Scale-dependent parameterization of groundwater – surface water interactions in a regional hydrogeological model

This is a post-print of a peer-reviewed article published in Journal of Hydrology volume 576. The published article can be found at <https://doi.org/10.1016/j.jhydrol.2019.06.072>.

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Highlights

- A new expression of the groundwater – surface water conductance is derived.
- It is function of the surface water network density and aquifer properties.
- It is dependent on the vertical and horizontal model discretization sizes.
- It is evaluated against 2D and 3D numerical experiments.
- It accurately captures the scale dependency of the conductance.

Abstract

In regional hydrogeological models groundwater – surface water interaction is generally represented with a Cauchy boundary condition, in which a conductance parameter governs the exchange flux rate. In some models, the conductance is controlled by the streambed properties, since it has generally a lower hydraulic conductivity than the aquifer. However, depending on the specific system and the spatial discretization of the hydrogeological model, aquifer conductance can be a limiting factor for groundwater – surface water interactions. The present study introduces a new expression to represent the aquifer conductance as a function of aquifer properties, surface water network density and model discretization. This expression is based on the Dupuit-Forcheimer theory, the Ernst equation and vertical 2D numerical experiments at the field scale. The main assumptions used to derive our formulation are the presence of a no-flow boundary at the bottom of the hydrogeological model and the homogeneity of the aquifer. The expression is evaluated using simulations with 3D hydrogeological models at different spatial resolutions and compared against previously published parameterization approaches. The results show that the new expression outperforms the other approaches by capturing accurately both the grid-size and the surface water network density dependency of the conductance, which is caused by pressure head losses due to flow within the aquifer grid cell to the surface water, without any additional numerical calculation. Moreover, the proposed expression can be implemented directly in hydrogeological models thereby

improving current approaches to represent groundwater – surface water interactions in regional hydrogeological models.

1. Introduction

Modelling water exchange between groundwater (GW) and surface water (SW) is of great importance at all scales for a variety of functions some including: river ecology (e.g., Boulton et al. (2010), Stegen et al. (2016)), water, mass and energy transfer at the catchment scale (e.g., Bailey et al. (2016), Rassam et al. (2008)) and even at the continental and global scales as a part of earth system modelling (e.g., de Graaf et al. (2017), Maxwell et al. (2015)).

In spatially distributed numerical models, the coupling of surface and ground water flows can be simulated in different ways. Recent advances in computational power and coupling methods have led to a new generation of models that solve differential equations of flow in the variably saturated subsurface and equations for SW flow in a fully coupled way (e.g., Camporese et al. (2010), Kollet and Maxwell (2006), Maxwell et al. (2014), Therrien and Sudicky (2006)). However, CPU-time and resources can still be a limiting factor depending on the spatial scale and the resolution that needs to be considered to simulate the processes in the system of interest. For instance, dense surface water networks (i.e. rivers and drains) of drained lowland areas cannot be spatially realistically resolved or represented to a sufficient degree in a regional hydrogeological model without an excessive computational cost. An implicit representation of surface water bodies appears, in this case, to be the only viable solution. In regional hydrogeological models, the GW-SW coupling is usually accomplished with a Cauchy boundary condition (head-dependent flux). In a model cell, the volumetric flow rate between GW and SW ($Q_{\text{GW-SW}}$, [$\text{L}^3.\text{T}^{-1}$]) is calculated as the product of the conductance ($C_{\text{GW-SW},3\text{D}}$ [$\text{L}^2.\text{T}^{-1}$]) and the difference between the hydraulic heads of groundwater (h_{GW} , [L]) and surface water (h_{SW} , [L]):

$$Q_{GW-SW} = C_{GW-SW,3D}(h_{GW} - h_{SW})$$

Equation 1

The inverse of the conductance is defined as the resistance ($\gamma_{GW-SW,3D}$, [T.L⁻²]). Therefore the resistance represents the whole head losses happening between the GW and the SW. For convenience, in this paper the resistance will be used and thus Equation 1 becomes:

$$Q_{GW-SW} = \frac{h_{GW} - h_{SW}}{\gamma_{GW-SW,3D}}$$

Equation 2

The volumetric flow between GW and SW can also be expressed per unit length of surface water, considering a vertical cross-section of the stream – aquifer system. In this case the 2D resistance ($\gamma_{GW-SW,2D}$) is equal to γ_{3D} multiplied by the length of surface water and has the dimension [T.L⁻¹].

The most commonly used parameterization of the resistance dates from the 1970's (Prickett and Lonquist 1971). The approach assumes that the flow mechanism between SW and GW is similar to the leakage through a low permeability layer and therefore depends only on the geometry and hydraulic properties of the streambed. It is used, for instance, in the MODFLOW family of codes, as highlighted by (Harbaugh 2005), relies on the assumption that a distinct interface of low permeability (i.e. streambed) between the surface and subsurface exists and that all the head losses occur in the streambed. The resistance of this low permeability layer can then be easily calculated using Darcy law and is referred to as streambed resistance (γ_{sb}). This conceptualization is convenient and might be correct in particular cases because γ_{sb} can indeed be an important parameter governing the GW – SW interactions. However, it suffers the major conceptual problem of not considering the head losses occurring in the aquifer (i.e. aquifer resistance, γ_{aqui}). This shortcoming was well known by the authors of the original MODFLOW code and discussed in McDonald and Harbaugh (1988). Assuming that the resistances of the streambed and of the aquifer act in series, the GW – SW resistance is related to these resistances as follows:

$$\gamma_{GW-SW} = \gamma_{sb} + \gamma_{aqui}$$

Equation 3

The importance of aquifer resistance relatively to the streambed resistance depends on the geometry and the hydraulic properties of the streambed and the aquifer. Some field studies have shown that a clear interface between the surface and subsurface is not always present (e.g., Bayani and A. (2003), Ghysels et al. (2018), Kollet and Zlotnik (2003), Kollet and Zlotnik (2007)). Including the aquifer resistance is essential in such conditions. Moreover, even if the hydraulic conductivity is lower in the streambed than in the aquifer, the aquifer resistance can still be important because the distance over which the flow takes place is usually much larger than the streambed dimensions. This problem is expected to be of particular importance when a low spatial resolution is used in the numerical model and the lateral extent of the grid cells over which pressure head differences are calculated is large compared to the dimension of the surface water bodies. A solution would be to keep the hydrogeological model cell size small enough for the aquifer resistance to be negligible. Haitjema et al. (2001) proposed a criteria, depending on the aquifer transmissivity and the streambed resistance, which should not be exceeded by the cell size. However, this is not always possible in practice and does not solve the conceptual problem.

Another issue lies in the fact that the GW – SW interactions can occur not only through the streambed but also through the river banks (e.g., Anibas et al. (2018)) especially in managed ditches, rivers or drains, which are deepened and in which the bed sediments are often removed. In this case, the resistance of the streambed would not be the main limiting factor for the water exchange between the groundwater and surface water. As a consequence, the resistance parameter, γ , is often perceived as a lumped fitting parameter that needs to be estimated during model calibration (e.g. Bencala (1984), Mehl and Hill (2010), Rushton (2007)). However, the calibration of γ is often an ill-posed problem given the data availability (Cousquer et al. 2018, Fleckenstein et al. 2010, Hunt et al. 2006). Moreover, when the resistance to flow in the aquifer plays an important role, the calibrated resistance parameters cannot be compared with direct measurements of streambed characteristics

(e.g. Foglia et al. (2013), Krause et al. (2007)). Another related issue, is the grid-size dependency of γ (Mehl and Hill 2010). This leads to the non-transferability of γ values between models with different grid-cell sizes. A detailed investigation of GW – SW resistance has been done in some studies using 2D mechanistic models of a stream – aquifer cross section. Rushton (2007) highlights the importance of aquifer hydraulic conductivity on the GW – SW resistance and provides guidance for estimating the aquifer resistance value. Furthermore, Cousquer et al. (2017) show that grid-cell size and aquifer hydraulic conductivity have a predominant role over streambed geometry and hydraulic properties. They suggest that 2D vertical models should be used for the prior estimation of the resistance as a step forward in the calibration and uncertainty analysis of surface – subsurface models. However, this implies a significant amount of additional 2D model runs in order to consider the specific characteristics of the GW – SW interfaces in a regional scale model.

Furthermore, analytical equations have been proposed to calculate the aquifer resistance for aquifer-river interactions (Anderson (2003a, 2003b, 2005), Morel-Seytoux (2009), Morel-Seytoux et al. (2014)). Progress towards the integration of an analytical solution within hydrogeological models have been achieved recently by Morel-Seytoux et al. (2017) and Morel-Seytoux et al. (2018). However, their results are limited to 2D field-scale models including a single surface water body only.

Another field dealing with GW – SW interactions is drainage design. In drainage design studies, the most widely used analytical solutions for the calculation of aquifer resistance are the Hooghoudt (1940) and Ernst (1962) equations. The theory behind these equations is detailed by Ritzema (1994). However, these equations were derived to help in the design of man-made drainage networks and not to be included in hydrogeological models. Consequently, their use to specify γ_{GW-SW} in hydrogeological models is impracticable because, among others, they require the GW hydraulic head between two SW bodies as input and estimate the aquifer resistance for the whole aquifer system.

Furthermore, De Lange (1999) presented an analytical solution for the interaction between any number of surface water bodies and a regional aquifer. This equation was evaluated and compared

to other approaches in a set of numerical experiments by Pauw et al. (2015). Their results showed that the equation of De Lange (1999) performed better than the other approaches. However, this equation was derived under the assumption that two distinct aquifers exist (the regional and the phreatic aquifer), separated by an aquitard. Consequently, when the hydrogeological settings deviate from this conceptualization, this equation is impracticable to use.

Finally, two alternatives to the classical MODFLOW approach were proposed by Mehl and Hill (2010), which aimed at including the resistance due to the vertical flow between the bottom of the streambed and the cell center. However, the results of their numerical experiments highlight that the resistance formulations considered, including the commonly used, fail to account for the grid-size dependency in the horizontal and vertical directions. Nonetheless, these two alternatives approaches remain today the only ones which have been implemented directly in a MODFLOW model.

It should be noted that all the approaches discussed in this introduction as well as the new approach proposed in this article are only meant to represent a connected GW – SW system (i.e. without an unsaturated zone below the SW body).

In this work, we propose a new model for the calculation of aquifer resistance that is required to simulate GW – SW interactions using a Cauchy boundary condition. A novel term is proposed for the horizontal resistance and the Ernst (1962) equation is used for the radial resistance, but with a correction derived from detailed numerical experiments of flow through a stream – aquifer cross sections at the field scale. Because of this empirical correction, we refer to our model as a metamodel.

The remainder of this paper is organized as follows. Section 2 presents the theoretical underpinnings of the proposed metamodel. Next, section 3 details both the detailed 2D numerical experiments that are used to define part of our metamodel and the upscaling experiment involving a 3D hydrogeological model that is used to demonstrate our proposed approach against the usual MODFLOW approach and the two alternatives proposed by Mehl and Hill (2010). This experiment is

similar to that used by Mehl and Hill (2010) but further includes ditch drains in addition to the river network. In section 4, our validation results are analyzed. Finally, section 5 discusses the advantages and limitations of our approach and outline possible future developments before section 6 concludes with a summary of the most important findings.

2. Aquifer resistance metamodel

2.1. Aquifer resistance in two dimensions (2D)

To derive the metamodel for aquifer resistance, we are considering parallel surface water bodies with a hydraulic head, h_{SW} [L], which are separated by a distance, L [L], and are located at a height, D [L], above the impermeable, horizontal bottom of the aquifer. A uniform recharge R [$L \cdot T^{-1}$] is assumed, and thus RL equates to $q_{2D,GW-SW}$ [$L^2 \cdot T^{-1}$], the groundwater discharge to the surface water. We resort to two additional variables to represent the discretization of the hydrogeological model in the horizontal and vertical directions: L_{avg} [L] and D_{avg} [L], respectively. They correspond to the distance over which the groundwater pressure head field is averaged in the horizontal and vertical directions, respectively. The datum is the aquifer bottom. A schematic view of this conceptualization is shown in Figure 1.

Figure 1: Schematic view of the model conceptualization where L [L] is the distance between two surface water bodies, D [L] is the thickness of the aquifer, h_{GW} [L] is the groundwater hydraulic head, h_{SW} [L] is the surface water hydraulic head, L_{avg} [L] is the horizontal discretization of a hydrogeological model, D_{avg} [L] is the vertical discretization of a hydrogeological model, x [L] is the horizontal coordinate, R [$L \cdot T^{-1}$] is recharge flux and $q_{2D,GW-SW}$ [$L^2 \cdot T^{-1}$] is the GW – SW flux

Similar to the approach of Ernst (1962), the flow domain is divided in a vertical, a horizontal and a radial component (Figure 1). A resistance is associated to each of these components and referred to as the vertical ($\gamma_{\text{aqui,ver}}$), horizontal ($\gamma_{\text{aqui,hor}}$) and radial ($\gamma_{\text{aqui,rad}}$) resistances. Assuming that these resistances act in series, the aquifer resistance (γ_{aqui}) is given by:

$$\gamma_{\text{aqui}} = \gamma_{\text{aqui,ver}} + \gamma_{\text{aqui,hor}} + \gamma_{\text{aqui,rad}}$$

Equation 4

The vertical resistance, which is due to the infiltration of water from the soil surface, is generally small compared to the horizontal and radial resistances (Ritzema 1994). This is because the vertical resistance only applies above the SW bottom level. Therefore the vertical flow domain is generally smaller than the horizontal and radial flow domain located below the SW bottom level (i.e. $h_{GW} \approx h_{sw} \approx D$). Therefore, in the derivation of the metamodel, the vertical resistance is neglected and Equation 4 becomes:

$$\gamma_{\text{aqui}} = \gamma_{\text{aqui,hor}} + \gamma_{\text{aqui,rad}}$$

Equation 5

For the derivation of the horizontal resistance, it is also assumed that $h_{GW} \approx h_{sw} \approx D$. This makes the resistance parameter independent of both the groundwater and surface water hydraulic heads and therefore facilitates its integration in hydrogeological models.

Two limitations following the above assumptions are that (i) in the case of a thin aquifer and/or highly incised channels, the simplification $h_{GW} \approx h_{sw} \approx D$ cannot be assumed; and (ii) in the case of an aquifer with a significant vertical anisotropy, the vertical resistance can be relatively important.

Hereafter, the derivation of the horizontal aquifer resistance is first presented, followed by the derivation of the radial resistance.

2.1.1. Horizontal aquifer resistance

The horizontal aquifer resistance is derived by considering only the horizontal flow processes. Thus, the Dupuit-Forcheimer theory applies that describes the hydraulic head at a given x ($h(x)$, [L]) as:

$$h_{GW}^2(x) = -\frac{R}{K_{hor}}x^2 + \frac{RL}{K_{hor}}x + h_{SW}^2$$

Equation 6

where K_{hor} [$L.T^{-1}$] is the horizontal hydraulic conductivity, R [$L.T^{-1}$] is the recharge flux, L [L] is the spacing between 2 surface water bodies, h_{SW} [L] is the surface water hydraulic head and x [L] is the horizontal coordinate.

From Equation 6 we can derive the mean hydraulic head ($h_{GW,mean}(x)$) over a given domain from 0 to x :

$$h_{GW,mean}^2(x) = -\frac{R}{3K_{hor}}x^2 + \frac{RL}{2K_{hor}}x + h_{SW}^2$$

Equation 7

Rearranging this equation and considering that $q_{2D,GW-SW} = RL$, we obtain the following relationship between the GW – SW exchange flux ($q_{2D,GW-SW}$) and the hydraulic heads, aquifer and geometric properties:

$$q_{2D,GW-SW} = \frac{\frac{h_{GW,mean}(x) + h_{SW}}{x} - \frac{h_{GW,mean}(x) - h_{SW}}{x^2}}{\frac{2}{K_{hor}} - \frac{3}{K_{hor}L}}(h_{GW,mean}(x) - h_{SW})$$

Equation 8

where x can be replaced by $\frac{L_{avg}}{2}$ which leads to:

$$q_{2D,GW-SW} = \frac{\frac{h_{GW,mean} + h_{SW}}{L_{avg}} - \frac{h_{GW,mean} - h_{SW}}{\frac{L_{avg}^2}{4}}}{\frac{4}{K_{hor}} - \frac{12}{K_{hor}L_{avg}}}(h_{GW,mean} - h_{SW})$$

Equation 9

where L_{avg} [L] is the horizontal averaging distance of the aquifer hydraulic head.

241 By analogy with Equation 1, the 2D horizontal aquifer resistance ($\gamma_{2D,aqui,hor}$ [T.L⁻¹]) can be expressed
 242 as:

$$243 \quad \gamma_{2D,aqui,hor} = \frac{\frac{L_{avg}}{4K_{hor}} - \frac{L_{avg}^2}{12K_{hor}L}}{h_{GW,mean} + h_{SW}}$$

244 Equation 10

245 Considering that $h_{GW,mean} \approx h_{SW} \approx D$ (the aquifer thickness, [L]), the horizontal aquifer resistance can
 246 be approximated as:

$$247 \quad \gamma_{2D,aqui,hor} = \frac{\frac{L_{avg}}{4K_{hor}} - \frac{L_{avg}^2}{12K_{hor}L}}{2D} = \frac{3LL_{avg} - L_{avg}^2}{24LK_{hor}D}$$

248 Equation 11

249 This equation relates the horizontal aquifer resistance ($\gamma_{2D,aqui,hor}$, [T.L⁻¹]) to the horizontal aquifer
 250 hydraulic conductivity (K_{hor} , [L.T⁻¹]), the aquifer thickness (D , [L]) and the horizontal discretization of
 251 the hydrogeological regional model (L_{avg} , [L]).

252 2.1.2. Radial aquifer resistance

253 The radial aquifer resistance is expressed in the same way as by Ernst (1962):

$$254 \quad \gamma_{2D,aqui,rad} = \frac{\ln(\frac{D}{u})}{\pi K_{rad}}$$

255 Equation 12

256 where u [L] is the wetted perimeter of the surface water body and K_{rad} [L.T⁻¹] the radial hydraulic
 257 conductivity calculated as $\sqrt{K_{hor}K_{ver}}$ (Kroes et al. 2008).

258 However, a correction to the radial aquifer resistance is needed to account for the cases where the
 259 vertical discretization of the hydrogeological model (D_{avg}) is smaller than the aquifer thickness (D).

260 Therefore, a correction factor α [-] is included, which should be equal to 1 when $D_{avg} = D$ and when

261 $L_{avg} = L$. It is applied as a weighting factor of the radial aquifer resistance and is written as:

$$\alpha = 1 - \beta \left(1 - \frac{L_{avg}}{L}\right) \left(1 - \frac{D_{avg}}{D}\right)$$

Equation 13

in which D_{avg} [L] is the vertical averaging distance of the aquifer hydraulic head. The β parameter is determined from the numerical experiments. It is fitted to the outputs of the mechanistic 2D numerical experiments (i.e. fitted resistances, see section 3.1).

2.1.3. Aquifer resistance

The total aquifer resistance is therefore given by:

$$\gamma_{2D,aqui} = \gamma_{aqui,hor} + \alpha \cdot \gamma_{aqui,rad}$$

Equation 14

$$\gamma_{2D,aqui} = \frac{3LL_{avg} - L_{avg}^2}{24LK_{hor}D} + \left[1 - \beta \left(1 - \frac{L_{avg}}{L}\right) \left(1 - \frac{D_{avg}}{D}\right)\right] \frac{\ln\left(\frac{D}{u}\right)}{\pi\sqrt{K_{hor}K_{ver}}}$$

Equation 15

The principle relationships outlined in equations 13-14 link the aquifer resistance to the aquifer hydraulic conductivities (K_{hor} , K_{ver}), the aquifer thickness (D), the wetted perimeter of the SW bodies (u), the distance between two SW bodies (L) and the horizontal discretization of the hydrogeological model in the horizontal (L_{avg}) and vertical (D_{avg}) direction. They apply to isotropic and anisotropic aquifers. In the application presented hereafter however, only isotropic aquifers are considered.

2.2. Aquifer resistance in three dimensions (3D)

In a model cell, the 3D aquifer resistance ($\gamma_{3D,aqui}$, [T.L⁻²]) is straightforward to calculate from the 2D aquifer resistance ($\gamma_{2D,aqui}$, [T.L⁻¹]). As the volumetric flow is an integration of the 2D fluxes over the length of surface water features within a cell (L_{sw} [L]), $\gamma_{3D,aqui}$ is equal to $\gamma_{2D,aqui}$ divided by L_{sw} :

$$\begin{aligned}
\gamma_{3D,aqui} &= \frac{\gamma_{2D,aqui}}{L_{sw}} \\
&= \frac{M}{K_{sb} W_{sb} L_{sw}} + \frac{3LL_{avg} - L_{avg}^2}{24LK_{hor}DL_{sw}} + \left[1 - \beta \left(1 - \frac{L_{avg}}{L} \right) \left(1 - \frac{D_{avg}}{D} \right) \right] \frac{\ln\left(\frac{D}{u}\right)}{\pi\sqrt{K_{hor}K_{ver}L_{sw}}}
\end{aligned}$$

Equation 16

However, in 3D, the distance between surface water bodies (L) is not a straightforward variable to specify because the surface water network can be far from the ideal case of parallel bodies assumed in the derivation of the metamodel. Therefore, the next section presents a method to calculate a representative L all over a 3D modelling domain.

2.2.1. Representative distance between surface water bodies (L) in 3D

To calculate $\gamma_{aqui,3D}$ a representative distance between surface water bodies (L) has to be determined in each model grid cell of size G ([L]) containing one or more surface water bodies. L is calculated as:

$$L = \frac{S_{cell}}{L_{sw}} = \frac{G^2}{L_{sw}}$$

Equation 17

where S_{cell} is the surface area of the grid cell [L²] and L_{sw} is the length of the surface water features in the cell [L]. However, if L_{sw} is smaller than the grid cell diagonal ($D = G\sqrt{2}$, [L]), we consider that only one surface water feature is present in the cell. Therefore L is calculated using a larger G . Therefore, the calculation of L should start with a fine-resolution grid (i.e. small G), then G should be increased iteratively until S_{cell} gets close to the whole domain area and/or a value of L has been assigned to all fine-resolution cells.

In this work, the modelling domain is a 2400 m by 2400 m square in the horizontal directions. Thus, grids with increasing horizontal resolutions are generated: 5, 10, 20, 50, 100, 200, 300, 400, 600, 800, 1200 and 2400 m. Starting at the finest resolution (5 m), the values of L calculated using Equation 17 are attributed to the corresponding cells where L_{sw} is larger than D . For grid cells where L_{sw} is

smaller than D , L is calculated using the next larger G value. This is done iteratively until the grid resolution reaches 2400 m. Thus, S_{cell} is equal to the whole modelling domain and a value of L has been assigned to all 5 m resolution cells.

3. Numerical experiments

Section 3 investigates the conditions of an unconfined aquifer, with rather high hydraulic conductivities, drained by a dense complex network of small rivers and ditch drains using the numerical experiments presented below. The variables are representative of the well-documented Nete catchment around Mol, Belgium (e.g. Batelaan and De Smedt (2007), Dams et al. (2012), Gedeon (2008), Gedeon et al. (2011), Leterme et al. (2013)).

3.1. 2D Field scale numerical experiments

3.1.1.1. *Modelling domains and boundary conditions*

The modelling domain consists of 2D vertical cross sections with half a surface water body at the left boundary and a drainage divide at the right boundary. The length of the modelling domain ($\frac{L}{2}$) is 100 m, representing half of the distance between two parallel surface water bodies. The surface slope, represented by the top boundary, is 1 %. The depth of the SW body (H_d) is 1 m. The bottom width of the SW body (W_d) is 1 m, and the top width is 2 m. Through symmetry, only half of the SW body needs to be represented. An example of the modelling domain is shown in Figure 2.

Figure 2: 2D field scale modelling domain where $\frac{L}{2}$ is the length of the modelling domain, D is the thickness of the aquifer, W_d and H_d are respectively the width and the height of the half surface water body

A three year model was simulated using the Hydrus 2D/3D software (Šimůnek et al. 2006), which solves Richards equation in variably saturated media. The mass balance error tolerance was 1 % as it is a standard use with this software. The soil hydraulic properties were specified using the model of Van Genuchten (1980) and textural classes were represented by the conclusions of Carsel and Parrish (1988).

The top boundary condition was set to atmospheric conditions using meteorological data recorded from September 1st, 2012 until the September 1st, 2015 at SCK•CEN (Belgian nuclear research center, Mol, Belgium). The side and bottom boundary conditions were set to no flow conditions to represent the drainage divide on the right, the middle of the drain on the left and the bottom of the aquifer at the bottom. In the surface water body, the boundary condition was set to a specific boundary condition, which is a variable head but switches to a seepage face when the pressure head at the boundary drops below zero. The variable head was specified using a piezometric time series from a drained area close to Mol (Belgium).

The soil textural class, aquifer hydraulic conductivity (K_s) and thickness of the aquifer (D) were varied between different scenarios and are summarized in Table 1. Combination of these variables led to 9 models.

Table 1: Investigated variables in the 2D field scale numerical experiments

Variable	Values
Soil textural class	Sand, loamy-sand, sandy-loam
Hydraulic conductivity K_s (m/d)	7.5, 3.5, 1
Thickness of aquifer D (m)	10, 30, 50

3.1.1.2. Averaging of groundwater hydraulic head

At each time step where water was present in the surface water body, the hydraulic head simulated in HYDRUS was averaged over different domains by varying the vertical (D_{avg}) and horizontal (L_{avg}) extents (Figure 3). The different values used for L_{avg} and D_{avg} are summarized in Table 2. Therefore, each model output time series is processed considering each value of L_{avg} and D_{avg} , resulting in 900 different domains (9 models x 5 D_{avg} x 20 L_{avg}).

Figure 3: Schematic view and examples of the averaging length (L_{avg}) and thickness (D_{avg})

Table 2: L_{avg} and D_{avg} values

Variable	Values
L_{avg} (m)	5, 10, 15, 20, ..., 100
D_{avg} (m)	0.1.D, 0.2.D, 0.4.D, 0.6.D, D

L_{avg} and D_{avg} allow evaluation of cases where the boundaries of a grid cell do not coincide with the no flow bottom or lateral boundaries, which respectively represents the bottom of the aquifer and the water divide between two surface water bodies. L_{avg} thus denotes the horizontal grid boundary and D_{avg} signifies the vertical grid boundary.

3.1.1.3. Aquifer resistance estimation

In the numerical experiments, for each time step, the surface water hydraulic head is known as it is one of the boundary conditions. The averaged groundwater hydraulic head was calculated by averaging the pressure field for the different averaging distances as described in the previous section, while the flux towards the surface water body is known as model output. Therefore, the only unknown in Equation 1 is the conductance. A conductance value representative of all time steps is estimated, for each numerical experiments, using a linear regression of the simulated fluxes against the pressure head differences. The resistances are then calculated as the inverse of the conductances. The resistances derived in this way are referred to as fitted resistances. The accuracy of these regressions is assessed by the coefficient of determination (R^2) and residual standard deviation (RSD). These statistics measure how the GW – SW water flow interaction can be approximated using a constant conductance approach in these transient numerical experiments.

The estimated resistances are only representative of the aquifer because no resistance of the streambed was considered. The simulations led to the creation of a dataset consisting of one value of resistance (and conductance) for each combination of averaging scales (i.e. L_{avg} and D_{avg}), soil hydraulic parameters, and aquifer thicknesses. The obtained resistances were compared with

resistances calculated from the properties of the aquifer, the drainage network and the spatial discretization of the hydrogeological model using the proposed metamodel (derived in section 2). This comparison considered two criteria, the root mean square error (RMSE) and the mean absolute percentage error (MAPE) between the two resistances.

3.2. Upscaling of a 3D hydrogeological model

A synthetic upscaling experiment was designed to evaluate the accuracy of the aquifer resistance provided by the proposed metamodel within a steady-state 3D hydrogeological model. The analysis consisted of simulating GW – SW interactions over the same domain but with various horizontal and vertical discretizations. The model with the finest horizontal and vertical discretizations is considered as a reference. The GW – SW fluxes simulated by the metamodel-based approach and the classical MODFLOW approach are evaluated against the reference model GW – SW fluxes outputs by calculating both the total and spatialized errors. Ultimately, the performance of the two approaches are compared.

3.2.1. Modeling domain

The MODFLOW modelling domain is a rectangular block of 2400 x 2400 x 40 m (Figure 4). The domain includes both rivers and ditch drains. The river and ditch drain networks are representative of the Kleine Nete catchment (Belgium) and are taken from, respectively, the Flemish hydrographic atlas (“Vlaamse Hydrografische Atlas”, VMM 2017) and the IGN/NGI dataset (IGN/NGI 2017). The topography was set to a linear slope between 25 m at the southern boundary and 20 m at the northern boundary. The elevation of the water level in the rivers was defined as the ground elevation minus 1.5 m and the water elevation in the ditch drains as the ground elevation minus 1 m. The wetted perimeter of the surface water bodies was set to their width.

Figure 4: Horizontal domain of the 3D hydrogeological upscaling study. HH stands for the hydraulic head boundary conditions defined at the northern and southern boundaries

The boundary conditions were set to “no flow” at the bottom and at the eastern and western boundaries, and to constant head at the northern and southern boundaries. Vertically, a homogenous layer with a hydraulic conductivity of 1 m/d was considered. The streambed properties for rivers and ditches were set to a thickness of 0.1 m, a hydraulic conductivity of 0.5 m/d and a width of 1 m and 0.5 m, respectively.

3.2.1. Reference and upscaled models

The model with the finest discretization (i.e. 5 m × 5 m × 5 m) was used as a reference to evaluate the upscaled models (with larger discretization in horizontal and vertical directions). This was done for four different ways of calculating the GW – SW resistance, γ : the classical MODFLOW approach ($\gamma_{3D,MODFLOW}$) which considers only the streambed resistance (Equation 18), our proposed

metamodel ($\gamma_{3D,metamodel}$) that accounts for the aquifer resistance as well (Equation 19) and the two approaches proposed by Mehl and Hill (2010) (Equation 20 and 21), which are referred to as Mehl_01 and Mehl_02.

$$\gamma_{3D,MODFLOW} = \gamma_{3D,sb} = \frac{M_{sb}}{K_{sb}W_{sb}L_{sw}}$$

Equation 18

$$\gamma_{3D,metamodel} = \gamma_{3D,sb} + \gamma_{3D,aqui,meta}$$

$$= \frac{M_{sb}}{K_{sb}W_{sb}L_{sw}} + \frac{3LL_{avg} - L_{avg}^2}{24LK_{hor}DL_{sw}} + \left[1 - 0.5 \left(1 - \frac{L_{avg}}{L} \right) \left(1 - \frac{D_{avg}}{D} \right) \right] \frac{\ln\left(\frac{D}{u}\right)}{\pi\sqrt{K_{hor}K_{ver}L_{sw}}}$$

Equation 19

$$\gamma_{3D,Mehl_01} = \gamma_{3D,sb} + \gamma_{3D,aqui,Mehl_01} = \frac{M_{sb}}{K_{sb}W_{sb}L_{sw}} + \frac{\frac{D_{avg}}{2} - M_{sb}}{K_{ver}L_{avg}L_{sw}}$$

Equation 20

$$\gamma_{3D,Mehl_02} = \gamma_{3D,sb} + \gamma_{3D,aqui,Mehl_02} = \frac{M_{sb}}{K_{sb}W_{sb}L_{sw}} + \frac{\frac{D_{avg}}{2} - M_{sb}}{K_{ver}W_{sb}L_{sw}}$$

Equation 21

where $\gamma_{3D,aqui}$ are the aquifer resistance calculated by the different models [$L^{-2}.T$], $\gamma_{3D,sb}$ is the streambed resistance [$L^{-2}.T$], M_{sb} is the streambed thickness [L], K_{sb} is the streambed hydraulic conductivity [$L.T^{-1}$], W_{sb} is the streambed width [L], L_{sw} is the length of surface water in a cell [L], L is the spacing between two surface water bodies [L], K_{hor} is the horizontal hydraulic conductivity [$L.T^{-1}$], K_{ver} is the vertical hydraulic conductivity [$L.T^{-1}$], u is the wetted perimeter of the surface water body [L], D is the aquifer thickness [L], L_{avg} is the horizontal discretization of the groundwater model [L] and D_{avg} is the vertical discretization of the groundwater model [L].

However, the aquifer resistance in the reference model needs to be considered in order to have a unique reference model for both resistance expressions and avoid any bias in the comparison. Still, this is not expected to have a major impact on the results. This is done using the metamodel and Equation 18 becomes:

$$\gamma_{3D,MODFLOW} = \frac{M_{sb}}{K_{sb} \cdot W_{sb} \cdot L_{sw}} + \gamma_{3D,aqui,ref}$$

Equation 22

where $\gamma_{aqui,3D,ref}$ is the aquifer resistance calculated by the metamodel in a 5 m × 5 m × 5 m grid (reference model) [L⁻².T].

This is done only for the model using the usual MODFLOW expression because the models Mehl_01 and Mehl_02 already account for the aquifer resistance. Consequently, the reference models of Mehl_01 and Mehl_02 have different values of GW – SW resistance.

Regarding the discretization of the upscaled models, grid with block side lengths of 5, 10, 50, 100, 200 and 400 m in the horizontal plane and thicknesses of 5, 10, 20 and 40 m were considered. All combinations of these 6 horizontal and 4 vertical discretizations were investigated, leading to 24 configurations.

3.2.2. Evaluation of the upscaled models

First, the sum of the GW – SW fluxes in all model cells were compared. Then, the total absolute error and the absolute value of the relative error were calculated as:

$$total\ error = Q_{tot_ref} - Q_{tot}$$

Equation 23

$$relative\ total\ error = \left| \frac{Q_{tot_ref} - Q_{tot}}{Q_{tot_ref}} \right|$$

Equation 24

with:

- $Q_{\text{tot_ref}}$ = sum of groundwater fluxes to rivers or ditch drains in all reference model cells

- Q_{tot} = sum of groundwater fluxes to rivers or ditch drains in all upscaled model cells

In a second step, the spatial distribution of the GW – SW fluxes was evaluated. The fluxes computed by the reference model were aggregated in the upscaled model cells and the errors per model cell were calculated as:

$$error_i = Q_{\text{ref},i} - Q_i$$

Equation 25

with:

- $Q_{\text{ref},i}$ = aggregated flux of the reference model in cell i of the upscaled model
- Q_i = flux in cell i in the upscaled model

In a third step, the GW – SW fluxes errors per cell are summarized by calculating the RMSE as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (error_i)^2}{n}}$$

Equation 26

with:

- n = number of cell with rivers or ditch drains in the upscaled model

The magnitude of the GW – SW fluxes in a grid cell generally increases with the horizontal area of the grid cell, resulting in an increase of the error. The RMSE was normalized using the spatial variation of the aggregated fluxes of the reference model (Equation 27) in order to compare the performance of the models with different horizontal discretizations. The normalization was done in this way because the GW – SW fluxes can be both positive or negative, therefore the normalization with the mean value could be problematic.

474

$$NRMSE = \frac{RMSE}{\sqrt{\frac{\sum_{i=1}^n (Q_{ref,i})^2}{n} - \frac{Q_{tot,ref}^2}{n^2}}}$$

475

Equation 27

476

4. Results

477

4.1. 2D field scale numerical experiments

478 This section presents the results of the 2D field scale numerical experiments. First, two examples of
 479 the regressions used to calculate the GW – SW resistances from the 2D numerical experiments (fitted
 480 resistances) as well as the accuracy of all of them are shown. Secondly, the results of the comparison
 481 between the fitted resistances and the metamodel resistances are highlighted.

482

4.1.1. Estimation of the GW – SW conductances

483 Two examples of the regression used to calculate the GW – SW conductance from the numerical
 484 experiments are presented in Figure 5. An overview of the coefficients of determination (R^2) and
 485 residual standard deviations (RSD) of all the regressions is displayed in Figure 6.

486

487 **Figure 5: Examples GW – SW flux ($q_{2D,GW-SW}$) as a function of the hydraulic gradient ($h_{GW,mean} - h_{SW}$) and**
 488 **the linear regressions associated, for models with (a) an aquifer thickness (D) = 10 m, a vertical averaging**
 489 **length (D_{avg}) = 10 m and a horizontal averaging length (L_{avg}) = 100 m (a) and (b) D = 50 m , D_{avg} = 50 m**
 490 **and L_{avg} = 100 m (b)**

Figure 6: Coefficient of determination (R^2) and residual standard deviation (RSD) of the linear regressions used to calculate the conductance, for different combinations of aquifer thicknesses (D) and hydraulic conductivity (K_s)

The results show that the resistance approach with a time constant parameter can appropriately represent GW – SW water flow exchanges in the numerical experiments. It can be noticed however, that the errors increase as D and L_{avg} decrease.

4.1.2. Comparison of the fitted and the metamodel resistances

The comparison between the resistance calculated from the 2D numerical experiments and the metamodel with and without the correction coefficient α is presented in Figure 7. The ratio between D_{avg} and D is set as color scale. When not using the coefficient α , a significant deviation of the metamodel is observed for cases with D_{avg} smaller than D . The RMSE and MAPE are respectively 0.20 d/m and 31.2 %. This deviation is corrected quite well by introducing α with a manually fitted β value of 0.5 (see section 2.1.2 for the equations associated). The fit is better and the RMSE and MAPE improve to 0.13 d/m and 16.8 %.

The metamodel presents a slight overall overestimation of the resistance, which could be due to an overestimation of the length over which the hydraulic resistance is calculated. Note that the use of the α coefficient is not aimed to correct this overestimation.

Figure 7: Comparison of the resistance calculated from the 2D numerical experiments (fitted resistances) and by the metamodel, without (a) and with (b) the correction coefficient α

4.2. 3D hydrogeological model upscaling study

This section presents the results of the 3D hydrogeological model upscaling study. First, examples of the calculation of L (the representative distance between surface water bodies) are given. Secondly, the comparison of the total GW – SW net fluxes is presented and finally the results of the evaluation of the upscaled models against the reference model are discussed. Note that in the results presented in this section, negative fluxes are from GW towards SW.

The results of the two approaches proposed by Mehl and Hill (2010) (Mehl_01 and Mehl_02) are not presented in details here because they show the same shortcomings as in the original study and are not commonly used approaches in hydrogeological models. These results are in line with the original study conclusions and are presented in the supplementary material available on-line. Briefly summarized, the Mehl_01 approach leads to a decrease of the GW – SW resistance when the horizontal discretization increases. However, the numerical experiments show that the GW – SW resistance and the horizontal discretization are positively correlated. Moreover, in the vertical direction, this approach overestimates the increase in GW – SW resistance due to the increase in vertical discretization. Concerning the Mehl_02 approach, the GW – SW resistance does not increase with the horizontal resistance and the overestimation of the GW – SW resistance when the vertical discretization increases is even more pronounced than for Mehl_01.

4.2.1. Representative distance between surface water bodies (L) in 3D

Figure 8 shows the map of the calculated representative distance between the rivers and ditch drains, for horizontal discretizations of 50 and 400 m as examples. The representative distance calculated for the two resolutions are similar because they are both an aggregation at different scales of the same 5 x 5 m grid data. L values for rivers vary over a large range; from 55 to approximately 3000 m. The smaller values are mainly located in the confluence areas. The distance between surface water bodies, represented by L , varies from 45 to 815 m.

Figure 8: Map of the calculated representative distance between surface water bodies (L) for a horizontal discretization of 50 m (a: rivers, b: ditch drains) and 400 m (c: rivers, d: ditch drains). Note the different color scales for the rivers (a and c) and drains (b and d)

4.2.2. Comparison of the total GW – SW net fluxes

The total net rivers – groundwater and ditch drains – groundwater fluxes in all the upscaled models as well as the associated mean 2D resistances considering the MODFLOW expression are shown in Figure 9. Figure 10 shows the results when considering the metamodel resistance expression.

Figure 9: Total net rivers – groundwater and ditch drains – groundwater fluxes ($q_{3D,GW-SW}$) in all the upscaled models considering the MODFLOW resistance and the associated mean 2D resistances (γ_{2D})

Figure 10: Total net rivers – groundwater and ditch drains – groundwater fluxes ($q_{3D,GW-SW}$) in all the upscaled models considering the metamodel resistance and the associated mean 2D resistances (γ_{2D})

The results reveal that with the MODFLOW expression, the total groundwater – rivers fluxes increase with the horizontal discretization. This leads to an important overestimation at the coarsest resolution of 400 m. To a lesser extent, an overestimation of the fluxes is also observed when the vertical discretization increases. This overestimation is more important for the smallest horizontal discretization and seems to be negligible when the horizontal discretization is superior to 100 m. The observed overestimation of groundwater – river fluxes by the MODFLOW parameterization as the horizontal and vertical discretization increase is because the MODFLOW resistance does not consider the aquifer resistance. Indeed, the 2D resistances remain constant as the discretization gets coarser.

Concerning the groundwater – ditch drains fluxes, the vertical discretization seems not to have any noticeable impact. However, when the horizontal discretization increases the fluxes are underestimated. This is because when fluxes to rivers are overestimated, hydraulic heads drop, and smaller fluxes to the ditch drains is computed. This behavior caused groundwater to drop below the

ditch drains, in a part of the domain, so fluxes were no longer computed in these cells. The inactivity of ditch drains in part of the domain increases with the horizontal discretization size in the MODFLOW resistance models as is illustrated in Figure 11 (a) for a vertical discretization of 5 m (the patterns are similar for the others vertical discretizations). This behavior is also observed, although less importantly, when the vertical discretization increases as shown in Figure 11 (b) for a horizontal discretization of 5 m.

Conversely, the fluxes to ditch drains and rivers simulated by the proposed metamodel remain rather constant as the horizontal discretization increases. This indicates that the metamodel properly upscales the resistances. Concerning the vertical discretization, some improvement can be seen for the rivers in models with horizontal discretizations smaller than 50 m. However, for the horizontal discretizations larger than 100 m, the metamodel slightly underestimates the fluxes. A small bias can also be seen when looking at the effect of the vertical discretization on the groundwater – ditch drains fluxes. This is most probably due to the α correction parameter in the metamodel expression, as it is the only term that accounts for the vertical discretization size. However these small inconsistencies cover a very low error range of a few percent.

Figure 11: Area of inactive drain cell plotted against the horizontal discretization size (a) and the vertical discretization size (b)

4.2.3. Evaluation of the upscaled models

The section discusses some selected results from the complete comparison between upscaled models and the reference model. For the horizontal upscaling, only the results of the models with a vertical discretization of 5 m are presented. For the vertical upscaling, only the results of the models with a horizontal discretization of 5 m are presented. The remaining results are presented as tables in the supplementary material available on-line.

4.2.3.1. Evaluation of total GW – SW fluxes

The absolute and relative total errors of the upscaled models, compared to the reference model, for the horizontal upscaling are presented in Figure 12. The same results for the vertical upscaling are presented in Figure 13.

Figure 12: Absolute and relative total errors of GW – SW total fluxes of the horizontal upscaling study for the models of 5 m vertical discretization

Figure 13: Absolute and relative total errors GW – SW total fluxes of the vertical upscaling study for the models of 5 m horizontal discretization

Concerning the horizontal upscaling, the total errors associated with the MODFLOW resistance expression reach more than 500 m³/d or 25 % of the total flux for the rivers and more than 300 m³/d or 10 % of the reference flux for the ditch drains. In contrast, total errors for the models using the proposed metamodel remain lower than 50 m³/d or 5% of the reference flux for both rivers and ditch drains. With respect to this criterion and as far as the ditch drains are concerned, the models based on the MODFLOW resistance expression perform better than the ones relying on our metamodel for horizontal discretizations from 5 to 100 m (Figure 12). However, this should be regarded with caution as two different sources of error are compensating each other here. The underestimated resistance leads on the one hand to an overestimation of the fluxes to the rivers, and on the other hand, this resistance underestimation causes a drop in groundwater head together with a subsequent underestimation of the fluxes to the ditch drains.

Regarding the vertical upscaling, the absolute and relative total errors observed are much smaller than for the upscaling in the horizontal direction, probably because the upscaling is done over a smaller scale range. The models using the MODFLOW expression show a total error of the groundwater – rivers fluxes of approximately 7 %, for a vertical discretization of 40 m. The relative total errors of the groundwater – ditch drains fluxes remain below 2%. In contrast, using the metamodel resistance expression, relative total errors remain below 3 % for the rivers and below 2 % for the ditch drains.

4.2.3.2. Evaluation of GW – SW fluxes per cell

To illustrate GW – SW fluxes model outputs, maps of the GW – SW fluxes for the models with a horizontal discretization of 400 m and a vertical discretization of 5 m are presented in Figure 14. On the Figure 15, the map of GW – SW fluxes errors between the reference model and the two upscaled models as well as the associated histograms are presented.

Figure 14: Map of the GW – SW fluxes for the model with a horizontal discretization of 400 m and a vertical discretization of 5 m. a and b are the outputs of the reference model for the rivers and ditch drains respectively, c and d the outputs of the model using the usual MODFLOW resistance, e and f the outputs of the model using the metamodel resistance

Figure 15: Map of errors between the GW – SW fluxes computed by the reference model and the upscaled models with a horizontal discretization of 400 m and histograms associated. a and b are the map of errors observed using the MODFLOW expression for the rivers and drains respectively, c and d the map of errors observed using the metamodel expression, e and f are the histograms associated for the rivers and drains respectively

Concerning the rivers, the distribution of the errors observed when using the MODFLOW expression

is spread between approximately 0 and 100 m³/d. In contrast, when using the metamodel, the

distribution of errors is spread approximately between -50 and 50 m³/d and is centered on 0. This shows that the use of the metamodel removes the bias observed when the aquifer resistance is neglected.

Concerning the drains, the distribution of errors when using the MODFLOW expression is approximately between -100 and 100 m³/d. In contrast, when using the metamodel, the distribution of errors approximately between -50 and 50 m³/d.

The RMSE of the GW – SW fluxes per cell of the upscaled models compared to the reference model are shown in Figure 16 and Figure 17 for the horizontal and vertical upscaling, respectively.

Figure 16: RMSE and NRMSE of GW – SW fluxes per cell for the horizontal upscaling for the models of 5 m vertical discretization

Figure 17: RMSE and NRMSE of GW – SW fluxes per cell for the vertical upscaling for the models of 5 m horizontal discretization

Regarding the upscaling in horizontal direction, the RMSE increases to approximately 50 m³/d for the MODFLOW expression, which represents around 0.5 of the standard deviation of the reference

model for the ditch drains and around 0.7 for the rivers. In the case of the metamodel expression, the RMSE remains below $20 \text{ m}^3/\text{d}$ or 0.25 of the standard deviation of the reference model fluxes for both the rivers and the ditch drains. These values remain relatively high for both approaches. However, the improvement of the metamodel approach can be clearly seen. It is likely that a part of the RMSE increase is not linked to the scale dependency of the resistance but to other upscaling issues (e.g. averaging of hydraulic head, boundary conditions effect).

For the upscaling in the vertical direction, this criterion shows also a much lower error range for the metamodel. The models with the MODFLOW approach show a RMSE increasing to around $0.12 \text{ m}^3/\text{d}$ for the ditch drains and around $0.25 \text{ m}^3/\text{d}$ for the rivers. These values represent 0.15 for the ditch drains and 0.25 for the rivers of the standard deviation of the reference models fluxes per cell. Concerning the models using the metamodel resistance expression, the values are always lower, with RMSE lower than $0.1 \text{ m}^3/\text{d}$ for both rivers and ditch drains, which is around 0.1 of the standard deviation of the reference models fluxes per cell.

5. Discussion

This work presents a new metamodel to estimate aquifer resistance in Cauchy boundary conditions used to simulate groundwater – surface water interactions in regional hydrogeological models. This is achieved by deriving a new analytical equation and empirically correcting the equation of Ernst (1962) to obtain an expression which can be used in hydrogeological models.

Our metamodel is tested against 2D and 3D numerical experiments and shows good estimation of the groundwater – surface water fluxes as it manages to capture the scale dependency of the resistance that is caused by the flow processes occurring in the aquifer. Moreover, the proposed metamodel can be directly implemented in hydrogeological models, for any number of surface water bodies within a groundwater model cell, without requiring any additional numerical calculation.

671 It is important to note that the metamodel expression is derived under the assumption of a
672 homogeneous aquifer. For a layered aquifer with horizontal layers, the derivation of representative
673 horizontal and vertical hydraulic conductivities should be straightforward and could be used in the
674 metamodel equation. For more general cases, the estimation of representative hydraulic
675 conductivities would be trickier. However it could be achieved, for instance, by upscaling the
676 hydraulic conductivities field and/or using inverse modelling.

677 A second assumption in our analysis is the presence of a no flow boundary at the bottom of the
678 hydrogeological model. It should be noted though that the approach can be used for vertical
679 discretizations that are less than the thickness of the aquifer so that vertical flow across the bottom
680 of a grid cell in contact with surface water is in fact accounted for. For other kinds of boundary
681 conditions at the bottom of the hydrogeological model, some additional testing would be required.

682 A third assumption made in the derivation of the metamodel is that the GW and SW hydraulic head
683 can be approximated by the aquifer thickness . This seems to be a reasonable simplification for the
684 majority of hydrogeological models because the aquifer thickness is usually much larger than the
685 hydraulic head above the surface water bottom. It simplifies greatly the use of the metamodel in
686 hydrogeological models. The resistance could be derived without this simplification but would then
687 depend on the hydraulic head in the groundwater and the surface water, which would result in a
688 nonlinear model and thus add complexity to the numerical modelling exercise. However, in the case
689 where this assumption is unrealistic (e.g. highly incised channels), a possible alternative would be to
690 choose a level above the surface water bottom for the calculation of the horizontal head losses. Note
691 that, for the calculation of the radial aquifer resistance the surface water bottom level should be
692 chosen anyway.

693 The metamodel still needs to be validated for losing rivers (i.e. flux from surface water toward
694 groundwater) because the examples presented in this paper mainly consider gaining surface water
695 features.

A next step would be to use the proposed metamodel in a real world case study, either to parameterize directly hydrogeological models or to be incorporated in a calibration framework. Because it improves the conceptualization of the GW – SW resistance approach by decoupling the aquifer resistance from the streambed resistance, our approach should allow for transferring calibrated values in models of different horizontal and vertical discretizations. Moreover, it could also help for the investigation of streambed resistances and hyporheic processes.

6. Conclusion

A new expression for groundwater – surface water interactions in regional hydrogeological models is proposed. The main new aspect is that it includes the aquifer conductance in Cauchy boundary conditions as usually implemented in such models. The expression takes into account site-specific factors such as the hydraulic conductivity, the aquifer thickness and the distance between two surface water bodies but also model factors as the grid-cell size in both vertical and horizontal directions. One of the main advantages is that streambed and aquifer resistances are transferable between different model discretizations. The expression is derived from the Dupuit-Forcheimer theory, the Ernst drain design equation and an empirical correction parameter derived from detailed 2D (cross sectional) numerical experiments at the field-scale. Our expression appears captures in a satisfactory way the relationship observed in the 2D numerical experiments between the aquifer conductance and the aquifer characteristics. It is also tested in a 3D hydrogeological model upscaling study (both in horizontal and vertical directions) and compared with the MODFLOW expression, which shows grid-size dependency problems, and the alternatives proposed by Mehl and Hill (2010). The proposed expression outperforms the other approaches and is found to capture a substantial part of the conductance grid-size dependency without adding any complexity to the numerical modelling exercise. We therefore argue that it is an improvement over the existing approaches for representing groundwater – surface water interactions in regional hydrogeological models.

7. Acknowledgments

The authors are grateful to ENGIE for sponsoring this work and to the two anonymous reviewers for their comments that substantially improved the manuscript.

Declarations of interest: none, the sponsor had no involvement in the research work presented in this article.

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