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Wolff and Kant on the Mathematical Method

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Abstract: Wolff advocates the mathematical method, which consists in chains of syllogisms that proceed from axioms and definitions to theorems, for achieving scientific certainty in branches of philosophy like ontology and physics. By contrast, in ‘The Discipline of Pure Reason in its Dogmatic Use’ Kant significantly limits the efficacy of this method in philosophy. In this paper I investigate an under-examined result of the Discipline: Kant’s claim that his system of philosophy does not contain “dogmata”. By identifying “dogmata” in Wolff’s system of physics, I argue that, for Kant, they are propositions that uncritically deploy ideas of reason. I conclude that the Discipline extends criticisms raised in the Transcendental Dialectic to any erroneous use of the mathematical method in philosophy.

Keywords: Kant, Wolff, Mathematical Method, Dogmata

1 Introduction

The success of mathematics has long tempted philosophers to try to achieve scientific certainty within their own domain by mimicking its method. Historically speaking, the mathematical method is generally viewed as the method of Euclid’s *Elements* or of Aristotle’s *Posterior Analytics* and consists in chains of valid syllogisms that proceed from axioms and definitions to theorems. As is well known, Christian Wolff advocates the mathematical method as the means to achieving scientific certainty in various branches of philosophy, e. g., ontology and physics. Immanuel Kant, by contrast, warns that unless one understands why this method is successful in mathematics proper, one cannot know whether it can be employed in other fields with the same efficacy and what its limits might be. In a section of the *Critique of Pure Reason* called ‘The Discipline of Pure Reason in its Dogmatic Use’ (henceforth: the Discipline), Kant investigates the methods proper to phi-

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losophy and mathematics, along with their respective limits, by distinguishing between philosophical and mathematical cognition.¹

While the Discipline has received much attention, especially since it is the best source for understanding Kant's philosophy of mathematics,² one of its most important results remains under-examined: Kant's claim that his critical system of philosophy does not contain dogmatic propositions, which he calls 'dogmata'.³ According to him, "a direct synthetic proposition from concepts is a dogma"⁴; it is a synthetic *a priori* judgement from categories, for example, 'the soul is a substance', that is used as a premise in what is viewed as an apodictic demonstration.⁵ While we already know from the Transcendental Dialectic that using such premises in demonstrations in rational psychology, rational cosmology, and rational theology erroneously deploys and hypostasises ideas of reason, I argue that the Discipline broadens the scope of this criticism to include the use of the mathematical method in philosophy in general. Thus, on my interpretation, any time the mathematical method is used in philosophy, dogmata, and therewith ideas of reason, must be uncritically employed.⁶

To establish this claim, I will focus on Kant's criticisms against using the mathematical method in philosophy. I will first lay out how this method was used

1 I will only discuss mathematical cognition and the method proper to it with respect to theoretical philosophy.

2 See, for example, Hintikka, Jaakko: "Kant on the Mathematical Method". In: *The Monist* 51 (3), 1967, 352–375; Martin, Gottfried: *Arithmetik und Kombinatorik bei Kant*. Berlin 1972; Young, Michael J.: "Kant on the Construction of Arithmetical Concepts". In: *Kant-Studien* 73, 1982, 17–46; Friedman, Michael: "Kant's Theory of Geometry". In: *Philosophical Review* 94, (4) 1985, 455–506; Parsons, Charles: "Kant's Philosophy of Arithmetic". In: *Kant's Philosophy of Mathematics: Modern Essays*. Hrsg. von Carl J. Posy. Dordrecht 1992, 43–79; Shabel, Lisa: *Mathematics in Kant's Critical Philosophy: Reflections on Mathematical Practice*. London 2003; Carson, Emily: "Kant on the Method of Mathematics". In: *Journal of the History of Philosophy* 37 (4), 1999, 629–652; Koriako, Darius: *Kants Philosophie der Mathematik: Grundlagen, Voraussetzungen, Probleme*. Hamburg 1999.

3 See KrV, A 736/B 764.

4 "Ein direct synthetischer Satz aus Begriffen ist ein Dogma [...]" (KrV, A 736/B 764). All quotations of the *Critique of Pure Reason* are taken from Kant, Immanuel: *Critique of Pure Reason*. Ed. and transl. by Paul Guyer and A.W. Wood. Cambridge 1998. All translations of untranslated texts are mine.

5 Despite the import of this claim, discussions of dogmata in the secondary literature so far are rather technical and abstract. See, for example, von Wolff-Metternich, Brigitta-Sophie: *Die Überwindung des mathematischen Erkenntnisideals*. Berlin 1995, 153; von Wolff-Metternich: "Dogma". In: *Kant-Lexikon*. Ed. Markus Willaschek, Jürgen Stolzenberg, Georg Mohr and Stefano Bacin. Berlin 2015, 435.

6 For Kant, general physics is an endeavour of mathematical cognition, and even though it is built on his metaphysical foundation for natural sciences, it nonetheless employs the mathematical method correctly; see MAN, AA 04: 472f.

in philosophy by Christian Wolff, who provides an example of how to apply it to physics (section 2).⁷ I will then discuss Kant's account of mathematical and philosophical cognition, as well as the methods proper to each, in sections 3 and 4. Finally, I will illustrate Kant's criticisms against using the mathematical method in philosophy with Wolff's example of this method applied to physics (section 5).⁸

Against reading Kant's criticisms in light of Wolffian physics, it may be argued that, in the *Discipline*, Kant does not address the method used by any branch of philosophy that engages with nature. However, on my view, Kant undertakes two tasks in this section of the *Critique*: (i) he provides a critical account of mathe-

7 Wolff states that there are three parts of philosophy: the study of God, human souls, and bodies or material things (*Preliminary Discourse on Philosophy in General* [1728]. Transl. by Richard Blackwell. Indianapolis. 1963, § 56. (Prelim. Disc.)). He calls the third 'physics' (Prelim. Disc. § 59). Wolffian physics investigates, for example, the mineral, vegetable, and animal kingdom (Prelim. Disc. § 75).

8 We know that Kant was familiar with Wolff's mathematical method as he taught mathematics using textbooks written by Wolff; see Shabel, Lisa: "Kant on the 'Symbolic Construction' of Mathematical Concepts". In: *Studies in History and Philosophy of Science* 29 (4), 1998, 589–621, 590. For Kant's teaching schedule, see Martin (1972, 12–14), and for the books on mathematics he had in his library, see Martin (1972, 15–17). Furthermore, Kant must have been familiar with Wolff's application of the mathematical method to philosophy, since the topic of the Berlin Academy essay competition, for which Kant wrote his 'Prize Essay' (*Untersuchung über die Deutlichkeit der Grundsätze der natürlichen Theologie und der Moral*, 1763), was whether Wolff's mathematical method could attain the same level of certainty in philosophy as it did in mathematics. See Dunlop, Katherine: "Arbitrary combination and the use of signs in mathematics: Kant's 1763 Prize Essay and its Wolffian background". In: *Canadian Journal of Philosophy* 44 (5–6), 2014, 658–685. École argues that Kant did not directly read any of Wolff's works on metaphysics, despite having Wolff's *Latin Metaphysics* in his library. École, Jean: "De la connaissance qu'avait Kant de la métaphysique wolffienne, ou Kant avait-il lu les ouvrages métaphysiques de Wolff?" In: *Archiv für Geschichte der Philosophie* 73, 1991, 261–276, 274–276. However, my discussion of Wolff's mathematical method mainly relies on the *Kurzer Unterricht von der mathematischen Methode oder Lehrart* (*Kurzer Unterricht*) (KUn). In: *Anfangsgründe aller mathematischen Wissenschaften* [1710]. *Gesammelte Werke* 12.I. Ed. by J. E. Hofmann. Hildesheim 1973. This is one of the works Kant taught, according to Martin. I also rely on Wolff's *Vernünfftige Gedanken von den Kräften des menschlichen Verstandes und ihrem richtigen Gebrauch in der Erkenntnis der Wahrheit* (*Deutsche Logik*) (DL) [1713]. In: *Gesammelte Werke* I.I. Ed. by H.W. Arndt. Hildesheim 1978. Translations of this text are taken from Wolff, Christian: *Rational Thoughts on the Powers of the Human Understanding* [1713]. Transl. by Barron Wolfius. London 1770. Schiffert shows Kant would have studied this work as a student and taught it as a professor, since it was part of the curriculum in Königsberg. See Schiffert, Christian: "Über das Königsberger Collegium Fridericianum." In: *Die Schule Immanuel Kants*. Ed. by Heiner F. Klemme. Hamburg 1994, 88–90. For clarification only, I occasionally refer to Wolff's *Vernünfftige Gedancken von Gott, der Welt und der Seele des Menschen, auch allen Dingen überhaupt* (*Deutsche Metaphysik*) (DM) [1719]. In: *Gesammelte Werke* I.2. Ed. by C.A. Corr. Hildesheim 1983, the *Prelim. Disc.*, as well as Wolff's *Philosophia rationalis, sive Logica* (*Latin Logik*) (LL) [1740]. In: *Gesammelte Werke* II.1.1–1.3. Ed. by C.A. Corr. Hildesheim 1983.

mathematical and philosophical cognition, and (ii) he argues, on the basis of (i), that the method proper to mathematics cannot be used with apodictic certainty in any branch of philosophy, including natural philosophy, as was the habit of many philosophers like Wolff prior to Kant.⁹ The somewhat surprising result will be that, for Kant, dogmata are not only used by thinkers like Wolff to transgress the bounds of experience in rational psychology, rational cosmology, and rational theology, but also in various branches of philosophy that deal with nature (section 5).

2 Wolff's Mathematical Method

The mathematical method, as Wolff applies it to philosophy, is basically Aristotle's axiomatic-deductive method, arguably the best example of which is found in Euclid's *Elements*. It consists of chains of valid syllogisms and produces a system of interconnected genera and species otherwise known as a Porphyrian Tree.¹⁰ Although this method is well known, the peculiarities of Wolff's version of it are not. Thus, I will detail its salient features for my discussion of Kant.

To this end, I will focus on two examples Wolff provides, one in which he illustrates how he employs the mathematical method in geometry, and the other in which he illustrates how he employs the same method in physics. Jaakko Hintikka has devised a helpful breakdown of the Euclidian method, which I will use to compare Wolff's two examples.¹¹ This framework, as Hintikka presents it, consists of the following five steps:¹²

9 Task (i) refers roughly to the text KrV, A 712/B 740–A 726/B 754, and task (ii), to the remaining text in the Discipline.

10 Syllogisms, which consist of propositions, produce a Porphyrian Tree, a hierarchy of concepts, because conclusions of syllogisms add new characteristics to the concepts in question. See further Dunlop (2014, 664).

11 While I agree with Hintikka's general breakdown of the mathematical method, in line with scholars like Emily Carson, I disagree with his account of the role of intuition in mathematical construction. See Carson, Emily: "Hintikka on Kant's mathematical method". In: *Revue internationale de philosophie* 250 (4), 2009, 435–449, 436. According to Hintikka, intuition is merely for the purpose of 'instantiation in the logical sense of the word' of pure mathematical concepts (2009, 467). However, on my view, serving as a field of instantiation for such concepts is not the only role intuition plays. *A priori* intuition and pure mathematical concepts are also interdependent in a way that, for Kant, is indispensable. On the one hand, mathematical construction determines space and time according to its pure concepts; on the other, intuition guarantees the real possibility of the pure concepts.

12 See Hintikka (1967, 361f).

1. *Enunciation*: statement of the general proposition about a figure in general (say a triangle) to be proved.
2. *Ecthesis*: exhibition of a particular instance of the figure in general.¹³
3. *Preparation*: exhibition of additions, e.g. points, lines, etc., to the figure exhibited in step (2), to prove the proposition enunciated in (1).
4. *Proof*: demonstration of the proposition enunciated in (1) by chains of syllogisms, which make use of axioms, propositions that are already known, and properties of the figure furnished by steps (2) and (3).
5. *Conclusion*: conclusion about the particular instance of the figure, which proves the proposition about the figure in general enunciated in (1) because the particular figure serves as a representative of the figure in general.

The first step provides the goal, namely, the proposition that is to be proved. The second and third steps are closely related to one another; the ‘preparation’ stage (3) involves making additions to the exhibited geometric figure of (2), such as adding lines to it. The preparation stage helps to draw out the properties of the exhibited figure not otherwise obvious without such additions. These latter two steps are essential to the mathematical method because its proof stage (4) must be carried out with recourse to experience of a particular geometric figure; that is, some key premises of the proof are about the particular figure. The demonstration (4) proves or disproves the enunciated statement with apodictic certainty; the statement enunciated in (1) is the demonstration’s conclusion (5). The particular figure, for example, a triangle, is viewed to represent a perfect model of all triangles; therefore, propositions about it used as premises in the demonstration are viewed as universal propositions. Thus, the demonstration proves (1) with apodictic certainty by appeal to the particular figure.

To get a better grasp of the Euclidian method, let us now consider an example to which this method is applied. Here we can turn to Wolff, who, in his *Deutsche Logik*, Chapter 4, § 23, uses the mathematical method to prove the proposition that ‘the internal angles of a triangle are equal to two right angles’. Using Hintikka’s five-step breakdown, Wolff’s proof can be presented as follows:

¹³ ‘*Ecthesis*’ is the Greek term Aristotle uses to describe this step within his own theory of scientific demonstration, by which he means making reference to an actual instance of the type of thing under discussion. For example, Aristotle uses *ecthesis* to complete some third figure moods of syllogism (e.g., Darapti and Datisi). See *Prior Analytics* 28a22–26; 28b14–15. He also uses it to prove two rules of conversion (*Prior Analytics* 25a14–26). Rules of conversion, in turn, are used to complete syllogisms of the second and third figures. See Ross, William D.: *Aristotle’s Prior and Posterior Analytics*. Oxford 1965, 32f; 412 ff.

1. *Enunciation*: the general proposition to be proved is “in every triangle ABC, all the three angles are equal to two right angles”¹⁴.
2. *Ecthesis*: A particular triangle is exhibited in the form of a picture of a triangle labelled ABC and its internal angles labelled 123.
3. *Preparation*: to the figure exhibited in step (2), lines and the appropriate labels are added (e. g., ‘DE’, as indicated below, and the extended base of the triangle) such that axioms and propositions already known can be used with reference to the figure when proving the proposition enunciated in step (1) by step (4). Wolff provides the following picture of a figure which corresponds to steps (2) and (3) combined.

4. *Proof*: Wolff uses the following string of syllogisms to demonstrate the proposition enunciated in step (1):
 - a. First syllogism
 - i. All alternate angles between parallel lines are equal [already known]
 - ii. Angles I and 1 are alternate angles [by experience of the figure exhibited in (3)]
 - iii. Therefore, angles I and 1 are equal
 - b. Second syllogism
 - i. All alternate angles between parallel lines are equal [already known]
 - ii. Angles II and 2 are alternate angles [by experience of the figure exhibited in (3)]
 - iii. Therefore, angles II and 2 are equal
 - c. Third syllogism
 - i. All angles standing on the same right line at the same point are equal to two right angles [already known]
 - ii. Angles I, 3 and II are standing on the same right line [DE] at point [C] [by experience of the figure exhibited in (3)]
 - iii. Therefore, angles I, 3 and II are equal to two right angles
 - d. Fourth syllogism
 - i. All equal angles may be substituted for one another [already known]

¹⁴ “In jedem Triangel ABC machen alle drey Winckel zusammen 180°” (DL, Ch. 4, § 23).

- ii. Angle 1 can be substituted for angle I, and 2 for II [by (di), (aiii), and (biii)]
 - iii. Therefore, angles 123 are equal to two right angles [by (ciii) and (dii)]¹⁵
5. *Conclusion:* So the proposition enunciated in (1) is true, Q. E. D.

The conclusion of the fourth syllogism (diii) is about the particular triangle exhibited in step (2), and prepared in step (3), and yet Wolff takes it to have proved the general proposition, enunciated in (1), that is, that in every triangle ABC, all three angles are equal to two right angles.¹⁶ Since Wolff understands the particular triangle as a perfect model which represents all triangles, any propositions about it will be the case for all triangles.¹⁷

15 “[...] alle Wechsels-Winckel an Parallel-Linien sind einander gleich. Die Figur giebet: I und 1 sind Wechsels-Winckel an Parallel-Linien AB und DE. Daher schliesset er die Winckel I und 1 sind einander gleich. [...] alle Wechsels-Winckel an Parallel-Linien sind einander gleich. Die Figur giebet auch hier: II und 2 sind Wechsels-Winckel an Parallel-Linien AB und DE. Daher schliesset er wieder: die Winckel II und 2 sind einander gleich. [...] alle Winckel, die an einem Punkte auf einer Linie stehen, machen 180°. Die Figure giebet: die Winckel I, 3, und II stehen an einem Punkte C auf Linie DE. Daher schliesset er: die Winckel I, 3, und II machen zusammen 180°. [...] gleiche Winckel können vor gleiche [...] gesetzt werden. Der Beweis giebet I und 1, ingleichen II und 2 sind gleiche Winckel. Daher schliesset er: der Winckel 1 kan vor I und 2 vor II gesetzt werden, [...] folgendes siehet er, daß auch I, 2 und 3 zusammen 180° machen” (DL, Ch. 4, § 23). The syllogisms in the main text are taken, almost verbatim, from the German text here cited. I indicate the source of each premise, in square brackets, according to Wolff’s descriptions and have organised each syllogism under structure headings, e.g., ‘a. First Syllogism’.

16 Wolff viewed the syllogisms comprising a geometric demonstration to be standard Aristotelian syllogisms (DL, Ch. 4, § 22). However, this is not the case. Aristotelian syllogisms only employ monadic predicates, that is, predicates which do not involve a relation, whereas the syllogisms of Euclidean proofs involve premises which contain polyadic predicates. Take, for example, premise (aii) above, “angles I and 1 are alternate angles”.

17 Wolff introduces many principles which are employed in the mathematical method. For example, the “art of invention” (*Erfindungs-Kunst*), which involves recalling propositions already known to be used in the demonstration and to decide how best to prepare the geometric figure (DL, Ch. 4, § 24); the “art of the combination of signs” (*Verbindungs-Kunst der Zeichen*) and the associated notion of “figurative knowledge” (*figürliche Erkenntniß*) – the art of the combination of signs is basically the connection of previously unconnected signs (which can be symbols or words, for Wolff (DM § 317)) by way of the middle term in a syllogism (DL, Ch. 4, § 22n) – this art makes use of figurative knowledge, which is the separation of the concept of a thing from its image such that its properties can be made explicit (DM § 324). Wolff also condones the use of “substitution”, as it is often used in algebra (DM § 324). Discussing all of these different aspects of the mathematical method is beyond the scope of this paper. For more on substitution in algebra, see Shabel (2003, 89). For more on Wolff’s art of the combination of signs, see Dunlop (2014, 663–666).

I now turn to Wolff's application of the mathematical method to physics, which can be found in the *Deutsche Logik*, Chapter 4, § 25. Since Wolff here too follows the Euclidean procedure closely, I will also present this example using Hintikka's five-step breakdown. Wolff begins by stating the general proposition to be proved, namely, "air has an elastic force" ("*daß die Luft eine ausdehnende Kraft habe*")¹⁸ (step 1). In order to prove this general proposition, Wolff takes a particular sample of air sealed in a bladder. On my view, this corresponds to the *ecthe-sis* step (2), which, in the geometrical demonstration considered above, involved appeal to a particular triangle. Next, Wolff describes a scientific experiment that one ought to conduct (and he himself in fact conducted) with this particular sample of air to conclude to its elastic force. He says that one ought to place it in a controlled environment where it is possible to remove all of the air surrounding the bladder. As soon as all the air surrounding the bladder is removed, Wolff reports that the air in the bladder expands. On my view, this nicely corresponds to the preparation stage (3). Just as in the geometrical proof points and lines are added to the particular figure exhibited in the previous step, so Wolff here 'adds' something to the sample of air serving as his particular example: he changes its environment. Now, Wolff thinks, the stage is set for the proof of the general proposition that air has elastic force, that is, for step (4), which, as Wolff explains, consists in the following string of syllogisms:

- a. First syllogism
 - i. That which begins to expand, on removing the resistance, has an expansive or elastic force [assumed, will be proved in the second syllogism]
 - ii. Air begins to expand, on removing the resistance [assumed, will be proved in the third syllogism]
 - iii. Therefore, air has an expansive force
- b. Second syllogism
 - i. That which exerts a continual endeavour or effort of expansion has an elastic force [already known, definition of elasticity]
 - ii. That which begins to expand, on removing the resistance, exerts a continual effort of expansion [assumed as principle (*Grundsatz*) "after the example of Euclid"]
 - iii. Therefore, whatever begins to expand, on removing the resistance, has an elastic force [proves 4ai]
- c. Third syllogism

¹⁸ DL, Ch. 4, § 25.

- i. Whatever expands in a bladder, upon removing the resistance, is itself expansive [assumed as principle (*Grundsatz*) “after the example of Euclid”]
- ii. The air expands the bladder, on removing the resistance [by experience of (3)]
- iii. Therefore, the air, on removing the resistance, is expansive [proves aii]¹⁹

As we can see, this chain of syllogisms is exactly like that of the example in mathematics. The general proposition enunciated in step (1) has now been proved in step (4) via syllogisms (4a), (4b), and (4c), with recourse to the individual sample of air exhibited in step (2) and further ‘prepared’ in a scientific experiment (involving a change of environment) in step (3). On this basis, Wolff takes himself to have proved the general proposition that air has an elastic force by means of the mathematical method. However, Wolff, in this proof, uses two important premises ‘after the example of Euclid’. These premises are: (bii) ‘that which begins to expand, on removing the resistance, exerts a continual effort of expansion,’ and (ci) ‘whatever expands in a bladder, upon removing the resistance, is itself expansive’. What does Wolff mean by this phrase?

On my interpretation, the propositions labelled ‘after the example of Euclid’, namely, (bii) and (ci), are ‘enthymemes’, which Wolff defines as “a syllogism,

19 “Was sich anfängt auszudehnen, wenn der Widerstand gehoben wird, das hat eine ausdehnende Kraft. Die Luft fängt sich an auszudehnen, wenn der Widerstand gehoben wird; also hat sie eine ausdehnende Kraft. Der Ober-Satz muß durch einen neuen Schluss erwiesen werden, dessen Mittel-Glied die Erklärung der ausdehnenden Kraft ist. [...] Was in steter Bemühung ist, sich auszudehnen, hat eine ausdehnende Kraft. Was sich anfängt auszudehnen, wenn der Widerstand gehoben wird, ist in steter Bemühung, sich auszudehnen; Also, was sich anfängt auszudehnen, wenn der Widerstand gehoben wird, hat eine ausdehnende Kraft. Der Ober-Satz ist eine Erklärung, und kan also nicht weiter erwiesen werden. Der Unter-Satz lässet sich zwar weiter beweisen: jedoch weil er aus der Erfahrung bekandt genug ist und der Beweiß einen zu lange aufhalten würde, kan man nach dem Exempel des Euclidis ihn ohne Beweiß als einen Grundsatz annehmen. Was eine Blase ausdehnet, wenn der Widerstand gehoben wird, das muß sich auch selber ausdehnen. Die Luft dehnet die Blase aus, wenn der Widerstand gehoben wird; Also muß sich die Luft selber ausdehnen, so bald der Widerstand gehoben wird. Der Ober-Satz kan, eben wie vorhin der Unter-Satz, als ein Grund-Satz ohne Beweiß nach dem Exempel Euclidis angenommen werden. Der Unter-Satz ist die Erfahrung. Und also haben wird alle Materie zu einem mathematischen Beweise” (DL, Ch. 4, § 25). The premises of the syllogisms are copied verbatim. I have added the justification of each premise, or what the premise proves, in square brackets, according to Wolff’s descriptions, as well as the structure headings, e.g., ‘a. First Syllogism’.

thus maimed of one of its premises”²⁰. However, in contrast to the Aristotelean enthymemes, which are only missing one of their premises, Wolff explicitly states that mathematicians use enthymemes maimed of both their premises in demonstrations.²¹ According to him, this is only the case where the syllogism is so obvious from the information given in the scientific experiment or in the exhibited geometrical figure that it can be left implicit. I interpret both premises ‘after the example of Euclid’ to be enthymemes missing both of their premises for two reasons. First, they are not self-evident; they are general propositions, and therefore, they cannot be warranted by experience alone. Hence, they require proofs. Second, Wolff does not explicitly include any such proofs in the above demonstration. Therefore, on my view, premises (bii) and (ci) are enthymemes missing both of their premises ‘after the example of Euclid’.²² I will take up why Wolff’s use of enthymemes is so important in section 5 when I discuss Kant’s philosophical account of the moves Wolff makes when he employs the mathematical method in philosophy.

3 Kant’s Account of Mathematical Cognition

Against Wolff’s view that the mathematical method can be used to achieve scientific certainty in philosophy, Kant argues for a strict separation between the methods proper to mathematical and philosophical cognition on the basis of his investigation of these two types of cognition. For Kant, mathematical cognition is

²⁰ “Wenn einer von den Förder-Sätzen so klar ist, daß ihn niemand in Zweifel ziehet; so pfleget man ihn wegzulassen, und *dann nennet man es einen verstümmelten Schluß*” (DL, Ch. 4, § 17).

²¹ DL, Ch. 4, § 21.

²² On my view, the conclusion of the enthymeme that Wolff employs in the example in question follows from what I call the ‘Generalisation Principle’, which Wolff explains as follows: any “natural thing under the same conditions always produces the same effect” (“[...] die natürlichen Dinge unter einerley Umständen immer einerley Würckung haben”) (KUn, § 35). On the basis of the Generalisation Principle, a proposition about an individual case can be generalised simply by including the condition under which the effect occurs in the general proposition (DL, Ch. 3, § 5). Take, for example, the singular proposition ‘this rock becomes wet in the rain’. According to the Generalisation Principle, this singular proposition can be turned into the following general and necessary rule: any rock that is sitting in the rain (barring all impediments) will become wet. On my interpretation, Wolff relies on the Generalisation Principle when he applies the mathematical method to natural philosophy. Indeed, Wolff recommends that the Generalisation Principle be used with the mathematical method (KUn, § 35).

“rational cognition from the construction of concepts”²³. There are two components to the explanation of this type of cognition.

The first is concept construction. To construct a concept, Kant explains, is to exhibit a concrete instance of it in *a priori* intuition, for example, by setting out the figure of a triangle. The exhibited triangle is an exact model of the pure geometrical concept because it embodies the characteristics contained in the concept perfectly, namely, being a space enclosed by three straight lines.²⁴

According to Kant, mathematical cognition is unique because its pure concepts are universals that can be thought in the particular, in the sense that the particular mathematical instance embodies all there is to know about the concept.²⁵ Since the exhibited figure is a perfect model of the concept, it is an individual that represents all objects of its kind.²⁶ Anything that is said of the exhib-

23 “[...] die mathematische [Erkenntniß ist] aus der Construction der Begriffe” (KrV, A 713/B 741).

24 Even though I side with Carson in her criticism of Hintikka, I also disagree with her account of the relation between pure mathematical concepts and *a priori* intuition. She claims that “the fundamental concepts of geometry are ‘derived from’ the ‘true nature of space’” (2009, 441). Thus, as I understand her view, the nature of space somehow determines the pure mathematical concepts. While I think that any real object of a pure mathematical concept must cohere with the nature of space and time, I contend that the forms contained in the pure mathematical concepts are in no way determined by the nature of space and time. On my view, although the real objects of pure mathematical concepts must cohere with the nature of space, they likewise determine that very nature. This can be seen in Kant’s discussion of generating the manifold of time by means of drawing a line (see KrV, A 162 f/B 203).

25 See KrV, A 714/B 742.

26 Against scholars like Shabel and Guyer, I argue that, for Kant, the exhibited figure can only be an individual that represents every instance of the pure concept because it is the schema of the pure concept: it determines the *a priori* forms of space and time such that any instances of the concept will necessarily be subsumed under it. Kant states that the exhibited figure is the schema of the pure concept through which images of the concept first become possible (see KrV, A 141 f/B 181). Therefore, it plays a central role in cognising *a posteriori* instances of the pure concept. Against Kant’s view, Shabel argues that mathematical schemata are superfluous for two reasons: (1) pure mathematical concepts are constructible, and thus they already contain the rules for representing their instances, and (2) pure mathematical concepts are homogeneous with both pure and empirical concepts, and therefore they are not in need of a ‘third thing’ to mediate between them and their empirical instances; Shabel, Lisa: “Kant’s Philosophy of Mathematics”. In: *The Cambridge Companion to Kant and Modern Philosophy*. Ed. by Paul Guyer. Cambridge 2006, 94–128, 111; see also Guyer, Paul: *Kant and the Claims of Knowledge*. Cambridge 1987, 159. However, I argue that this view does not take Kant’s general account of cognition into consideration. Without mathematical schemata, pure mathematical concepts would not be brought to the synthetic unity of apperception. Therefore, they would not form part of the fabric of nature. On my view, this is achieved by what Kant identifies as their schemata, namely, the constructed figure, and explains why anything that holds of the constructed figure holds of any instances appearing in space and time. My view is similar to that of, for example, von

ited figure must also be said of any other instances of the concept, including *a posteriori* instances. As we will see, Kant's view that the object of mathematical cognition is a particular that represents the universal lies at the heart of how Kant explains that the mathematical method generates knowledge in the strict sense.

The second component that Kant invokes in his explanation of mathematical cognition concerns the scope of this cognition. Kant states that it is *rational cognition from (aus)* such constructed concepts. On my interpretation, 'rational cognition' is to be understood in the general sense: it encompasses cognition generated by reason and the understanding. This interpretation is supported by the fact that Kant argues that only mathematical cognition involves definitions, axioms, and demonstrations in the strict sense.²⁷ As is well known, on the Kantian framework, the first two are produced by the faculty of understanding, and the latter is produced by reason.

To clarify the method proper to mathematical cognition, Kant provides an example. He shows how a mathematician would prove that the sum of the internal angles of a triangle is equal to 180 degrees. We will see in the next section that, for Kant, the mathematician's way of proving this proposition differs considerably from the way in which a philosopher would prove it.

According to Kant, the mathematician would immediately construct the pure concept of a triangle in *a priori* intuition.²⁸ By recalling pertinent principles, the mathematician could add lines or shapes to the triangle as needed in order to prove the proposition at hand.²⁹ For example, as we saw above, one could add a line parallel to the base such that the triangle is enclosed within parallel lines. The principle that internal angles between parallel lines are equal would then be at one's disposal in the demonstration, under which a universal premise about the constructed triangle could be subsumed. Finally, the mathematician is, on the basis of the information produced by the exhibited and prepared triangle, able to prove the general proposition in an apodictically certain scientific demonstration.³⁰

From this example, we see that Kant has a five-step method similar to Wolff's in mind, as laid out in the previous section, when he refers to the method proper

Wolff-Metternich; she argues that mathematical schemata are necessary, on Kant's view, in order to subsume the manifold of intuition under pure mathematical concepts; on this view, it would be impossible to cognise instances of, for example, triangles, in the world without mathematical schemata; see von Wolff-Metternich (1995, 48 f). However, as far as I know, she does not mention the synthetic unity of apperception in this context.

²⁷ See KrV, A 727/B 755–A 735/B 763.

²⁸ See KrV, A 716/B 744.

²⁹ See KrV, A 716/B 744.

³⁰ See KrV, A 717/B 745.

to mathematical cognition, with the exception that Kant radically breaks with the way in which a mathematical figure is exhibited and prepared in stages (2) and (3). Instead of simply claiming that the figure is drawn, he provides a philosophical account of how it is constructed in *a priori* intuition. This is the result of Kant's effort to philosophise about mathematics, a task that, as he explains, has hardly ever been undertaken.³¹

By adding a philosophical account of concept construction to mathematical cognition, Kant makes no small adjustment to the traditional mathematical method. With it, Kant draws a clear line between the methods proper to mathematical and philosophical cognition, as we will see. Since anything said of the constructed triangle must hold of every instance of a triangle, any conclusion derived from synthetic judgements about it is necessary and augments knowledge of the pure concept of a triangle. Accordingly, for Kant, the mathematical method obtains apodictically certain conclusions when used in mathematical cognition. As will be seen, on his view, philosophical cognition cannot construct its concepts. Therefore, it cannot employ the mathematical method with apodictic certainty.

4 Philosophical Cognition According to Kant

In contrast to mathematical cognition, Kant states that philosophical cognition is “rational cognition from concepts”³². In short, to Kant's mind, philosophical cognition does not involve construction. On his view, this brief explanation of philosophical cognition, as I will discuss, provides the confines within which any method used in philosophy must operate if it is to remain apodictically certain and yield *objectively real* knowledge.

In his account of philosophical cognition, the concepts that Kant first and foremost has in mind are the categories, such as the relational ones of substance and accident or of cause and effect. The result of the Transcendental Analytic is that each category contains a rule of synthesis that is applied in judgements of the understanding and that only has objective reality when used to synthesise given intuition. The *a priori* instances of the categories are time-determinations, that is, various ways of synthesising the manifold of intuition in a *possible* object of experience; they are not *real* objects constructed in intuition, as is the case with the concepts of mathematical cognition. Accordingly, for Kant, the concepts

³¹ See KrV, A 725/B 753.

³² KrV, A 713/B 741.

of philosophical cognition depend on *a posteriori* intuition for objective reality: they are discursive. They are not intuitive like the concepts of mathematical cognition; they cannot exhibit to themselves a real object in *a priori* intuition.

As was the case with mathematical cognition, for Kant, the scope of philosophical cognition is reason in the general sense. We see this when Kant considers whether the philosopher can also make use of the tools that the mathematician requires in order to employ the mathematical method with apodictic certainty. Recall that the mathematician's tools are definitions, axioms, and demonstrations in the strict sense, all of which, for Kant, are dependent upon concept construction. He finds that instead of having such tools at her disposal, the philosopher can only employ explanations, discursive principles, and discursive demonstrations, as I will shortly discuss, in philosophical cognition.³³ Explanations and principles, he thinks, are produced by the understanding, and demonstrations, by reason. Therefore, on my interpretation, 'philosophical cognition' must refer to reason in the general sense.

Kant illustrates the method proper to philosophy when he considers how a philosopher would proceed if asked to prove that the sum of the internal angles of a triangle is equal to 180 degrees, keeping in mind that, for the purpose of the example, she is restricted to philosophical cognition. Her mode of going about this differs considerably from the mathematician's method considered in the previous section. According to Kant, the philosopher can only investigate the particular triangle by means of its concept: she can only analyse its concept and the concepts of its marks.³⁴ The result is distinct concepts of, for example, a straight line and an angle.³⁵ The philosopher would not be able to construct the triangle, and therewith appeal to an instance of her object of investigation in *a priori* intuition, for philosophical cognition is unable to exhibit to itself a real object in *a priori* intuition, but rather only a representation of a rule of synthesis, which must still be applied to given intuition if it is to have objective reality. For this reason, she cannot generate new, apodictically certain knowledge in a scientific demonstration, as the mathematician can.

In this example, Kant claims that the philosopher is restricted to investigating both the concept in question and the concepts of its marks. As I understand Kant, she can proceed in two ways. First, she can analyse the concept under investiga-

³³ See KrV, A 727/B 755–A 735/B 763.

³⁴ See KrV, A 716/B 744. The philosopher could also draw the triangle. However, according to Kant, the process would then be an empirical one, and the philosopher could only measure the angles of the drawn triangle, arriving at results which only pertain to the individual triangle (KrV, A 718/B 746).

³⁵ See KrV, A 716/B 744.

tion, for example, the concept of a triangle, and in this way attempt to adduce all of its marks, such as ‘lines’, ‘angles’, etc. Second, she can connect marks of the concept’s marks with the concept in question in syllogisms of the first figure; in this way, she can bring marks implicitly contained in the concept under investigation to light.³⁶ Take, for example, the concept ‘bachelor’. We can add marks implicitly contained in it by analysing the concepts of its marks and connecting the results with it in a syllogism as follows:

All bachelors are unmarried men
 Unmarried men do not have legally recognised life partners
 Therefore, bachelors do not have legally recognised life partners³⁷

Employing a syllogism is not an instance of conceptual analysis, according to Kant, because conceptual analysis is only expressed in judgements. A syllogism, by contrast, consists of three judgements and is an instance of connecting a mark with a concept by means of a middle term.³⁸ This is, however, on Kant’s terms, an appropriate *a priori* and apodictically certain use of philosophical cognition because it remains on the level of concepts; it simply unpacks the marks already logically contained in the concept in question.

One immediately sees that this use of syllogising, which Kant deems appropriate to philosophical cognition, does not make reference to an individual in intuition: it skips the *ecthesis* step of the traditional mathematical method. What happens if the proof were to integrate *ecthesis* (step 2)?

We could, for example, subsume ‘Kant’ under ‘bachelor’ in the minor premise. In this case, the conclusion ‘therefore, Kant is an unmarried man’ follows necessarily from the premises. Although this example maintains the correct logical form of a *Darii* syllogism, of which Kant approves for philosophical cognition, and its conclusion is an instance of real knowledge gained completely *a priori*, it is merely a particular proposition. Furthermore, it only serves to make a mark explicit that is already logically contained in the concept ‘Kant’. The result is the

³⁶ See DfS, AA 02: 49. An example of a first figure syllogism accepted by Kant is Aristotle’s *Barbara* syllogism, for example, ‘all animals are mortal’, ‘all men are animals’, ‘therefore, all men are mortal’. See further Vanzo, Alberto: “Kant’s ‘False Subtlety of the Four Syllogistic Figures’ in its Intellectual Context”. In: *The Aftermath of Syllogism*. Ed. by Marco Sgarbi and Matteo Cosci. London 2018, 157–190, 175.

³⁷ For an example employing categories instead of words denoting empirical concepts, see section 5.

³⁸ The conclusion follows necessarily from the two premises by subsuming the condition of the minor premise under a given rule in the major premise; see further Log, AA 09: 120.

same if we consider a syllogism with a major premise that has been generated from the categories.³⁹

Thus, any time *ecthesis* is used in philosophical cognition, as is required in the traditional mathematical method, philosophical cognition can only make reference to a real object in a *posteriori* intuition, not in a *priori* intuition. Therefore, although the act of inference carried out by reason is *a priori*, the minor premise must be about a particular in experience, and any conclusion that follows must likewise be particular. Thus, it does not allow us to generate new, necessary and universal knowledge about a real object, as is the case with the mathematical method. What happens if one employs *ecthesis* in conjunction with the mathematical method in philosophy with a universal concept that is viewed as denoting a real object? I will provide the answer to this question in section 5.

Thus, the appropriate use of syllogistic reasoning within philosophical cognition does not follow the five-step mathematical method laid out in section 2. On my interpretation of Kant, the difference between the use of syllogistic reasoning in mathematical and philosophical cognition is that if it is to be about reality and it is to remain universal, then it skips the *ecthesis* and preparation stages (steps 2 and 3); however, if it employs the *ecthesis* stage, then knowledge is real, but only of a particular. In the next section, I will analyse the way in which Wolff uses the mathematical method in philosophy on the Kantian picture just laid out, focusing on the premises Wolff takes himself to have at his disposal. The result will be that, for Kant, Wolff employs ‘dogmata’ that involve ideas of reason in his example of the mathematical method at work in philosophy.

5 ‘Dogmata’ in Wolff

Unlike Kant, Wolff recommends the mathematical method to augment knowledge in philosophy.⁴⁰ As I have shown in section 2, Wolff thinks that conducting a scientific experiment on a thing in nature fulfills the same function as the *ecthesis* and preparation stages in a mathematical proof (steps 2 and 3). For Kant, by contrast, as I have argued in section 4, *ecthesis* can only be employed with recourse to a par-

³⁹ Consider the following example: ‘Everything that happens has a cause’, ‘the existence of this thing with its various marks is something that happens’, ‘therefore, the existence of this thing with its various marks has a cause’. The minor premise, and therewith the conclusion, is a particular proposition.

⁴⁰ Indeed, in the *Deutsche Logik*, against the trend of his contemporaries to view the syllogistic method as incapable of generating new knowledge, Wolff claims that discoveries are made in mathematics and all other sciences by the syllogistic method alone (Ch. 4, § 22).

ticular thing in intuition in philosophical cognition, from which only knowledge about the particular can follow. In this final section, I will discuss where Wolff goes wrong, on the Kantian framework. I will show that, on the Kantian account, if one thinks that one can use the mathematical method in philosophy with the same efficacy as in mathematics, as Wolff does, then any premise obtained with recourse to such a thing in nature is ‘a dogma’; it is a discursive proposition in philosophy that is mistakenly viewed as self-evident. As I will discuss, for Kant, this mistake involves denoting the thing in nature by an idea of reason.

Kant’s exposition of mathematical and philosophical cognition comprises what I have identified as the first task of the Discipline in the Introduction. In what I have called the second task of the Discipline, Kant’s goal is to preclude the use of definitions, axioms, and demonstrations, in the strict sense, from philosophical cognition.⁴¹ In so doing, Kant argues that these tools that thinkers like Wolff thought they had at their disposal cannot have the same apodictically certain status as they do in mathematics. I will focus on Kant’s arguments to exclude axioms, in the strict sense, from philosophical cognition.

A dogma, as I interpret Kant, is a proposition that is mistakenly taken for an axiom in philosophy. Accordingly, the task of excluding axioms from philosophy is also expressed in terms of an important aim of the Discipline: to show that the critical system of philosophy cannot contain a single dogma.⁴² In order to understand the magnitude of this claim, we must understand what Kant means by ‘dogma’:

I divide all apodictic propositions (whether they are demonstrable or immediately certain) into *dogmata* and *mathemata*. A direct synthetic proposition from concepts is a *dogma*; such a proposition through construction of concepts, on the contrary, is a *mathema*.⁴³

⁴¹ See KrV, A 726 f/B 755 f.

⁴² See KrV, A 736/B 764. Kant equates metaphysics with the speculative cognition of reason (KrV, B XIV).

⁴³ “Ich theile alle apodiktische Sätze (sie mögen nun erweislich oder auch unmittelbar gewiß sein) in Dogmata und Mathemata ein. Ein direct synthetischer Satz aus Begriffen ist ein Dogma; hingegen ein dergleichen Satz durch Construction der Begriffe ist ein Mathema” KrV, A 736/B 764. Kant repeats these explanations in the *Logik Hechsel*: “All apodictische Sätze sind 1) Entweder Mathemata, und gehören zur Nothwendig Mathematic 2) Oder dogmata und gehören zur philosophie. Alle Vernunft Erkenntnisse aus Begriffen sind dogmata, alle Vernunft Erkenntnisse aber aus der construction der Begriffe sind Mathematha. In Methmatic giebt es keine dogmata, aber wohl apedictische Sätze: dogmata aber haben wir in Philosophischen Dingen. Da nun die Dogmata aus Begriffen, so ist ein wesentlicher Unterschied darin. Der Autor erklärt in 104 § ein system, und versteht dadurch den Inbegrif dogmatischer Wahrheiten” (Kant, Immanuel: “Logik Hechsel”. In: *Logik-Vorlesung Unveröffentlichte Nachschriften II*. Ed. by Tillman Pinder. Hamburg 1998, 40).

The distinction between dogmata and mathemata is a reformulation of Kant's definitions of philosophical and mathematical cognition. While a mathema is a proposition brought forth by mathematical cognition, a dogma is one brought forth by philosophical cognition.

A mathema is produced when the understanding forms a synthetic *a priori* judgement on the basis of a constructed figure. These judgements are better known as axioms because they are self-evident, necessary, and intuitive.⁴⁴ Necessary and intuitive propositions can also be proved, on Kant's view, but then they must follow from self-evident axioms. Kant gives the following proposition as an example of an axiom: 'three non-linear points always lie on a plane'.⁴⁵ The mathematician can construct three non-linear points and immediately see that they are strewn about on a plane. Axioms are, thus, self-evident because one can see that the judgement is true by simply looking at the exhibited figure. They are necessary because they are synthetic *a priori* judgements. Finally, they are intuitive because the exhibited figure, upon which the judgement is based, is a perfect model of the pure concept in intuition. These characteristics allow for mathemata to stand at the head of apodictically certain demonstrations.

Kant defines 'dogmata' as *necessary* and *direct* synthetic *a priori* propositions. However, instead of being formed on the basis of constructed concepts, they are produced by philosophical cognition and are comprised of concepts that Kant has shown can only be employed discursively. There is, thus, a problem with the definition of dogmata; for Kant, a discursive principle can be necessary, but not self-evident:

A synthetic principle, on the contrary, e.g., the proposition that everything that happens has its cause, can never be immediately certain from mere concepts, because I must always look around for some third thing, namely the condition of time-determination in an experience, and could never directly cognize such a principle immediately from concepts alone. Discursive principles are therefore something entirely different from intuitive ones, i.e., axioms.⁴⁶

⁴⁴ See KrV, A 732/B 760.

⁴⁵ See KrV, A 732/B 761.

⁴⁶ "Dagegen kann ein synthetischer Grundsatz bloß aus Begriffen niemals unmittelbar gewiß sein, z.B. der Satz: alles, was geschieht, hat seine Ursache; da ich mich nach einem Dritten herumsehen muß, nämlich der Bedingung der Zeitbestimmung in einer Erfahrung, und nicht direct, unmittelbar aus den Begriffen allein, einen solchen Grundsatz erkennen konnte. Discursive Grundsätze sind also ganz etwas anderes als intuitive, d.i. Axiomen [...]" (KrV, A 732f/B 761).

Kant contends that principles in philosophical cognition can be produced from the categories, but they are not self-evident. Take his example: ‘everything that happens has a cause’. We cannot simply construct the concept of ‘everything that happens’ in *a priori* intuition in order to connect to it the mark ‘has its cause’ in an immediate synthetic *a priori* judgement. Thus, the necessity of such a proposition cannot be self-evident, for Kant, but rather must be the conclusion of a deduction.⁴⁷

Yet many pre-critical philosophers like Wolff seem to have an endless supply of necessary and self-evident propositions in philosophy. Where does Wolff go wrong, according to Kant, when he views a proposition in philosophical cognition as self-evident? As I will now argue, Kant’s answer is that he, and philosophers like him, employ ideas of reason.

Recall Wolff’s above-discussed example of how the mathematical method applies to philosophy, in which he claims that some premises are obtained by following ‘the example of Euclid’. Take the major premise of the third syllogism: “Whatever expands in a bladder, upon removing the resistance, is itself expansive”.⁴⁸ Recall that, on my interpretation of Wolff, ‘the example of Euclid’ means that this principle is the conclusion of a hidden syllogism, or more precisely, an ‘enthymeme’, that is missing both of its premises. Therefore, it is necessary because it is proved on the basis of a syllogism of reason, which is always *a priori*. However, for him, the principle is sufficiently self-evident because we already know how it works on the basis of the scientific experiment; thus, its premises can be left out.

From the *Jäsche Logic* and the *False Subtlety of the Four Syllogistic Figures*, we know that there is a problem with the logical form of a syllogism if one of its premises is the conclusion of a hidden syllogism: there is no middle term, properly speaking.⁴⁹ Thus, it is an invalid syllogism. We now have a logical account of where, on Kant’s view, Wolff errs: he recommends the use of premises that are grounded on hidden syllogisms in scientific demonstrations, which for Kant, renders the syllogisms invalid.

However, what is Kant’s assessment of where he thinks Wolff errs in terms of transcendental logic? On the Kantian account, Wolff’s major premise of the third syllogism is a proposition in philosophical cognition. Therefore, on his view, it can be synthetic *a priori*, but not *self-evident*. How then might Kant account for Wolff’s view that it is like a mathematical axiom? As I see it, Kant would char-

⁴⁷ See KrV, A 733/B 761.

⁴⁸ DL, Ch. 4, § 25.

⁴⁹ See Log, AA 09: 124 f; see DfS, AA 02: 55.

acterise it as a dogma that employs ideas of reason.⁵⁰ An idea of reason has a unity and extension beyond the bounds of possible experience and, accordingly, cannot possibly correspond to an object given in experience.⁵¹ It is a “*focus imaginarius*”, as Kant puts it, that is, a concept that has been inferred by reason in order to organise knowledge, but through which the understanding does not really constitute objects when determining cognition.⁵² Upon first glance, it may seem that there are glaring issues with characterising Wolff’s premise, which he takes to be analogous to a mathematical axiom, as what Kant would call a dogma that employs ideas of reason. I will attempt to address these issues in what follows.

By means of an enthymeme that has recourse to a *particular* sample of air, Wolff arrives at the conclusion “Whatever expands in a bladder, upon removing the resistance, is itself expansive”. In this way, he introduces a *general* bearer of an expansive force into the demonstration. He postulates a new subject-term, namely, ‘whatever expands in a bladder, upon removing the resistance’. To this concept, by way of the hidden syllogism, he adds the mark ‘expansive’, which should be understood as being the cause of the effected expansion. On his view, the conclusion of the hidden syllogism explicitly contains the condition under which any sample of air would expand, namely, ‘upon removing the resistance’.⁵³ Therefore, the conclusion of this enthymeme is a universal proposition in which the effect of something happening in nature is connected with its cause.

To my mind, for Kant, Wolff’s general bearer of an expansive force must be classified as an idea of reason. More specifically, as Kant discusses in the first section of the ‘Appendix to the Transcendental Dialectic’, it is an idea that is postulated to guide an investigation of nature. Ideas that guide one’s investigation of nature are, for Kant, ideal versions of composites found in nature. He gives the examples of ‘pure air’, ‘pure earth’, and ‘pure water’.⁵⁴ In one’s investigation,

⁵⁰ Kant clearly explains that dogmata are made up of ideas of reason, for the first sentence of the following passage states that the system of philosophical knowledge does not contain a dogma, keeping in mind that Kant equates metaphysics with the speculative cognition of reason (KrV, B XIV), and the second sentence provides the reason why, namely, because this system cannot contain ideas of reason if it is to be necessary and have objective validity: “now all of pure reason in its merely speculative use contains not a single direct synthetic judgment from concepts. For through ideas, as we have shown, it is not capable of any synthetic judgments that would have objective validity” (“Nun enthält die ganze reine Vernunft in ihrem bloß speculativen Gebrauche nicht ein einziges direct synthetisches Urtheil aus Begriffen. Denn durch Ideen ist sie, wie wir gezeigt haben, gar keiner synthetischer Urtheile, die objective Gültigkeit hätten, fähig [...]” (KrV, A 736/B 764).

⁵¹ See KrV, A 327/B 383.

⁵² See KrV, A 644/B 672.

⁵³ See note 22.

⁵⁴ KrV, A 645 f/B 673 f.

impure versions of these composites in nature are compared to them to see in what way the particulars instantiate them.⁵⁵

For Kant, ideas of reason guiding investigations of nature serve the more general idea of systematic unity; it is a necessary *logical principle of reason* to organise empirical cognition that it has received from the understanding into a systematic whole.⁵⁶ This principle is necessary because it serves the transcendental idea of a highest reason that authors teleology in nature.⁵⁷ However, it is merely logical because it does not determine empirical objects such that they relate to one another according to systematic unity.⁵⁸ This general rule of systematic unity, for Kant, is fulfilled by the necessary logical laws of reason that order the manifold of empirical concepts into a united system of genera and species, otherwise known as a Porphyrian Tree.⁵⁹

On my interpretation of Kant, the concepts that guide scientific investigations, like ‘pure air’, or in Wolff’s case, ‘the general bearer of an expansive force’, are ideas of reason that are produced by these necessary logical laws of reason; more specifically, they are species.⁶⁰ Since, however, on Kant’s view, genera and species are only postulated according to logical laws of reason, these ideas can only be considered *as if* they were the grounds of the connection of things in nature; but, for Kant, they are merely regulative, not constitutive of systematic unity of the manifold of empirical concepts.⁶¹ However, when Wolff employs the notion of a general bearer of expansive form in the mathematical method, he views it as constitutive of nature, as is clear from his claim that the demonstration that employs it tells us something necessary and real about the essence of air.

Calling ‘pure air’, ‘pure water’, and Wolff’s concept of a general bearer of an expansive force ‘ideas of reason’, and any other concepts used in the mathematical method as Wolff suggests, fits with my initial characterisation of ideas as having an extension beyond the bounds of possible experience. However, it is incompatible with the criterion of being an ‘absolute’, or the highest condition of anything that is conditioned within a branch of philosophy, which seems to be the general interpretation of ‘unconditioned’ and seems to be commonly attrib-

⁵⁵ See KrV, A 645 f/B 673 f.

⁵⁶ See KrV, A 645/B 673.

⁵⁷ See KrV, A 686/B 714. See further Horstmann, Rolf-Peter: “Der Anhang zur transzendentalen Dialektik”. In: *Kritik der reinen Vernunft*. Ed. by Georg Mohr and Marcus Willaschek. Berlin 1998, 525–545, 537.

⁵⁸ See KrV, A 643 f/B 671 f.

⁵⁹ See KrV, A 657 f/B 685 f. See further Horstmann (1998, 532 f).

⁶⁰ An implication of my view is that all natural kinds are, for Kant, ideas of reason.

⁶¹ See Horstmann (1998, 536).

uted to ideas of reason in general.⁶² For example, according to Kant, the object of rational theology, namely, ‘God’, is the highest condition of all conditioned beings in the world, and is thus absolutely unconditioned. Yet Kant himself calls ‘pure air’, ‘pure water’, and ‘pure earth’ ideas. They are not *absolutely* unconditioned, because they can be subsumed under a higher condition like ‘composites in nature’. Thus, ‘absolutely unconditioned’ cannot be a criterion of all ideas of reason, but only of the transcendental ideas of reason: God, world, and soul.

On my view, the difference lies in Kant’s view that the ideas of reason that guide scientific investigation of nature, like ‘pure air’ and Wolff’s ‘general bearer of an expansive force’, are not the products of *transcendental* principles of reason, but merely of necessary *logical* principles of reason. While the products of transcendental principles are not revisable on the basis of experience, the latter are.⁶³ Accordingly, products of logical principles of reason merely fulfill the more general criterion for being an idea of reason, that is, of having an extension beyond the bounds of possible experience.⁶⁴

Even if one accepts this explanation, one might still object that ‘pure air’, ‘pure water’, and Wolff’s general bearer of a force cannot be ideas of reason because they are in some sense generated on the basis of experience. As Kant explains it, all matter is reduced to (bringen auf) salts, water, air, etc.⁶⁵ Yet, Kant insists that ‘pure water’ is ‘pure’ and that it is an idea.⁶⁶ On my view, this can only be explained by the speculative nature of such ideas of reason as ‘pure water’:

⁶² See for example KrV, A 322/B 379. See further, for example, Bickmann, Claudia: “Zur systematischen Funktion der Kantischen Ideenlehre”. In: *Perpektiven der Transzendentalphilosophie im Anschluß an die Philosophie Kants*. Ed. by Reinhard Hiltcher and André Georgi. Freiburg 2002, 55 ff; Grier, Michelle: *Kant’s Doctrine of Transcendental Illusion*. Cambridge 2004, 263 f.; Klimmek, Nikolai: *Kants System der transzendentalen Ideen*. Berlin 2005, 22. It seems to me that ‘unconditioned’, has at least three meanings: (i) a series in its totality (KrV, A 417/B 445), (ii) a maximum, that is, the highest of any given chain of conditioned-condition (V-Met/Mron, AA 29: 834; KrV, A 326 f./B 383 f.), and (iii) an absolute, e. g., the transcendental ideas of God, world, and soul (KrV, A 417/B 445).

⁶³ See Zuckert, Rachel: “Kantian Ideas of Reason and Empirical Scientific Investigation”. In: *Kant and the Laws of Nature*. Ed. by Michela Massimi and Angela Breitenbach. Cambridge 2017, 89–107, 101.

⁶⁴ Examples of scholars who only list ‘being a concept from categories that has an extension beyond the bounds of possible experience’ as the criterion of ideas of reason in general are Karasek, Jindrich: “Idee”. In: *Kant-Lexikon*. Ed. by Markus Willaschek, Jürgen Stolzenberg, Georg Mohr and Stefano Bacin. Berlin 2015, 1114 ff; and Horstmann (1998, 535).

⁶⁵ KrV, A 646/B 674.

⁶⁶ See KrV, A 646/B 674. This explanation seems to contradict certain claims Kant makes about ‘pure’ meaning absolutely no admixture from experience, for example, “I call all representations pure (in the transcendental sense) in which nothing is to be encountered that belongs to sen-

If one infers from the existence of things in the world to their cause, this does not belong to the natural but to the speculative use of reason [...].⁶⁷

Since the speculative use of reason infers from the existence of things in the world to their super-sensible causes, for example, their essences (which for Wolff is a type of cause), there is no corresponding object to these concepts in experience.⁶⁸ If we look at Wolff's general bearer of an expansive force, reason has inferred this idea as the cause or ground of air inflating the bladder. Thus, Wolff has used the category of causality in isolation from any possible experience; he has used it to generate an idea to which no possible object of experience can correspond. Thus, although the predicates of the idea may have been in some sense gained from experience, postulating the idea as the *cause* of objects in experience detaches the idea from any original connection to intuitions. In this sense, the idea is 'pure'.⁶⁹

On my interpretation, the same type of inference on the basis of a category like causality is employed whenever reason produces a dogma. The major premise of Wolff's third syllogism again provides a helpful illustration. The subject of 'whatever expands in a bladder, upon removing the resistance, is expansive' is inferred as the cause of the instance of expanding air in Wolff's scientific experiment. In this way, Wolff's premise is "from concepts", that is, inferred on the basis of the categories, as Kant states is the case for all dogma.

On my interpretation of the Kantian distinction between mathematical and philosophical cognition, when Wolff uses the mathematical method in philosophy, he postulates an idea of reason by inferring the cause of an event in nature and uses it in a dogma, from which he takes apodictically certain conclusions

sation" ("Ich nenne alle Vorstellungen rein (im transscendentalen Verstande), in denen nichts, was zur Empfindung gehört, angetroffen wird"[...] (KrV, A 20/B 34)). However, Kant's claim that ideas like 'pure water' are indeed pure force us to reconsider its meaning and instead consider it in light of the speculative use of reason.

67 "Wenn man nun vom Dasein der Dinge in der Welt auf ihre Ursache schließt, so gehört dieses nicht zum natürlichen, sondern zum speculativen Vernunftgebrauch [...]" (KrV, A 635/B 663).

68 See KrV, A 634/B 664.

69 Alfredo Ferrarin and Rachel Zuckert likewise hold that, for Kant, the logical principles of reason organise nature into Porphyrian Trees comprised of ideas of reason; see Ferrarin, Alfredo: *The Powers of Pure Reason*. Chicago 2015, 52ff; Zuckert (2017, 101). However, Zuckert claims that there is a difference in the degree of purity between species of natural kinds like 'jade' and pure composites like 'pure water' (2017, 101f.). As I understand her claim, while the former are revisable on the basis of experience, the latter are not. Against this view, I argue that, for Kant, we must understand both as products of the same necessary logical laws of reason to organise empirical concepts into a system of interconnecting genera and species; both are inferred on the basis of the same function of reason.

to follow. Since Wolff takes the Porphyrian Tree produced by such scientific demonstrations to represent things in nature as they are in their essences, on the Kantian account, he employs dogmata, and therewith hypostasised ideas of reason, to transgress the bounds of reason, even when reasoning about nature.

6 Conclusion

I have argued that Wolff's application of the mathematical method to natural philosophy provides an example of what Kant describes as a philosopher hoping to expand her base of knowledge by employing a method which has proved itself effective in its own domain. Kant, by contrast, on the basis of his distinction between mathematical and philosophical cognition, argues that the method of mathematics cannot be employed in philosophy, as Wolff uses it, with apodictic certainty. The result of my investigation is: if the mathematical method is employed in philosophy, as Wolff recommends, then on Kant's view, it must involve ideas of reason to generate premises that are used in what are viewed as scientifically certain demonstrations. I have argued that these premises, which Kant calls 'dogmata', are used to reason beyond the bounds of experience, even when doing natural philosophy, by assuming that they denote causes of things in the world. Thus, on my view, Kant's criticisms in the Discipline extend the claims about fallacies in reasoning that he makes in the Transcendental Dialectic to any erroneous use of the mathematical method in philosophy.⁷⁰

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