Optimization of Light Emitting Diodes based on Bipolar Double-Barrier Resonant-Tunneling Structures

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A bipolar double-barrier resonant-tunneling light emitting diode will have maximum emitted light intensity if (1) electrons and holes simultaneously resonant-tunnel into the quantum well, (2) the charge carriers are entirely trapped in the well as the only light source, and (3) the device operates at nearly zero-field condition. We have performed a theoretical modeling, by solving selfconsistently the Schrödinger equation and the Poisson equation, to optimize such Resonant-tunneling light emitting diodes with respect to the geometric structures, the chemical compositions, and the doping profiles. All physical parameters are calculated from first principle, except the electron-hole recombination rate which is treated as an input parameter. We have also optimized the device performance with respect to the temperature stability, such that the profile of the electroluminescence spectrum is insensitive to the temperature. A completely automated computer code has been constructed for such theoretical modeling of light emitting diodes.

I. INTRODUCTION

Modern semiconductor technology makes it possible to fabricate low-dimensional nanostructures with controllable chemical composition and geometric structure. This opened an extremely important frontier field of research for both fundamental physics and quantum electronic devices. Deep physical insight into such novel systems provides ideas for fabricating new devices, which help to create new challenging physical problems. The milestones in the development of quantum devices include the resonant tunneling diodes,¹ the avalanche photodiodes,² the quantum well laser,³ and the resonant tunneling bipolar⁴ and field-effect⁵ transistor.

Among various interests, one very active field is the double-barrier resonant-tunneling (DBRT) structures, which exhibit the characteristic *negative differential resistance* (NDR). In a unipolar DBRT structure, the range of NDR directly affects the maximum operating frequency of a resonant-tunneling oscillator,⁶ and is essential for the more reliable switches.⁷ Using a bipolar DBRT structure as a *light emitting diode* (LED),^{8,9} it has the advantage of ultrafast tunneling of charge carriers into the well, and of sharp spectrum of emitted light. However, its drawback is the weak intensity of the emitted light. What kind of theoretical guideline one

may provide to the design of an intensity-optimized LED based on the bipolar DBRT structure is the central theme of this paper.

The theoretical modeling of a bipolar DBRT LED consists of two parts: the quantum transport of high resonant-tunneling current and the radiative recombination of electrons and holes. Almost all existing theoretical works on resonant-tunneling are for unipolar DBRT structures. In this case, because there is only one type of charge carrier, either in conduction band or in valence band, the effect of electron-electron interaction can be treated beyond the Hartree-type approximation,¹⁰⁻¹³ such as the local density functional approximation,^{14,15} the Wigner distribution function,^{16,17} and the random phase approximation.¹⁸ Results were also obtained with time-dependent simulations,¹⁹⁻²¹ which reproduce experimental observation. To study the resonant-tunneling of both electrons and holes in a bipolar DBRT LED, including the electron-electron interaction even at the level of Hartree approximation, requires a very tedious self-consistent numerical calculation. There exists almost no such calculation in the literature, but we will present one here. Incorporating our quantum transport to the electron-hole recombination process,²² we will provide a theoretical modeling for optimizing the performance of a bipolar DBRT LED.

In Sec. II we will define our model system and outline the standard self-consistent approach. The result will be used in Sec. III for detailed numerical calculations. Some remarks will be given in the last Sec. IV.

II. THEORETICAL MODEL

We assume a bipolar DBRT structure with perfect interfaces, so the scattering due to the interface roughness will be ignored. In this case all physical quantities depend only on the z-axis which is defined perpendicular to interfaces. We will restrict ourselves to direct bandgap materials with negligible lattice mismatch. To be realistic, the electron emitter and the hole emitter are assumed highly doped such that all electrons (or holes) form a well defined Fermi sea. As an example, the structure of one sample used in our calculation is specified in Table 1. Because of diffusion, the impurity concentration varies continuously across the interface between the emitter and the spacer. To take this effect into account, we introduce a very narrow Error-function tail of impurity concentra-

tion. The width of this tail is typically about 10 Å, and has negligible effect on final numerical results.

Table 1. Sample structure for Figs. 1, 2, 3 and 4.

Electron Emitter	$2 \times 10^{18} \text{ n}^+ \text{-Al}_{0.11}\text{Ga}_{0.89}\text{As}$	400Å
Spacer	un-doped $\text{Al}_{0.11}\text{Ga}_{0.89}\text{As}$	50Å
Barrier	$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$	40Å
Well	GaAs	60Å
Barrier	$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$	40Å
Spacer	un-doped $\text{Al}_{0.0775}\text{Ga}_{0.9225}\text{As}$	50Å
Hole Emitter	$4 \times 10^{18} \text{ p}^+ \text{-Al}_{0.0775}\text{Ga}_{0.9225}\text{As}$	400Å

We will solve the Schrödinger equation within the effective mass approximation. In principle we should include both the heavy and the light hole band which are coupled together. The self-consistent numerical computation for such three-band system is extremely heavy and we decide to leave it as a future work. The essential physics and the principle of optimizing the LED will be clear even if we decouple the heavy and the light hole band. In this paper we will consider only the conduction band and the heavy hole band. We then solve self-consistently the Schrödinger equation

$$\left\{ -\frac{\hbar^2}{2} \frac{d}{dz} \frac{1}{m^*(z)} \frac{d}{dz} + V(z) \right\} \psi(z) = E\psi(z) \quad (1)$$

for both electrons and heavy holes, together with the Poisson equation

$$\frac{d}{dz} \epsilon(z) \frac{dV(z)}{dz} = -\rho(z) \quad (2)$$

for the potential. The solutions of (1) include all quasi-bound states and extended states.

The matching conditions for (1) are the continuity of wave functions and the continuity of current across each interface. Let the far end of the electron emitter be located at $z=z_0$, and the far end of the hole emitter at $z=z_0+L$. We set the zero reference energy $V(z_0)=0$ at the bottom of the conduction band of the electron emitter. If we define $E_{e,F}>0$ as the Fermi energy in the electron emitter measured from the bottom of the conduction band, and $E_{h,F}<0$ as the Fermi energy in the hole emitter measured from the top of the valence band, then when solving (2) for the potential of electrons in the conduction band, the boundary conditions are $V(z_0)=0$ and

$$V(z_0+L) = E_{e,F} - E_{h,F} + E_{h,g} - V_b,$$

where $E_{h,g}$ is the bandgap in the hole emitter. We have used the convention that the bias V_b is positive for a forward bias. When solving (2) for the potential of holes in the valence band, the boundary conditions are $V(z_0)=-E_{e,g}$ and $V(z_0+L)=E_{e,F}-E_{h,F}-V_b$, where $E_{e,g}$ is the bandgap in the electron emitter.

The contribution to the charge density $\rho(z)$ comes from both the impurity ions and the electrons and holes in all eigenstates of the Schrödinger equation (1). At finite temperature T , the electron (or hole) occupation probability in each eigenstate follows the Fermi distribution $f_e(\vec{k}, T)$ [or $f_h(\vec{k}, T)$]. At zero bias, for the sample specified by Table 1, even at room temperature there is no tunneling current. However, in order to show the self-consistent solution of (1) and (2) with a well-defined Fermi energy, we plot the zero temperature potential profile in Fig. 1. To obtain these numerical results, we have used the empirical values of the relevant materials parameters.²³⁻²⁶

Fig. 1. Self-consistent solution for the sample specified in Table 1, at zero bias and at zero temperature.

When resonant-tunneling occurs at sufficiently large bias, charges will accumulate in the well, as well as in any possible quasi-bound state. If both electrons and holes are present, they will recombine and emit light. To include the light emission in our self-consistent scheme, let us describe the calculation in three steps. Since in our final optimized DBRT LED the well is the only light source, we will treat this case in order to simplify the description.

Let us first consider the various tunneling currents. For either the electron process or the hole process, there are two tunneling currents through each barrier: one into the well and the other out of the well. If we assume a quasi-Fermi energy ϵ_e for the electrons in the well, then we can use the Tsu-Esaki formula¹ to calculate the net electron current from the electron emitter into the well

$$J_{e,e \rightarrow w} = \frac{2e}{(2\pi)^3} \int d\vec{k} \tilde{v}(\vec{k}) |t_e(E, E_{e,F} - \epsilon_e)|^2$$

$$\times \{N_e(E) - N_w(E + E_{e,F} - \epsilon_e)\}. \quad (3)$$

In the above equation, E is the energy of the tunneling electron, which is a function of $\vec{k}$, $t_e(E, \Delta)$ is the transmission coefficient, and $N_e(E)$ and $N_w(E)$ are, respectively, the density of states in the electron emitter and the well. We can similarly calculate the other electron current $J_{e,w \rightarrow h}$ from the well to the hole emitter, and the two corresponding hole currents $J_{h,h \rightarrow w}$ and $J_{h,w \rightarrow e}$. The net electron current and the net hole current into the well are

$$\begin{aligned} J_e^{net} &= J_{e,e \rightarrow w} - J_{e,w \rightarrow h} \\ J_h^{net} &= J_{h,h \rightarrow w} - J_{h,w \rightarrow e}. \end{aligned} \quad (4)$$

The electrons and holes accumulated in the well due to these two net currents recombine to emit light. The number of emitted photons $W(\omega)$ of frequency ω per unit area per second for radiative recombination is given by²²

$$\begin{aligned} W(\omega) &= \frac{2e^2}{m_e^2} \int \frac{d^3q}{(2\pi)^3 \omega} (\vec{\xi} \cdot \vec{p}_{eh})^2 \int d^2k f_e(\vec{k}, T) f_h(\vec{k}, T) \\ &\times \delta\{E_{w,g} - \hbar\omega + \frac{(\hbar k)^2}{2\mu}\}, \end{aligned} \quad (5)$$

where m_e is the electronic mass, μ the effective reduced mass of the electron-hole system, $E_{w,g}$ the two-dimensional bandgap in the well, $\vec{\xi}$ the polarization of the emitted photon, and $\vec{p}_{eh}$ the interband momentum matrix element. In the above equation, the integration over k is restricted to $0 < k < k_m$, where k_m is either the Fermi vector of the electrons or the Fermi vector of the holes in the well, whichever is smaller. (5) can be expressed in a compact form

$$\begin{aligned} W(\omega) &= \frac{1}{\tau(\omega)} \times \{2 \int \frac{d^2k}{(2\pi)^2} f_e(\vec{k}, T) f_h(\vec{k}, T)\} \\ &\equiv \mathcal{N}/\tau(\omega). \end{aligned} \quad (6)$$

The relaxation time $\tau(\omega)$ has a complicated expression and is difficult to calculate accurately. In the present work we will treat it as an input parameter determined from experiments. Now we can set up the rate equations for the current-light equilibrium. When the system reach its steady state, we have

$$\begin{aligned} J_e^{net} &= \mathcal{N}/\tau(\omega) \\ J_h^{net} &= \mathcal{N}/\tau(\omega). \end{aligned} \quad (7)$$

Since $\mathcal{N}$ can be easily calculated and $\tau(\omega)$ is an input parameter, we can solve (7) for the electron quasi-Fermi energy ϵ_e and the hole quasi-Fermi energy ϵ_h in the well.

Our computation scheme is then to solve (1), (2), (3), (4), (6), and (7) self-consistently. Except the input empirical parameter $\tau(\omega)$, all physical quantities are calculated from the first principle. It is important to point out that since the residual impurity concentration in the well is about 10^{14} - 10^{15} cm⁻³, which is much less than

the concentration of conduction band electrons (or valence band holes) accumulated in the well under steady state, the radiative decay of free electrons and free holes is the dominating recombination process in the well. In this respect, in our self-consistent scheme the negligible contribution of non-radiative decay is ignored.

III. OPTIMIZATION PROCEDURE

The goal of our theoretical modeling of the bipolar DBRT LED is to optimize the electroluminescence spectrum with respect to both its intensity and its shape. The intensity is directly proportional to J_e^{net} and J_h^{net} . In this respect, there are two factors which we must take into account. The first one is to achieve simultaneous resonant-tunneling of both electrons and holes into the well. The second factor is to trap all electrons and holes in the well once they tunnel in. That is, in (4) we must have $J_{e,w \rightarrow h} = 0$ and $J_{h,w \rightarrow e} = 0$. The electroluminescence intensity also depends on the oscillator strength. To increase the oscillator strength we need to maximize the overlap of the electron wave function and the hole wave function in the well. Since electrons and holes are spatially polarized away from each other under an electric field, in our optimization procedure, inside the well we have to reach the nearly zero-field condition.

Since electrons and holes can recombine and emit light at any part of the DBRT structure provided that they are close enough spatially, in order to produce a narrowly peaked electroluminescence spectrum, it is essential to make sure that the well is the only source of light emission. That is, besides the conditions $J_{e,w \rightarrow h} = 0$ and $J_{h,w \rightarrow e} = 0$, there should exist no quasi-bound state in the spacer regions.

In a bipolar DBRT LED there are several features which affect the tunneling currents of electrons and holes, and so can be adjusted to achieve the optimal electroluminescence spectrum. After examining almost all possible variations, we found the simple and effective optimization procedure by adjusting the local bandgaps of both emitters, as well as the thicknesses of the spacers. The resonant levels in the well are mainly determined by the barrier-well-barrier structure, and are insensitive to the chemical compositions of the emitters, which determine the local bandgaps. Therefore, if we increase the local bandgap of the electron (or hole) emitter by a proper amount, we can control the range of bias in which electrons (or holes) tunnel resonantly. When the bias range of electron resonant-tunneling overlaps with the bias range of hole resonant-tunneling, we then tune the system into simultaneous resonant-tunneling of both electrons and holes. With such adjustment of local bandgaps, we can also keep the electron (or hole) resonant level in the well below the bottom (or top) of the conduction (or valence) band in the hole (or electron) emitter. In this situation, the desired conditions $J_{e,w \rightarrow h} = 0$ and $J_{h,w \rightarrow e} = 0$

are satisfied. Furthermore, the increase of local bandgaps in the emitters will raise the quasi Fermi energies of the emitters towards their respective resonant levels in the well. A smaller bias is then required to create the simultaneous electron resonant-tunneling and hole resonant-tunneling. Consequently, the electric field in the well becomes weaker, allowing the possibility to operate the bipolar DBRT LED with nearly zero-field in the well.

Finally, we have to prevent the appearance of quasi-bound state in the spacer regions. We remind the reader that the potential drop across each spacer is already small, because of the larger local bandgaps in the emitters and the smaller operating bias. The strength of the triangular potential well in each spacer can be further reduced by decreasing the thickness of the spacer. In this way we can remove the quasi-bound states in both spacer regions.

Fig. 2. Self-consistent potential profile and resonant levels for the DBRT LED sample specified in Table 1, at the bias of 1.683 Volts and at zero temperature.

When all above listed conditions are satisfied, the peak value of the electroluminescence spectrum increases with $J_e^{net} = J_{e,e \rightarrow w} = N/\tau(\omega) = J_{h,h \rightarrow w} = J_h^{net}$. Since $J_{e,e \rightarrow w}$ (or $J_{h,h \rightarrow w}$) approaches its maximum when the electron (or hole) resonant level is in the very near vicinity of the bottom (or top) of the conduction (or valence) band in the electron (or hole) emitter, consequently, we should bias the bipolar DBRT LED into simultaneous resonant-

tunneling with the resonant level in the conduction (or valence) band quantum well just above (or below) the bottom (or top) of the conduction (or valence) band in the electron (or hole) emitter. The bipolar DBRT LED sample specified in Table 1 is optimized in this way, and will be used here as an illustrating example. Starting from the zero temperature zero bias potential profile shown in Fig. 1, when the applied bias increases to 1.683 Volts, the self-consistent potential profile develops into the form plotted in Fig. 2, together with the two resonant levels in the well. In this calculation, as well as in the rest of this paper, we have set the relaxation time $\tau = 1$ ns, within the range of measured value. It is important to point out that the conclusion derived from our theoretical modeling in this paper does not depend on the precise value of τ used in the calculation.

Fig. 3. Temperature-dependence of some physical quantities of the bipolar DBRT LED sample specified in Table 1. The curve *Bias* represents the bias $V_{bm}(T)$ for peak current, *Photon Energy* the energy separation $\hbar\omega(T)$ between the electron resonant level and the hole resonant level, and *Band Gap* the bandgap $E_{w,g}(T)$ of the well material GaAs.

A striking feature of the bipolar DBRT LED sample specified in Table 1 is its temperature-insensitive electroluminescence spectrum under normal operation condition. To clarify this point, let us fix a temperature T and calculate the I - V curve. The current $J_e^{net}(T) = J_h^{net}(T)$ increases monotonically from zero at the threshold bias $V_{b0}(T)$ to its peak value $J_{max}(T)$ at the bias $V_{bm}(T)$, and then drops to zero very rapidly. The functional dependence of $V_{bm}(T)$ on the temperature T is shown in Fig. 3 as the curve *Bias*. Corresponding to each $V_{bm}(T)$, we define the phonon energy $\hbar\omega(T)$ as the energy separation between the electron resonant level and the hole resonant level in the well, which is also plotted in Fig. 3. In

our calculation we have used the empirical temperature-dependence of the local bandgaps $E_{e,g}$, $E_{h,g}$ and $E_{w,g}$, as well as of the bandgaps of the barrier materials. In Fig. 3 the curve *Band Gap* represents $E_{w,g}(T)$. The three curves in Fig. 3 are almost parallel to each other, strongly suggesting that the temperature effect on the electroluminescence has its origin in the temperature-dependence of the bandgaps.

Fig. 4. The I - V characteristics of the bipolar DBRT LED sample specified in Table. 1, for temperature at 10 K, 50 K, 100 K, 200 K and 300 K. Each lower temperature I - V curves is shifted such that the position of its peak current coincides with that of the 300 K I - V curve. The curve marked as *Overlap* is for $T=300$ K.

In order to confirm our conjecture, in Fig. 4 we first plot the I - V curve for room temperature $T=300$ K, which exhibits a sharp peak at $V_{bm}(T=300)=1.554$ Volts. The I - V curves at other temperatures, for example, at $T=10$ K, 50 K, 100 K and 200 K have almost exactly the same shape as the one at 300 K, but with different positions $V_{bm}(T)$ for the peak currents $J_{max}(T)$. These positions $V_{bm}(T)$ are plotted in Fig. 3 as curve *Bias*. If we shift each lower temperature I - V characteristics such that $V_{bm}(T)$ coincides with $V_{bm}(T=300)$, then all these I - V curves collapse into one, as shown in Fig. 4.

According to (7) the current $J_e^{net}(T)=J_h^{net}(T)$ is a direct measure of the electroluminescence intensity. From the collapsed I - V characteristics shown in Fig. 4, it is clear that the bipolar DBRT LED sample specified in Table 1 will produce the same electroluminescence spectrum regardless the operation temperature. When the temperature varies, all one needs to do to recover the same spectrum is to shift the bias according to the curve *Bias* or the curve *Band Gap* in Fig. 3. Within the whole range of the I - V curve, the overlap

$$S = \int |\psi_e(z)|^2 |\psi_h(z)|^2 dz$$

between the electron wave function $\psi_e(z)$ and the hole wave function $\psi_h(z)$ of resonant levels remains almost constant. As an illustrating example, in Fig. 4 we have plotted the overlap S for the temperature $T=300$ K.

Table 2. Sample structure for Figs. 5 and 6.

Electron Emitter	$2 \times 10^{18} \text{ n}^+ \text{-Al}_{0.1125}\text{Ga}_{0.8875}\text{As}$	400Å
Spacer	un-doped $\text{Al}_{0.1125}\text{Ga}_{0.8875}\text{As}$	150Å
Barrier	$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$	40Å
Well	GaAs	40Å
Barrier	$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$	40Å
Spacer	un-doped $\text{Al}_{0.095}\text{Ga}_{0.905}\text{As}$	100Å
Hole Emitter	$4 \times 10^{18} \text{ p}^+ \text{-Al}_{0.095}\text{Ga}_{0.905}\text{As}$	400Å

Fig. 5. Self-consistent potential profile and resonant levels for the DBRT LED sample specified in Table 2, at the bias of 1.601 Volts and at zero temperature.

This special temperature-insensitive feature is due to the fact that we have biased the DBRT LED into simultaneous resonant-tunneling with the resonant level in the conduction (or valence) band much lower than the quasi Fermi energy of the electron (or hole) emitter. Therefore, thermal excitation at room temperature has negligible effect on the resonant-tunneling currents. The

complicated many-body effect also plays an important role in this case, because the accumulated charges in the well affect the electron resonant-tunneling and the hole resonant-tunneling by different amounts.

Intuitively, one often tends to tune the system into simultaneous resonant-tunneling at the threshold bias, because such a situation is much easier to reach. However, this incorrect optimization, besides many other errors, is unstable against the temperature variation. As an example, let us consider another bipolar DBRT LED sample specified in Table 2. At zero temperature and at a bias of 1.601 Volts, the simultaneous resonant-tunneling starts as shown in Fig. 5. Here the resonant level in the conduction (or valence) band matches the quasi Fermi energy of the electron (or hole) emitter. One would expect a significant temperature effect on the tunneling current and so on the electroluminescence spectrum. The tunneling current $J_e^{net}(T) = J_h^{net}(T)$, as a function of temperature, of this bipolar DBRT LED at the bias 1.601 Volts is shown in Fig. 6. We have analyzed the current and found out that the increase of current at higher temperature has a large contribution from the thermal excitation around the quasi Fermi energies in both emitters.

Fig. 6. The tunneling current as a function of temperature for the bipolar DBRT LED sample specified in Table 2, operating at a bias 1.601 Volts.

To close this section, we would like to emphasize the complication of the optimization procedure. Besides many parameters which should be adjusted, for each fixed set of parameter values, to obtain the self-consistent solution itself is a very tedious numerical process. This can be handled only with a completely automated computer code, with the sample structure as the input. It took us almost one year to construct such automated computer code.

IV. DISCUSSION

In this paper we have presented the theoretical modeling of an optimized bipolar DBRT LED. Let us first make a few remarks. The electron emitter and the hole emitter are considered as particle reservoirs. Therefore, in theoretical modeling the reservoirs can not have only finite strength. This has to be tested before performing a full scale self-consistent calculation. In our model, the effect of electron-phonon interaction is to bring the electron system into thermal equilibrium. We did not touch the problem of phonon-replica in the I - V characteristics, because we are working in the bias region before reaching the NDR. For the same reason, our bipolar DBRT LED operates in the bias region separated from that in which the possible bistability may occur. Finally, the density of charges accumulated in the well is so high that the exciton recombination channel can be safely ignored.

While our self-consistent scheme is already extremely complicated, we have yet neglected several important aspects. Some of them will only slightly modify our final result, but others require further investigations which will be done in the future. The mixing of heavy and light holes will definitely change the hole resonant-tunneling current significantly. In almost all theoretical modeling, the common tendency is to avoid the ΓX tunneling channel, which is also the case in the present work. However, one should consider this effect positively as an additional degree of freedom in the optimization process, since relevant device has already been studied experimentally.²⁷ The same situation is our attitude toward the effect of strain in our search for better devices.

The self-consistent scheme presented here in this paper can be easily extended to study the light emission from interwell recombination in multiple quantum wells, made of both Type-I and Type-II materials. Nevertheless, in closing this paper, we would like to raise one more question: Will it be a straightforward generalization from our optimized bipolar DBRT LED to an optimized quantum well laser?

- ¹ R. Tsu and L. Esaki, *Appl. Phys. Lett.* **22**, 562 (1973).
- ² N. Holonyak, R. M. Kolbas, R. D. Dupuis, and P. D. Dapkus, *IEEE J. Quantum Electron* **QE-16**, 170 (1980).
- ³ Y. Arakawa and H. Sakaki, *Appl. Phys. Lett.* **40**, 939 (1982).
- ⁴ F. Capasso and R. Kiehl, *J. Appl. Phys.* **58**, 1366 (1985).
- ⁵ T. K. Woodward, T. C. McGill, H. F. Chung, and R. d. Burnham, *Appl. Phys. Lett.* **51**, 1542 (1987).
- ⁶ T. C. L. G. Sollner, W. D. Goodhue, P. E. Tannenwald, C. D. Parker, and D. D. Peck, *Appl. Phys. Lett.* **43**, 588 (1983).

- ⁷ T. C. L. G. Sollner, P. E. Tannenwald, D. D. Peck, and W. D. Goodhue, Appl. Phys. Lett. **45**, 1319 (1984).
- ⁸ C. van Hoof, J. Genoe, R. Mertens, G. Borghs, and E. Goovaerts, Appl. Phys. Lett. **60**, 77 (1992).
- ⁹ C. R. H. White, H. B. Evans, L. Eaves, P. M. Martin, M. Henini, G. Hill, and M. A. Pate, Phys. Rev. B **45**, 9513 (1992).
- ¹⁰ H. Onishi, T. Inata, S. Muto, N. Yokoyama, and A. Shibatomi, Appl. Phys. Lett. **49**, 1248 (1986).
- ¹¹ M. Cahay, M. McLennan, S. Datta, and M. S. Lundstrom, Appl. Phys. Lett. **50**, 612 (1987).
- ¹² K. Brennan, J. Appl. Phys. **62**, 2392 (1987).
- ¹³ W. Potz, J. Appl. Phys. **66**, 2458 (1989).
- ¹⁴ R. K. Mains, J. P. Sun, and G. I. Haddad, Appl. Phys. Lett. **55**, 371 (1989).
- ¹⁵ E. Gawlinski, T. Dzurak, and R. A. Tahir-Kheli, J. Appl. Phys. **72** 3562 (1992).
- ¹⁶ N. C. Kluksdahl, A. M. Krizan, D. K. Ferry, and C. Ringhofer, Phys. Rev. B **39**, 7720 (1989).
- ¹⁷ W. R. Frensley, Rev. Mod. Phys. **62**, 745 (1990).
- ¹⁸ N. Zou, M. Willander, I. Linnerud, U. Hanke, K. A. Chao, and Yu. M. Galperin, Phys. Rev. B **49**, 2193 (1994).
- ¹⁹ K. L. Jensen and F. A. Buot, Phys. Rev. Lett. **66**, 1078 (1991).
- ²⁰ A. Nakano, P. Vashishta, and R. K. Kalia, Phys. Rev. B **43**, 9066 (1991).
- ²¹ A. Nakano, P. Vashishta, and R. K. Kalia, Appl. Phys. Lett. **64**, 2569 (1994).
- ²² G. D. Mahan and L. E. Oliveira, Phys. Rev. B **44**, 3150 (1991).
- ²³ S. Adachi, J. Appl. Phys. **58**, R1 (1985).
- ²⁴ J. S. Blakemore, in *Key Papers in Physics: Gallium Arsenide* (American Institute of Physics, New York, 1987).
- ²⁵ S. M. Sze, *Semiconductor Devices: Physics and Technology* (Wiley, New York, 1985).
- ²⁶ S. Tiwari and D. J. Frank, Appl. Phys. Lett. **60**, 630 (1992).
- ²⁷ E. C. Larkins, H. Schneider, S. Ehret, J. Fleissner, B. Dischler, P. Koidl, and J. D. Ralston, IEEE Trans. Electron Devices, **41**, 511 (1994).