

Transfer From Instruction on Pedagogical Content Knowledge About Fractions in Sixth-
Grade Mathematics to Content Knowledge and Pedagogical Knowledge

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Abstract

German pre-service teacher education aims to foster the concurrent formation of content knowledge, pedagogical knowledge, and pedagogical content knowledge. Accordingly, the coordination, sequencing, and prioritization of learning opportunities for the three areas of professional knowledge represent vital issues of the organization of teacher education at German universities. In this context, reanalyzing selected data of a previous experiment, we examined transfer from instruction on pedagogical content knowledge about fractions in sixth-grade mathematics to the formation of corresponding content knowledge and generic pedagogical knowledge. Totalling 59 pre-service elementary school teachers, three groups had received seven hours of intervention on either content knowledge, pedagogical knowledge or pedagogical content knowledge. Analyses of video recordings, ratings of treatment quality, and tests of professional knowledge revealed strong internal validity of the data. We found small transfer effects of instruction on pedagogical content knowledge on the formation of both content knowledge and pedagogical knowledge. However, contrary to our expectations, transfer to pedagogical knowledge was more consistent and statistically robust than transfer to content knowledge. For the development of pedagogical knowledge in teacher education, this finding highlights the potential of parallel or integrated coursework, in which pre-service teachers use specific instructional strategies and student conceptions as examples to derive generic principles of teaching and learning.

keywords: pedagogical content knowledge, pedagogical knowledge, transfer, mathematics, elementary education, teacher education

Transfer von fachdidaktischer Lehre zur Bruchrechnung der sechsten Jahrgangsstufe zu
Fachwissen und pädagogischem Wissen

Zusammenfassung

Das Lehramtsstudium in Deutschland zielt darauf ab, Fachwissen, fachdidaktisches Wissen und pädagogisches Wissen zu vermitteln. Entsprechend stellt die Koordinierung, Sequenzierung und Gewichtung von Lerngelegenheiten für diese drei Facetten des Professionswissens eine entscheidende Frage für die Organisation des Lehramtsstudiums dar. Vor diesem Hintergrund haben wir ausgewählte Daten eines früheren Experimentes reanalysiert, um Transfer von fachdidaktischer Lehre zur Bruchrechnung der sechsten Jahrgangsstufe zur Bildung zugehörigen Fachwissens und generischen pädagogischen Wissens zu untersuchen. Drei Gruppen, mit insgesamt 59 Studierenden des Primarstufenlehramts, erhielten Interventionen von sieben Stunden in Fachwissen, pädagogischem Wissen oder fachdidaktischem Wissen. Auswertungen von Videoaufnahmen, Beurteilungen der Lehrqualität und Testungen des Professionswissens offenbarten eine starke interne Validität der genutzten Daten. Wir fanden kleine Transfereffekte fachdidaktischer Lehre auf den Aufbau sowohl von Fachwissen als auch von pädagogischem Wissen. Entgegen unserer Erwartungen war der Transfer zu pädagogischem Wissen allerdings konsistenter und statistisch besser abgesichert als der Transfer zu Fachwissen. Dieses Ergebnis hebt das Potential paralleler oder integrierter Kurse, in welchen Lehramtsstudierende konkrete Instruktionsstrategien und Schülervorstellungen als Beispiele zur Ableitung generischer Prinzipien des Lehrens und Lernens nutzen, für die Entwicklung pädagogischen Wissens im Lehramtsstudium hervor.

Schlüsselwörter: fachdidaktisches Wissen, pädagogisches Wissen, Transfer, Mathematik,
Primarstufe, Lehrerbildung,

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1 Introduction

Following Shulman's (1987) taxonomy, researchers distinguish the broad categories of content knowledge, pedagogical knowledge, and pedagogical content knowledge within teachers' professional knowledge (e.g., Baumert & Kunter, 2013). Content knowledge contains the academic contents of a discipline. This includes understanding discipline-specific methods for generating knowledge and understanding discipline-specific core concepts (Gess-Newsome, 2015). Independent of specific content, pedagogical knowledge covers generic principles of teaching, learning, assessment, and classroom management (König, Blömeke, Paine, Schmidt, & Hsieh, 2011). Pedagogical content knowledge, finally, at the intersection of content knowledge and pedagogical knowledge, constitutes teachers' unique professional expertise: knowledge relevant for making specific content accessible to students (Depaepe, Verschaffel, & Kelchtermans, 2013). In this context, content knowledge is often considered a necessary prerequisite for pedagogical content knowledge (Ball, 1991). Field studies have shown that teachers' content knowledge, pedagogical knowledge, and pedagogical content knowledge are positively associated with instructional quality and student outcomes (e.g., Baumert et al., 2010; Brühwiler & Blatchford, 2011; Gess-Newsome et al., 2017; Keller, Neumann, & Fischer, 2017; Sadler, Sonnert, Coyle, Cook-Smith, & Miller, 2013; Voss, Kunter, Seiz, Hoehne, & Baumert, 2014). So, both theoretical and empirical arguments support widespread consensus that all three main areas of professional knowledge constitute relevant targets for teacher education.

The organization of pre-service teacher education at German universities reflects the assumed tripartition of teachers' professional knowledge. Specialized faculty provides prospective teachers with instruction on content knowledge, pedagogical knowledge, and

pedagogical content knowledge in separated courses and lectures. In this context, faculty teaching content knowledge and pedagogical content knowledge is organized in discipline-specific departments, while faculty teaching pedagogical knowledge resides in departments of education and psychology (König et al, 2018). This organizational framework affords decisions regarding the coordination, sequencing, and prioritization of courses in the three main areas of professional knowledge. To inform such decisions with empirical evidence, we were interested in potential transfer from instruction on pedagogical content knowledge to pre-service teachers' content knowledge and pedagogical knowledge. Specifically, we selected and reanalyzed data from an experimental study on the development of pedagogical content knowledge about fractions in sixth-grade mathematics (Author, 2017; Author, 2018).

1.1 Transfer From Instruction on Pedagogical Content Knowledge?

To our knowledge, with regard to the development of teachers' professional knowledge, currently no model of professional knowledge includes explicit assumptions about transfer from instruction on pedagogical content knowledge to content knowledge and pedagogical knowledge. Listing relevant components of teachers' professional knowledge, models have been predominately taxonomic in nature (e.g., Baumert & Kunter, 2013; Magnusson, Krajcik, & Borko, 1999; Shulman, 1987). Recently, efforts have been made to devise models that chart the intermediate cognitive representations between general declarative professional knowledge and actual classroom practice (e.g., Blömeke, Gustafsson, & Shavelson, 2015; Gess-Newsome, 2015). However, when taking a developmental perspective, theoretical thought and empirical research has concentrated on pedagogical content knowledge as the central outcome of teacher education. This entailed a focus on the contributions of content knowledge and pedagogical knowledge to the formation of pedagogical content knowledge, and neglect of the investigation of transfer in the reversed direction (e.g., Schneider & Plasman, 2011; Van Driel & Berry, 2010; Author, 2018).

Are there theoretical considerations that support the idea of potential transfer from instruction on pedagogical content knowledge to teachers' content knowledge and pedagogical knowledge? In Shulman's (1987) initial conceptualization of the construct, pedagogical content knowledge constitutes a blend of content and pedagogy. With this definition in mind, it appears straightforward to assume that it should be possible to retrieve content knowledge and pedagogical knowledge from instruction on pedagogical content knowledge. This general assumption gains additional plausibility when pedagogical content knowledge is viewed as a transient phenomenon resulting from the simultaneous activation of cognitive representations of content knowledge and pedagogical knowledge (Gess-Newsome, 1999).

In line with very strong correlations between content knowledge and pedagogical content knowledge (Baumert et al., 2010; Depaepe et al., 2015), some researchers in the field of mathematics education consider the two constructs inseparable. For instance, the concept of mathematical knowledge for teaching combines content knowledge and pedagogical content knowledge in a single superordinate construct (Ball, Thames, & Phelps, 2008; Hill, Schilling, & Ball, 2004). According to this understanding, it should be impossible to provide instruction on pedagogical content knowledge without implicitly also providing some instruction on corresponding content knowledge. However, in any case, pedagogical content knowledge and content knowledge are closely related because they both constitute forms of subject-specific knowledge (Baumert & Kunter, 2013). In other words, they occupy the same level of abstraction so that, for transfer to occur, learners should only have to salvage the inherent content knowledge. Moreover, instruction on pedagogical content knowledge should trigger learners to reflect on their own previous learning of content. For instance, instruction on subject-specific student misconceptions should help pre-service teachers to correct their own misunderstandings of content.

At first glance, transfer from pedagogical content knowledge to pedagogical knowledge appears less straightforward than transfer to content knowledge as it requires pre-service teachers to derive generic principles of teaching and learning from specific examples of instructional strategies, learning difficulties, and student conceptions. However, psychological research on the acquisition of formal principles has shown that learners can derive abstract principles successfully from the comparison of multiple examples (Schalk, Saalbach, & Stern, 2016). In a similar vein, research in pre-service teacher education has uncovered that the acquisition of generic pedagogical knowledge is fostered by the integration of examples from pedagogical content knowledge into learning materials (Harr, Eichler, & Renkl, 2014, 2015).

1.2 The present study

To examine potential transfer effects of instruction on pedagogical content knowledge on the formation of content knowledge and pedagogical knowledge, we resorted to data of an experiment that had investigated the development of pre-service elementary school teachers' pedagogical content knowledge (Author, 2017; Author, 2018). The original objective of the study had been to test three hypotheses: the amalgamation hypothesis (i.e., content knowledge and pedagogical knowledge amalgamate to form pedagogical content knowledge), the sufficiency hypothesis (i.e., content knowledge is sufficient to generate pedagogical content knowledge), and the facilitation hypothesis (i.e., prior content knowledge facilitates learning from instruction on pedagogical content knowledge). Participants completed two-day interventions featuring combinations of instruction in selected areas of professional knowledge. Specifically, the study design comprised three experimental groups, each embodying one of the hypotheses about the formation of pedagogical content knowledge, and two control groups that received exclusively instruction on either pedagogical content knowledge or pedagogical knowledge. The topic of fractions, essential for elementary and

lower secondary mathematics, had been selected for the design of the interventions on and tests of content and pedagogical content knowledge. While students struggle to build a suitable understanding of fractions, teachers, particularly elementary school teachers, possess restricted content and pedagogical content knowledge about fractions (Depaepe et al., 2015).

For the original study, a sample of 100 pre-service elementary school teachers starting their second year at university had been recruited in Berlin and Potsdam. This was because, on the one hand, in contrast to the majority of German federal states, the federal states of Berlin and Brandenburg offer a six-year elementary school. Accordingly, fractions are part of the elementary school mathematics curriculum in these federal states. On the other hand, it had been intended to recruit pre-service teachers with limited proficiency in all three areas of professional knowledge. At the same time of the study, the universities in Berlin and Potsdam offered a variety of undergraduate programs that formally qualified for future careers at both elementary and non-academic secondary schools. These programs differed in terms of their focus on elementary or lower secondary education.

For the present investigation, we selected those three groups from the study design that had received exclusive treatment in one area of professional knowledge; the group exploring the sufficiency hypothesis, which had received instruction on content knowledge and the two control groups, which had received instruction on either pedagogical knowledge or pedagogical content knowledge. As we were interested in transfer from instruction on pedagogical content knowledge, the latter formed the experimental group for the present investigation, while the two other groups served as control groups. When we examined transfer to content knowledge, we used the group receiving instruction on content knowledge as a strong control group defining an upper bound of growth in content knowledge, while we used the group receiving instruction on pedagogical knowledge as a weak control group not inducing instruction related growth in content knowledge. When we explored transfer to

pedagogical knowledge, we alternated the roles of the control groups.

Besides its direct effect on the formation of pedagogical content knowledge, we expected instruction on pedagogical content knowledge to induce growth in content knowledge and pedagogical knowledge. However, because both content knowledge and pedagogical content knowledge constitute subject-specific areas of teachers' professional knowledge, we anticipated transfer to content knowledge to be easier than transfer to pedagogical knowledge. Thus, we hypothesized that growth due to instruction on pedagogical content knowledge should be larger for content knowledge than for pedagogical knowledge.

2 Method

2.1 Design

For the present investigation, we selected three groups from the original experimental design: the group that received exclusively instruction on pedagogical content knowledge, the group that received exclusively instruction on content knowledge, and the group that received exclusively instruction on pedagogical knowledge. The interventions comprised two treatment sessions, which were conducted at two consecutive days. Each session consisted of two treatment blocks. We included three assessments from the original design in our investigation: a pretest at the outset of the interventions, an intermediate test before the start of the second treatment sessions, and a posttest at the end of the interventions.

The content of the interventions was closely aligned. The interventions on both content knowledge and pedagogical knowledge had specific overlap with the intervention on pedagogical content knowledge. There was no overlap between the interventions on content knowledge and pedagogical knowledge. For example, the intervention on pedagogical content knowledge covered enactive and iconic representations that illustrated the technique of expanding and reducing fractions. The intervention on pedagogical knowledge introduced the generic classification of enactive, iconic, and symbolic representations without mathematical

content, while the intervention on content knowledge presented the technique of expanding and reducing fractions without enactive and iconic representations.

2.2 Participants

The three groups selected for the present analysis comprised 59 pre-service teachers (53 were female). Their age ranged from 19 to 44 years; median age was 21 years. They were enrolled in undergraduate programs that qualified them for prospective careers at elementary and non-academic secondary schools. One participant completing a program with a focus on lower secondary education, the vast majority of participants completed programs with a focus on elementary education. Except for one participant, which had been enrolled already for four years, participants were just starting their second year at university. Four participants were majoring in mathematics education, 48 participants were minoring in mathematics education, and four participants were not studying mathematics education at all.

Aiming for a sample of prospective elementary school teachers at the beginning of their university training, for the original study a pool of 165 applicants from the universities in Berlin and Potsdam had been recruited for potential participation. From this pool, 30 applicants were randomly invited to each group of the original design. The recruitment procedure and dropout just before the start of the study resulted in group sizes of 23, 16, and 20 participants for the groups receiving instruction on content knowledge, pedagogical knowledge, and pedagogical content knowledge, respectively. Participants were paid 200 Euros (or 160 Euros when missing the follow-up of the original study).

2.3 Intervention Procedures

The two-day interventions had a common time schedule. The days started with assessments (i.e., the 120-minute pretest and the 60-minute intermediate test, respectively), which were followed by 30-minute breaks. These breaks were followed by two 105-minute treatment blocks, which were separated by 60-minute lunch breaks. Additionally, the second

day ended with another 30-minute break and the 90-minute posttest. In total, each group received seven hours of treatment.

The treatments were administered by an experienced lecturer in elementary mathematics education. To prevent teaching to the test, he was oblivious to the items of the tests of professional knowledge. The instructor used presentation slides designed by the second author. Various activities ranging from short questions to role play ensured active learning. Participants produced a written summary of the central points of preceding instruction at the end of each treatment block.

The groups included in the present investigation provided instruction on the same area of professional knowledge on both days. Thus, they featured a basic and an advanced session. Apart from straightforward repetition, the advanced sessions added further examples and perspectives to the scope set by basic sessions. To test fidelity of treatment implementation, the interventions had been recorded on video. However, mishandling of equipment had prevented the recording of the advanced session on pedagogical knowledge.

Pedagogical content knowledge. The first block of the basic session commenced with an introduction to the importance of student conceptions for teaching mathematics (Padberg, 2009). In line with this, the first block covered primarily conceptual understanding of fractions, for instance, the part–whole concept and the operator concept (e.g., Wartha, 2009). In addition, the first block introduced techniques for demonstrating the density property of rational numbers and the correspondence of a class of fractions with a specific rational number. The second block was dedicated to teaching operations with fractions. This included methods that support the flexible use of strategies for comparing the size of fractions and teaching approaches that adequately deal with common arithmetical errors in addition and multiplication involving fractions (e.g., Clarke & Roche, 2009; Cramer & Bezuk, 1991; Siegler & Lortie-Forgues, 2015). The second block ended with a summary of the fundamental

changes in conceptual understanding necessary when transitioning from the set of natural numbers to the set of rational numbers (Prediger, 2008).

The advanced session started with a brief review of these fundamental conceptual changes. The rest of the first block covered teaching operations, which included comparison of, multiplication with, and division with fractions. The second block was dedicated to topic-specific representations. This included enactive, iconic, and symbolic representations for clarifying addition with fractions, multiplication with fractions, and expanding and reducing fractions.

Content knowledge. The first block of the basic session began with the introduction of the terms numerator, denominator, and vinculum. The instructor derived the set of positive rational numbers from the set of natural numbers by defining fractions as equivalence classes of linear equations (Reiss & Schmieder, 2005). Furthermore, he pointed out that equivalent fractions represent the same rational number; expanding and reducing were presented as techniques for converting equivalent fractions into each other. The second block was dedicated to arithmetic with fractions. Relying on the definition of fractions as linear equations, participants examined properties of addition, multiplication, and division. This included a discussion of closure. Finally, participants were guided to the discovery of the density property of rational numbers by ordering fractions according to size.

To start the advanced session, the instructor repeated the derivation of the set of positive rational numbers from the set of natural numbers. The remainder of the first block covered the differentiation of fractions from rational numbers, expanding and reducing fractions, and the cardinality and density property of the set of positive rational numbers. Beyond mere repetition, the second block included demonstrations of the validity of the distributive, commutative, and associative laws within the set of rational numbers.

Pedagogical knowledge. The first block of the basic session defined teachers as

creators of learning opportunities for their students (Seidel, 2014). Additionally, from a constructivist perspective, the first block covered generic principles of learning, that is, learning was framed as conceptual change necessitating the enrichment and revision of student conceptions (e.g., Vosniadou & Brewer, 1987). The second block addressed generic principles of teaching. This included adequate handling of student conceptions, constructive feedback, scaffolding, and the use of representations to foster understanding (Bruner, 1966; Oser & Spychiger, 2005; Reiser, 2004).

The advanced session started with a repetition of generic principles of learning. In addition, the first block trained participants' capacities for diagnosing student conceptions. Likewise, apart from reviewing principles of teaching, the second block concentrated on reducing complexity and structuring as domain-general methods for scaffolding.

2.4 Measures

Pedagogical content knowledge. The measurement of pedagogical content knowledge featured 40 items. In eight instances, two to four items were nested under a common stem. Items were rotated unsystematically across assessments. Participants completed 36, 29, and 38 items at pretest, intermediate test, and posttest, respectively. For the subsample of this reanalysis, corresponding values of Cronbach's alpha were .62, .63, and .76, respectively. We presented 23 anchor items across all three assessments and showed 17 items twice. We adopted 27 items from previous research on German teachers' professional knowledge (Heinze, Dreher, Lindmeier, & Niemand, 2016; Kleickmann et al., 2014). Additionally, previous research inspired the design of nine further items (Depaepe et al., 2015; Hill et al., 2004).

On the one hand, the test featured topic-specific student conceptions and learning difficulties. On the other hand, the test covered topic-specific representations and instructional strategies. This paralleled the coverage of generic aspects of learning and generic aspects of

teaching in the test of pedagogical knowledge. Moreover, in parallel to the test of content knowledge, the test addressed conceptual understanding of fractions and arithmetic with fractions. In particular, the test included elementary school students' topic-specific concepts (e.g., the part-whole concept), topic-specific representations for fostering understanding of fractions and arithmetic involving fractions, typical arithmetical errors, and instructional strategies relevant for arithmetic with fractions and the ordering of fractions.

Content knowledge. The test of content knowledge relied on a pool of 27 items. In four instances, two to three items were clustered under a common stem. The test had an incomplete item design. Participants completed 20, 19, and 24 items at pretest, intermediate test, and posttest, respectively. For the subsample of this reanalysis, corresponding values of Cronbach's alpha were .81, .82, and .85, respectively. We showed 11 anchor items across all three assessments, used 15 items twice, and presented one item once. We borrowed two items from previous research (Depaepe et al., 2015). We based another five items on released items from the Trends in International Mathematics and Science Study (Mullis et al., 2005; Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009).

The test probed both conceptual understanding of fractions and arithmetic with fractions. On the one hand, this included equivalence of fractions and linear equations, properties of the set of rational numbers, and understanding the size of fractions (e.g., placing fractions on a number line). On the other hand, the test covered conversion of decimals into fractions, and vice versa, as well as addition, subtraction, multiplication, and division involving fractions. For the most part, these arithmetic tasks were embedded in word problems. Generally, the test featured proper fractions, improper fractions, and mixed numerals.

Pedagogical knowledge. The test of pedagogical knowledge featured a pool of 40 items. In 10 instances, two to four items were nested under a common stem. Items were

partially rotated across assessments. Participants completed 29, 27, and 34 items at pretest, intermediate test, and posttest, respectively. For the subsample of this reanalysis, corresponding values of Cronbach's alpha were .54, .73, and .82, respectively. We included 16 anchor items across all three assessments, used 16 items twice, and showed eight items once. We adapted seven items from previous research on German teachers' professional knowledge (Kleickmann et al., 2014; Linninger et al., 2015; Lohse-Bossenz, Kunina-Habenicht, Dicke, Leutner, & Kunter, 2015).

With regard to generic aspects of learning, the test covered principles of socio-constructivism, the relevance of student conceptions, and the effects of prior knowledge on learning outcomes. With regard to generic aspects of teaching, the test addressed the productive handling of student errors, modes of representation, and scaffolding. Due to the alignment of the three tests of professional knowledge, yet contrary to other instruments (e.g., König et al., 2011; Voss, Kunter, & Baumert, 2011), the test did not address classroom management.

2.5 Implementation Check

To check the integrity of treatment implementation, we explored participants' perception of treatment quality, observer ratings of video segments, and participants' performance on the test of pedagogical content knowledge. On four items with a four-point Likert-style response format, participants rated the quality of preceding instruction for each session. Scores from 0 to 3 were assigned to the response categories. The interventions on pedagogical content knowledge, pedagogical knowledge, and content knowledge receiving average ratings of 2.65 ($SD = 0.31$), 2.69 ($SD = 0.29$), and 2.79 ($SD = 0.26$), respectively, perception of treatment quality was very positive. According to a univariate analysis of variance with group as a between-subjects factor, perceived treatment quality did not vary significantly across interventions, $F(4, 56) = 1.43$, $p = .25$, $\eta^2 = .05$.

Two undergraduate research assistants rated a stratified sample of one-minute segments from the video recordings of the treatment blocks. First, they produced a disjunctive categorization of the video segments (“Is this segment an example of instruction on content knowledge, pedagogical knowledge, or pedagogical content knowledge?”). For the three groups included in the present investigation, averaged over both raters, agreement in terms of Cohen’s kappa between raters’ classifications and the intended treatment was .90. Second, on a four-point Likert scale, the raters provided a non-disjunctive evaluation of the extent to that each area of professional knowledge was evident in the video segments (e.g., “Is instruction on content knowledge present in this video segment?”). Scores from 0 to 3 were assigned to these evaluations. The sessions on pedagogical content knowledge included in the present investigation received average evaluations of 0.65, 0.03, and 2.26 for the presence of instruction on content knowledge, pedagogical knowledge, and pedagogical content knowledge, respectively. The central test of treatment integrity, performance on the test of pedagogical content knowledge, is reported in the results section.

2.6 Statistical Analysis

We estimated explanatory item response models to analyze performance on the tests of professional knowledge, that is, participants’ responses were modeled with generalized linear mixed models featuring a logistic link function and crossed random effects for persons and items (De Boeck & Wilson, 2004). Ensuring simultaneous generalizability of the results to new persons and new items, these random effects accounted for variance in overall person ability and overall item difficulty. To gauge the effects of the interventions, we incorporated dummy-coded variables for group, assessment, and their interaction as fixed effects in the models. We computed odds ratios as measures of effect size for fixed effects. For model estimation, we used the lme4-package (Bates, Maechler, & Bolker, 2011) for the statistical computing environment R 3.0.0 (R Core Team, 2013).

3 Results

3.1 Implementation Check: Growth of Pedagogical Content Knowledge

To test the integrity of treatment implementation, we investigated if instruction on pedagogical content knowledge actually produced growth in participants' pedagogical content knowledge. We estimated an explanatory item response model for responses on the test of pedagogical content knowledge with crossed random effects for persons and items. We included dummy-coded variables for group and assessment as fixed effects into the model. The reference categories for group and assessment were group receiving instruction on pedagogical knowledge and pretest, respectively.

There were no statistically significant main effects for group, that is, the groups did not differ substantially in pedagogical content knowledge when the interventions started (see Table 1). There were no statistically significant main effects for assessment either; in other words, the group receiving instruction on pedagogical knowledge did not gain pedagogical content knowledge over the course of the interventions. However, statistically significant interactions indicated that the group receiving instruction on pedagogical content knowledge outperformed the group receiving instruction on pedagogical knowledge at both intermediate test, $B = 0.66$, $SE = 0.19$, $OR = 1.93$, $p < .001$, and posttest, $B = 1.04$, $SE = 0.18$, $OR = 2.83$, $p < .001$. Obviously, our intervention caused growth in pedagogical content knowledge as intended. Moreover, for the group receiving instruction on content knowledge, we observed enhanced pedagogical content knowledge at posttest, $B = 0.48$, $SE = 0.17$, $OR = 1.61$, $p < .01$. Apparently, instruction on content knowledge was sufficient to induce the formation of a small amount of pedagogical content knowledge (see Author, 2018).

3.2 Effects on Content Knowledge

To examine if instruction on pedagogical content knowledge caused gains in content knowledge, we estimated an explanatory item response model for responses on the test of

content knowledge. Besides crossed random effects for persons and items, as fixed effects, we incorporated dummy-coded variables for group and assessment into the model. The reference categories for group and assessment were group receiving instruction on pedagogical knowledge and pretest, respectively.

The absence of statistically significant main effects for group indicated that there were no substantial disparities in content knowledge between groups at the outset of the interventions (see Table 1). Regarding the main effects for assessment, we found that performance at posttest significantly exceeded performance at pretest, $B = 0.44$, $SE = 0.19$, $OR = 1.55$, $p = .02$. In other words, at posttest, the group that received instruction on pedagogical knowledge showed a substantial growth in content knowledge (see Figure 1). In comparison to the group that received instruction on pedagogical knowledge, implied by statistically significant interactions, the group that received instruction on content knowledge displayed enhanced content knowledge at intermediate test, $B = 1.28$, $SE = 0.25$, $OR = 3.60$, $p < .001$, and posttest, $B = 1.78$, $SE = 0.25$, $OR = 5.94$, $p < .001$. Apparently, the intervention targeting content knowledge produced knowledge gains in the intended area. Finally, relative to the group receiving instruction on pedagogical knowledge, for the group receiving instruction on pedagogical content knowledge a marginally significant interaction hinted at enhanced content knowledge at posttest, $B = 0.42$, $SE = 0.25$, $OR = 1.52$, $p = .09$. So, there was small, statistically marginally significant, transfer from instruction on pedagogical content knowledge to the formation of content knowledge (see Figure 1).

3.3 Effects on Pedagogical Knowledge

To explore if instruction on pedagogical content knowledge produced growth in pedagogical knowledge, we estimated an explanatory item response model for responses on the test of pedagogical knowledge. It contained crossed random effects for persons and items. We included dummy-coded variables for group and assessment as fixed effects in the model.

The reference category for group was group receiving instruction on content knowledge. The reference category for assessment was pretest.

The absence of statistically significant main effects for group showed that the groups did not vary systematically in pedagogical knowledge when the interventions began (see Table 1). According to the main effects for assessment, the group receiving instruction on content knowledge displayed a significant decrease in pedagogical knowledge at intermediate test, $B = -0.47$, $SE = 0.16$, $OR = 0.62$, $p < .01$, and no growth of pedagogical knowledge at posttest, $B = 0.01$, $SE = 0.14$, $OR = 1.01$, $p = .95$. In contrast to this, for the group receiving instruction on pedagogical knowledge, statistically significant interactions indicated a substantial increase in pedagogical knowledge at intermediate test, $B = 1.32$, $SE = 0.22$, $OR = 3.75$, $p < .001$, and posttest, $B = 1.62$, $SE = 0.21$, $OR = 5.07$, $p < .001$. Similarly, for the group receiving instruction on pedagogical content knowledge, statistically significant interactions revealed an increase in pedagogical knowledge at intermediate test, $B = 0.45$, $SE = 0.22$, $OR = 1.57$, $p = .04$, and posttest, $B = 0.65$, $SE = 0.20$, $OR = 1.92$, $p < .001$. Apparently, the intervention on pedagogical knowledge generated the intended growth in pedagogical knowledge. Moreover, we observed small, yet consistent and statistically significant, transfer from instruction on pedagogical content knowledge to the formation of pedagogical knowledge (see Figure 2).

4 Discussion

So far, in research on teacher education, transfer from instruction on pedagogical content knowledge to the formation of content knowledge and pedagogical knowledge has received limited attention. Reanalyzing data of an experimental study, we found some evidence for a transfer effect on content knowledge; though, at posttest, transfer to content knowledge was only of marginal statistical significance, the small effect was approximately as large as the statistically significant transfer to pedagogical knowledge at intermediate test.

(Please also note that one-sided statistical testing would have resulted in the observation of statistically significant transfer to content knowledge.)

Contrary to our anticipations, we found transfer to pedagogical knowledge to be more consistent and statistically robust than transfer to content knowledge; both at intermediate test and at posttest, there was small, statistically significant and continuously increasing transfer from instruction on pedagogical content knowledge to pedagogical knowledge. Apparently, participants were capable to retrieve generic knowledge about teaching and learning from specific examples in elementary mathematics education; in this context, it is plausible that cognitive processes of comparison and reflection were prompted by the repeated testing of pedagogical knowledge (Schalk et al., 2016). Moreover, our evidence in favor of transfer from instruction on pedagogical content knowledge to pedagogical knowledge is in line with previous occasional observations of such transfer in field studies (König et al., 2018).

The data possessed excellent internal validity: At the outset of the interventions, the groups did not vary substantially in professional knowledge, participants' perceptions of treatment quality were similar across groups, and observers' ratings of videotaped instruction underscored that the groups had received the intended treatment. In support of external validity, the interventions had been conducted by an experienced lecturer in elementary mathematics education. So, with respect to teaching methods, pacing, and learning activities, the interventions resembled instruction offered in conventional courses of German pre-service teacher education.

Which limitations do apply to the present investigation? Our findings rely on a study with brief interventions and a small sample. The interventions covered only a single topic of elementary mathematics education. Naturally, this constrains the generalizability of results. Thus, for future research, it is essential to replicate the present observations for other topics. Moreover, exploring only immediate effects, the intervention study featured massed practice

and intense testing. This is atypical for pre-service teacher education at German universities. It is likely that the wider time frame in natural teacher education, that is, delays between courses in the different areas of professional knowledge as well as delays between instruction and testing, hamper the spontaneous induction of comparison processes necessary for transfer. Therefore, future research should investigate the effectiveness of explicit prompts for transfer in more natural settings (see Graichen, Wegner, & Nückles, this issue).

Our findings expand the results of a recent quasi-experimental study that uncovered transfer effects of marginal statistical significance from instruction on pedagogical content knowledge to content knowledge and pedagogical knowledge in a four-week follow-up (Evens, Elen, Larmuseau, & Depaepe, 2018): In an experimental setting, spontaneous transfer from instruction on pedagogical content knowledge to content knowledge and pedagogical knowledge is possible. Why was transfer to content knowledge small and relatively unstable? From a theoretical perspective, transfer from instruction on common arithmetic errors involving fractions to performance on arithmetic tasks involving fractions is straightforward. However, presumably, spontaneous transfer to only slightly more formalized aspects of content knowledge about fractions, for instance, construing fractions as equivalence classes of linear equations, is too difficult for prospective elementary school teachers. So, our observation of relatively elusive transfer to content knowledge can be interpreted as evidence in favor of the necessity of explicit instruction on content knowledge in pre-service elementary school teacher education.

With regard to the development of pedagogical knowledge, the results entail the following tentative conclusions for pre-service teacher education: First, when prioritizing courses and lectures for content knowledge, pedagogical knowledge, and pedagogical content knowledge in pre-service teacher education, transfer effects from instruction in pedagogical content knowledge to the other areas of professional knowledge could be taken into account.

Second, coursework on pedagogical knowledge, on generic principles of teaching and learning, has not necessarily to precede coursework on pedagogical content knowledge, on subject-specific instructional strategies and student conceptions. Rather, parallel coursework, which offers examples from pedagogical content knowledge to derive generic pedagogical knowledge, might prove fruitful for the formation of pedagogical knowledge, in particular, when pre-service teachers receive explicit prompts. This might also be realized via blending instruction on pedagogical content knowledge and pedagogical knowledge into integrated courses (Evens et al., 2018; Harr et al., 2014, 2015).

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Table 1

Effects of Explanatory Item Response Models for the Prediction of Pedagogical Content Knowledge, Content Knowledge, and Pedagogical Knowledge

| Effects | Dependent variable | | | | | | | | |
|----------------|--|------|------|--------------------------------|------|------|------------------------------------|------|------|
| | Pedagogical content knowledge ^a | | | Content knowledge ^a | | | Pedagogical knowledge ^b | | |
| | Est. | SE | OR | Est. | SE | OR | Est. | SE | OR |
| Fixed effects | | | | | | | | | |
| Intercept | -0.21 | 0.25 | 0.81 | -0.85* | 0.39 | 0.43 | -1.63*** | 0.28 | 0.20 |
| CK-CK | -0.02 | 0.21 | 0.98 | 0.28 | 0.37 | 1.33 | --- | --- | --- |
| PK-PK | --- | --- | --- | --- | --- | --- | 0.24 | 0.21 | 1.27 |
| PCK-PCK | 0.05 | 0.22 | 1.05 | 0.09 | 0.38 | 1.10 | 0.11 | 0.20 | 1.11 |
| Inter | 0.01 | 0.14 | 1.01 | 0.29 | 0.20 | 1.34 | -0.47** | 0.16 | 0.62 |
| Post | -0.02 | 0.13 | 0.98 | 0.44* | 0.19 | 1.55 | 0.01 | 0.14 | 1.01 |
| CK-CK*Inter | 0.20 | 0.19 | 1.22 | 1.28*** | 0.25 | 3.60 | --- | --- | --- |
| PK-PK*Inter | --- | --- | --- | --- | --- | --- | 1.32*** | 0.22 | 3.75 |
| PCK-PCK*Inter | 0.66*** | 0.19 | 1.93 | -0.10 | 0.27 | 0.90 | 0.45* | 0.22 | 1.57 |
| CK-CK*Post | 0.48** | 0.17 | 1.61 | 1.78*** | 0.25 | 5.94 | --- | --- | --- |
| PK-PK*Post | --- | --- | --- | --- | --- | --- | 1.62*** | 0.21 | 5.07 |
| PCK-PCK*Post | 1.04*** | 0.18 | 2.83 | 0.42 [†] | 0.25 | 1.52 | 0.65*** | 0.20 | 1.92 |
| Random effects | | | | | | | | | |
| Participants | Var. | | | Var. | | | Var. | | |
| | 0.27 | | | 0.98 | | | 0.18 | | |
| Items | 1.48 | | | 1.92 | | | 2.39 | | |

Note. Reference category for assessment is pretest. CK-CK = group receiving instruction on content knowledge; PK-PK = group receiving instruction on pedagogical knowledge; PCK-PCK = group receiving instruction on pedagogical content knowledge; Inter = intermediate test; Post = posttest; SE = standard error; OR = odds ratio; Var. = variance.

^aReference category for group is group receiving instruction on pedagogical knowledge.

^bReference category for group is group receiving instruction on content knowledge.

[†] $p < .10$. * $p < .05$. ** $p < .01$. *** $p < .001$.

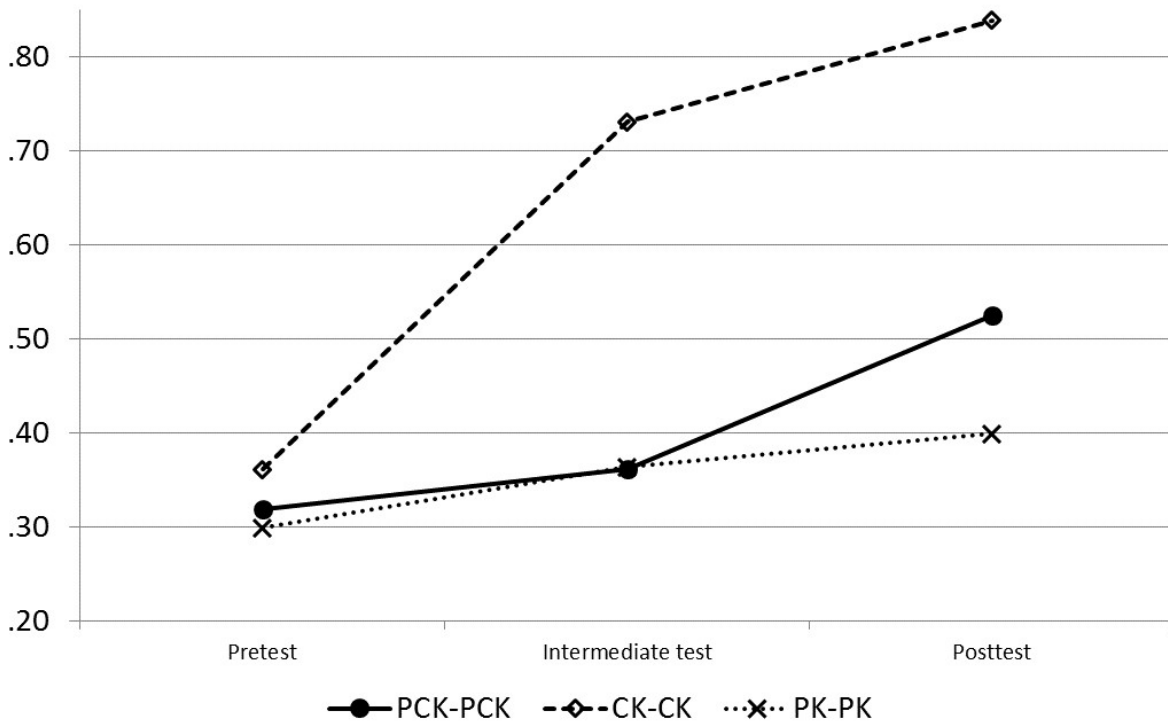


Figure 1. Estimated average probabilities of correct responses for the test on content knowledge by group across assessments. Average probabilities were derived from estimates of fixed effects of the corresponding explanatory item response model. PCK-PCK = group receiving instruction on pedagogical content knowledge; CK-CK = group receiving instruction on content knowledge; PK-PK = group receiving instruction on pedagogical knowledge.

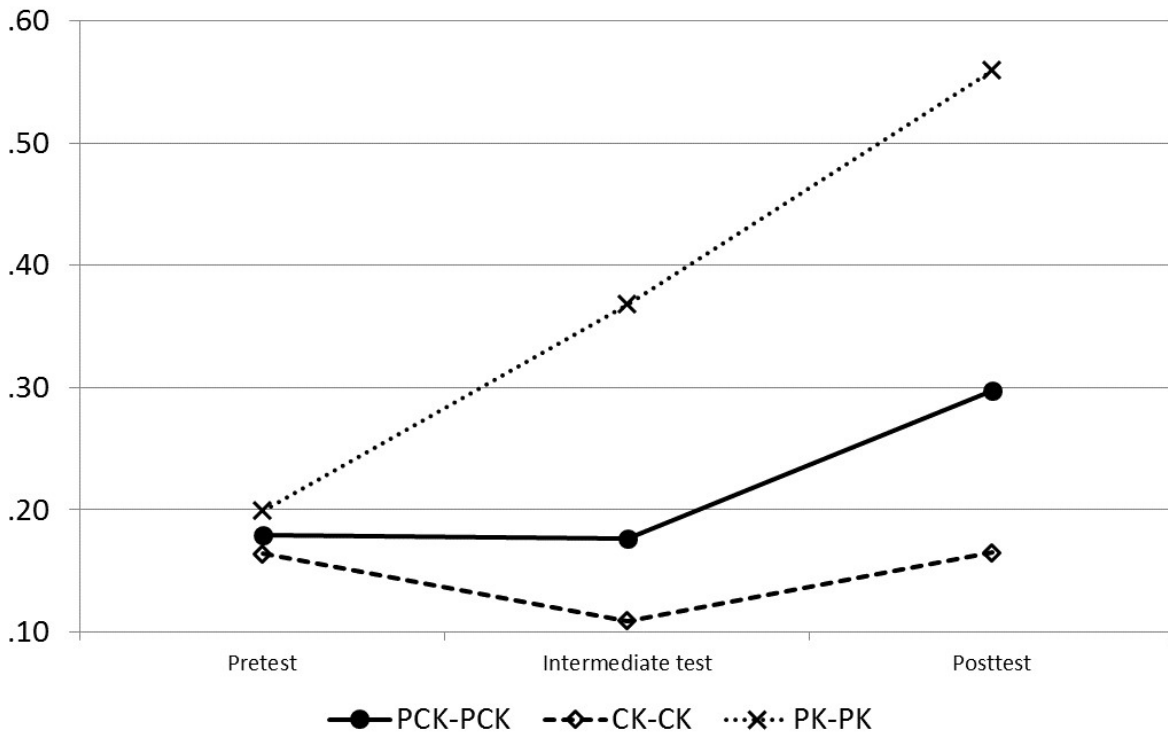


Figure 2. Estimated average probabilities of correct responses for the test on pedagogical knowledge by group across assessments. Average probabilities were derived from estimates of fixed effects of the corresponding explanatory item response model. PCK-PCK = group receiving instruction on pedagogical content knowledge; CK-CK = group receiving instruction on content knowledge; PK-PK = group receiving instruction on pedagogical knowledge.