.

At all frequencies, the measured signal space S_{Φ}^{meas} contributes information on the scatterers' locations, while the error is contributing arbitrarily at different frequencies. Therefore, in average over multiple frequencies, the product in equation (11) is no longer 0, but still smaller than the one in equation (12). Applying equation (15) to this scenario allows to filter the coherent parts of the image between different frequencies and is thus a way to enable the detection of scatterers although the conditions required according to the theoretical derivation of the algorithm are not strictly fulfilled. The example for this case is presented in figure 2d, where all 6 scatterers are correctly indicated. The choice for $M = R_0^{meas} = 4$ is somewhat arbitrary and should be understood as an illustration. The performance of the algorithm gradually increases the closer S_{Φ}^{meas} comes to S_{Φ} , and hence depends on the choice of R_0^{meas} compared to R_0 .

A similar discussion treating the advantages of multi-frequency MUSIC (without an explicit link to TR) and in particular the interferometric MUSIC approach for the identification of scatterers in a high-frequency scenario, where there is no modal behaviour expected can be found in Solimene et al. [36]

In literature, multiple authors report on the trouble finding the correct separation between signal and noise space [20, 19]. Re-interpreting the findings presented in this section in the context of that problem, it follows for the same reasons that a good detection can still be achieved, although the separation between signal and noise space is slightly incorrect.

Furthermore, also the influence of random measurement noise on the detection quality decreases when using an interferometric TR-MUSIC algorithm as compared to a single frequency case. The reason for this is similar to what is explained so far in this section. The overlapping parts of the true and the measured signal space contribute coherent information of the defect location over all frequencies, while this is not the case for the erroneous contribution.

2.3. Frequency selection for the TR-MUSIC algorithm

As described in the previous section, adding multiple frequencies to the construction of the pseudospectrum has various advantages. However, if too many frequencies are included in the calculation of the denominator in equation (15) and the denominator for

most of these frequencies is smaller than 1, numerical issues may arise, since equation (15) runs the risk of becoming infinite, even if z is not a scatterer location. It is proposed to cope with this challenge by including only those frequency lines in the algorithm that add valuable information.

In order to evaluate whether the information carried at a specific frequency is valuable for a scatterer localisation based on equation (15), certain requirements must be fulfilled from experimental/measurement side (Φ_n) as well as from the model's side ($\mathbf{g}_f(z)$). All criteria used in this publication to distinguish between frequencies adding valuable information that should be kept and those frequencies that can be discarded are listed below and explained in more detail subsequently:

- Model: $\mathbf{g}_f(z)$ must be as different as possible for all z investigated. This is because if $\mathbf{g}_f(z)$ was similar for all z , all $\mathbf{g}_f(z)$ would be close to orthogonal to N_Φ . Therefore, even a small error would cause the product $\mathbf{g}_f(z)^H \Phi_n$ in the denominator of equation (15) to be comparable for all z , independent of the location of z . As such, no information would be added in this case.
- Measurement: The amplitude of the scattered wave field $\Psi_s(x_n, x_m)$ (equation (2)) should be sufficiently large to ensure that it can be measured under the given environmental conditions and with the measurement equipment in use.

In order to evaluate whether $\mathbf{g}_f(z)$ is sufficiently different for different z , first the MAC-value is calculated as [37]

$$MAC_f(z_1, z_2) = \frac{|\mathbf{g}_f(z_1)^H \mathbf{g}_f(z_2)|^2}{|\mathbf{g}_f(z_1)^H \mathbf{g}_f(z_1)| |\mathbf{g}_f(z_2)^H \mathbf{g}_f(z_2)|}. \quad (16)$$

$MAC_f(z_1, z_2)$ is equal to 1 if the vectors $\mathbf{g}_f(z_1)$ and $\mathbf{g}_f(z_2)$ are equal and 0 if the two vectors are orthogonal. The values $MAC_f(z_1, z_2)$ are collected in a lower triangular matrix $\mathbf{MAC}_f$ of dimension $Z \times Z$, where Z is the number of potential scatterer locations / pixels in the image z_n . Due to the symmetry of the MAC matrix, all the diagonal and upper triangular elements do not need to be calculated.

Desirable is a MAC matrix with as small as possible off-diagonal elements indicating

that the vectors $\mathbf{g}_f(z_n)$ are in average as far as possible from being equal. This results in the first selection criterion, being: a frequency is included if

$$\frac{\sum \mathbf{MAC}_f}{\frac{(Z-1)Z}{2}} < \mathbf{MAC}_{max}, \quad (17)$$

325 where the denominator is equal to the number of non zero elements in $\mathbf{MAC}_f$. The threshold $\mathbf{MAC}_{max}$ must be set by the user.

Since $\mathbf{g}_f(z_n)$ is dependent on the sensor location and the location of potential scatterers z , also the question whether a frequency is included in the TR-MUSIC algorithm or not depends on these parameters. In NDT&E applications, the potential
330 scatterers are the inspected zones, where defects are expected to occur. Consequently they are defined by the test object and purpose and cannot be influenced. On the other hand, in most applications, the possible sensor positions are not as restricted, providing an opportunity to find more frequencies adding valuable information. However, the criterion in equation (17) is built on an average value and can as such not guarantee that
335 all potential scatterers locations can be distinguished by the TR-MUSIC algorithm. A-priori finding a sensor array guaranteeing that all pre-defined potential defect locations can be discriminated is subject of ongoing research.

In order to develop the second selection criterion, first the amplitude of the scattered wave at all sensors N is calculated

$$\Psi_{s,f} = \log (|\Psi_{meas,f} - \Psi_{init,f}|), \quad (18)$$

340 where $\Psi_{meas,f}$ and $\Psi_{init,f}$ are vectors containing the vibration measured at the object with scatterers and the vibration of the intact object, respectively, with dimension $1 \times N$.

In order to be able to set a dimensionless threshold for the amplitude of the scattered field, $\Psi_{s,f}$ is set in relation to the amplitude measured at the defected and the intact test
345 object:

$$\Psi_{s,f}^{norm} = \frac{\Psi_{s,f}}{RMS_{\Psi}}, \quad (19)$$

with $\bar{\bullet}$ indicating the mean value and the matrix

$$RMS_{\Psi} = \begin{bmatrix} RMS(\Psi_{meas}) & RMS(\Psi_{init}) \end{bmatrix}. \quad (20)$$

The abbreviation RMS in equation (20) indicates the calculation of the root means square of a matrix, which is a positive, scalar value.

Based on the definitions of equation (19), a frequency is selected if the condition

$$\frac{\sum_N \Psi_{s,f}^{norm}}{N} > \Psi_{s,min}, \quad (21)$$

350 with the user-set threshold $\Psi_{s,min}$ is met.

Note that the selection of frequencies from sensor side is done separately for each excitation. Therefore, for small M , it is likely that R_0^{meas} varies with frequency.

Frequency selection for the transmitter side usage of the TR-MUSIC algorithm can be derived similarly.

355 2.4. TR-MUSIC with different sensor-types in one array

As mentioned in the introduction, under certain conditions, it might be beneficial or simply necessary to combine data retrieved from sensors measuring different quantities. In principle, the derivation of the TR-MUSIC algorithm presented previously holds, independent of the quantities measured by the sensor array. However, if the values
360 measured by different sensors are significantly different (orders of magnitude), the corresponding rows of $\mathbf{K}$ would be significantly larger than others. This can cause rank deficiency of $\mathbf{K}$ and as a consequence the dimension of S_{Φ} would decrease.

In order to avoid this happening, it is proposed to condition $\mathbf{K}$ according to the following routine. Again it is intended to facilitate reading by implicitly assuming that
365 all variables and matrices in this subsection relate to one frequency.

First the sensors in the sensor array are grouped according to the physical quantity they measure. E.g. in the case of an array composed of accelerometers and microphones, these two types of sensors would form the two groups. Then the average Frobenius norm of each group is calculated:

$$\hat{q}_k = \frac{\left\| \begin{bmatrix} \mathbf{g}_k(z_1) & \mathbf{g}_k(z_2) & \dots & \mathbf{g}_k(z_Z) \end{bmatrix} \right\|_{Fro}}{N_k}, \quad (22)$$

where k is the indicator for one of the K groups $\hat{q}_k$ relates to and N_k represents the number of sensors in this group.

The matrix $\mathbf{K}$ is scaled by a left multiplication with the diagonal scaling matrix $\mathbf{B}$ containing the scaling factors $\hat{q}_k$ of equation (22) on the diagonal elements.

$$\hat{\mathbf{K}} = \mathbf{B}\mathbf{K} = \begin{bmatrix} \text{diag}(\hat{q}_1^{-1}) & & 0 \\ & \text{diag}(\hat{q}_2^{-1}) & \\ & & \ddots \\ 0 & & & \text{diag}(\hat{q}_k^{-1}) \end{bmatrix} \mathbf{K}. \quad (23)$$

As this change influences the singular vectors Φ_n , the vectors $\mathbf{g}(z)$ need to be adapted accordingly (proof in Appendix A):

$$\hat{\mathbf{g}}(z) = \mathbf{B}\mathbf{g}(z). \quad (24)$$

Although the physical meaning of $\mathbf{g}(z)$ gets partially lost due to this operation (e.g. it cannot be seen anymore that the acoustically radiated energy is a lot lower than the energy in the structural vibration due to the impedance mismatch), it allows to combine different sensor types, since the amplitude of each group of sensors in the array is now in a comparable range. Furthermore, additional freedom for the user is generated as no care needs to be taken to work in a consistent unit system between sensors of different groups.

The normalisation of $\mathbf{g}(z)$ as mentioned in Appendix B, should be done after this step.

There are two reasons to perform the conditioning of $\mathbf{K}$ based on $\mathbf{g}(z)$, rather than on the values in $\mathbf{K}$ itself. The first one is that $\mathbf{g}(z)$ needs to be measured only once for the intact structure. Therefore, without losing time in the actual localisation process, this measurement can be repeated multiple times, ensuring a high reliability

and at convenient moments, ensuring low-error measurements. Furthermore, since $g(z)$
390 remains unchanged for all measurements at the defected structure, $\hat{q}_k$ can be computed
a-priori, further minimising the computation time.

3. Experimental validation

In order to validate the developments of the previous section, a validation case with
high complexity is considered. Furthermore, it is investigated whether a sensor array
395 composed of sensors measuring different physical quantities can overcome limitations
set by noisy measurements on one of the physical channels.

Therefore, first the measurement setup is explained, then the equations of the
previous section are applied and artificial measurement errors are added on some
sensors.

400 3.1. Experimental setup

The test object under investigation is an assembly of multiple components (figure
3). The base frame (figure 3b) is composed of aluminium beams. Three vertically
aligned beams with a C-cross-section and 4 mm wall thickness are joined with two
horizontally aligned beams with I-cross-section and 3 mm wall thickness. The I-beams
405 are constructed by merging two C-sections with rivets. This construction results in six
connection points ($Z = 6$) between the beams, where each connection is fixed with two
bolts (figure 3c). The frame is a replica of a section used in commercial air planes.

The I-section is covered by a honeycomb panel (10 mm core thickness, skins on
both sides with 0.3 mm thickness) of $1.4 \text{ m} \times 0.5 \text{ m}$. The connection between the
410 honeycomb panel and the I-beam is realised via double-sided tape and bolts. Looking
at the honeycomb panel, large parts of the frame, including all six connections are
hidden. This view is referred to as 'front' in the following and all directions - top,
bottom, left, right - are given with reference to this view.

The aim of this experiment is to find at which connection there is a defect, where a
415 defect is simulated by loosening or removing bolts. In order to investigate if there is a
dependency of the algorithm on the severity of a defect, different levels of severity are

(a) Front view on the setup.

(b) Back view on test object.

(c) Detail view on bottom right connection.

Figure 3: Measurement setup, test object and detail view on one potential defect location.

Figure 4: Example for one loose bolt (left), one removed bolt (centre) and two removed bolt (right).

introduced by either loosening one bolt at a connection or removing one or both bolts, respectively at the same connection. Whether the TR-MUSIC algorithm is able to point out defects at different locations is tested by loosening or removing bolts at different connections (multi-defect scenario). A complete list of all tested cases is given below:

1. One loose bolt at the top central connection (invisible with the bare eye, figure 4 left);
2. One removed bolt at the top central connection;
3. One loose bolt at the bottom left connection (invisible with the bare eye);
4. One removed bolt at the bottom left connection (figure 4 centre);
5. Two removed bolts at the bottom left connection (figure 4 right);
6. One loose bolt at the top right connection (invisible with the bare eye);
7. One removed bolt at the top right connection;
8. One removed bolt at the top right connection and one removed bolt at the top central connection.

For the localisation of these defects, in total $N = 13$ sensors are installed, of which seven are microphones and six are 1-D accelerometers (PCB 352a24), measuring the out-of-plane displacement on the frame. All microphones (1/2", PCB 426E01 and Bruel & Kjaer Type 2671) are installed in a plane at 0.5 m distance and parallel to the honeycomb panel. Three of the accelerometers are installed at the front side of the frame and three on the vertical C-sections. None of the defects is in the direct line of

sight for any of the sensors. The test object is freely supported, so that the lower end is in a height of 0.9 m over the floor. The excitation is applied with an impact hammer (PCB 086C03) at $M = 3$ locations on the front side of the frame. All experiments are repeated 5 times and the results are averaged in order to suppress random measurement errors. The good repeatability of the experiment is verified by the comparison of two successive measurements in the beginning of the test campaign at the healthy structure and the measurement of the completely repaired structure at the end of the measurement campaign. The setup is installed in a semi-anechoic chamber (reflecting floor; absorbing walls and ceiling, designed for a cut-off frequency of 200 Hz), due to its lower background noise level as compared to other areas in the lab. Other test-setups being installed simultaneously in the same semi-anechoic chamber cause that the assumption of a semi-free-field (full reflection at the floor; ideally radiation to infinity in all other directions) does not hold during the measurements, making the obtained results applicable to a more general environment. The measurement system is designed to capture a frequency range from 0 Hz to 900 Hz in steps of 0.5 Hz. The actual measurement is acquired with a LMS SCADAS Mobile at a sampling frequency of 2048 Hz and triggered by the impact hammer.

It should be noted that due to the frequency selection procedure, at many frequency lines not all of the excitations are included in the algorithm, which is why $M = 3$ is the maximum value and best possible case. At the same time, there are cases, where either defects are introduced at two different connections or both (all) bolts at one connection are removed. Certainly, both damage scenarios cannot be simplified by the assumption of a point scatterer. Therefore, in most of the cases studied, the data is undersampled ($R_0^{meas} = M < R_0$).

The vectors $\mathbf{g}(z)$, needed to compute the pseudospectrum $I(z)$, are measured using the same equipment and settings as described in the paragraphs above. For the measurement of $\mathbf{g}(z)$ for one potential defect location z , the respective bolt head is excited with an impact hammer-hit in direction of the bolt's symmetry axis and the response is measured by all sensors. The resulting frequency response function is used to define $\mathbf{g}(z)$.

3.2. Localisation under ideal conditions

In this section only the results for the localisation of defect cases 1, 5, 7 and 8 based on the actually measured data are presented in detail. Since the outcome of the localisation of all other defects is in accordance with what is presented and all conclusions apply, the other 4 cases are only presented in an overview to avoid redundancy. The localisation results for these four cases are shown in figure 5. On the x-axis, the potentially defected connections are listed, where TL refers to the top left connection, BL to the bottom left connection and so on. In the figures at the left, for each connection, the calculated pseudospectrum $I(z)$ normalised by the largest $I(z)$ is presented, so that the y-axis ranges from 0 to 1. The figures at the right illustrate the logarithm of the left figures.

Especially when analysing the figures at the left, it is apparent that in all cases, one location is uniquely highlighted as defected. This includes all different types of defects introduced at only one connection (figure 5a-c), which are uniquely highlighted by the TR-MUSIC pseudospectrum at the correct location. This observation demonstrates that the developed algorithm is a strong tool for the position determination of a variety of defects, even in complex assemblies.

Figure 5d shows the result for defects at two different connections. Clearly only one of these connections is correctly highlighted, but uniquely, which means that also in this case no false positive detection is given. Furthermore, after fixing the highlighted connection and repeating the test in this condition, only the second defected connection is indicated by the pseudospectrum. Therefore, it can be concluded that the TR-MUSIC algorithm as defined in this publication is not able to locate multiple defects at once. Nevertheless this example indicates the potential of the algorithm for the sequential determination of multiple defects.

The large difference of multiple orders of magnitude between the pseudospectrum at the defected location and at other locations comes to the fore at the graphs at the right in figure 5.

In figure 6, the detection results for all 8 defect locations using different settings and sensor arrays are presented. On the x-axis, the different defect cases 1 - 8 are listed in accordance with the list given in the section 'Experimental setup'. Each number on

(a) One loose bolt at the top central connection (case 1).

(b) Two removed bolts at the bottom left connection (case 5).

(c) One removed bolt at the top right connection (case 7).

(d) One removed bolt at the top right connection and one removed bolt at the top central connection (case 8).

Figure 5: Normalised pseudospectrum $I(z)$ for all six connections (left) and the logarithmic illustration $\log(I(z))$ (right).

Setup number	Selection criterion	Sensors	Scaling
1	$MAC_{max}, \Psi_{s,min}$	all	yes
2	$MAC_{max}, \Psi_{s,min}$	Mic.	not applicable
3	$MAC_{max}, \Psi_{s,min}$	Accel.	not applicable
4	$MAC_{max}, \Psi_{s,min}$	all	no
5	$\Psi_{s,min}$	all	yes
6	MAC_{max}	all	yes

Table 1: Overview of setup configurations for the results presented in figure 6.

Figure 6: Localisation results for 8 different defect cases, 6 different configurations (see table 1) and $\vartheta = 100$. Black indicates successful localisation, white indicates a failed localisation.

the y-axis represents one of the setup configurations listed in table 1. A localisation is assessed as 'successful', if the pseudospectrum at the correct location is at least a factor ϑ higher than the pseudospectrum at any other location (due to the previous discussion, case 8 is also regarded as 'located successfully' if only one of the two defects is highlighted). Successful localisations are indicated with a black field, failed ones with a white field.

As already indicated at the beginning of this section, if all described selection criteria and the scaling for different sensor types is applied to an array composed of microphones and accelerometers, the localisation of all defects under investigation is possible (figure 3, setup 1). If the array is built by only microphones (setup 2) or only accelerometers (setup 3), respectively, then not all defects can be located correctly.

Here, it should be noted that the size of a purely structural or acoustic array is smaller
 510 than that of a combined array. Also if the scaling for different sensor types (setup 4)
 or one of the selection criteria (setup 5 and 6) are not applied, the localisation fails -
 underlining the necessity of all previously described steps.

3.3. Localisation in noisy environment

As demonstrated in the previous section, the TR-MUSIC algorithm reliably locates
 515 defects based on data measured in an environment with low background noise. However,
 such close to ideal measurement conditions rarely exist in real-life measurements. In
 many test environments, e.g. outdoor or in a production plant, especially the pollution of
 the data measured by acoustic sensors with ambient noise is not uncommon. Therefore,
 in this section the robustness of the algorithm in face of measurement errors acting
 520 mainly on one group of sensors is investigated.

In order to perform this investigation, artificial, random measurement noise with
 zero mean is added on the measured data. The singular value decomposition of $\mathbf{K}$ in
 the TR-MUSIC algorithm is then replaced by the decomposition of $\mathbf{K}^\epsilon$ containing the
 erroneous measurement of the scattered field. It is calculated as:

$$\mathbf{K}^\epsilon = \mathbf{K} + \begin{bmatrix} \boldsymbol{\epsilon}_1^T \sqrt{\beta_1} & \boldsymbol{\epsilon}_2^T \sqrt{\beta_2} & \cdots & \boldsymbol{\epsilon}_K^T \sqrt{\beta_K} \end{bmatrix}^T, \quad (25)$$

525 where β_k is a scalar factor defining the amplitude of the artificially applied error
 on the group of sensors k and $\boldsymbol{\epsilon}$ is a matrix of the same dimension as $\mathbf{K}$ but filled with
 random complex numbers following a standard normal distribution. Correspondingly,
 $\boldsymbol{\epsilon}_k$ refers to the error added to the k -th group of sensors. β_k depends on the requested
 error level per group of sensors SNR_k in the following way:

$$\beta_k = \frac{\sum_F |\mathbf{K}_{f,k}|^2}{10^{\frac{SNR_k}{10}} \sum_F |\boldsymbol{\epsilon}_{f,k}|^2}. \quad (26)$$

530 As can be seen from equation (26), when SNR_k tends towards infinity, β_k tends to
 0, which means that there is no error added to the respective group of sensors.

In order to give a tangible impression on the meaning of the SNR values, the frequency response function measured by one representative microphone at defect case 5 is shown in figure 7. The plot at the bottom is a representation of the scattered wave field, computed according to equation (2), with noise added according to different SNR values. This is the quantity, which is directly used by the algorithm. The top of figure 7 shows the same absolute errors, now added to the pressure directly measured by the microphone. This comparison is of interest, since here the influence of measurement noise in relation to the actually measured quantity is visualised. In both cases it can be seen that $SNR = 20$ and $SNR = 10$ represent significant measurement errors. In the case of $SNR = 10$, even in the frequency response function of the measurement at the defected test object, apart from the resonances, only noise, no signal pattern can be seen.

In tables 2 to 5, the detection results for all 8 defect locations under different error conditions are presented. For each case, the simulation is repeated 100 times with the same SNR value applied to the same group of sensors. However, due to the random nature of the error, results are not necessarily equal. In each table, the number of correct localisations is presented in the first content row. It should be emphasised that the term 'correct' depends on the choice of ϑ , which is given in the table caption. Furthermore, in each table, the number of simulations, where none of the connections is highlighted as defected and if applicable the number of false positive localisations is shown. The sum of each column individually equals 100, the total number of simulations.

Tables 2 and 3 show results, where the artificial error is only applied to the microphone signals, while the measurement values provided by the accelerometers remain untouched. In table 2a, it can be seen that even with $SNR_{mic} = 20$ in most simulations, the defect is correctly localised applying a factor of $\vartheta = 100$. In this scenario, the algorithm only struggled in 65 simulations with the localisation of a defect at the top right position (cases 6 and 7), while in 735 simulations the localisation worked successfully. Increasing the error level to $SNR_{mic} = 10$ (table 2b), still the defects at the majority of the locations are located correctly. With $SNR_{mic} = 10$, still all defects at the top centre (case 1 and 2) and at the bottom left (case 3, 4 and 5) are always located correctly, but the number of false localisations for defects at the top right position increased to

Figure 7: Frequency response function at one microphone with different artificial error levels. Top: influence of the error on the pressure measured directly at the defect case 5. Bottom: The influence of the error on the corresponding scattered field.

Defect case	1	2	3	4	5	6	7	8
Correct localisation	100	100	100	100	100	36	99	100
No unique localisation						64	1	

(a) $SNR_{mic} = 20, SNR_{acc} = \text{Inf}$

Defect case	1	2	3	4	5	6	7	8
Correct localisation	100	100	100	100	100	27	77	100
No unique localisation						73	23	

(b) $SNR_{mic} = 10, SNR_{acc} = \text{Inf}$

Table 2: Localisation results for 8 different defect cases and 100 repetitions with different noise levels added to the microphones. The threshold for a successful localisation is set at $\vartheta = 100$.

Defect case	1	2	3	4	5	6	7	8
Correct localisation	100	100	100	100	100	99	100	100
No unique localisation						1		

(a) $SNR_{mic} = 20, SNR_{acc} = \text{Inf}$

Defect case	1	2	3	4	5	6	7	8
Correct localisation	100	100	100	100	100	74	99	100
False positive at BR						5		
No unique localisation						21	1	

(b) $SNR_{mic} = 10, SNR_{acc} = \text{Inf}$

Table 3: Localisation results for 8 different defect cases and 100 repetitions with different noise levels added to the microphones. The threshold for a successful localisation is set at $\vartheta = 2$.

96. Since the amplitude of the scattered wave field for defects at this position is lower than for defects at any other connection, it is assumed that due to the placement of excitations and sensors defects at the top right are more difficult to detect than defects at other places and thus the TR-MUSIC algorithm fails first for the localisation of defects at this location. As a consequence, it is expected that the localisation of defects at the top right could be more accurate by adding sensors and excitations or by changing their

location. As mentioned previously, the optimal placement of sensors and excitations is
570 subject of ongoing research.

If ϑ is decreased from before 100 to $\vartheta = 2$, all simulations, except one, are successful
for the error level of $SNR_{mic} = 20$ added to the microphones (table 3a). This points out
that the failures in table 2a are not due to wrongly highlighting a non-defected connection
as 'defected', but simply due to a too small difference between the pseudospectrum at
575 the correct connection and at other connections. Analysing the corresponding results
for $SNR_{mic} = 10$ (table 3b), it can be seen that the number of correct detections
improves as compared to table 2b, but still in 27 cases a defect at the top right location
is not found. In all these cases, the reason is a too large pseudospectrum at the bottom
right location, resulting in either a false positive localisation of the bottom right or in
580 a scenario, where no unique localisation is possible because the pseudospectrum at the
bottom right and at the top right are approximately equal. This observation can be
explained, as the factor between the pseudospectrum at the correctly pointed out top
right and that at the bottom right under perfect measurement conditions is 4421 and as
such many orders of magnitude smaller than the factor to other connections (10^{17} and
585 higher). Perturbing parts of the measurement further decreases that factor and leads to
false detections first, where the difference is smallest.

While in tables 2 and 3 the artificial measurement error was only added to the
microphones, in tables 4 and 5, the same levels of measurement error are added to all
sensors. The conclusions that can be drawn for an error level of $SNR = 20$ (tables 4a
590 and 5a) are similar to those discussed in connection with the results shown in tables 2
and 3 for both error levels. If the highest error level under investigation of $SNR = 10$
is applied to all sensors (tables 4b and 5b), the localisation of a loose bolt at the bottom
left connection (case 4) becomes the most challenging case. This statement is only
weakly dependent on the chosen value for ϑ , as can be seen in the comparison of table
595 4b with $\vartheta = 100$ and table 5b with $\vartheta = 2$. It follows that most localisations did not fail
due to an insufficiently high difference between the pseudospectrum at the bottom left
and another connection, but rather due to a false positive localisation of a non-defected
connection. The reason for this and the seeming discrepancy to the results shown in
table 4a are explained in the following paragraphs.

Defect case	1	2	3	4	5	6	7	8
Correct localisation	100	100	100	100	100	17	100	100
No unique localisation						83		

(a) $SNR_{mic} = 20, SNR_{acc} = 20$

Defect case	1	2	3	4	5	6	7	8
Correct localisation	100	100	100	33	100	100	100	100
False positive at TR				12				
No unique localisation				55				

(b) $SNR_{mic} = 10, SNR_{acc} = 10$

Table 4: Localisation results for 8 different defect cases and 100 repetitions with different noise levels added to all sensors. The threshold for a successful localisation is set at $\vartheta = 100$.

Defect case	1	2	3	4	5	6	7	8
Correct localisation	100	100	100	100	100	87	100	100
False positive at BR						2		
No unique localisation						11		

(a) $SNR_{mic} = 20, SNR_{acc} = 20$

Defect case	1	2	3	4	5	6	7	8
Correct localisation	100	100	100	63	100	100	100	100
False positive at TR				28				
No unique localisation				9				

(b) $SNR_{mic} = 10, SNR_{acc} = 10$

Table 5: Localisation results for 8 different defect cases and 100 repetitions with different noise levels added to all sensors. The threshold for a successful localisation is set at $\vartheta = 2$.

600 Looking closer at these results, it is observed that every false detection in tables 4b and 5b highlights the top right connection instead of the bottom left. This can be explained by analysing the vectors $\mathbf{g}(z)$. Summing the absolute values of all entries of $\mathbf{g}(z)$ over all frequencies, the value obtained at $z = \text{'bottom left'}$ is slightly smaller than

the values for the other connections. Since at this high noise level the separation between
605 measured noise space and real signal space is rather inaccurate, the theoretically existing
orthogonality of the noise space singular vectors to the signal space cannot be properly
abused anymore. Therefore, $I(z)$ in equation (15) naturally tends towards the top
right location if the entire measurement does not carry sufficient physical information.
This statement is bagged up by replacing Ψ^{meas} with vectors, where all elements are
610 given a value of 1 or are randomly generated, respectively. In both cases the highest
pseudospectrum occurs at the top right position.

Other then in the case where noise is only added to the microphones, while the ac-
celerometers still deliver physically meaningful signals, here all signals are close to pure
randomness at $SNR = 10$. Therefore, if only the microphone data is extremely noisy,
615 a false localisation is still dependent on the difference between the pseudospectrum at
the correct connection and at other potential defect locations in the ideal scenario. On
the other hand, if all data consists mainly of noise, all physical meaning gets lost and
the algorithm tends towards one location, which is defined by the mathematics of the
algorithm itself, but has no physical interpretation. This explains the difference between
620 table 3b and table 5b.

The amplitude of the scattered field is lowest for defects introduced at the top
right and second lowest for defects introduced at the bottom left. For that reason, the
algorithm is still able to locate defects at the top centre correctly, but fails at the bottom
left. As the pseudospectrum naturally tends to be maximum at the top right for random
625 Ψ^{meas} , the defects at the top right appear to be located correctly, although the amplitude
of the scattered field for defects at this connection are actually lowest.

Depending on the choice of ϑ , the number of unique localisations changes. At
small values of ϑ , only a small difference between the highest pseudospectrum and the
pseudospectrum computed for other potential defect locations is required to classify a
630 localisation as unique. As a consequence, both, the number of correct localisations, but
as well the number of false positive localisations increases with decreasing ϑ . On the
other hand, the higher ϑ , the more reliable is a localisation result. The comparison of
table 2b with 3b, table 4a with 5a and table 4b with 5b support this expectation.

The results presented in this section show that the localisation of defects provided by

635 the TR-MUSIC algorithm is functioning, even if parts of the sensor-array are exposed to heavy noise. Furthermore, it can be stated that the localisation with measurement errors on only one group of sensors in the array is superior to a scenario, where all sensors are exposed to noise. For the vibro-acoustic array with erroneous microphone measurements, at the error level of $SNR_{mic} = 20$ still a reliable localisation is obtained
640 (with difficulties to localise one loose bolt at the top right location), while for $SNR_{mic} = 10$ the procedure for more cases in this example starts to fail. However, with reference to table 7, even the noise level $SNR_{mic} = 20$ is a severe perturbation of the correct measured signal. In conclusion, the initial assumption 'a vibro-acoustic array can be used in environments with high background noise' can be confirmed.

645 Also the claim of a speed-up of the overall test time seems plausible since the test was performed using only an impact hammer, seven microphones and six accelerometers. An additional advantage could be that this equipment is low priced as compared to highly specialised excitation systems in combination with a huge number of sensors, Laser-vibrometers or IR-cameras as they are in use for other inspection techniques.
650 Also the simplicity of the test, which is characterised by hammer impact measurements at only three excitation locations and the fact that the microphones do not need to be physically attached to the test object leads to further benefits in time.

4. Conclusion

In order to contribute to the development of a NDT&E system for the automated
655 inspection of complex mass produced products, it is proposed to use a TR-MUSIC algorithm in combination with a vibro-acoustic sensor-array composed of microphones and enhanced by structural sensors. A proposal on how to handle different sensor types in one array is described in this publication. Using such a setup in combination with impact-hammer excitation, the shortcomings of traditional technologies, such as
660 long-lasting inspection time due to the coupling of a large number of sensors to the test object or the sensitivity of purely acoustic arrays to ambient noise can be overcome.

It is, furthermore, shown how to further improve the inspection time by reducing the number of excitations. It is demonstrated that the number of excitations can be

pushed even below the theoretically necessary limit. In addition, a strategy to select
665 the frequency bands carrying the most information is presented in order to improve
the robustness and quality of the localisation in simple application cases and to enable
defect localisation in complex structures.

The proposed method is experimentally applied to the detection of defects in a
structure of industrial complexity, composed of aluminium bars, which are connected
670 by bolts and rivets. One side of the test object is covered with a honeycomb panel,
so that the bolts are not visible for the sensor array. The structure is excited one after
another at three locations with an impact hammer and the vibrations are measured by
an array of six 1-D accelerometers and seven microphones. The experiment showed
that the TR-MUSIC algorithm is able to locate defects of different severity at different
675 locations of the test object, where a defect is defined as loosened or removed bolt(s). It
could also be shown that the method is able to locate multiple defects successively using
the vibro-acoustic array. Based on a purely structural or acoustic array, the localisation
was not possible.

By adding artificial noise on the microphone measurement data, the applicability
680 of the technique under rough environmental conditions is simulated. It could be shown
that the algorithm still provides very good localisation results, even at presence of severe
background noise, supporting the assumption that a vibro-acoustic array in combination
with a TR-MUSIC algorithm could be one valuable contribution to a fast, but yet reliable
NDT&E method.

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Appendix A: Proof for retained orthogonality of singular vectors after scaling

For the TR-MUSIC algorithm, the orthogonality between signal and noise space is used. But since the signal and noise space spanned by the singular vectors of $\mathbf{K}$ is not necessarily equal to that spanned by the singular vectors of $\hat{\mathbf{K}}$, proof needs to be given that the scaling done in equations (23) and (24) preserves the required orthogonality between the noise space resulting from the singular value decomposition of $\hat{\mathbf{K}}$ and the test vector $\mathbf{g}(z)$ for a point z on a scatterer's surface.

Assumed is a matrix $\mathbf{A}$, which can be decomposed into its singular vectors and values

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^H, \quad (\text{A.1})$$

where $\mathbf{U}$ and $\mathbf{V}$ are square matrices, whose columns are orthonormal vectors and $\mathbf{S}$ is a matrix containing the singular values on its diagonal elements.

Logically, the matrix $\mathbf{A}$ scaled with the diagonal scaling matrix $\mathbf{B}$ writes as

$$\mathbf{BA} = \mathbf{BUSV}^H = \mathbf{U}_{\text{scaled}}\mathbf{S}_{\text{scaled}}\mathbf{V}_{\text{scaled}}^H. \quad (\text{A.2})$$

Decomposing the product $\mathbf{BU}$ into the product of an orthogonal matrix $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$ applying a QR factorisation and introducing this into equation (A.2) results in

$$\mathbf{BA} = \mathbf{QRSV}^H. \quad (\text{A.3})$$

Writing $\mathbf{RS}$ as the product of its singular values and vectors $\mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^H$ gives

$$\mathbf{BA} = \mathbf{QU}_1\mathbf{S}_1\mathbf{V}_1^H\mathbf{V}^H, \quad (\text{A.4})$$

Since the product of two unitary matrices gives another unitary matrix (if $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$ and $\mathbf{U}_1^H\mathbf{U}_1 = \mathbf{I}$, then $(\mathbf{QU}_1)^H(\mathbf{QU}_1) = \mathbf{U}_1^H\mathbf{Q}^H\mathbf{QU}_1 = \mathbf{U}_1^H\mathbf{IU}_1 = \mathbf{I}$), it follows that $\mathbf{QU}_1$

and $V_1^H V^H$ are the unitary matrices. Comparing equations (A.2) and (A.4), it can be concluded that the columns of QU_1 are orthogonal to the columns of U_{scaled} and the rows of $V_1^H V^H$ are orthogonal to the rows of V_{scaled}^H . However, since it cannot be guaranteed that $S_1 = S_{\text{scaled}}$, no statement can be made regarding the orthogonality between the rows of QU_1 and U_{scaled} or the orthogonality between the columns of $V_1^H V^H$ and V_{scaled}^H . This relation holds for all $K \in \mathbb{C}$.

Since a basic assumption of the TR-MUSIC algorithm is the rank deficiency of the data matrix, in the following, rank deficiency of A is assumed, so that the singular values can be split into signal and noise space. In order to prove that the necessary orthogonality is preserved after the scaling, it needs to be shown that the noise space singular vectors of U_{scaled} are orthogonal to the signal space singular vectors of BU .

This orthogonality relation can be written as

$$U_{\text{scaled}}^H QU_1 = D, \quad (\text{A.5})$$

where D is a diagonal matrix.

Introducing the relation previously given by the QR-factorisation of BU and the singular value decomposition of RS into equation (A.6) results in:

$$U_{\text{scaled}}^H BU = DS_1 V_1^H S^{-1}. \quad (\text{A.6})$$

In the following, it is assumed, that the singular vectors are ordered such that the signals space singular vectors are first, followed by the noise space singular vectors ($U_{\bullet} = \begin{bmatrix} U_{\bullet}^{\text{signal}} & U_{\bullet}^{\text{noise}} \end{bmatrix}$ and correspondingly for all other vectors). Due to the rank deficiency, the product $S_1 V_1^H$ has only non-zero entries in the elements corresponding to the signal space

$$S_1 V_1^H = \begin{bmatrix} \text{diag}(P_1) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} P_2 & \mathbf{0} \\ \mathbf{0} & N_1 \end{bmatrix} = \begin{bmatrix} P_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad (\text{A.7})$$

where P_n and N_n indicate matrices relating to the signal or noise space, respectively.

Equation (A.7) shows that the desired orthogonality between signal and noise space is not violated due to the product $\mathbf{S}_1 \mathbf{V}_1^H$.

The inverse of the last element of the right hand side of equation (A.6) $\mathbf{S}^{-1}$ is not defined due to the rank deficiency of $\mathbf{A}$ and the consequential full-zero rows and columns in $\mathbf{S}$. However, since the diagonal of the signal space of matrix $\mathbf{S}$ is fully populated, also its inverse is uniquely defined, linking the signal space singular vectors of $\mathbf{U}$ and $\mathbf{V}$ and being independent of the noise space components. Therefore, some elements of $\mathbf{S}^{-1}$ are known and the product

$$\mathbf{D} \mathbf{S}_1 \mathbf{V}_1^H \mathbf{S}^{-1} = \mathbf{D} \begin{bmatrix} \mathbf{P}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \text{diag}(\mathbf{P}_4) & ? \\ \mathbf{0} & ? \end{bmatrix} = \mathbf{D} \begin{bmatrix} \mathbf{P}_5 & ? \\ \mathbf{0} & ? \end{bmatrix} = \begin{bmatrix} \mathbf{P}_6 & ? \\ \mathbf{0} & ? \end{bmatrix} \quad (\text{A.8})$$

can be partially defined. Although this operation is not fully defined, the $\mathbf{0}$ in the lower left of equation (A.8) is sufficient to prove that the noise space singular vectors of $\mathbf{U}_{\text{scaled}}$ are orthogonal to the signal space singular vectors of $\mathbf{B}\mathbf{U}$.

Appendix B: Normalisation of test vectors

The vectors $\mathbf{g}_f(z)$, contain the transfer functions connecting the scatterer(s) to the sensors. In the low and mid frequency region, these test vectors can have very different amplitudes for different points $z \in D$ due to nodal points, lines and maxima of the dominating modes.

Errors in the measured matrix $\mathbf{K}$ directly lead to an error in the singular vectors Φ_n . In the following, the influence of such measurement errors is explained at the example of a sensor side defect localisation. However, what is described holds directly also for the computation of the pseudospectrum from excitation side by replacing all parameters belonging to the sensor side with their counter parts from the excitation side (e.g. replace Φ_n with Ψ_n).

Most obvious is the influence of errors, when only a single frequency is considered.

755 Then equation (13) changes to

$$\begin{aligned} I(z) &= \left(\left| \mathbf{g}(z)^H (\mathbf{\Phi}_n + \boldsymbol{\epsilon}) \right| \right)^{-1} \\ &= \left(\left| \mathbf{g}(z)^H \mathbf{\Phi}_n + \mathbf{g}(z)^H \boldsymbol{\epsilon} \right| \right)^{-1}, \end{aligned} \quad (\text{B.1})$$

where the erroneous change of $\mathbf{\Phi}_n$ is expressed by the addition of the random error vector $\boldsymbol{\epsilon}$. For equation (B.1) and all following expressions relating to the pseudospectra, it is implicitly assumed that n is part of the noise space. This is $R_0 < n \leq M$ for the excitation side and $R_0 < n \leq N$ for the sensor side.

760 If, as described above, the absolute value of $\mathbf{g}(z)$ fluctuates significantly for different z , then also the contribution of the error to the image $\mathbf{g}(z)^H \boldsymbol{\epsilon}$ is significantly different for each z .

In order to keep the contribution of the error as constant as possible, it is beneficial to normalise $\mathbf{g}(z)$ at every single image point, so that $\|\mathbf{g}(z)\| = 1$ holds for all z , where
765 $\|\bullet\|$ denotes the 2-norm of a vector.

Due to this normalisation, the fluctuation of $\mathbf{g}(z)^H \boldsymbol{\epsilon}$ is minimized for a given error $\boldsymbol{\epsilon}$. In this case, the influence of the error on the image comes closer to adding a constant to the pseudospectrum, which is proposed in [19] in order to avoid singularities. Adding a constant greater than 0 to the image function does reduce the contrast between $I(z \in D_0)$
770 and $I(z \notin D_0)$, but does not change the location of the maximum of the image function, as:

$$\left(\left| \mathbf{g}(z \in D_0)^H \mathbf{\Phi}_n + \text{const} \right| \right)^{-1} > \left(\left| \mathbf{g}(z \notin D_0)^H \mathbf{\Phi}_n + \text{const} \right| \right)^{-1}. \quad (\text{B.2})$$

However, as the elements of $\boldsymbol{\epsilon}$ are generally different from each other, the influence of $\boldsymbol{\epsilon}$ on $\mathbf{g}_{\Psi,x}^H \boldsymbol{\epsilon}$ will be different for each image point. In addition to that, due to $\boldsymbol{\epsilon} \in \mathbb{C}$, it cannot be guaranteed that $\mathbf{g}_{\Psi,x}^H \boldsymbol{\epsilon}$ is real and greater than 0. Therefore the error will
775 always add a random contribution to $I(z)$. Nevertheless, the normalisation guarantees that the influence of the error on each image point becomes more comparable.

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