Numerical Modeling of a Church Nave Wall Subjected to Differential Settlements: Soil-Structure Interaction, Time-Dependence and Sensitivity Analysis

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Abstract

Historic masonry structures are particularly sensitive to differential soil settlements. These settlements may be caused by deformable soil, shallow or inadequate foundation, structural additions in the building and changes in the underground water table due to the large-scale land use change in urban areas.

This paper deals with the numerical modeling of a church nave wall subjected to differential settlement caused by a combination of the above factors. The building in question, the church of Saint Jacob in Leuven, has suffered extensive damage caused by centuries-long settlement. A numerical simulation campaign is carried out in order to reproduce and interpret the cracking damage observed in the building.

The numerical analyses are based on material and soil property determination, the monitoring of settlement in the church over an extended period of time and soil-structure interaction. A sensitivity study is carried out, focused on the effect of material parameters on the response in terms of settlement magnitude and crack width and extent. Soil consolidation over time is considered through an analytical approach. The numerical results are compared with the in-situ observed pathology and with an analytical damage prediction model.

Keywords

Masonry; soil-structure interaction; historic structures; finite element modeling; settlement-induced damage

39 **Highlights**

- 40 • A masonry church nave wall subjected to differential settlements is numerically modeled, considering
41 time-dependent material properties and changes in geometry and loading
- 42 • The foundation and the soil properties are considered
- 43 • A sensitivity analysis highlights the parameters affecting the cracking pattern and extent
- 44 • The phased analysis results in a much more accurate representation of the observed damage compared
45 to a single-phase model

46 **Notation**

47	E	Young's modulus
48	G	shear modulus
49	ν	Poisson's ratio
50	ρ	mass density
51	σ_t	tensile stress
52	ε_{cr}	crack strain
53	ε_u	ultimate strain
54	f_c	compressive strength
55	f_t	tensile strength
56	G_f	tensile fracture energy
57	h	characteristic finite element length
58	L^j	footing half-length

59	B^j	footing half-width
60	D^j	footing embedment depth
61	A_f^j	footing area
62	A_w^j	footing side-wall contact area
63	K_n^j	footing vertical elastic stiffness
64	k_n^j	footing modulus of subgrade reaction
65	k_n	wall/colonnade modulus of subgrade reaction

66 **1. Introduction**

67 **1.1 State of the Art**

68 The analysis of large monumental structures subjected to differential ground movement is a challenging
69 subject of study. The challenge mainly arises from geometric complexity and sheer size, problems only partially
70 alleviated by a detailed geometric survey, material property determination and the definition of the applied
71 deformation load profile through concerted monitoring efforts [1]. It is, however, a worthwhile endeavor in service
72 of estimating the risk of damage or collapse and designing effective intervention strategies for repair and
73 strengthening.

74 The discretization of monumental church structures in macro-elements with different stiffness is often
75 considered conceptually and empirically valid. Macro-elements, such as façades, towers, apses and single naves,
76 are often separated through insufficient tying and the presence of structural cracking. They are further
77 characterized by different stiffness, weight and internal stresses. Namely, the effects of earthquake action and
78 differential settlement loading affect different parts of the structure in distinct ways. Therefore, the problem of
79 structural analysis of large churches can be simplified by pursuing it on an individual macro-element basis [2]. It

80 is both practical and valid to study certain components of monumental church structures, such as single bays,
81 naves, façades or towers individually.

82 In a nonlinear finite element (FE) analysis framework, the large dimensions of monumental church structures
83 built in masonry can render detailed modeling computationally prohibitive and model preparation effort excessive.
84 Macro-modeling, therefore, which consists in the homogenous modeling of the masonry composite, becomes a
85 suitable alternative. Despite the assumption of homogeneity, nonlinear macro-modeling of large masonry
86 structures can provide insight into the mechanisms through which damage arises and expands [3,4].

87 Predictive models for damage estimation and categorization in masonry structures subjected to differential
88 ground movement have been proposed in the literature [5]. These models rely on the determination of parameters
89 related to the material, geometric and foundation properties of the structure and are mostly used to evaluate the
90 effects of tunneling-induced settlements. The analytical and parametric basis of these models allows their adoption
91 and modification according to the requirements of a variety of loading scenarios and foundation types. This basis
92 further allows the parallel application with, and direct comparison to, numerical modeling of damage induced by
93 differential ground settlement.

94 In this paper both numerical and analytical models are applied for the analysis of a large monumental structure.
95 The church of Saint Jacob in Leuven has been the subject of wide and inclusive study over the preceding decades
96 [6–8]. It is characterized by extensive and developing pathology due to differential ground movement. This
97 pathology is well documented and extensive data is available on the profile of ground movement over different
98 periods. Finally, studies have been performed on its material properties, the stress state of its structural elements
99 and the properties of the foundation soil. It is therefore a prime candidate for investigation through analytical
100 simulation and numerical analysis for the purpose of interpreting the damages present in the fabric and the
101 development of ground movement.

102 **1.2 Objectives**

103 The focus of the present paper is the numerical reproduction of the damage patterns observed through time in
104 the church of Saint Jacob in Leuven, the investigation of their underlying cause, namely the differential settlements
105 at the site, and the study of the effect of material and numerical analysis parameters in the obtained results. This
106 is accomplished through a sensitivity study involving the material properties of the masonry structure, the
107 application of different loading patterns in the form of settlement profiles and the variation of the boundary
108 conditions as affected by the passage of time.

109 The interpretation of the results of the numerical analyses, coupled with the assembly and evaluation of historic
110 data, on-site observations and structural monitoring aim at providing insight into the manifestation and
111 development of structural pathology in the monument. A quantitative assessment of the development in time of
112 soil consolidation under the effect of gravity loads is given, thus outlining the behavior of the monument over an
113 extended period.

114 Further interpretation and quantification of the numerical results is provided through their comparison with
115 simplified analytical damage assessment models. This comparison allows the evaluation of the applicability of
116 analytical models in monumental masonry structures and demonstrates their potential for the interpretation of
117 current and the prediction of future structural damage.

118 **2. The Case Study**

119 **2.1 Layout and Brief Historic Outline**

120 Details on the history of the construction of the church may be found in [6], with the main points repeated here
121 for clarity. Construction of St. Jacob's church began around 1220, with the erection of the tower over the remnants
122 of an existing Romanesque church. The initial plan called for the church to have a flat timber ceiling, which was
123 later substituted for a timber barrel vault. The main nave, at its originally intended height, was completed in the
124 14th century, along with the side naves and their stone vaults, and the bell tower over the crossing was added in
125 the 15th. During the period 1534-1535, an additional level over the main nave was added and masonry vaults were

126 added in place of the timber vault, which was complemented by the addition of two, possibly four, pairs of flying
127 buttresses. These alterations resulted in the addition of self-weight not originally anticipated in the construction of
128 the foundations.

129 First mention of structural problems stemming from differential settlement dates back to at least the 15th
130 century. These problems led to the reconstruction of the side nave vaults. The timber bell tower over the crossing
131 was dismantled in 1735 due to concerns over its decay. The development of vertical cracking in the pillars led to
132 the installment of confining steel rings in the early 19th century, still present. Further consolidation measures were
133 taken in the early 20th century due to severe cracking in the west wall of the northern transept. In 1963, the entire
134 church was definitively closed for the public. During the partially executed consolidation works of 1965-1971, the
135 structure was internally shored using massive reinforced concrete elements and steel profile braces. While only
136 foreseen as a temporary measure, the shoring members are still present today. Additionally, the side nave vaults
137 were dismantled for weight reduction. In 2000 the flying buttresses were removed due to their being severely out-
138 of-plumb, which raised concerns of sudden collapse, and were replaced by temporary steel tie-rods. A
139 comprehensive structural intervention project, including localized repairs on the masonry a micro-piling
140 reinforcement of the foundations and reconstruction of the dismantled elements (side nave vaults and flying
141 buttresses) and soil consolidation was launched in September 2018 [9].

142 A floor plan of the church, along with the designation of the construction phases, can be seen in Figure 1. The
143 construction process, beginning with the tower and following with the arcade, naves, transept, chapels and finally
144 the choir are indicated.

Figure 1 Church floor plan and construction phases. Adapted from [6].

2.2 Damage Survey

The differential settlements in the church, which are the cause of the clear majority of structural damage, are caused by the building being erected in a swamp area near the river Voer, coupled with the initially unplanned addition of a second and third level, resulting in a severe increase in self-weight. The substitution of the original flat timber ceiling with a wooden barrel vault and, later, a masonry vault resulted in further increase in the self-weight.

The present study focuses on the damage documented in the northern wall of the main nave. An elevation view, the main structural elements and damage, along with the notation used for their designation, is shown in Figure 2. The nave wall measures approximately 26 m in length and 21 m in height. The pathology of the nave, consisting primarily of cracks caused by differential settlement, has been documented both with hand drawings and photographically during site visits, and more recently using semi-automated point-cloud data processing [10].

Tower Pillar 1 Pillar 2 Pillar 3 Pillar 4 Crossing

Figure 2 Northern wall of main nave: structural element designation and documented major cracks indicated in red (adapted from a hand-drawn survey of the building's geometry and pathology carried out by students of the Raymond Lemaire International Centre for Conservation, 1983-1984 [11]).

The arithmetically designated cracks 2 through 4 of the nave have been photographically documented and are shown in Figure 3. Continuous visibility of cracks 1 through 4 is not possible due to the obstruction caused by the organ loft near the western tower. Despite the time passed between the survey in 1983-1984 (Figure 2) and the photographic survey in 2018 (Figure 3), there does not appear to be any lengthening of the major cracks. The same cannot be said with certainty about the crack widths, however, since these were not measured in the prior case.

Figure 3 Photographic survey of northern nave wall cracks. View of cracks above nave pillars (photo by Pepijn Szekér, 2018).

The widths of the cracks above the nave pillars have been measured by hand. While the external plaster presents a crack width of a few mm, the crack width on the masonry behind the plaster is roughly between 10mm and 20mm (i.e. eroded mortar joints), Figure 4. The cracks appear to mostly pass through the mortar joints rather than splitting the masonry stone units in the investigated area.

Figure 4 Close-up of cracks 4 on main nave wall (photo by Els Verstrynge, 2018).

2.3 History of Settlements: Estimation and Geomatic Data Processing

An estimation of the soil settlements for different parts of the structure, under different calculated loads, has been carried out in the extensive studies of the church. These results are presented in internal technical reports,

179 property of KU Leuven [11]. These settlements are presented for the tower, crossing and pillars in Figure 5 and
 180 have been calculated based on cone penetration tests and evaluation of the soil consolidation progress according
 181 to Terzaghi, Buisman and Koppejan [12]. The results are not differentiated between individual pillars, thus
 182 rendering the calculation of the differential settlement between pillars impossible. Since the tower was completed
 183 before the beginning of the construction of the nave, an additional graph of the development of the tower settlement
 184 is provided. This graph ignores the settlements occurred before the completion of the nave wall (see grey line in
 185 Figure 5).

186

187 **Figure 5 Calculated settlements for different structural components of the tower and nave from 1220**
 188 **to 1970 (adapted from [11]).**

189 Concerning more recent ground movement and the resulting differential settlement of the pillars, levelling
 190 surveys conducted over the previous two decades provide info on the development of differential settlements over
 191 selected periods of measurements. Contour plots of the settlements over the period 1994-2005 are presented in
 192 Figure 6. These geodetic survey results are presented in terms of settlement relative to a point in the choir which
 193 is considered, due to the absence of apparent damage, stable. The maximum settlement over this period was
 194 measured at the area around pillars 1 and 2 of the northern nave, with areas at the southern nave and northern
 195 transept presenting some uplifting. The settlement at pillars 1 and 2 is consistent with cracks 1, 2 and 3 as indicated

196 in Figure 2, although the time of the first appearance of these cracks is not known with certainty. It is interesting
 197 to note that all points along the nave exhibited a rather uniform settlement in the period 1994-2000, while in the
 198 period 2000-2005 the settlement of pillar 2 appeared to increase at a faster rate than the other points in the nave.

199
 200 **Figure 6 Contour plot of ground settlement in mm during the period 1994-2005 against church**
 201 **architectural outline. Location of northern nave from tower pilaster to crossing indicated by solid hatch**
 202 **circles.**

203 An additional third approach may be adopted for the evaluation of the differential settlements over the entire
 204 history of the church. This is accomplished through the study of the geometric disposition of characteristic
 205 architectural features present on the structural elements. This approach is based on detailed laser scanning data
 206 acquired in the context of this investigation [10]. The downside of this rough approach is the inability to evaluate
 207 the total settlement that each element has undergone: only the final differential settlement can be estimated. Five
 208 reference features are chosen for this approach, from top to bottom: 1) the top of the third level arches' voussoirs,
 209 2) the base of the pilaster abacus of the third level, 3) the top of the first level arches' voussoirs, 4) the base of the
 210 pillar abacus of the first level and 5) the top of the first level pillar pedestals. Assuming that features 1-2, and
 211 similarly 3-4-5, were built at the same period, it follows that they were, in all probability, vertically level at the
 212 time of construction.

213 The measured vertical displacements relative to pillar 1 (lines 2, 4-5) or arch 2 (lines 1, 3) are shown in Figure
 214 7. All architectural features are present in pillar 1 and its neighboring arch span, hence the choice of these elements
 215 as a reference, instead of, for example, the crossing column. The obtained profile is similar to the profile found in
 216 the recent leveling measurements, with the settlement being mostly concentrated around pillars 1 and 2. This is
 217 consistent with the formation of cracks 2 and 3 (Figure 2). Differences in the profile at different heights of the
 218 structure are expected for two reasons: a) differential settlement generally affects the lower parts of the building
 219 more severely and b) the second and third levels of the nave wall were constructed at a later phase, when part of
 220 the settlements of the colonnade had already occurred.

222 **Figure 7 Differential settlements relative to pillar 1 (position: 6.99 m) measured from point cloud**
 223 **data architectural feature analysis. Position distance measured from tower pilaster.**

224 3. Analysis Procedure

225 3.1 Modeling the Nave Wall

226 The geometry of the nave is derived from an idealization of the in-situ geometry in its undeformed state. A
 227 distinction is made between the three-leaf masonry of the nave wall and the solid stone masonry of the lower part
 228 of the pillars, each with its own set of material properties. The basic values of the material properties used for the

numerical analyses, in part determined in previous experimental efforts [6,7] and in part assigned nominal or empirical values as proposed in the relevant literature [13,14], are summarized in Table 1.

Table 1 Basic material properties used in numerical analysis.

	E [N/mm ²]	ν [-]	ρ [kg/m ³]	f_c [N/mm ²]	f_t [N/mm ²]	G_f [N/mm]
Wall masonry	3000 ^c	0.15 ^c	1920 ^b	6.99 ^a	0.10 ^c	0.012 ^d
Pillar masonry	15700 ^b	0.20 ^b	2360 ^b	11.95 ^a	1.00 ^d	0.075 ^d
a: experimentally derived value [6]				c: estimated value [13]		
b: experimentally derived value [7]				d: estimated value [14]		

The masonry walls and columns of the nave are modeled using 8-node quadrilateral and 6-node triangular plane stress elements, an approach suited to the geometric arrangement, element thickness and load orientation. A macro-modeling approach is adopted for the model, in which the masonry composite is treated as a homogenous continuum, with no distinction between units, mortar and the unit-mortar interface. The nonlinearity in tension is modeled using a multi-directional fixed crack model. A Rankine-type tension cut-off is used in pure or biaxial tension, while the influence of lateral compression is accounted for through a Mohr-Coulomb-type criterion. Nonlinear tension softening is assumed, governed by fracture energy, according to the expression:

$$\sigma_t = \begin{cases} f_t \left(1 - \frac{\varepsilon_{cr}}{\varepsilon_u}\right)^{0.31} & \text{for } 0 \leq \varepsilon_{cr} \leq \varepsilon_u \\ 0 & \text{for } \varepsilon_u \leq \varepsilon_{cr} < \infty \end{cases} \quad (1)$$

where σ_t is the tensile stress, ε_{cr} is the crack strain and ε_u is the ultimate strain, calculated according to the expression [15]:

$$\varepsilon_u = 4.226 \frac{G_f}{f_t h} \quad (2)$$

where h is the characteristic length of the finite element.

242 3.2 Foundation and Soil-Structure Interaction

243 Soil-structure interaction is directly considered through the introduction of linear elastic structural interfaces
 244 at the base of the masonry pillars, capable of accounting for normal and shear deformation. The normal stiffness
 245 may be determined according to two distinct approaches: (a) from a calibration effort targeted at reproducing the
 246 settlement profile measured over a given time period and (b) directly from the geometric characteristics of the
 247 pillar footing and the elastic properties of the soil.

248 Both approaches are adopted and compared in the present paper. The values obtained from approach (a) are
 249 presented in the results section. For approach (b), the vertical elastic spring constant for a single rigid arbitrarily
 250 shaped footing j circumscribed in a rectangle with dimensions $2L^j \cdot 2B^j$ and embedded in the ground at a depth
 251 of D^j is equal to [16]:

$$K_n^j = \frac{2GL^j}{1-\nu} (0.73 + 1.54x^{0.75}) \left[1 + \frac{1}{21} \frac{D^j}{B^j} (1 + 1.3x) \right] \left[1 + 0.2 \left(\frac{A_w^j}{A_f^j} \right)^{2/3} \right] \quad (3)$$

252 where G is the shear modulus of the soil, ν is the Poisson's ratio of the soil, A_f^j is the area of the footing, A_w^j
 253 is the total sidewall-soil contact area (equal to the perimeter of the footing times the embedment depth D^j in case
 254 of a foundation with constant cross-section) and $x = A_f^j / (2L^j)^2$. Division of the spring constant by A_f^j produces
 255 the modulus of subgrade reaction for a single footing:

$$k_n^j = \frac{K_n^j}{A_f^j} \quad (4)$$

256 This value is used for the normal stiffness of the interfaces below the pillars, adjusted according to the ratio of
 257 the base area of the footing over the cross-sectional area of the pillar at ground level. The settlement d_n^j of the
 258 foundation for a given normal force F_n^j is:

$$d_n^j = \frac{F_n^j}{K_n^j} \quad (5)$$

259 The value for the shear stiffness of the interface is calculated by the expression:

$$k_s^j = \frac{k_n^j}{2(1 + \nu)} \quad (6)$$

260 according to the Poisson's ratio ν of the stone masonry foundation. This value is not determined
 261 experimentally but is consistent with the material properties of the stone masonry and is numerically more stable
 262 and less arbitrary than the use of a dummy value that precludes shear slipping at the foundation. This slipping
 263 mode is, in any case, constrained by the high compressive stress applied on the interface by the self-weight and
 264 the lack of horizontal loads. Therefore, the shear stiffness is calculated for a Poisson's ratio of 0.20 without further
 265 investigation.

266 The soil beneath the church foundations has been investigated and found to be generally composed of, from
 267 the surface advancing in depth: a) sandy clay (± 2.0 m thick), b) highly compressible peat (± 2.0 m thick), c) sandy
 268 clay (± 2.6 m thick), d) quaternary clay-containing sand (± 5.4 m thick) and e) tertiary highly consolidated clay-
 269 containing sand (unknown thickness) [6]. The foundation bases of the main pillars are roughly in the middle of
 270 the peat layer. In the present case study, the settlements have been calculated prior to this investigation. Therefore,
 271 the interface normal stiffnesses can be directly calculated from eq. (5) and (4). Using eq. (3) the apparent Young's
 272 modulus of the homogenized foundation soil can be back-calculated. In the case of the nave pillars at the final
 273 settlement (as shown in Figure 5) this apparent Young's modulus is equal to 0.597 N/mm^2 or 2.400 N/mm^2
 274 when taking into account or disregarding the effect of embedment respectively. The former value is representative
 275 of peats, while the latter is rather low for the all the soils in the other layers. This indicates the major contribution
 276 of the peat layer to the total settlements and the potential primary cause of the excessive settlements of the church
 277 at the nave.

278 For all analysis cases, the structure is let to deform under its self-weight and the extra load applied at various
 279 levels of the nave wall from other elements present in the structure but not explicitly modeled, such as the timber
 280 roof or the stone masonry vaults.

281 All finite element calculations were carried out using the DIANA FEA package [17]. The geometric layout
 282 and the boundary conditions applied are illustrated in Figure 8. The different phases are colored in shades of grey
 283 and the areas where pillar masonry material is assigned are given a reddish overlay. It is assumed that the tower to
 284 the west of the nave (left side in the illustration) provides a rigid lateral support to the nave. The average element
 285 length is roughly 166 mm, resulting in a total of 40308 nodes and 13029 continuum and 37 interface elements.
 286 The roof loads associated with each of the three phases is applied at the top of every model.

288 **Figure 8 Geometric layout of church nave wall. Structural phase designation, boundary conditions**
 289 **and material assignment.**

290 3.3 Definition of Time Periods for Phased Analysis

291 Three different time periods are defined for the investigation of the time-dependent behavior of the building:
 292 period A (1300-1316), period B (1317-1534) and period C (1535-1970), which coincide with the construction
 293 phases illustrated in Figure 8. The beginning of period A corresponds to the completion of the first nave wall level.

294 The end of period A corresponds to the initiation of the increase of the height of the nave and period B starts upon
 295 its completion. The end of period B corresponds to the construction of the crossing and the third level of the nave
 296 wall. Period C brings us near to the present period, at the time of major temporary shoring of the building.

297 Through use of the equations (3) to (5), and for known values of the applied force and displacement, one may
 298 calculate the normal interface stiffness, and also back-calculate the apparent Young's modulus of the soil at given
 299 time instants. Both the forces and the settlements have been estimated for various points in the history of the
 300 building, as shown in Figure 5. The calculated normal interface stiffness for different structural parts through time
 301 is shown in Figure 9. This value is proportional to the apparent Young's modulus of the soil. One can differentiate
 302 between a short-term Young's modulus, governed by immediate settlements due to a change in load, and a long-
 303 term Young's modulus, governed by settlements due to, for example, consolidation. In the case of the pillars, the
 304 short-term Young's modulus generally exhibits a decreasing trend, whereas the long-term modulus exhibits an
 305 increase. The two curves appear to converge near the end of the measurement period, indicating that processes
 306 causing settlement under sustained loads have been halted. The consolidation being further completed accounts
 307 for the latter phenomenon, but the continuation of the settlements cannot be entirely excluded.

308
 309 **Figure 9** Calculated pillar spring stiffness for estimated loads and settlements. Development of
 310 instant and long-term response.

311 **3.4 Analysis Approaches**

312 Three different approaches will be adopted for the analysis of the nave: a single-phase analysis, a phased
313 analysis and a parametric study.

314 Firstly, in the single-phase analysis, the whole structure is taken in its entirety and the foundation interfaces
315 are assigned their final values according to the estimated settlements of each structural part and the dimensions of
316 the footings.

317 Secondly, in the phased analysis, three major phases are considered, with each one decomposed into two parts.
318 The major phases correspond to the phase designation indicated in Figure 8 and the decomposition of each phase
319 is based on the differences between the short-term (immediate) and the long-term response of the structure under
320 the sustained loads of each phase. Both the single-phase analysis and phased analysis make use of approach (b)
321 for determining the interface normal stiffness as explained in Section 3.2.

322 Thirdly, a parametric analysis is performed to address uncertainties in the mechanical properties of the
323 materials. In order to reduce the computational cost, these analyses are only carried out for the settlement profile
324 obtained from the levelling surveys of the period 1994-2005. Such analysis is defined in Section 3.2 as approach
325 (a) for determining the normal stiffness. In the following section, this choice of loading is motivated further,
326 through a discussion of the obtained failure patterns. The material parameters included in the investigation are the
327 Young's modulus, the tensile strength and the tensile fracture energy of the masonry composing the nave. The
328 variation of the parameters ranged from 50% to 200% of the initial values indicated in Table 1. The model using
329 the initial values will be henceforth referred to as the reference model.

330 **4. Analysis Results**

331 **4.1 Single-Phase Analysis**

332 As a first approach, the self-weight of the complete structure along with the final additional roof weights is
333 applied in a single phase. The final value for the stiffness of the foundation interfaces is used (see Figure 9),
334 corresponding to the long-term soil modulus of phase C. The obtained crack pattern is shown in Figure 10. The

335 obtained damage pattern presents several differences from the actual structure. Due to the settlement towards the
 336 crossing, the response is dominated by the separation cracks between the nave and the tower. Crack 2 above arch
 337 2 (Figure 2) is entirely absent. A single crack is formed between pillars 2 and 3 (crack 3, but inclined in the other
 338 direction) and the crossing itself remains intact, as does its connection with the wall. Therefore, application of the
 339 deformation loads in a single analysis step reveals only part of the response of the building and is not indicative
 340 of its behavior throughout its history.

341

342 **Figure 10 Crack patterns for application of load in single analysis phase.**

343 **4.2 Phased Analysis**

344 The phased analysis of the nave provides a much more complete and detailed illustration of the development
 345 of damage on the building throughout its history (Figure 11). The obtained settlement profiles derived from the
 346 phased analysis, as well as from the single-phase case, are presented in Figure 12.

347 During phase A, only minor damage is registered at arches 1 and 5, see Figure 11a. There is some widening
 348 of the cracks under long-term loading, but no formation of new major cracks, see Figure 11b. The response is
 349 mostly of a sagging type due to the tower and the crossing exhibiting only minor settlement. Due to the numerical
 350 cracks not having sufficiently developed in extent, no clear comparison between the numerically derived cracks
 351 and the existing damage in the building can be made.

352 The situation changes significantly during phase B with the increase of the height of the nave wall above the
353 colonnade. A major separation crack is formed in the short-term phase between the tower and the nave (cracks 1
354 and 5). Additionally, cracks 2 and 3 are formed above pillar 3. At the end of the long-term loading, cracks 1 and
355 4 have emerged above the main colonnade. While the settlement profile is mostly of a sagging type at the end of
356 the short-term phase (Figure 12), a mixed profile with a significant tilting component is obtained at the end of the
357 long-term phase. The increase of the weight at the crossing is substantial and unable to be borne by its foundations.

358 Phase C witnesses the formation of the new cracks above and beside the third level windows (cracks 6 and 7
359 in Figure 2). The extent of the previously formed cracks is increased without, however, significant widening,
360 indicating the activation of the cracks at the new parts of the structure. As shown in Figure 12, the obtained profile
361 resembles the tilting-dominated response at the end of phase B, but of a larger magnitude.

362 In Figure 12, the maximum settlement obtained at the end of the phase C closely resembles that of the single-
363 phase analysis, albeit with a slightly smaller magnitude. Nevertheless, the cracking pattern is significantly different
364 in the two approaches. The resulting cracking pattern from the phased analysis resembles in a higher degree the
365 actual pattern (compare Figure 2 and Figure 11). The cracking pattern resulting from the phased analysis resembles
366 much more closely the actual crack pattern compared to the single-phase analysis (compare Figure 10 and Figure
367 11). The complexity of the model, as the outcome of soil-structure interaction, is underlined by the substantially
368 different settlements obtained between the four pillars. This is despite the fact that they are of the same cross-
369 section with identical foundations (meaning equal interface stiffness) and bearing roughly the same vertical loads.

370 Overall, as time progresses, the deflection ratio of the structure tends to decrease, despite the increase in the
371 overall settlements (Figure 12). Cracks 2 and 3, caused by the sagging of the center are nearly fully developed by
372 the end of phase B, after which newly arising damage is possibly associated with tilting of the nave towards the
373 crossing. The maximum width of the cracks in the first level of the wall in fact decreases from phase B to phase C
374 due to the stabilizing effect of the added stiffness of the second level and despite the increase in weight.

Figure 11 Crack patterns for phased analysis of nave: a) phase A short-term, b) phase A long-term, c) phase B short-term, d) phase B long-term, e) phase C short-term, f) phase C long-term.

Figure 12 Settlement profiles obtained from finite element analysis: phased analysis and single-phase approaches.

The numerically obtained cracks have been matched to cracks documented in the structure (Figure 2 and Figure 11). Rather than integrating the obtained crack strains over the continuum to calculate the crack width, the opening of the cracks is indirectly calculated through measurement of the horizontal relative displacements of nodes on either side of the smeared crack mouth. Displacements due to elastic stress are minimal compared to displacements due to crack opening. In cases where the cracks in the actual structure are composed of more than one distinct branch, this measured numerical crack width is divided by the number of branches in order to obtain the magnitude of a single crack branch. The development of the normalized crack width, defined as the sum of the crack width divided by the number of crack branches in the actual structure, is presented in Figure 13. All cracks tend to increase with the passage of time, except crack 3, which is reduced in width after the construction of the second and third level of the nave.

Figure 13 Normalized crack width for various analysis phases.

4.3 Parametric Investigation

The damage pattern obtained from the reference model is shown in Figure 14. This is the outcome of the application of the settlement pattern measured in the period 1994-2005. Rather than the displacement being applied to the supports directly, the stiffness of the interfaces was calibrated in order to match this settlement profile. This approach allows the evaluation of the soil-structure-interaction by altering the settlements from a variation of the stiffness of the superstructure.

Figure 14 Crack pattern obtained from the reference model of the parametric investigation.

401 Despite the narrow extent of this measurement period compared to the entire history of the building, the
402 pattern closely resembles the damage present in the structure, both in location and extent, albeit with a much
403 smaller magnitude in terms of crack width. This constitutes an indication that the emergency measures taken from
404 1965 onwards may have not completely halted the progress of settlement in the structure. Nevertheless, this
405 resemblance motivates the use of this measured profile as a basis in the parametric investigation carried out in this
406 section.

407 The results of the parametric investigation are illustrated in Figure 15. They are presented in terms of the width
408 of cracks 2 and 3 vs. settlement of pillar 2, above which the cracks in question are situated. Initially, the Young's
409 modulus of the pillar was investigated, due to the initially determined value being higher than expected given the
410 compressive strength of the pillar masonry (see Table 1). The change in the Young's modulus of the pillar does
411 not significantly affect the response of the nave, due to the limited extent of the nave area in which it is encountered.
412 However, the reduction of the Young's modulus of the wall increases both the width of the cracks and the amount
413 of settlement of pillar 2. Interestingly, the reduction of the Young's modulus causes crack 2 to increase in width
414 and crack 3 to be severely reduced, owing to the redistribution of forces in the wall. Cracks 2 and 3 are differently
415 affected by changes in the tensile strength of the wall masonry as well. The width of crack 2 slightly increases for
416 any change in the parameter, while crack 3 decreases, practically disappearing for a decrease in the tensile strength.
417 Finally, the response was not particularly sensitive to changes in the tensile fracture energy of the wall masonry.
418 However, a slight increase in the total settlement of the pillar is registered for a decreased value of this parameter.

Figure 15 Results of parametric investigation for a) crack 2 and b) crack 3. Crack widths vs. variation of material parameters of masonry.

The results are also tabulated in Table 2, with the addition of the results of the horizontal movement and vertical settlement of pillar 2. The changes in the horizontal movement of the pillar, measured at the capital and indicating tilting rather than whole body slipping, are either associated with the crack widths above the pillar or with the capacity of the wall for post-cracking deformation (as influenced by an increase in the fracture energy).

Crack 2 did not present strong sensitivity to the material properties. Crack 3 presented some sensitivity to the tensile strength of masonry. The dependence on the Young's modulus of masonry is partially related to the increase of crack width due to reduced stiffness of the superstructure. The tensile fracture energy had only a marginal effect on either crack. The crack arrangement was not sensitive to the material properties. These observations suggest that crack formation and development are more sensitive to the applied deformation profile. This outcome illustrates the importance of an accurate calculation of the properties of the soil and a good measurement of the historic settlement profile in order to achieve meaningful analysis results. In light of the envisaged intervention project, consolidation of the wall masonry, which would lead to some degree of increase of the tensile strength, fracture energy and Young's modulus of the material, can be beneficial in itself for maintaining the integrity of the church and limiting the effects of possible future differential settlement. This can act as a complement to the foundation strengthening and soil consolidation underway.

Table 2 Results of parametric investigation. Percentile differences from reference model results in crack width and displacement (vertical and horizontal) of pillar 2.

	Crack width 2	Crack width 3	$\Delta x_{pillar\ 2}$	$\Delta y_{pillar\ 2}$
Reference	100	100	100	100
$1/2 f_t$	146	32	190	105
$2f_t$	129	77	85	93
$1/2 G_f$	129	137	76	99
$2G_f$	83	105	271	91
$2/3 E_{pillar}$	85	111	97	103
$1/2 E_{pillar}$	80	136	102	107
$1/2 E_{wall}$	218	29	352	134

4.4 Discussion on Model Results

A final appraisal of the obtained results from the modeling approaches is warranted. In particular, the results from the phased analysis need to be contrasted with the monitoring and survey data. The final deformation profile obtained for the phased analysis (Figure 12) relies on the estimation of the behavior of the soil through time. The profile estimated from the architectural features of the building (Figure 7) depicts the effects of soil-structure interaction more directly. The latter profile compares favorably with the deformation profile obtained from

445 monitoring data over the period 1994-2005. The normalization of the estimated settlements by disregarding the
446 tower movement prior to the construction of the nave wall makes a direct comparison difficult. The lack of
447 measurements in the period 2005-2018 further complicates matters, which the new intervention project will help
448 clarify. Finally, the analysis approach does not take into account the full three-dimensional geometry effects of the
449 structure. These are potentially more acute near the crossing pillar due to the presence of the transept. Nevertheless,
450 the numerical analysis approach adopted here is able to capture the response of the nave with significant fidelity.

451 The assumption of a uniform interface stiffness below all the pillars in the nave is a necessary simplification
452 due to the lack of more detailed data. Despite resulting in an accurate crack pattern, the final disposition is not in
453 complete agreement with the settlement as estimated from the architectural feature analysis. However, a more
454 clear relation between the numerical and measured deformation profile is obtained when comparing deflection
455 ratios. The deflection ratio is defined as the ratio of the relative settlement to the length of the deflected part. In
456 Figure 16, the deflection ratios along feature Line 4 (base of the pillar abacus of the first level) are presented, based
457 on the measurements shown in Figure 7. The deflection ratio along this line could be more clearly defined along
458 a larger portion of the structure compared to other lines and is sufficiently close to the base of the pillars to provide
459 an indication of the settlement. As Figure 16 illustrates, the deflection ratios between the two approaches are quite
460 similar, with the sagging of the colonnade between the tower and the crossing pillar being clearly indicated.

461

462

463 **Figure 16 Comparison of deflection ratio from finite element analysis and architectural feature**
464 **measurement on the point cloud (Line 4).**

465 **5. Damage Calculation Using an Analytical Damage Function**

466 **5.1 Calculation of Model Parameters**

467 The model proposed by Giardina et al [5], which introduced an analytical relation between settlement and
468 damage index for masonry buildings, will be adopted for the present study. The model relates the deflection ratio
469 $\hat{\Delta}$ due to sagging or hogging ground deformation to a damage level of the structure according to the classification
470 proposed by Burland & Wroth [18]. The damage level, linked to the severity of damage and the means required
471 for its repair, is quantitatively expressed in terms of crack width, thus directly comparable to both documented
472 pathology and nonlinear finite element analysis results. The damage classes are outlined in Table 3. The damage
473 model for two-dimensional structures is a function of several geometric and material parameters expressed in a
474 polynomial equation as follows:

$$d'_{2D}(\hat{\Delta}, \bar{x}) = d_{2D,ref}(\hat{\Delta}) + \sum_{i=1}^6 a_i \bar{x}_i = b_1 + b_2 \hat{\Delta} + b_3 \hat{\Delta}^2 + b_4 \hat{\Delta}^3 + \sum_{i=1}^6 a_i \bar{x}_i \quad (7)$$

where $d_{2D,ref}$ are the selected reference values, a_i and b_i are fitted polynomial coefficients and $\bar{x}$ contains the normalized values of the model parameters x_i . The x_i model parameters, along with their reference values $x_{i,ref}$ are given in Table 4. All values for the polynomial coefficients and the normalization process for the model parameters are detailed in [5]. From the value of the damage level calculated from the model, one can calculate the corresponding crack width through linear interpolation based on the values found in Table 3.

Table 3 Damage classification for masonry structures subjected to differential settlements [18].

Damage Level	Damage Class	Crack Width [mm]
1	Negligible	0.0 – 0.1
2	Very Slight	0.1 – 1.0
3	Slight	1.0 – 5.0
4	Moderate	5.0 – 15.0
5	Severe	15.0 – 25.0
6	Very Severe	>25.0

Table 4 Damage model reference values $x_{i,ref}$ [5] and input for current analysis x_i .

Openings	G_f	E	f_t	k_n	Interface shear behavior/Trough shape
[%]	[N/m]	[N/mm ²]	[N/mm ²]	[N/mm ³]	[-]
$x_{1,ref} = 30$	$x_{2,ref} = 10$	$x_{3,ref} = 3000$	$x_{4,ref} = 0.10$	$x_{5,ref} = 0.7 \times 10^9$	$x_{6,ref} = 1$
$x_1 = 31.92 \div 33.88$	$x_2 = 12$	$x_3 = 3000$	$x_4 = 0.10$	$x_5 = [\text{See Table 5}]$	$x_6 = 1$

Parameters x_1 to x_4 are derived according to the material properties used in the finite element analysis and the geometry of the nave. Parameter x_6 is assigned its reference value according to [5]. Special attention is drawn to the parameter x_5 related to the normal stiffness of the soil-structure interface. The reference value for x_5 has been calculated according to typical Dutch pile foundation systems distributed along the façade of brick masonry structures [19]. In the present research, this parameter is calculated from the modulus of subgrade reaction under vertical loading of the foundation system of the nave, an approach that generalizes the applicability of the damage function to other foundation and soil types. This approach additionally allows for taking into account foundation strengthening, micro-piling and foundation soil improvement directly in the damage function.

490 For strip foundations, the modulus of subgrade reaction k_n is directly equivalent to the parameter x_5 and can
 491 be applied to continuous shallow foundations of masonry walls. Equations for its calculation are available in the
 492 literature (e.g. [20,21]). Some further manipulation is required in the case of colonnades founded on individual
 493 footings, as is the case with the nave pillars of the present case study. The x_5 parameter for a series of m single
 494 footings j is then calculated as follows:

$$k_n = \frac{\sum_{j=1}^m k_n^j A_f^j}{\sum_{j=1}^m A_f^j} = x_5 \quad (8)$$

495 where k_n^j is the modulus of subgrade reaction of footing j . This can be calculated from equations (3) and (4)
 496 or from equations (4) and (5) if the settlement has been pre-estimated. This averaging approach to the subgrade
 497 reaction modulus is similar to the one followed for the allocation of the stiffness provided by the distributed piles
 498 according to Rots [19], but can be generalized as shown for continuous footings of walls or isolated footings of
 499 colonnades. This parameter becomes significant in light of the results shown in Figure 12. Despite the decrease of
 500 the deflection ratio as the phases progress, the damage, in terms of crack width, increases. This is captured by the
 501 damage model through the change in the x_5 parameter due to soil-structure interaction (decrease of the apparent
 502 Young's modulus of the foundation soil within a single phase). The disposition of the data points, capable of being
 503 approximated by a third order polynomial fit, suggests that the damage model can be successfully adapted to this
 504 case.

505 In addition to adjusting the reference value for the vertical interface stiffness (x_5), the a_5 coefficient associated
 506 with the interface stiffness is also adapted. These parameters are modified in order to fit the available numerical
 507 results of the phased analysis (phase B and C) and the reference model. The material properties of masonry are
 508 taken as equal to those of the wall masonry (Table 1), which comprises most of the structure and on which the
 509 majority of the damage is accumulated. The percentage of openings varies between 33.88% in phase B and 31.92%
 510 in phase C and the reference model.

511 **5.2 Results**

512 Using equation (8) the values for the parameter x_5 are found for the colonnade. These are shown in Table 5,
513 where it becomes apparent that the range of the parameter values varies within the range initially investigated in
514 [5], with the exception of the reference model case. Nevertheless, it is expected that the calibrated values for the
515 numerical model parameters be significantly different from those initially proposed. This is due to the fact that the
516 initial model was calibrated against a finite element benchmark where the interface stiffness was not extensively
517 investigated. A complete recalibration of the model is bound to alter the numerical parameters to some degree.

518 The slight increase of x_5 during the transition from long-term phase B to short-term phase C is contrary to the
519 progress of soil consolidation, which decreases the Young's modulus of the soil. However, depending on the
520 calculation method elected (such as for the estimated settlement method employed here or for continuous footings),
521 the x_5 parameter may depend directly or indirectly on the stiffness of the superstructure as well. This stiffness is
522 increased by the addition of the second level of the nave wall. This fact clearly illustrates the significance of soil-
523 structure interaction in the study of differential settlement damage problems.

524 The analytical model, when used with its initial reference values and normalization process, greatly
525 exaggerates the damage corresponding to the reference model and underestimates the damage in the phased
526 analysis (phases B and C). The calibration of the new parameters is performed by a simple minimization process,
527 during which the linear regression between FE and analytical crack widths is required to be a unitary slope curve.
528 The normalization of the x_5 parameter in [5] is carried out according to:

$$\overline{x_5} = \frac{\log_{10} x_5 - \log_{10} x_{5,ref}}{2} \quad (9)$$

529 A new normalization of the parameters is proposed here in order to match the trend of interface stiffness to
530 damage level, according to which the $\overline{x_5}$ parameter is equal to:

$$\overline{x_5} = \left(\frac{\log_{10} x_{5,ref}}{\log_{10} x_5} \right)^{1.5} \quad (10)$$

531 The results of the modified analytical model are compared with the finite element analysis results in terms of
 532 the main cracks of the first level of the nave wall, following their calculation as described in the phased analysis
 533 section. The minimization process for the available data set produces a value for the a_5 parameter equal to -1.1722.
 534 The comparison of the results is plotted in Figure 17a, in which satisfactory agreement is found throughout the
 535 range of available data. Despite some discrepancy in the results in terms of crack width, the damage level is well
 536 approximated by the calibrated model.

537 **Table 5 x_5 parameter results for phase B model, phase C model and reference model.**

		Reference	B short	B long	C short	C long
x_5	$[N/mm^3]$	1.43E+10	2.79E+07	9.63E+06	9.70E+06	6.86E+06

538 The envelope indicating the change in the polynomial model curve, almost entirely due to alterations in the
 539 spring stiffness and to a very minor extent due to changes in the opening percentage, is presented in Figure 17b.
 540 The upper envelope curve corresponds to the maximum apparent stiffness associated with the reference model and
 541 the lower curve corresponds to the minimum apparent stiffness of the long-term part of phase C. This fact
 542 illustrates the influence of the foundation stiffness, and, by extension, the properties of the soil, on the behavior of
 543 complex structures.

Figure 17 a) Comparison of analytical model with FE analysis results. Dashed diagonal indicates line of equality between FE analysis and analytical modeling. b) Crack width according to damage prediction model and comparison with FE analysis results.

The initial results of this approach towards the extension of the predictive analytical model are promising. The generalization of this extension requires robust verification in order to recalculate the numerical coefficients of the model. Coupled experimental tests and parametric numerical analyses need to be developed, along the same lines of the prior development of the analytical model but attempting to include variations in the vertical stiffness of the foundation. This is likely to lead to modifications to all numerical parameters, unlike the simplified calibration approach adopted in this paper, where only the parameters linked with the vertical stiffness were modified.

6. Conclusions

In this study the behavior of a church nave wall subjected to ground deformation over an extended period of time is investigated. The problem is approached through single-phase and multi-phase finite element analysis. The paper demonstrates the importance of detailed modeling of the soil behavior over time, the soil-structure interaction and the accurate measurement of settlements for the analysis of complex structures subjected to soil movement. Through this investigation it is shown that the application of a single-phase analysis does not reliably provide the cracking pattern observed in the actual structure. The need to take into account construction phases

561 and changes in the soil stiffness is clearly shown, even when studying individual element ensembles, such as the
562 nave wall here investigated. The importance of detailed geometric and damage survey is also demonstrated.

563 The sensitivity analysis illustrates the predominance of the deformation profile, as influenced mainly by the
564 soil-structure interaction, in the disposition of the cracking pattern on the structure. Nevertheless, the width of the
565 cracks is strongly influenced by the material properties of the nave wall, as are the obtained settlements, although
566 to a lesser degree.

567 The phased analysis, taking into account changes in the behavior of the soil and alterations in the geometry of
568 the structure, provides a complete picture of the history of the nave's pathology. Different major cracks appear
569 and develop at different phases of the building, due to redistribution of the forces and changes in the stiffness of
570 the foundations.

571 An analytical model for the prediction of the damage level in masonry structures subjected to differential
572 settlements is adapted and expanded. Moving beyond the initial formulation of the model, a method for the direct
573 calculation of the normal stiffness of the structure is proposed. Following calibration of the numerical coefficients
574 linked to this stiffness, the results of the model are consistent with the numerical analysis results and the crack
575 state of the real structure. Further expansion of the model along the lines pursued here can greatly enhance the
576 potential for accurate analytical modeling of complex masonry structures.

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