CALCULATION OF MEASUREMENT UNCERTAINTY FOR MULTI-DIMENSIONAL MACHINES, USING THE METHOD OF SURROGATE DATA

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When a measurement has to be traceable, it is required to know the measurement uncertainty. For measurements on coordinate measurement machines (CMM's), it can be difficult to calculate the measurement uncertainty. A method is needed that can deal with the complex error structure of a CMM and with complex measurement tasks. Among other things, this method has to deal adequately with the autocorrelation behaviour of the error structure. The method of surrogate data, applied to multiple dimensions, seems a suitable way to deal with this problem.

1 Traceability and measurement uncertainty

In current measurement technology, traceability is of great importance (ISO 14253²). To obtain traceability of a measurement, it is required that the measurement uncertainty is stated. However, for coordinate measurement machines (CMM's), and other multi-dimensional machines, there is no adequate method available to evaluate the measurement uncertainty of every possible measurement. Currently, the standard work on how to state measurement uncertainty in general, is the ISO "Guide to expression of Uncertainty in Measurement"¹. A way has to be found that allows the measurement uncertainty of a specific measurement task on a CMM to be evaluated.

2 Measurement uncertainty and multi-dimensional machines

One method to obtain a traceable measurement on a coordinate measurement machine is to compare the measurement of the object with a measurement of a reference artefact that has a known measurement uncertainty. This artefact should roughly have the same size and shape as the object to be measured. This method is known as the comparator approach. The problem of this method is that the versatility of the CMM is not used anymore.

A second method is the error synthesis approach. This means that from calibration data of the machine a measurement uncertainty of other measurement tasks can be derived. Usually, a Monte Carlo method is used to combine

the errors, such a method is called a virtual CMM³. For every measurement point, a number of simulated measurement points are generated, these simulated measurement points represent possible measurements on a real CMM. From these simulations, the uncertainties of the calculated object parameters are then calculated.

In recent years, some attempts have been made to create such methods, but these models are not adequate in the treatment of autocorrelations of error signals. In this article, an approach is proposed to create a model that does take the autocorrelations into account.

2.1 The role of autocorrelation in relative measurements

A CMM measurement task always consists of multiple points. The machine measures the coordinates of several points, and from these coordinates, the parameters of interest are then calculated. This implies that a CMM always performs relative measurements. We are interested only in the value of a coordinate relative to another coordinate.

The commonly used formula for propagation of uncertainties, for a measurement task $L = x_2 - x_1$, with uncertainties $u_1 = u(x_1)$ and $u_2 = u(x_2)$ is:

$$u_L^2 = \left(\frac{\partial L}{\partial x_1}\right)^2 u_1^2 + \left(\frac{\partial L}{\partial x_2}\right)^2 u_2^2 + 2 \left(\frac{\partial L}{\partial x_1}\right) \cdot \left(\frac{\partial L}{\partial x_2}\right) \cdot \langle u(x_1) \cdot u(x_2) \rangle \quad (1)$$

The term $\langle u(x_1) \cdot u(x_2) \rangle$ represents the autocorrelation of the signal.

Absolute errors are not of importance when calculating measurement uncertainty. Very often, when the error on a certain point of the axis is stated, this is treated as if it were an absolute error. This is confusing, because it is actually the error relative to an arbitrary point. For example, the error relative to the origin of the machine.

A method that quantifies the errors of a CMM should not depend on some arbitrary point like the origin of the machine. A general method is needed that only takes into account the errors relative to each other. To put it another way, what has to be quantified is how much information about the error is preserved when moving along the axis. The autocorrelation function of the error profile seems to be a suitable tool. A Monte Carlo simulation technique, has to be arranged in such a way that the simulated errors of the machine have the same autocorrelation behaviour as the real machine. A Monte Carlo method that preserves the autocorrelation of the errors would allow us to both treat complicated measurement tasks, and correctly take into account the machine behaviour.

One way of generating simulations that have the same autocorrelation as the measured error profile is the method of surrogate data⁶. In short, the method of surrogate data uses a phase randomization scheme to generate alternative signals with the same autocorrelation function.

The amplitude spectrum of the original data is calculated using a Fourier transform. Then a phase spectrum, using a random generator, is produced. When this spectrum is transformed back to the position domain, we have a signal that has the same autocorrelation function as the original signal,

2.2 Tests on actual measurement data

To test the proposed approach, linearity and straightness errors of a Zeiss Prismo coordinate measurement machine have been investigated. In this section, the results of the linearity error measurements are discussed. A calibrated step gauge, with a pitch of 10 mm, was used to measure linearity errors of the machine along two axes of the machine. In figure 1 the results of a measurement along the x -axis are shown.

Figure 1. Measurement data (solid line) along x -axis and calibration data (star marks)

The solid line represents the measurement data. Data are available only on discrete points, with a sample width of 10 mm. The stars represent the calibration data of the used step gauge, these data are traceable to national standards. When the calibrated data are subtracted from the measurement data, the data represented in figure 2 are obtained.

Tests on these machine data have been performed to see if the linearity error profile of the machine can be modelled as a linear process. Two important conditions for linear behaviour are the variance and mean being independent of the position on the axis. The variance being independent of the position

Figure 2. Linearity error along x -axis

can easily be verified from these measurements. However, the data clearly show a trend, which suggests that the mean is depending on the position. Before verifying if this is the case in a mathematical sense, it is first checked if this could be explained from the measurement setup. There are some sources that can introduce a linear length-dependent effect on the measurement data, which can not be eliminated. For example, when the step gauge is measured, the temperature of both step gauge and machine rulers are measured. The CMM software corrects for thermal expansion of both gauge and rulers, using the linear expansion coefficients. Errors in these temperature measurements, and errors in the values of the expansion coefficients result in a linear trend in the observed data. It is impossible for the observer to separate these effects from other possible effects. The design of the machine does not suggest other substantial sources that may introduce errors that are linear depending on the position. Therefore, it is assumed that it is appropriate to subtract any observed linear effects, and then investigate the remaining errors. This means that a straight line is fitted on the data from figure 2, after which this line is subtracted from the data. Afterwards, the amplitude spectrum can be calculated.

Of course, linear effects still have to be taken into account, because they do contribute to the uncertainty of the measurement. This is done by adding a linear effect to the generated signals, using a random generator.

In figure 3, two simulated error signals are shown. These were generated from the data in figure 1, using the method of surrogate data, with the addition of a random linear effect. Furthermore, the calibration uncertainty of the step gauge itself has been taken into account when generating these signals. This is necessary to guarantee traceability of the results. These data represent the

Figure 3. Example of two simulated error signals, with random linear effect added

error profiles of non-existing CMM's, but they are, however, realistic CMM's. They have error profiles which would allow a measurement to be performed with the same uncertainty as with the CMM that provided the measurement data.

This approach can be extended to higher sampling rates per unit distance. Currently a device is developed that can calibrate the linearity errors of a CMM using any desired spatial sampling frequency. In this way, all short and long wave effects can be studied, making it easier to compare the quality of multiple CMM's. This is needed because currently, CMM's are used for tasks which involve higher sampling rates per unit distance, such as scanning.

2.3 Simulating a specific measurement task

Using the described method, it's possible to perform uncertainty calculations for one dimensional measurements. This is done by generating a number of sets of simulated errors. These data are used to perform virtual measurements. For example, a length measurement consisting of two points can be simulated by calculating the distance between two virtual points. By doing this a number of times, and observing the distribution of the values, the measurement uncertainty can be determined.

Figure 4 is a graphical representation of length measurements of several lengths. The vertical lines represent the positions of the simulated points on the signal. Two simulations are shown in the graph. Every simulation is used to simulate several measurements of different lengths. By doing this, we can calculate the length dependent measurement uncertainty of the CMM for this measurement task. A graph of these results is shown in figure 5. The

Figure 4. Simulation of length measurement of different lengths

measurement uncertainty decreases at large distances. This can be explained from the fact that the examined error signal has a dominant harmonic with a wavelength of roughly 850 mm, as can be readily seen from figure 1. This implies that the autocorrelation signal is periodic with this wavelength as well.

Figure 5. Length dependent measurement uncertainty of a length measurement

3 A two dimensional machine

A two dimensional example of a CMM is provided in figure 6. A close look has been taken at the correlation between the error in the x coordinate of measurement point P_1 and the error in the x coordinate of measurement point P_2 . The following effects have to be taken into account: the linearity error in x direction on position P_1 , linearity error in x direction on position

P_2 and straightness errors when moving in y direction from P_1 to P_2 .

Figure 6. Example of a simple two dimensional coordinate measurement machine

The required calibrations for this action are shown in figure 7. These three calibrations provide all information needed to calculate the measurement uncertainty for this task.

Within one simulation, one set of random phases for the generation of the straightness errors is used, and another one is used for the linearity errors. The total error of a measurement point is calculated by superposition of all errors. In this way, the autocorrelation of all machine errors is preserved. For the y -coordinate of the points P_1 and P_2 , a similar approach has been used. This approach can be extended to three dimensional machines as well.

The rotational errors of the machine will be calculated from the measured linearity and straightness errors. In this example, the rotational errors can be determined from the difference between the spectra on calibration line 1 and calibration line 2.

The uncertainty in the squareness errors of the machine is taken into account by adding a random squareness error to each set of simulated points.

4 Conclusions

This method will result in a calculated measurement uncertainty that takes into account the actual short and long wave behaviour of all machine errors. The method omits the need to measure the rotational errors of the machine, but it does take them into account. The method is straightforward, and can be applied to machines of virtually every configuration. Preliminary results have

Figure 7. Power spectrum of linearity and straightness errors, measured along three lines

shown that it is possible to apply this approach to determine the measurement uncertainty of multi-dimensional machines, using only one dimensional calibration data.

References

1. International Organization for Standardization, *GUM — Guide to the Expression of Uncertainty in Measurement*, 1993
2. International Organization for Standardization, *ISO 14253 — Geometrical Product Specifications*, 1998
3. Trapet, E. et al. *Development of Methods and Guidelines to Establish Traceability for Measurements with Coordinate Measuring Machines*, Physikalisch-Technische Bundesanstalt, Braunschweig, August 1998
4. S.D. Philips "A General Quantative Method to Validate Instrument Calibration Techniques", *Proceedings of ASPE 14th Annual Meeting*, 525-528, 1999
5. A. Balsamo et al. "Evaluation of CMM Uncertainty Through Monte Carlo Simulations", *Annals of the CIRP*, Vol. 48(1), 425-428, 1999
6. Theiler, J. "Testing for non-linearity in time-series", *Physica D*, Vol. 58, 77-94, 1992