New closed-form proximity-effect complex permeability expression for characterizing Litz-Wire windings

Korawich Niyomsatian^{*,†}, Johan. Gyselinck[†], and Ruth V. Sabariego^{*}

^{*}EnergyVille, KU Leuven, Belgium, [†]BEAMS, Université Libre de Bruxelles, Belgium

Keywords: Eddy currents, proximity effect, litz-wire windings, homogenization, complex permeability

Abstract

The eddy current losses in litz-wire windings are mostly due to proximity effects, effects that can be macroscopically modelled via a complex frequency-dependent reluctivity. In this digest, we propose a new closed-form expression for this complex reluctivity. Results are compared with approximations found in the literature.

1 Introduction

Accurately considering eddy-current effects in the winding of electromagnetic devices working at higher and higher frequencies implies the fine discretisation of each turn, which is unworkable in practice, particularly in litz-wire windings. With the advent of wide-bandgap semiconductors, able to operate at even higher frequencies, litz wires are increasingly present in e.g. switch mode supplies as they reduce the skin-effect losses to a minimum. At the strand level, the proximity-effect losses cannot be neglected, though [1].

Frequency-domain homogenization techniques provide a closed-form continuous representation of the homogenized winding, see e.g. [2, 3] and references herein. Eddy-current effects are accounted for by a complex impedance in the supply electrical circuit (skin effect) and a complex reluctivity in the homogenized winding window (proximity effect). These two frequency-dependent complex parameters can be identified by means of an elementary 2D finite element (FE) model, as proposed in [4]. The approach has been successfully applied to litz-wire windings in [5].

In this work, we present an alternative closed-form representation of the proximity-effect complex permeability. The values obtained by means of the elementary 2D FE model are fitted to Ollendorff's mixing formula [6] and compared to previous results available in the literature.

2 Proximity-effect complex permeability

Let us define the reduced frequency X as the ratio of a strand radius r to the skin depth $\delta = \sqrt{2/(\omega\sigma\mu_0)}$, i.e.

$$X = \frac{r}{\delta} = \sqrt{f} \cdot r \sqrt{\pi\sigma\mu_0}, \quad (1)$$

with f the working frequency (or pulsation $\omega = 2\pi f$), non-magnetic strand with permeability $\mu_0 = 4\pi \cdot 10^{-7}$ H/m and conductivity σ .

We obtain the reference proximity-effect complex permeability by solving an elementary FE model with a central cell (conductor plus insulation) and a single layer of surrounding cells [5]. A pure proximity-effect excitation is achieved by imposing a unit horizontal (or vertical) induction and zero net current through appropriate boundary conditions. By integrating over the central cell $\sigma^{-1} \mathbf{j} \mathbf{j}^* / 2$ with $\mathbf{j}$ the current density and $\omega \mathbf{b} \mathbf{b}^* / 2$ with $\mathbf{b}$ the flux density (r.m.s. values), we quantify the complex power absorbed by the central cell (complex numbers in bold, i imaginary unit). The complex proximity-effect reluctivity $\nu_{\text{prox}}(X) = 1/\mu_{\text{prox}}(X)$, at reduced frequency X , can be written in terms of two dimensionless coefficients [4]. Such a model with hexagonal packing ($r = 0.1$ mm, fill factor $\lambda = 0.69$, $X = 2$) and imposed unit horizontal flux is represented in Fig. 1. The real and imaginary parts of the flux are depicted as well. Note that the packing type of litz-wire strands

Figure 1. Elementary FE model with hexagonal packing and circular cross-section conductors. Proximity-effect flux pattern—real and imaginary components.

inside a bundle does not significantly affect the complex permeability [1], only the fill factor λ (conductor to bundle area ratio) does. Hexagonal packing is herein adopted.

As in Igarashi's studies [6], we choose the Ollendorff formula as ansatz to fit the reference FE solutions from the elementary problem [4]. It reads as follows

$$\frac{\mu_{\text{prox}}}{\mu_0} = 1 + \frac{2\eta \left(\frac{\mu_w}{\mu_0} - 1 \right)}{2 + (1 - \eta) \left(\frac{\mu_w}{\mu_0} - 1 \right)}, \quad (2)$$

where μ_w is the macroscopic complex permeability of an iso-

lated round wire [6]. It is given by

$$\frac{\mu_w}{\mu_0} = \frac{J_1(z)}{zJ_0(z) - J_1(z)}, \quad (3)$$

with $z = (1 - \imath)X$ the complex reduced frequency, J_0 and J_1 , the zero- and first-order Bessel functions. A corrected complex fill factor η is further introduced to match the FE results:

$$\eta = \lambda(1 + \Theta(\lambda)\Gamma(z)) \quad (4)$$

with $\Theta(\lambda) = \sum_0^{n_\Theta} h_j \lambda^j$ and $\Gamma(z) = \sum_0^{n_\Gamma} g_j z^j$ the complex polynomials with orders n_Θ and n_Γ , respectively.

This approximation is fitted and then compared with Meeker's one [3], which is the most accurate in the literature. Indeed, a comparison with results in [7] and [1] can be found in [3]. For fair comparison, our fitting matches thus the range of frequencies and the order of the polynomials in [3], e.g. $X \in [0, 30]$. Note that increasing the order of the polynomials in (4). The fitted coefficients h_j and g_j are displayed in Table 2.

h_0	$(-2.5159 - \imath 0.1316) \times 10^{-2}$
h_1	$(1.9870 + \imath 0.6130) \times 10^{-1}$
h_2	$(-5.1038 - \imath 1.8358) \times 10^{-1}$
h_3	$(4.3864 + \imath 1.4787) \times 10^{-1}$
g_0	$(-1.1318 + \imath 0.8916) \times 10^{-1}$
g_1	$(1.8429 + \imath 3.0917) \times 10^{-2}$
g_2	$(2.1330 - \imath 3.7990) \times 10^{-4}$

Table 1. Coefficients h_j of $\Theta(\lambda)$ with $n_\Theta = 3$; and coefficients g_j of $\Gamma(z)$ with $n_\Gamma = 2$.

Figure 2. Relative complex permeability μ_{prox}/μ_0 vs X obtained with the reference FE problem, this work in (2), Igarashi's [6] and Meeker's expressions [3]. Results with fill factor $\lambda = 0.1 : 0.2 : 0.9$.

The results of our new expression (2) are compared with Meeker's [3] and Igarashi's [6] approximations in Fig. 2. An

excellent agreement is observed. We compute the relative L_2 -norm error with regard to the reference FE solution of the three approximations. Meeker's approximate error is 1.3%, our new expression (2) error is 0.3% and Igarashi's proximate error is 4.2%. It is worth mentioning that we (2) and Igarashi [6] use the same accurate ansatz from Ollendorf's mixing formula, but with different fill factor. We incorporate a corrected complex fill factor Λ while Igarashi's uses $\Lambda = \lambda$. The deviations of both Meeker's and Igarashi's expressions can be clearly seen from the phase response at high fill factor ($\lambda > 0.7$) and high frequencies ($X > 3$).

3 Conclusions

We have proposed a new complex frequency-dependent relativity expression for homogenizing litz-wire windings in FE-type models. The agreement with the brute-force FE reference model is excellent in a wide frequency range, much wider than the practical range of working frequencies in classical electrotechnical problems, i.e. $X = r_s/\delta \in [0, 2]$. Most available formulae available [1, 6, 3] cover accurately this practical frequency range. For the sake of comparison and in view of the current frequency trend in power devices, we have chosen to study the wide band $X = r_s/\delta \in [0, 30]$

References

- [1] X. Nan and C. R. Sullivan, "An Equivalent Complex Permeability Model for Litz-Wire Windings," *IEEE Trans. Ind. Appl.*, vol. 45, no. 2, pp. 854–860, 2009.
- [2] G. Meunier, V. Chamoille, C. Guérin, P. Labie, Y. Maréchal, "Homogenization for Periodical Electromagnetic Structure: Which Formulation?," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 3409–3412, 2010.
- [3] D. C. Meeker, "An improved continuum skin and proximity effect model for hexagonally packed wires," *J. Comput. Appl. Math.*, vol. 236, no. 18, pp. 4635–4644, 2012.
- [4] J. Gyselinck, P. Dular, "Frequency-domain homogenization of bundles of wires in 2D magnetodynamic FE calculations," *IEEE Trans. Magn.*, vol. 41, no. 1, pp. 1416–1419, 2005.
- [5] K. Niyomsatian, J. Van den Keybus, R. V. Sabariego, J. Gyselinck, "Frequency-domain Homogenization for Impedance Characterization of Litz-Wire Transformers in 2-D Finite Element Models," 11th International Conference on Ecological Vehicles and Renewable Energies (EVER), Monte Carlo, Monaco, 6-8 April, 2016
- [6] H. Igarashi, "Semi-analytical approach for finite-element analysis of multi-turn coil considering skin and proximity effects," *IEEE Trans. Magn.*, vol. 53, no. 1, 2017.
- [7] H. Rossmannith, M. Albach, J. Fischer, and A. Stadler, "Improved Characterization of the Magnetic Properties of Hexagonally Packed Wires," *EPE J.*, vol. 22, no. 4, pp. 5–10, 2012.