AN ADJOINT APPROACH TO THERMAL NETWORK TOPOLOGY OPTIMIZATION

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ABSTRACT

On the road towards the large-scale implementation of efficient district heating systems and waste heat recovery, tools are needed to assist the design of thermal networks. In this paper, an adjoint-based approach to thermal network optimization is presented for the first time. In contrast to mixed-integer programming techniques, the adjoint-based approach is able to simultaneously deal with nonlinear models and many degrees of freedom. First, a thermal network model based on conservation of mass, momentum, and energy is presented. It is then used to optimize a fictitious thermal network for a given cost function. The results intuitively show the correct functioning of the adjoint-based algorithm. The impact of omitting energy modelling in the optimization on the obtained solution is finally discussed.

KEY WORDS: Network optimization, adjoint optimization, district heating, thermal networks, topology optimization

1. INTRODUCTION

Thermal networks, able to distribute waste and renewable energy heat for building heating, are an important means to decarbonize our energy consumption. The high and upfront infrastructure costs associated to thermal networks, however, require a careful design to maximize feasibility, profits and efficiency. Yet, given the many design variables involved the many potential routes for network piping, the location and dimensions of heat storages, and network operation parameters manual or intuitive designs will most often lead to suboptimal solutions.

A variety of methods has been developed to optimize district heating networks [9, 15]. Yang et al. [17] describe the design of a district-scale distributed energy resource system that is optimized. Not taken into account are the topology of the city or network, conversion losses, conversion, storage, redundancy and multi-stage networks. Dorfner and Hamacher [6] and Bordin et al. [2] both describe a Mixed Integer Linear Programming (MILP)-model which optimizes the topology of a district heating network towards certain objectives, e.g. operational costs or maximal potential consumers. In [8] different optimization techniques for hybrid energy systems are reviewed. In [12] a combined analysis of electricity and heat networks is presented. Considering the complexity and large number of variables, optimization of a network typically requires linearised techniques that involve assumptions, e.g. constant temperature drops over the DH substations, that possibly flaw how optimal the obtained results are. Furthermore, intrinsically nonlinear features of the thermal network such

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Fig. 1 A graph representation of an example district heating network, consisting of nodes and arcs. Producer, consumer, and internal nodes are denoted by hexagons, squares, and circles, respectively. Dark full arrows represent feed arcs, while light full arrows represent return arcs. dotted arrows represent producer (red) and consumer arcs.

as pressure drops for pump dimensioning can only approximately be accounted for through piecewise linear approximations [2]. Furthermore, with thermal networks evolving to its *fourth generation* [13], with suppliers at different temperatures, accurate modelling of the energy flows becomes even more important.

Almost all reported DH network optimization studies use MILP techniques, for which the computational cost scales badly with the network size and therefore limits the scale of the network to be optimized, in addition to a loss of detail that is intrinsic to the linear approximation. In this paper, we therefore resort to a model-based optimization strategy that is inspired by adjoint techniques from the domain of optimization constrained by nonlinear PDEs (Partial Differential Equations) [4]. These techniques are able to deal with strongly nonlinear equations while featuring a computational cost scaling that is roughly independent of the number of design variables in the optimization problem and is therefore very attractive for topology optimization applications (see e.g. [3]). In contrast to MILP techniques, these methods rely on decreasing the cost function in each optimization step by making use of the cost function gradient. Gradient-based algorithms have been applied in the past to optimization of hydraulic flow networks [7, 11]. In this paper, an adjoint-based optimization technique is applied for the first time to district heating network optimization. This allows us to base our optimization on a model in which mass, momentum, and energy flows are evaluated within the optimization loop.

The paper is outlined as follows. In section 2, a model for network optimization and its solution strategy are presented. Next, in section 3 the network design problem is formulated as a constrained model-based optimization problem. Finally, results on a first test-case are shown.

2. THERMAL NETWORK MODEL

Thermal networks are typically designed for worst-case situations. In this perspective, a steady-state thermal network model will be developed here that is able to assess the energy flows and pumping requirements in a worst-case scenario. In the coming subsections, both hydraulic and thermal models will be considered. We first introduce the necessary notations to represent a thermal network as a mathematical graph.

2.1 Graph representation

We represent the thermal network by a directed graph $G(N, A)$, with N the set of nodes and A the set of arcs. Following Bordin et al [2], the graph is built up of a feed and a return line with arcs of opposite directions, as

illustrated in Fig. 1. The set of nodes N consists of three subsets $N_p \cup N_c \cup N_i = N$, denoting the producer, consumer, and internal nodes, respectively. Similarly, the arcs A are partitioned in subsets $A_p \cup A_c \cup A_i = A$ denoting the producer, consumer, and internal arcs, respectively. Note that internal arcs can be interpreted as the physical geometry of the network piping, while the other arcs rather represent state transitions at the producer and consumer ends, but at the same physical location. In what follows, we can then denote a certain node as $n \in N$ and we can denote arcs as $(i, j) \in A$, or succinctly as $ij \in A$.

2.2 Flow modelling

Given the graph representation of the thermal network, we will now formulate a set of model equations to determine the pressures p in the nodes N and the volumetric flow rates q in the arcs A . Mass conservation in the nodes leads to a first equation for the flow rates q , i.e.

$$\sum_{a=(i,n) \in A} q_a - \sum_{a=(n,j) \in A} q_a = 0, \quad \forall n \in N. \quad (1)$$

These equations are complemented with momentum equations in the arcs. For the pipes, we use the Darcy-Weisbach equation that relates the friction in a pipe of length L to the mean flow velocity. Formulating this equation as a function of the flow rate q and the internal volume of the pipe V , the pipe equations become

$$p_i - p_j = \frac{\rho \sqrt{\pi}}{4} \frac{L_{ij}^{7/2}}{V_{ij}^{5/2}} f_{ij} |q_{ij}| q_{ij}, \quad \forall ij \in A_i, \quad (2)$$

with ρ the constant density of the carrier (water). The friction factor f_{ij} in this expression is modelled by the explicit Haaland correlation [10]

$$f_{ij}^{-1/2} = -1.8 \log \left[\left(\frac{\epsilon/d}{3.7} \right) + \frac{6.9}{\text{Re}} \right], \quad (3)$$

with ϵ the absolute pipe roughness and Re the Reynolds number.

For the pressure drop over the consumer heating system, the following relation is assumed

$$p_i - p_j = \zeta \frac{|q_{ij}| q_{ij}}{\alpha_{ij}^2}, \quad \forall ij \in A_c, \quad (4)$$

with ζ a constant determined from nominal operating conditions [14]. The variable $\alpha_{ij} \in [0, 1]$ is a flow control at consumer arc ij that we deliberately introduce for later use in optimization, allowing consumers to control their heating. In the producers arcs, a fixed input flow q_b is imposed as boundary condition for this system of equations, i.e. $q_a = q_{b,a}, \forall a \in A_p$. To uniquely define the pressures and flow rates in the graph, a reference pressure is finally imposed in one of the nodes.

2.3 Energy modelling

As already stated in the introduction, an energy model that accounts for pipe heat losses and the temperature dependence of the heating system efficiency at the consumer ends is essential to assess the potential of so-called *fourth generation* thermal network designs. In this subsection, we propose a set of model equations to calculate the temperatures T throughout the network.

First let us consider the heat losses in the pipes. Assuming axial convection of heat through a pipe line and radial diffusion of heat to the environment, an explicit relation between inlet and outlet temperatures can be obtained [16]. Let $\theta = T - T_\infty$ be the temperature difference with the outside air temperature T_∞ and consider

the pipe inlet temperature difference θ_i in node i and outlet temperature difference $\theta_{i,j}$ at the outlet of pipe ij . The equation for heat losses along the insulated pipe can then be formulated as

$$\theta_{ij} = \theta_i \exp\left(\frac{-L_{ij}}{\rho c_p |q_{ij}| R_{t,ij}}\right), \quad \forall ij \in A_i, \quad (5)$$

with c_p the specific heat capacity of the energy carrier. In the above equation, R_t represents the thermal resistance between the energy carrier and the environment for a pipe with a unit length, and is given by [5]

$$R_t = \frac{\log 4h/D_o}{2\pi\lambda_g} + \frac{\log D_o/D_i}{2\pi\lambda_i}, \quad (6)$$

with λ_i and λ_g the thermal conductivity of the insulation and the surrounding ground, respectively, h the depth at which the pipe is located, and D_i and D_o the inner and outer diameters of the ground. The above relations for the pipe temperature drop are validated with experimental data in Ref. [16].

In the nodes of the graph, energy conservation is used to model the effect of flow mixing. This leads to

$$\sum_{a=(i,n) \in A} (\max(q_a, 0) \theta_a + \min(q_a, 0) \theta_n) - \sum_{a=(n,j) \in A} (\max(q_a, 0) \theta_n + \min(q_a, 0) \theta_a) = 0, \quad \forall n \in N, \quad (7)$$

where the min and max functions are used to select the arc or node temperature depending on whether the flow is entering or leaving the node, respectively.

Using energy conservation on the consumer arcs gives

$$\rho c_p q_{ij} (\theta_i - \theta_{ij}) = Q_{ij}, \quad \forall ij \in A_c. \quad (8)$$

The heating power at the consumer arcs Q_{ij} is hereby modelled in the form of the characteristic equation for radiators [1]:

$$Q_{ij} = \xi (0.5\theta_i + 0.5\theta_{ij} - \theta_{c,ij})^n, \quad \forall ij \in A_c, \quad (9)$$

with $\theta_{c,a}$ the temperature difference between indoor and environment temperature at consumer arc a . Values of the coefficients ξ and n are tabulated for individual radiators, according to the EN 442-2 norm. Finally, the network inflow temperatures at the producer arcs are fixed as $\theta_a = \theta_{b,a}, \forall a \in A_p$.

2.4 Solution procedure

Remark that the above equations form a strongly nonlinear set of equations that have to be solved in an iterative approach. Because the flow equations are not depending on temperatures, hydraulic and thermal equations can hereby be decoupled and solved sequentially. Both sets of equations are then solved with a Newton solver to obtain efficient second order convergence. The linearizations of the governing equations with respect to volumetric flows, pressures, and temperatures are hereby obtained by linearizing the governing equations by hand.

3. OPTIMIZATION OF THERMAL NETWORK CONFIGURATIONS

3.1 Formulation as an optimization problem

In order to optimize thermal network configuration, we now formulate the design problem formally as a mathematical minimization problem. To this end, we define a general cost function $\mathcal{J}(\boldsymbol{\varphi}, \boldsymbol{q})$ that we will try to minimize. Here, $\boldsymbol{\varphi}$ is a vector with all design variables, i.e. what needs to be optimized, and $\boldsymbol{q}$ is a vector with all state variables, including flow rates, pressures, temperatures, and heat fluxes. Representing the model equations discussed in section 2 by $\boldsymbol{c}(\boldsymbol{\varphi}, \boldsymbol{q}) = 0$ and additional design constraints by $\boldsymbol{h}(\boldsymbol{\varphi}) < 0$, the optimization

problem is formulated as

$$\begin{aligned}
 \min_{\boldsymbol{\varphi}, \mathbf{q}} \quad & \mathcal{J}(\boldsymbol{\varphi}, \mathbf{q}) \\
 \text{s.t.} \quad & \mathbf{c}(\boldsymbol{\varphi}, \mathbf{q}) = 0, & \text{model equations} \\
 & \mathbf{h}(\boldsymbol{\varphi}) \leq 0. & \text{design constraints}
 \end{aligned} \tag{10}$$

Note that the model equations uniquely determine the state variables $\mathbf{q}$ for a set of design variables $\boldsymbol{\varphi}$, so that we in practice can consider the cost function a function of the design variables only, $\hat{\mathcal{J}}(\boldsymbol{\varphi}) = \mathcal{J}(\boldsymbol{\varphi}, \mathbf{q}(\boldsymbol{\varphi}))$ as long as $\mathbf{q}$ is evaluated by performing a model simulation in each optimization iteration.

This cost function might for example attempt minimizing total costs or optimize the thermal efficiency. For illustrational purposes, we will choose a cost function that searches a trade-off between minimizing the pressure losses through the network (and therefore pump capacity) and satisfying the consumers heat demand as good as possible. Formally we then obtain an objective function $\mathcal{J} = \beta \mathcal{J}_{\text{CH}}(\mathbf{q}) + (1 - \beta) \mathcal{J}_{\text{P}}(\mathbf{q})$, with β the trade-off parameter, $\mathcal{J}_{\text{CH}}$ the consumer heat objective

$$\mathcal{J}_{\text{CH}}(\mathbf{q}) = \lambda_{\text{CH}} \sqrt{\sum_{a \in A_c} (Q_a - Q_{d,a})^2}, \tag{11}$$

and $\mathcal{J}_{\text{P}}$ the pump objective

$$\mathcal{J}_{\text{P}}(\mathbf{q}) = \lambda_{\text{P}} \sum_{ij \in A_c \cup A_i} (p_i - p_j) q_{ij}. \tag{12}$$

Here, $Q_{d,a}$ represents the heat demand of consumer a and λ_{CH} and λ_{P} are appropriate normalization factors.

This function we now minimize as a function of the pipe volumes V_a throughout the network, as well as the consumer controls α_a and the producer outflows $q_{b,a}$, all combined in the vector $\boldsymbol{\varphi}$. By controlling producer outflows and consumer controls, we make sure that the network is dimensioned for a realistic (optimal) co-ordination of the producer pumps. In order to keep the network costs approximately fixed, $\mathbf{h}(\boldsymbol{\varphi})$ includes a limitation of the total pipe volume in the network, i.e. $\sum_{a \in A_i} V_a \leq V_{\text{tot}}$. Furthermore, the maximal value of the producers input power is constrained by limiting $q_{b,a}$ at fixed input temperature. Also pipe volumes and consumer controls can be attributed a minimal and maximal value and included into the design constraint vector $\mathbf{h}$.

The optimization problem 10 is then optimized using an iterative gradient-based optimization approach with adjoint gradient calculations (see e.g. Ref. [4]).

3.2 Test case

To discuss the influence of energy modelling on optimal thermal networks, a test case is set-up that is optimized first excluding and then including the energy model. We consider the network depicted in Fig. 2, with a single producer at feed node 1 and 5 consumers at feed nodes 2-6. The corresponding return nodes can be denoted similarly with an accent, i.e. 2'-6'. The parameters of the test case are shown in Fig. 2. The temperature at the producer is chosen equal to 80°C and the network is designed for a worst-case outside air temperature of -8°C.

We assume that the 5 consumers are relatively old houses with a demand of 20 kW each and an indoor temperature of $T_c = 20^\circ\text{C}$. The heating system characteristics are then determined to match the heat demand under nominal design conditions, denoted with subscript “des”. The design inflow temperature of water $T_{\text{in,des}}$ is chosen to equal 80°C. Furthermore, a temperature drop ΔT_{des} of 30°C and pressure drop Δp_{des} of 50 kPa over the heating system are supposed at nominal conditions. We can then obtain the heating system characteristics

Fig. 2 Set-up of a test case with one producer. The controlled arcs are indicated in grey. For node symbols, cfr. Fig. 1, with feed nodes indicated in red and return nodes in blue.

from

$$\xi \approx \frac{Q_d}{(T_{in,des} - T_c)^n} = 97.6 \frac{\text{W}}{\text{K}^{1.3}}, \quad (13)$$

$$\zeta = \frac{\Delta p_{des}}{q_{des}^2} = \frac{\Delta p_{des} * (\rho c_p \Delta T_{des})^2}{Q_d^2} = 1.9 \cdot 10^{12} \frac{\text{Pa s}}{\text{m}^3}, \quad (14)$$

using a coefficient $n = 1.3$ for old heating systems, as suggested in Ref. [1].

We start by designing the thermal network without making use of the energy equation. We follow the typical assumption of fixing the temperature drop over the consumer node to ΔT_{des} , neglecting all heat losses in the thermal network. This effectively reduces the simulation problem to that of an hydraulic network simulation. Starting from a network with equal pipe diameter sizes, we then optimize the thermal network with a limit on the total pipe volume of $V_{tot} = 1 \text{ m}^3$. The resulting pipe diameters and flow magnitudes are shown in Fig. 3. Our intuition directly confirms that the optimization procedure has found a good configuration to limit pressure drops. Indeed, pipes that bear a lot of flow are shaped with larger diameters and the pipes in arcs (2,4) and (3,4), are not used, since they would lead to a detour for the hydraulic flow. All consumers get 73.5% of their heat demand in this worst-case situation, since the cost function $\mathcal{J}$ balances decreasing the pump dimensions in $\mathcal{J}_p$, by decreasing the net inflow, against matching the consumer demand entirely, as aimed for with $\mathcal{J}_{CH}$. If we, however, re-evaluate the network solution including the energy equation, we notice that this heat demand is overestimated. Since heat is lost on the way to the consumers, the real ΔT at the consumers differs from ΔT_{des} . Especially for the consumer quite far away from the producer at node 3, where the temperature has dropped from $T = 80^\circ\text{C}$ to $T = 73.5^\circ\text{C}$, the effective heat delivered is only 60.2% (instead of 73.5%) of his heat demand.

Therefore, we now perform some more optimization iterations with the full network model, starting from the configuration found with the hydraulic model. Since the changes of pipe diameters only rang up to about 5%, the network optimization configuration is not visualized in this paper. Yet, some clear trends can be identified in the optimal network solution. First of all, all pipe sizes in the feed network, except for that of arc (2,3) have been slightly decreased. In turn, the mean pipe volume at the return side was increased. This is because small pipes lead to relatively lower heat losses. Therefore, decreasing the pipe sizes at the feed side increases the temperatures at the consumers, while adding the pipe volume to the return side decreases the pressure drop without influencing the heat delivered to the consumers. Secondly, the pipes in arcs (2,3), (3',2'), and (2',1') were increased to have relatively more flow to the consumer at node 3 to compensate for the lower feed temperature. Furthermore, the gain of increasing the inflow at the producer is relatively lower than in the hydraulic case because of thermal losses. Therefore, the balance between pump dimensioning and consumer

Fig. 3 Left: Relative pipe sizes of the optimized network, using the hydraulic thermal network model. In the consumer vertices, the width of the parameter α_{ij} is represented in purple. Right: Relative flow magnitudes in the different pipes and consumer vertices.

demand further changed and the network inflow dropped with another 30%. As such, the heat supplied to consumer 3 dropped to 50% of his demand.

4. DISCUSSION AND FUTURE PERSPECTIVES

In this paper, we applied adjoint-based optimization for the first time to the optimal design of thermal network. This allows us to use strongly nonlinear models in the optimization. Therefore, we presented a model based on mass, momentum, and energy conservation. Adjoint-based optimization was then applied to a fictitious test case, where producer pump design was balanced against meeting consumer demand in a worst-case situation. The method successfully retrieved optimal configurations. While the results of the hydraulic optimization are readily interpretable as minimization of pressure losses, the picture becomes more complex when adding the energy equation. This also leads to an increased sensitivity of this gradient-based optimization approach to so-called *local optima*. In future work, we will amongst others explore alternative parametrizations improve the conditioning of this nonlinear optimization problem. Furthermore, the optimization tool should now be further developed to account for other cost functions, accounting for e.g. investment costs. A tool should hereby be envisaged that is able to explore optimal configurations for practical 4th generation thermal networks.

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