Design and experimental validation of a metamaterial solution for improved noise and vibration behavior of pipes

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Abstract. This paper investigates a metamaterial solution for efficient vibration attenuation and acoustic radiation reduction of an aluminum pipe. To this end, using unit cell predictions, locally resonant structures are designed to have a pronounced flexural resonance frequency at the vicinity of a dominant vibration mode of the pipe. A direct approach of the Bloch-Floquet theorem is adopted to provide the dispersion relation representing wave motion in an infinite metamaterial pipe. Using these wave dispersion relations, the frequency range of the stopband zone created by the metamaterial solution is predicted. The dynamic behavior of the finite counterpart is predicted using the Finite Element Method (FEM). The resonant structures are produced from polymethyl methacrylate (PMMA) panels and are added to the host structure. In order to properly characterize both the vibrational behavior of the metamaterial pipe and the acoustic radiation from its wall, impact tests using roving hammer technique is performed on the pipe and both accelerations and acoustic pressures are measured at different locations. The experimental results show a pronounced stopband zone created by the addition of a few rows of resonant structures. Moreover, comparisons between the measurements and numerical predictions show a good agreement.

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1 INTRODUCTION

Improving Noise, Vibration and Harshness (NVH) behavior of mechanical systems has been a concern of many sectors of industry for a long time. The increasing trend of lightweight design has resulted in different families of novel lightweight materials and structures which often show unfavorable NVH behavior; considering that as opposed to their conventional counterparts, they tend to have much less effective mass, but the same order of stiffness. Therefore, there is a need for novel NVH solutions which also satisfy the lightweight criterion. In addition, it is imperative that these solutions do not interfere with the primary function of the system or reduce its structural integrity.

In the search for efficient NVH solutions, locally resonant vibro-acoustic metamaterials come to the fore as possible solutions for vibration attenuation and sound insulation problems [1, 2, 3, 4, 5]. These metamaterials have been shown to improve the NVH behavior of a host structure, at least in targeted frequency ranges known as stopbands [6, 7]. A stopband is a frequency range within which free wave propagation in the structure is hindered. The stopband behavior in metamaterials is obtained due to the addition of local resonators to the host structure on a subwavelength scale, i.e. a scale smaller than the governing wavelength of the waves propagating in the host structure. The destructive interference between the waves radiating in the structure and those re-radiated from the local resonators results in a stopband zone in vicinity of the resonance frequency of the resonators. This type of interference is referred to as Fano-type interference and it was first explained in analysis of inelastic autoionizing resonances in atoms [8] and later extended to locally resonant metamaterials by Goffaux et al. [9]. In this sense, the metamaterial solution does not need to be periodic in order to achieve stopband behavior in the host structure and the criteria which should be met are the subwavelength distribution of the resonant structures on the host structure and that the net sum of the forces on the host structure exerted by the resonators needs to be non-zero [6, 10, 11]. It has also been shown that addition of resonant structures with different resonance frequencies can result in metamaterials with multiple stopband zones [12, 13, 14]; as long as the subwavelength criterion is met.

As a novel solution for NVH problems, locally resonant metamaterials have been widely investigated by many researchers in recent years. Fundamental academic studies in which local resonators are modelled as grids of mass-spring have been conducted to provide a better insight on the dynamic behavior of metamaterials [6, 15, 16, 3]. However, recently, there have been examples of actual metamaterial designs which are examined numerically and produced as demonstrators [4, 14]. These studies, respectively, prove the increased acoustic insertion loss of a finite enclosure and increased vibration attenuation in a finite duct due to the addition of the resonant structures. These metamaterial demonstrators can serve as a valuable step towards application of metamaterials as an engineering solution for real NVH problems.

Whether metamaterials are represented by grids of mass-springs or detailed realizable designs, there is a need for numerical design tools, which can reliably predict stopband zones. Metamaterials can be modelled using Finite Element Method (FEM) as fully detailed finite structures under real boundary conditions. However, these Finite Element (FE) models are often computationally expensive, especially in early stages of design. Therefore, in order to reduce the required computational effort, it has been shown that assuming periodicity in an infinite structure, wave motion in the structure can be characterized by investigation of a unit cell and applying the Bloch-Floquet theorem [17, 18, 19, 20]. A unit cell is defined as the smallest portion of a periodic structure which contains all the essential information required to represent wave motion in the structure. In this manner, wave motion in the infinite periodic structure is usually described using dispersion curves, i.e. curves which relate wavenumbers

of waves to the frequency at which they propagate. The Bloch-Floquet theorem provides the dispersion relation as an eigenvalue problem in terms of frequency and wavenumber vector. This eigenvalue problem can be solved at given propagating wavenumbers to obtain the frequencies of free wave propagation, called the inverse approach [21] or alternatively, it can be solved at given frequencies for complex wavenumbers, called the direct approach, providing information on both propagating and attenuated waves [22]. In an undamped infinite metamaterial structure, stopband zones appear as gaps in the dispersion curves in the frequency range around the resonance frequency of the local resonators. Although, this stopband prediction is primarily accurate for an infinite periodic metamaterial structure, it has been shown that it can also give a good approximation of the stopband zone in the finite structure with aperiodic distribution of the resonators [10].

Numerical and analytical investigation of wave motion in cylindrically curved structures has been widely performed. In simpler cases, analytical formulations can be derived to describe wave motion in thin cylindrical shells and simple cylindrical structures [23, 24] and in some cases, these formulations can be extended to three-dimensional elasticity [25, 26]. In addition to general analytical solutions, several simplifying assumption and approximation theories can be utilized to represent wave motion in certain cylindrical structures analytically [27, 28, 29]. The FEM and Wave Finite Element Method (WFEM) have also been used to analyze wave dispersion relations in cylindrical shells and pipes [30, 31]. Recently, Silva et al. have utilized WFEM to analyze the forced response of coupled flat and cylindrical systems based on substructuring [32]. They have further investigated the use of WFEM in passive vibration control of 1D academic rod and 3D aircraft cylindrical fuselage-like structures [33]. Such numerical methods can facilitate the analysis of curved structures with more geometrical and mechanical complexities. In general, it is observed that the dynamics of a cylindrically curved structure is considerably different from its flat counterpart [31]. It is observed that, especially at frequencies below the ring frequency, motion of different types of waves are coupled. Therefore, starting from the metamaterial design proposed by Claes et al. [10], the previous findings of stopband behavior in locally resonant metamaterials was extended to the case of cylindrically curved metamaterial shells and pipes by the authors [34, 35]. In these studies, the Bloch-Floquet theorem was utilized to predict stopband zones in infinite periodic cylindrical metamaterials and the results indicated that due to the existence of the ring frequency, the stopband behavior is considerably different from that of a flat metamaterial. It was observed that tuning the local resonators to have a resonance frequency in vicinity of the ring frequency can result in a more pronounced stopband zone. Investigation of cylindrical metamaterials and stopbands are not limited to these studies; wave propagation and dispersion analysis in compound pipes with various curvatures with and without internal fluid loading have been investigated by Se-Knudsen and Sorokin [36]. They have suggested that the curved inserts can be used to tailor the desirable stopband characteristics.

In this paper, numerical findings of the work performed by Nateghi et al. [35] are utilized to design and produce a metamaterial solution which improves the NVH behavior of an aluminum pipe. While works by Nateghi et al. [35, 34] emphasize on the application of unit cell modelling to study vibrational behavior in infinite cylindrically curved metamaterial panels and infinite cylindrical metamaterial pipes, the current study concentrates on the experimental analysis and validation of the vibro-acoustic behavior of a finite metamaterial pipe. Given the importance of flexural waves in the acoustic radiation from panels and shells, the resonator is designed to have a pronounced flexural resonance at 800 Hz which is in vicinity of a dominant vibration mode of the pipe. Afterwards, assuming a periodic distribution of the local resonators on an infinite pipe, the effects of adding these resonators on the infinite structure is investigated using the Bloch-Floquet theorem and unit cell

modelling. In this step, the dispersion curves of the host and metamaterial structures are obtained and compared and the stopband zone is numerically predicted. In the next step, the metamaterial solution, i.e. the local resonators, are produced from PMMA panels using a laser cutter and applied to the aluminum hosting pipe. Finally, experimental modal analysis of the metamaterial pipe is performed in a semi-anechoic chamber using roving hammer technique and the structural acceleration of the pipe and the acoustic radiation from it are measured. The results of the experimental campaign prove the existence of a stopband zone which is also in good agreement with the numerical predictions and validate the existence of the stopband zone in the frequency response of a finite metamaterial pipe as was predicted for an infinite metamaterial pipe [34].

While for flat finite plates the acoustic and vibrational efficiency of metamaterial solutions have already been investigated numerically [6, 7], and for finite ducts with flat sides the structural behaviour has been studied both experimentally and numerically [14], the current paper studies, both numerically and experimentally, the vibro-acoustic behavior of a finite cylindrical metamaterial pipe, in which acoustic radiation is not only the result of the flexural bending of the pipe, but also due to the higher circumferential deformation modes of the pipe. Moreover, the experiment is repeated for pipes with different numbers of added resonators to depict the gradual build-up of the vibration attenuation and the reduction in acoustic radiation in the stopband zone. In this manner, the stopband behavior can be related to the amount of added mass due to the addition of resonators. The gradual addition of the resonators provides extra information on the build-up of the attenuation in the stopband zone which adds to the findings by Claeys et al. [14]. That study demonstrated the effectiveness of the metamaterial solution on a finite flat duct, while in the current study the cylindrical curvature of a finite pipe is tackled. Moreover, their experimental set-up did not include the measurement of acoustic radiation from the metamaterial duct.

The paper is structured as follows. Section 2 introduces the host structure as the application case study in more details and provides a FE simulation of the dynamic behavior of the host structure. Section 3 describes the design of the local resonators as the metamaterial solution. Section 4, at first, provides an overview of unit cell modelling and its application to metamaterials in order to detect stopband zones using dispersion curves and then predicts the stopband zones achieved by the current metamaterial solution. Section 5 describes the test set-up and the experimental results are discussed and compared with those obtained from numerical simulations in section 6. The paper ends with a summary of the main conclusions of the research.

2 UNIT CELL MODELLING

As stated in the introduction, in a metamaterial solution, resonant structures do not need to be distributed on the host structure periodically; however, at least in the early stages of the design, assuming a periodic distribution of the resonators can reduce the required computational effort for the numerical prediction of the stopband zone through the application of the unit cell modelling and Bloch-Floquet theorem [17, 18].

Considering the pipe-like structure used in the current study, the reader is referred to [34] for the detailed formulations of unit cell modelling in infinitely long cylindrical pipes. In such a pipe-like structure, the circumferential component of the wavenumber k_ϕ needs to be an integer value to account for the axisymmetric geometry; as also shown in Fig. 1. In this manner, by setting integer values to k_ϕ and solving the remaining quadratic eigenvalue problem for the axial component of the wavenumber k_z at fixed frequencies, one can obtain the dispersion curves representing wave propagation of the different circumferential modes of the pipe.

Fig. 1. Representation of the wavenumber vector in a pipe as shown in [34].

In general, solving the dispersion relation at fixed values of frequencies and k_ϕ provides complex solutions for the axial component of the wavenumber with real and imaginary parts representing propagating and attenuated parts of waves, respectively. However, in the following sections, the effects of damping on the stopband behavior are neglected and therefore, the wavenumber solution contains only real values corresponding to undamped propagating waves. Moreover, it is usually more convenient to analyze the wave propagation vector $\mu = k \cdot d$ instead of the wavenumber vector k ; in which d is the lattice basis vector. The wave propagation vector describes phase shift of the propagating wave per unit cell. Therefore, in the dispersion figures presented in this section the axial component of the wave propagation vector μ_z is presented instead of k_z .

Considering that Bloch-Floquet boundary conditions are applied to the unit cell, solving the wave dispersion relation results in a periodic set of solutions in the wavenumber domain. This periodic set of solutions has the interval of $\left[-\frac{\pi}{L_z}, \frac{\pi}{L_z}\right]$ for k_z in which L_z is the length of the unit cell in the axial direction while the interval of solution for μ_z is $[-\pi, \pi]$.

3 HOST STRUCTURE

In this section, the host structure is introduced and its dynamic behavior is predicted using FEM. Moreover, wave propagation in infinite structure is also analyzed using unit cell modelling. The numerical results can provide a good insight into the behavior of the structure in different frequency ranges.

3.1 Geometry and FE analysis of the host structure

In this work, an aluminum pipe is chosen as case study for the metamaterial treatment. Given the wide range of usages of lightweight pipes in piping networks amongst other applications on one hand and their possible high noise radiation due to their low mass, hollowness and small wall thickness on the other hand, such a structure is an excellent candidate to show the effectiveness of a metamaterial solution. The material properties of the aluminum pipe are provided in Table 1. The density is calculated through a weight measurement. The Young's modulus and Poisson ratio are obtained based on the correlation between the eigenfrequencies of the pipe calculated with the FE model of the bare pipe and experimentally measured ones on the bare pipe. For the FE model shell elements are used and the pipe is considered in free-free conditions. For the experimental test, the pipe is freely suspended and a roving hammer approach with 4 lightweight accelerometers, type PCB 352A24, are applied to perform the experimental modal analysis. The difference between the

eigenfrequencies and the frequency response of the pipe calculated with the FE model and measured is less than 1%.

The length of the pipe is 1 m, it has a wall thickness of 0.0021 m and its inner radius is 0.048 m.

Table 1. Material properties of the aluminum pipe and the PMMA resonant structures.

Material	Young's modulus [GPa]	Poisson ratio [-]	Density [kg/m ³]
Aluminum	68	0.36	2696.2
PMMA	4.84	0.31	1188.38

An important step in the design of the metamaterial solution is the analysis of the dynamic behavior of the host structure in order to determine the frequency ranges with unfavorable NVH behavior. To this end, the aluminum pipe is modelled in NX 11 [37] under a point excitation at 5 mm from the end of the pipe and free boundary conditions as shown in Fig. 2, and its dynamic frequency response is evaluated over the length of the pipe.

Fig. 3 provides the frequency response of the aluminum pipe in the form of RMS acceleration. RMS response of the pipe is calculated according to the following formula:

$$|H|_{\ddot{x}_{F_e},rms} = \sqrt{\frac{1}{N} \sum_{n=1}^N |H|_{\ddot{x}_{n/F_e}}^2}, \quad (1)$$

in which $|H|_{\ddot{x}_{n/F_e}}$ indicates the amplitude of the acceleration frequency response in point n for an excitation in point e , N is the number of measurement points on the pipe. Please note that from here on acceleration refers to radial acceleration of the pipe.

Fig. 2. The aluminum pipe as modelled in Siemens NX 11.

Fig. 3. Numerically predicted frequency response of the aluminum pipe.

Fig. 4. Magnitude of the radial displacement of the aluminum pipe under concentrated load at 800 Hz.

As shown in Fig. 3, there is a pronounced peak occurring around 800 Hz. The vibration pattern of the structure at this frequency is given in Fig. 4 in the form of magnitude of the radial displacement. Considering the radial motion of the structure, this mode can result in a considerable acoustic radiation. Therefore, this frequency range is targeted in the design of the metamaterial solution.

3.2 Wave propagation in the host structure

Fig. 5(b) shows the representative unit cell of the bare aluminum pipe. Given that the pipe is homogeneous, the choice of unit cell is arbitrary and in this case, the angular size of $\pi/4$ and axial length of 0.04 m are considered.

Fig. 5. Representative of the FE unit cell model used for (a) the metamaterial pipe and (b) the host pipe.

The unit cells of Fig. 5 are modelled in COMSOL Multiphysics following the formulations presented at [34] and the dispersion relation is solved for values of μ_z at fixed values of k_ϕ and frequency. Quadratic tetrahedron solid elements of maximum length 0.0015 m are used to discretize the model of the host structure, in Fig. 5(b), resulting in 11751 elements and 58164 degrees of freedom. The results of these dispersion analyses are used to study wave propagation in the Aluminum pipe.

Fig. 6. Dispersion curves representing wave motion in the infinite bare aluminum pipe with circumferential wavenumbers numerically equal to (a) 0, (b) 1 and (c) 2, respectively.

Fig. 6 shows the dispersion curves describing the propagating parts of waves with the circumferential wavenumber $k_\phi=0, 1$ and 2 rad/rad in the infinite aluminum pipe represented by the unit cell model of Fig. 5(b). The three graphs represent $k_\phi=0$ (breathing mode), $k_\phi=1$ (beam bending mode) and $k_\phi=2$ (ovalling mode), respectively. Fig. 7 shows the deformation of a cross-section of the pipe with respect to each of these modes. As mentioned, only the real part of μ_z as the propagating part of the waves is provided in the figure as it suffices for the purpose of numerical prediction of the stopband zone. It is worth mentioning that negative and positive values of μ_z corresponds to forward and backward propagating waves, respectively.

Fig. 7. Deformation of the cross-section of the pipe in breathing, beam bending and ovalling modes with circumferential wavenumbers numerically equal to (a) 0, (b) 1 and (c) 2, respectively.

In the dispersion curves for $k_\phi=0$ rad/rad, two pairs of waves which correspond to longitudinal and shear waves appear in the frequency range of 0 Hz - 1000 Hz. Considering the goal of the research which is the improvement of NVH behavior of the pipe by decreasing its acoustic radiation, these waves are not of interest and would not be affected by the addition of the metamaterial solution. In this frequency range, the flexural wave which contributes to the acoustic radiation does not propagate in the pipe as it is only propagative at 17417 Hz.

However, the dispersion curves of beam bending and ovalling modes show pairs of propagating flexural waves in the frequency range of targeted stopband zone around 800 Hz. Given the current design of the metamaterial solution, these waves are expected to be affected by the addition of the resonators. Wave dispersion of higher circumferential modes is not investigated in this section as in the frequency range of interest, flexural waves of these modes are not propagative yet.

4 LOCALLY RESONANT METAMATERIAL PIPE

The design of the metamaterial solution is explained in this section and its resonance frequency is evaluated using COMSOL Multiphysics® [38]. In addition, unit cell modelling is performed during the design process to choose the resonant structure which creates the desired stopband zone. The analysis of the final resonant structure is provided in this section.

4.1 Geometrical design and FE analysis of the resonant structure

The main goal of the current research is to improve the NVH behavior of the structure; therefore, given the importance of the flexural waves in acoustic radiation of structures, the metamaterial solution is designed to introduce a stopband zone in the propagation of flexural waves in the host structure. To this end, the local resonators are tuned to have a pronounced

low-frequency flexural mode and they are distributed on the host structure on a subwavelength scale to conform to the rules of metamaterial design [35]. Such a metamaterial solution should be able to deal with the dominant mode of the pipe occurring at 800 Hz as discussed in the previous section.

The metamaterial structure is composed of the aluminum pipe of section 2 as the host structure and resonant structures produced from a PMMA panel using a laser cutter as the metamaterial solution. In the current work, a similar design as used by [14] is adopted for the local resonators as shown in Fig. 8. This resonant structure is chosen due to the following three main reasons:

- Proven functionality in terms of stopband creation in similar flat structures [14].
- Ease of production of the resonant structure from flat PMMA panels through laser cutting.
- Ease of resonance frequency tuning as the resonant structure resembles a cantilever beam.

The material properties of the PMMA resonators are given in the last row of Table 1. The material properties are updated from the nominal properties through correlation of measured and numerically predicted resonance frequencies of the resonators.

Fig. 8. Representation of the resonant structures.

The dimensions of the resonant structure shown in Fig. 8 should be chosen in a way that the first flexural eigenfrequency of the resonator occurs around the frequency where the stopband effect is desired. Moreover, it is intended to keep the added mass of the resonator cells to the host structure as low as possible in an attempt to meet the lightweight criterion of the metamaterial solution. The geometrical parameters shown in Table 2 are chosen. All of the resonators are afterwards cut from PMMA panels of 0.008 m thickness; therefore, the thickness of the resonators are kept constant to facilitate its production through laser cutting.

Table 2. Geometrical parameters of the resonant structure shown in Fig. 8.

Dimensions [*10 ⁻³ m]	L ₁	L ₂	L ₃	L ₄	L ₅	L ₆	L ₇	L ₈
Designed value	3.12	3.54	3.00	12.00	6.5	12.00	13.51	16.49

The resonant structure is modelled using the properties and the dimensions stated in Tables 1 and 2. The corresponding FE model is discretized using 9536 quadratic hexagonal elements

resulting in 44271 degrees of freedom. Fixed boundary conditions are considered at the base of the resonator to imitate the condition in which resonators are glued to the aluminum pipe. The first two modes of the resonant structure are shown in Fig. 9. As it can be seen the current design has resulted in a pronounced flexural mode at 803.18 Hz while the second eigenfrequency of the resonator occurs at 1123.5 Hz.

Fig. 9. Mode shapes of the first and second eigenfrequencies of the resonant structures. The wireframe represents the undeformed state of the resonant structures.

4.2 Wave propagation in the metamaterial pipe

It is assumed that 8 resonators are distributed around the circumference of the aluminum pipe with equal angular distances, this row of resonators is then infinitely repeated with the equal distance of 0.04 m between each two adjacent resonators in the axial direction. The representative unit cell of the metamaterial pipe is shown in Fig. 5(a). The unit cell is modelled using quadratic tetrahedron solid finite elements of maximum length 0.0015 m resulting in 21600 elements and 103122 degrees of freedom.

(c) Owalling mode

Fig. 10. Dispersion curves representing wave motion in the infinite metamaterial pipe with circumferential wavenumbers numerically equal to (a) 0, (b) 1 and (c) 2, respectively.

Fig. 10 presents the dispersion curves corresponding to the propagating parts of the waves with $k_\phi=0, 1$ and 2 rad/rad based on the unit cell model of the metamaterial pipe as shown in Fig. 5(a). As it can be seen, the types of the waves which appear in these curves are comparable to those observed in subsection 2.2. However, the addition of the resonator has clearly affected the dispersion curves of the flexural waves in the beam bending and owalling modes by causing discontinuities in these curves. These gaps in the dispersion curves are referred to as stopbands and define a frequency range in which there is no free flexural wave propagation. The predicted stopband zones due to the addition of the metamaterial solution are $800 \text{ Hz} - 840 \text{ Hz}$ and $800 \text{ Hz} - 885 \text{ Hz}$ for beam bending and owalling modes, respectively. Therefore, based on these curves, an overall stopband zone between 800 Hz and 840 Hz is expected in the propagation of the flexural waves in the metamaterial pipe. However, in the frequency range of $800 \text{ Hz} - 885 \text{ Hz}$ a considerable amount of attenuation can be obtained by applying this metamaterial solution to the aluminum pipe.

Dispersion curves of beam bending and owalling modes also show the influence of the resonant structures on the cut-on frequency of the flexural waves. It is observed that below the resonance frequency where the motion of the resonant structures and the host structure are in-phase, the cut-on frequency of the flexural is lowered while above the stopband zone this frequency is increased. Above the stopband zone, the motion of the host and the resonators are out-of-phase which results in a lower dynamic mass and therefore a higher cut-on frequency. Readers are referred to [35] for more details. The effect of the metamaterial solution on the cut-on frequency of higher order waves shows a major difference between wave propagation in flat metamaterials and curved metamaterials.

Dispersion curves for $k_\phi=0$ rad/rad show that although the flexural wave is not propagative below 1000 Hz , longitudinal and shear waves are still affected by the addition of the resonant structure. As discussed in [35], below the cut-on frequency of the flexural wave for $k_\phi=0$ rad/rad, which is known as the ring frequency, waves are not of pure flexural, shear and longitudinal nature and therefore, the out-of-plane component of the waves can interact with the motion of the resonant structures. Therefore, the effect which is observed in the figure is related to the coupled nature of the waves. This behavior is not observed in flat metamaterials as investigated by Claeys et al. [6, 14].

5 EXPERIMENTAL SET-UP

This section describes the specifications of the test set-up and provides information on the production aspect of the metamaterial solution.

5.1 Production of the resonators

As mentioned, the resonant structures are produced from PMMA panels as shown in Fig. 11. The resonators are all cut from the same panel and using the same machine to increase the consistency of the production process. However, due to production variability, a slight change in the actual resonance frequency of the resonant structures is expected. Table 3 provides the measured dimensions of the produced resonant structures. These values are obtained by averaging the dimensions of 5 random resonant structures. The average mass of the produced resonant structure is measured to be 2.2 g with a standard deviation of ± 0.02 g.

Fig. 11. Produced PMMA resonators next to a coin.

Fig. 12 shows the test set-up which has been used to measure the actual resonance frequency of the produced resonant structures. To this end, a Polytec PSV-500 Mobile Scanning Laser Doppler Vibrometer (SLDV) is used to measure the acceleration of the tip of the resonators under shaker excitation. The resonators are glued to a rigid thick aluminum plate and the plate is excited by the shaker. In order to have a better contact surface between the resonators and the rigid plate, the design of the resonant structure is slightly changed to have a flat base. Numerical simulations show that the effect of this change in the design of the resonator on its flexural resonance frequency is negligible and below 1 Hz; given that the change is just limited to the part which is glued to the rigid plate. Ten resonators with flat base are cut from the same PMMA panel and their frequency responses are measured using the SLDV.

Table 3. Geometrical parameters of the produced resonant structure.

Dimensions [*10 ⁻³ m]	L ₁	L ₂	L ₃	L ₄	L ₅	L ₆	L ₇	L ₈
Measured value	2.97±0.03	3.52±0.03	2.74±0.03	11.91±0.06	6.41±0.02	11.71±0.07	13.56±0.08	16.07±0.06

Fig. 12. SLDV set-up for flexural eigenfrequency measurement of 5 of the resonators.

Fig. 13 shows the measured acceleration of the resonators in the frequency range of 0 Hz to 1000 Hz. Only the response of 5 samples are shown in this figure. Based on the measurement results, there is a clear resonance frequency between 789 Hz – 810 Hz, which is seen as a peak in the figure. Based on the measurements performed on 10 samples, the average resonance frequency of the produced part lies at 796 Hz which is in an acceptable agreement with the resonance frequency predicted by the numerical simulations. In addition, based on these measurements the difference between highest and lowest values of resonance frequencies of the produced resonant structures is 21 Hz with a standard deviation of 6.3 Hz with respect to the average value of 796 Hz. The scatter in the resonance frequencies could influence the performance of the metamaterial solution in the stopband zone; however, it can also be beneficial in the sense that it can broaden the frequency range of the stopband zone while reducing the maximum amount of attenuation [10]. The current scatter of 21 Hz between the resonance frequencies with a maximum relative error of 1.76% with respect to the average resonance frequency is in an agreeable range for the purpose of stopband creation as discussed in [10].

Fig. 13 can also be used to verify the material properties of the PMMA resonators including their damping parameter. To this end, the half-power bandwidth method is utilized to estimate the damping parameters of the resonators [39]. Based on this method, loss factor η can be evaluated using the following equation:

$$\eta = \frac{\omega_2 - \omega_1}{\omega_n}, \quad (2)$$

in which ω_n is the measured resonance frequency, ω_1 and ω_2 are frequencies at which the measured acceleration is $1/\sqrt{2}$ times the measured acceleration at the resonance frequency. Using these ten measurements, the average loss factor of the resonators is evaluated as 0.05.

Fig. 13. Frequency response of the produced PMMA resonators under shaker excitation measured by SLDV.

5.2 Test set-up

The specifications of the test set-up used for experimental characterization of the frequency response of the metamaterial pipe are discussed in this section. Fig. 14 shows the test set-up which is used to mount the metamaterial pipe. The pipe is freely suspended using thin guitar strings to limit the effects of boundary conditions on its dynamic behavior. Given that in addition to the dynamic behavior of the pipe, the acoustic radiation is also of interest, the test is performed in a semi-anechoic chamber to reduce the effects of background noise on the acoustic measurements.

Fig. 14. Test set-up.

Given the low mass of the pipe, hammer excitation is chosen instead of shaker excitation for the test. Four lightweight accelerometers of 0.8 g are glued to the pipe to measure the dynamic frequency response of the pipe under hammer excitation. Given the low mass of the accelerometers as compared to the mass of the pipe, their influence on the dynamic behavior of the pipe is negligible. In addition, seven microphones type PCB 378B02 are used to measure the acoustic pressure around the pipe and characterize the noise radiated from it. The exact location of the microphones and the accelerometers are given in Tables 4 and 5. The

coordinate system for these tables is centered at the centerline of the pipe and the zero of the axial axis (X-direction) is set at one of the ends of the pipe, as shown in Fig. 15.

In order to fully depict wave propagation through the metamaterial pipe there is a need for various measurement points on the pipe. This is achieved by using roving hammer technique in order to avoid increasing the number of required accelerometers. Based on the linearity of the system, the reciprocity principle can be utilized, which states that in a linear system, the response and input points can be interchanged. This means that the excitation point is varied and the measurement points are kept constant. Using this principle, the vibration behavior of the entire pipe can be characterized with only four lightweight accelerometers and hence reduce the mass loading effect of the added accelerometers.

Table 4. The location of accelerometers on the metamaterial pipe.

Acceleration number	Radius [$\times 10^{-2}$ m]	Axial location [$\times 10^{-2}$ m]	Angle [rad]
Point 1	5.01	0	0
Point 2	5.01	36	π
Point 3	5.01	64	$\pi/2$
Point 4	5.01	92	$\pi/4$

Table 5. Microphone locations around the metamaterial pipe.

Microphone number	X [$\times 10^{-2}$ m]	Y [$\times 10^{-2}$ m]	Z [$\times 10^{-2}$ m]
1	99	0	0
2	31	0	36
3	49	0	-29
4	47.5	-20	6
5	73	21	-11
6	25	19	18
7	67	0	34

Fig. 15 schematically represents the metamaterial pipe, its 26 excitation sections, the locations of the accelerometers and the locations of the resonance cells. On each excitation section, 8 equidistant points around the circumference are chosen, corresponding to a total of 208 excitation locations on the pipe. In addition, as discussed in section 4, 8 resonators are glued around the circumference of the pipe in each section and the distance between two consecutive resonators is 0.04 m in the axial direction.

To fully comprehend the effects of adding different rows of resonators on the vibration attenuation and acoustic radiation reduction in the stopband zone, the measurement is repeated 14 times. First, the response of the bare aluminum pipe is measured to serve as the reference case; afterwards, a row of resonators is glued to the left side of the pipe and the response is measured again. This procedure is repeated for the next 9 rows, where the rows are added consecutively from left to right. After the addition of the tenth row, the measurement is repeated adding 5 resonator rows each time to the pipe, given the fact that the reduction in the stopband zone is already pronounced when 10 resonator rows are added to the pipe.

As mentioned before, the averaged measured mass of each resonator cell is 2.20 g, which means that each resonator row adds 0.13% to the mass of the host structure. In this sense, a fully treated metamaterial pipe with 200 glued resonant structures has 25.21% added mass as compared to the bare aluminum pipe. It is worth investigating the minimum number of resonators needed to achieve a pronounced stopband zone as a lightweight design is aimed for. Given the test procedure, the experimental results of the frequency response of the pipe with different numbers of added resonators can give a good insight on the minimum number of needed resonators.

Fig. 15. Excitation sections, measurement points and the location of resonators.

6 MEASUREMENT RESULTS AND DISCUSSION

This section studies the experimental results of the modal tests performed on the pipe to establish the stopband behavior in the metamaterial pipe. Some of the experimental results are compared to numerical FE ones to validate the accuracy of numerical tools in predicting the frequency response of the metamaterial pipe.

The results are given in the form of averaged measured RMS accelerations of the entire pipe (in that case evaluated in 208 measurement points along the pipe) and of specific sections with respect to the location of different accelerometers as the corresponding reference positions. The RMS acceleration of the entire pipe can be calculated using Eq. (1), while the RMS acceleration of different sections of the pipe can be calculated using:

$$|H|_{\ddot{x}_{F_e},rms} = \sqrt{\frac{1}{8} \sum_{k=1}^8 |H|_{\ddot{x}_{k/F_e}}^2}, \quad (3)$$

in which $|H|_{\ddot{x}_{k/F_e}}$ is the measured acceleration at point k on the section due to an excitation at point e . In the following sub-sections, RMS acceleration of the pipe is represented in terms of g/N unit as a commonly used unit in experimental vibration analysis; in this unit, 1 N is equal to 1 kg m/s² and g as the standard gravitational acceleration is equal to 9.80665 m/s².

6.1 Numerical prediction of the frequency response of bare and metamaterial pipes

Fig. 16 compares measured frequency responses of a metamaterial pipe with four resonator rows with those predicted by FEM using NX 11. The same material properties used for unit cell modelling are applied in the FE model of the metamaterial pipe. All resonators in the FE model are modeled to be identical. RMS acceleration of the pipe due to the excitation at the location of accelerometer 1 is measured and evaluated both, over the entire pipe, Fig. 16(a), as

in different sections, Figs. 16(b-d). The numerically predicted stopband zone is represented by dashed lines in the figures. As can be seen, numerical and experimental frequency responses of the metamaterial pipe show a good agreement, especially given that the resonators are assumed to be identically the same and hence geometrical and mass variations between the resonators due to the production process are not taken into account in the numerical model. The numerical prediction captures the overall frequency response of the pipe, the frequency range of the stopband zone and the order of the attenuation due to the addition of the resonators both within and outside the stopband zone. The FE tools can thus with trust be utilized to design new metamaterial solutions for similar structures both in the form of unit cell analysis to evaluate the frequency range of the stopband zone and fully detailed simulations of the metamaterial structure to predict the detailed frequency response of the finite structure.

Fig. 16. Frequency response of the RMS acceleration of the points (a) all over the pipe and in sections (b) 4, (c) 15 and (d) 21 of the pipe with 4 rows of resonators based on the excitation at point 1. Dashed lines indicate the numerically predicted stopband zone.

6.2 Stopband zone in the frequency response of the metamaterial pipe

Fig. 17 depicts the frequency response of the full metamaterial pipe in the form of RMS acceleration. Figs. 17(a-d) show the frequency response due to the excitation of the pipe at the location of accelerometer 1 to 4, respectively, as shown in Fig. 15 and compares the response of the pipe with different numbers of added resonant structures.

As can be seen in Fig. 17, the first ovalling mode of the bare pipe occurs at 581 Hz while due to the addition of resonators, this mode has shifted to lower frequencies in metamaterial pipes. This observation is in line with the results observed in the unit cell modelling of the infinite metamaterial pipe in Section 4 where the cut-on frequency of the ovalling mode is

1 shifted to lower frequencies compared to that of the bare pipe. This reduction of the cut-on
2 frequency is due to the in-phase motion of the pipe and resonators below the stopband zone.

3 The comparison of the responses of the metamaterial pipes with that of the bare pipe in the
4 frequency range of predicted stopband zone shows a clear reduction in the RMS acceleration
5 of the pipe. As it can be seen, not only has the acceleration level significantly decreased in the
6 stopband zone, but also the dominant peak related to the vibrational mode of the pipe at 800
7 Hz (as discussed in section 2) has completely disappeared.

8
9 An interesting remark can be made with respect to the gradual build-up of the attenuation
10 in the stopband zone with the addition of extra resonator rows. It can be observed that the
11 attenuation due to the addition of resonators increases as more rows of resonators are added to
12 the pipe, resulting in a more pronounced stopband zone. This effect is due to the increase of
13 the effective mass of the resonators which are functioning against the motion of the pipe. This
14 behavior can be better seen in Fig. 18, which shows the frequency response of pipes with
15 different number of resonator rows in a narrow frequency range around the stopband zone. It
16 is clear that upon addition of 7 resonator rows the acceleration has already significantly
17 dropped; while additional resonator rows result in lower acceleration values, the amount of
18 reduction is not considerable compared to that caused by the addition of the first few rows.
19 These results add to those obtained by Claeys et al. [14], proving that even a partial treatment
20 of the structure can result in significant vibration attenuation in the stopband zone.

21 The addition of resonators creates a local minimum in the frequency response of the pipe
22 which indicates the onset of the stopband zone. Two extra peaks are also detected in the
23 frequency response of the metamaterial pipe. The first peak at a frequency lower than the
24 stopband zone is related to the shift to a lower frequency of the mode at 800 Hz of the bare
25 pipe. This effect is caused by the dynamic effective mass carried out by the resonators [35].
26 The second peak after the stopband zone is due to the out-of-phase motion of the resonators
27 and the pipe [7]. Comparison of the different figures representing the response with respect to
28 different excitation locations shows similar behavior. The radial displacement field of the pipe
29 at these peaks at 750 Hz and 880 Hz and at the onset of the stopband zone at 800 Hz for the
30 metamaterial pipe with 4 rows of resonators are shown in Figs. 19(a-c). In these figures, the
31 displacement of local resonators are not shown, allowing a better visualization.

32 The results presented in Fig. 17 show that the attenuation at the other resonance
33 frequencies of the pipe especially below the stopband zone is also significantly increased
34 when adding resonator rows. As below the stopband zone the motion of the pipe and
35 resonators are in-phase the overall mass of the structure is higher. This effect is also partially
36 related to the high loss factor of the resonators as compared to the aluminum pipe (0.05
37 compared to 0.003). This broadband reduction of the acceleration level of the pipe adds to the
38 benefits of the metamaterial solution in the stopband zone, improving the overall NVH
39 behavior of the pipe.

40 An additional observation from Fig. 17 is that the excitation location plays also a role on
41 the resulting averaged RMS acceleration levels, especially when only a few resonator rows
42 are added to the pipe; it seems that the attenuation in the stop band zone is more pronounced
43 when the excitation is located between resonator rows. This can be explained as follows, if
44 there is no resonator row between an end of the pipe and the excitation point, reflections can
45 occur between the end of the pipe and the resonators rows on that section of the pipe, leading
46 to a vibration pattern that is built up due to the excitation. It should be noted that this is
47 not the case for excitation point 1: the excited section of the pipe between first resonator row and
48 the end of the pipe is too small to build up a large vibration pattern.

Fig. 17. Frequency response of the RMS acceleration averaged over all of the points of the pipe with various numbers of resonator rows based on excitation at point (a) 1, (b) 2, (c) 3 and (d) 4. Dashed lines indicate the numerically predicted stopband zone.

Fig. 18. Gradual build-up of the stopband zone due to the addition of extra resonators. Dashed lines indicate the numerically predicted stopband zone.

Fig. 19. Radial displacement of the metamaterial pipe with 4 rows of resonators at (a) 750 Hz, (b) 800 Hz and (c) 850 Hz.

Fig. 20 compares the RMS acceleration measured based on excitation at different points for a fully treated metamaterial pipe to assess the impact of the excitation point on the response. Results due to the excitation at different points are clearly in line with each other. As it can be seen, the attenuation representing the stopband zone can be clearly detected in all measured responses. As expected, the acceleration levels are not coinciding for different excitation points due to the difference in the longitudinal locations of these points; however, these levels are very close in the cases of excitation points 2 and 3 due to their relatively symmetrical axial locations.

Fig. 20. Frequency response of the RMS acceleration averaged over all of the points of the metamaterial pipe based on different points of excitation. Dashed lines indicate the numerically predicted stopband zone.

6.3 Effect of the metamaterial solution on the acoustic radiation from the pipe

Fig. 21 shows the acoustic radiation of the pipe with different numbers of resonator rows in the form of RMS sound pressure level measured by all 7 microphones. The peaks in the figure correspond to the structural eigenfrequencies observed in previous sections and the acoustic modes within the pipe. The acoustic modes of the hollow space inside the pipe are numerically predicted at 171, 343, 514, 686 and 857 Hz for an open ended pipe with a length of 1 m. These acoustic modes are indicated by the arrows and the numbers from 1 to 5. As it

can be seen, the reduction in the acoustic radiation due to the addition of the metamaterial solution follows the same trend as the vibration attenuation within the pipe. In the range of the predicted stopband zone, a pronounced attenuation is observed with the minimum amount of RMS sound pressure level occurring at the onset of the stopband zone, which gradually increases to reach its peak value.

Fig. 21. RMS Acoustic radiation from the pipe measured by all of the microphones. Dashed lines indicate the numerically predicted stopband zone.

It is observed that with the addition of the first resonator row, the peak corresponding to the vibrational mode of the pipe at 800 Hz disappears resulting in a decrease of 39 dB in the sound pressure level of the radiated sound from the pipe at that specific frequency. At the same frequency, a reduction of 55 dB is obtained when all 25 resonator rows are added to the structure. These results prove that the findings by Nateghi et al. [34] for vibration attenuation in infinite metamaterial pipes can be extended to vibration attenuation in finite metamaterial pipes, resulting in a considerable amount of acoustic radiation reduction.

Sound pressure level measured by 4 different microphones in the locations reported in Table 5 are shown separately in Figs. 22(a-d). These figures prove that the metamaterial solution, especially in the case of partial treatment of the pipe affects the acoustic radiation from different regions of the pipe in a similar fashion. All of the microphones have clearly captured the reduction in the sound pressure level due to the addition of the resonators in the stopband zone. Microphone 1 in Fig. 22(a) was positioned inside the pipe and picked up the acoustic modes of the pipe more clearly than the other microphones. The first acoustic modes at 170 Hz and 340 Hz are clearly observable in Fig. 22(a) and are not influenced by the number of resonators rows added to the pipe. The other acoustic modes are masked by the presence of the structural modes in the frequency response of the pipe.

Similar to the acceleration level, while a broadband reduction of the sound pressure level is observed, the results show a peak right after the stopband zone. Although at the frequency range around this peak the sound pressure level of the metamaterial pipe is higher than that of the bare pipe, at only 6 dB of difference at its maximum level, it is negligible compared to the maximum reduction occurring in the stopband zone.

Fig. 22. RMS Acoustic radiation from the pipe measured by microphones (a) 1, (b) 2, (c) 4 and (d) 7, respectively. Dashed lines indicate the numerically predicted stopband zone.

6.4 Impact of the added mass and damping of resonators on the frequency response of the pipe

Fig. 23 compares the frequency responses at different sections of the pipe due to an excitation at point 1 for pipes with 5, 15 and 25 rows of resonators, in Figs. 23(b-d) respectively. The frequency response of the bare pipe is shown to serve as a reference case (Fig. 23(a)).

As it can be seen for a fully treated pipe the stopband attenuation becomes more pronounced as the waves travel through different sections.

For partially treated pipes, the gradual decrease of the acceleration level is clearly observed in sections 1, 7 and 10 in which resonators are attached; however, frequency response of other sections also indicate a clear reduction in the acceleration level, which proves that the metamaterial solution is affecting the dynamic behavior of the entire pipe. As long as there is a sufficient number of resonator rows between the location of the excitation and that of the acceleration measurement the stopband zone caused by the metamaterial solution is captured. However, it is observed that after the addition of 10 resonator rows, the attenuation level in the stopband zone is almost saturated, meaning that the effect of any extra added resonator rows on the acceleration level is less significant.

Fig. 23. Frequency response of the RMS acceleration of different sections of pipes with (a) 0, (b) 5, (c) 15 and (d) 25 rows of resonators based on the excitation at point 1. Dashed lines indicate the numerically predicted stopband zone.

Fig. 24 compares the RMS frequency response of the pipes treated with a different number of resonator rows measured at various sections based on excitation at point 1. It is observed that in section 25, Fig. 24(d), which is right before the location of the 25th resonator row, the addition of each extra resonator row gradually decreases the minimum value of the acceleration in the beginning of the stopband zone. This observation is in line with our expectation as the level of attenuation is dependent on the number of resonators which travelling waves pass through between the excitation point at section 1 and the measurement points at section 25; while in sections 5 in Fig. 24(b), and 10 in Fig. 24(c), any extra resonators added after these sections have less impact on the attenuation as they only influence the frequency response by adding to the mass and the overall damping of the structure. The frequency response of section 1, in which both excitation and measurement points are before the first resonator row still shows a considerable reduction in the acceleration level. In this case, travelling waves which are reflected at the opposite end of the pipe pass through the resonator rows twice to reach section 1, Fig. 24(a), therefore, the stopband zone is still recognizable in this section. Note that with the addition of extra rows of resonators, not only the added mass to the pipe is increased, but also additional local damping is introduced to the structure which, in return, results in extra reduction of the response level.

Fig. 24. Frequency response of the RMS acceleration of pipes with different resonator rows based on the excitation at point 1 at sections (a) 1, (b) 5, (c) 10 and (d) 25, respectively. Dashed lines indicate the numerically predicted stopband zone.

Fig. 25(a) shows the average level of RMS acceleration along the pipe and Fig. 25(b) the average sound pressure level of the sound radiated from the pipe with respect to the ratio of the added mass of the resonators to that of the pipe. The frequency response is averaged in three different frequency ranges:

- 400 Hz – 1000 Hz: as a broadband frequency range to depict the overall NVH behavior of the pipe due to the addition of the resonators;
- 750 Hz – 900 Hz: as the broad range of attenuation in the pipe with 25 added resonator rows;
- 800 Hz – 840 Hz: as the numerically predicted stopband zone.

Given that pipes with different numbers of resonator rows have different frequency ranges of attenuation due to the addition of resonators, these three frequency ranges are chosen to provide a more clear comparison between the different cases. The first observation is the gradual reduction in both acceleration and sound pressure level of the pipe with the addition of extra resonators. It is observed that the reduction of the acceleration level almost follows an exponential function in the sense that the addition of the first few resonator rows results in a considerable reduction in the acceleration level; afterwards, the reduction of the acceleration level is less pronounced. To demonstrate this behavior an exponential fit is added to Fig. 25(a) with the parameters provided in Table 6. Note that in this table, x refers to the ratio of resonator mass over host mass. It can be noticed that already after 4 rows of resonators it is possible to obtain a considerable attenuation in terms of structural response and sound pressure level of the pipe, by the addition of only 4% of mass. Therefore, if the source of

excitation in the structure is known, a good trade-off between the amount of added mass and the level of desired attenuation can be obtained with the addition of a few resonators.

Fig. 25. Effect of the number of resonators (added mass) on the frequency response of the pipe in terms of (a) averaged RMS acceleration based on the excitation at point 1 and (b) average sound pressure level measured by all microphones.

Table 6. Parameters of the exponential fit used in Fig. 25(a).

Frequency range [Hz]	Fitting curve = $a \cdot e^{b \cdot x} + c \cdot e^{d \cdot x}$			
	a	b	c	d
400 – 1000	1.009e-9	-19.72	1.369	-0.1126
750 – 900	1.149e-8	-18.10	0.890	-0.3422
800 – 840	2.519e-5	-10.83	0.403	-0.2451

In both acceleration and sound pressure level results, when the results are averaged over the stopband frequencies, in some cases pipes with more resonators show a worse behavior than pipes with less resonators; e.g. pipes with 5 and 15 resonator rows. This observation is less relevant when results are averaged in broadband. To explain this behavior, it should be noted that these results are obtained by measuring the response at a limited number of points; moreover, the frequency range of the stopband zone is different in pipes with different number of resonator rows and does not fully coincide with the numerical prediction. Given the low level of acceleration in the stopband zone, the response at the end of this zone dominates the average value, therefore, a broader frequency zone (i.e. 750 Hz - 900 Hz) can be a better representative of the behavior of different pipes.

While the acceleration level of the metamaterial pipe between 800 Hz – 840 Hz is clearly below the acceleration level in 750 Hz – 900 Hz, the average sound pressure levels of the pipe in these frequency ranges do not show much difference. This effect can be explained due to the existence of the fifth acoustic mode of the pipe in this frequency range. In a bare pipe, this acoustic mode is masked by the more dominant structural modes of the pipe at 800 Hz, while in the metamaterial pipe the structural mode disappears allowing the acoustic mode to appear in the frequency response as can be seen in Fig. 22.

7 CONCLUSIONS

Overall, the results studied in the paper show the potential of the metamaterial treatment as an efficient lightweight solution for vibration attenuation and sound radiation reduction of vibrating pipes. First, flexural wave propagation in an aluminum metamaterial pipe is investigated. The dynamic frequency response of the bare aluminum pipe is studied using FEM, identifying a frequency zone with unfavorable NVH behavior. Afterwards, a metamaterial solution in the form of local resonators is designed and the frequency range of the stopband zone is numerically predicted using the dispersion curves obtained from unit cell modelling. The resonators are produced from PMMA panels applying laser cutting and added to the pipe in the form of rows of 8 resonators around the circumference while the axial distance between two adjacent resonators is kept constant and subwavelength targeting the selected frequencies. In order to characterize the NVH behavior of the pipe, modal analysis is performed on the pipe in a semi-anechoic chamber following roving hammer technique. Structural vibration and acoustic radiation from the pipe are measured using various accelerometer and microphone locations. Moreover, resonant structures are added to the aluminum pipe gradually to characterize the decay in the wave propagation in the pipe. In this sense, first, the bare aluminum pipe is tested and then, after the addition of each resonator row, the test is repeated. The comparison of the stopband zone predicted using the dispersion analysis for the infinite metamaterial pipe and FEM for the finite pipe with four rows of resonators with the measured stopband zone show a good agreement. In addition to the clear reduction in the frequency response of the metamaterial pipe in the frequency range of the predicted stopband, an overall increase of vibration attenuation in the broadband frequency response of the metamaterial pipe is also observed. The impact of the metamaterial solution on the measured radiated acoustic pressure from the pipe around the frequency range of the stopband is clearly observed. The experimental results show that if the source of the excitation in the structure is known, it is possible to reduce the amount of required added mass for a certain amount of attenuation by choosing the location of the resonators appropriately with respect to the type and location of external forces. In this manner, a lower number of resonators and therefore added mass is required to achieve a considerable level of attenuation.

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