MAGNETOHYDRODYNAMICS
of Laboratory and Astrophysical Plasmas

With 90% of visible matter in the Universe existing in the plasma state, an understanding of magnetohydrodynamics is essential for anyone looking to understand solar and astrophysical processes, from stars to accretion discs and galaxies; as well as laboratory applications focused on harnessing controlled fusion energy.

This introduction to magnetohydrodynamics brings together the theory of plasma behavior with advanced topics including the applications of plasma physics to thermonuclear fusion and plasma-astrophysics. Topics covered include streaming and toroidal plasmas, nonlinear dynamics, modern computational techniques, incompressible plasma turbulence and extreme transonic and relativistic plasma flows. The numerical techniques needed to apply magnetohydrodynamics are explained, allowing the reader to move from theory to application and exploit the latest algorithmic advances. Bringing together two previous volumes: *Principles of Magnetohydrodynamics* and *Advanced Magnetohydrodynamics*, and completely updated with new examples, insights and applications, this volume constitutes a comprehensive reference for students and researchers interested in plasma physics, astrophysics and thermonuclear fusion.

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To Antonia, 陆蓉 (Rong Lu) and Małgosia

Contents

Preface

This book describes the two main applications of plasma physics, laboratory research on thermonuclear fusion energy and plasma-astrophysics of astronomical systems, from the single viewpoint of magnetohydrodynamics (MHD). This provides effective methods and insights for the interpretation of plasma phenomena on virtually all scales, ranging from the laboratory to the Universe. The key issue is understanding the complexities of plasma dynamics in extended magnetic structures. In the first half of the book, based on a revision of the previous volume *Principles of Magnetohydrodynamics* [1], the classical MHD model is developed in great detail without omitting steps in the derivations. This necessitated restriction to ideal dissipationless plasmas, in static equilibrium and inhomogeneous in one direction. In the second half of the book, based on a revision of the previous *Advanced Magnetohydrodynamics* [2], these restrictions are relaxed one by one: introducing stationary background flows, dissipation, two-dimensional toroidal geometry, linear and nonlinear computational techniques, turbulence, transonic flows and relativity. These topics transform the subject into a vital new area with numerous applications in laboratory, space and astrophysical plasmas. It is impossible to treat all topics that actually belong to the field of advanced MHD. Fortunately, books or chapters of books exist on some of those topics, like *dynamos* [444, 475, 174, 533], *solar magnetohydrodynamics* [510], *chaos* [649], *stellarators* [185], *spheromaks* [48] and *anomalous transport* [33, 662].

Inevitably, with the distinction between topics for Chapters 1–11 (mostly ideal linear phenomena described by self-adjoint linear operators) and Chapters 12–22 (mostly non-ideal and nonlinear phenomena), the difference between 'basic' and 'advanced' levels of magnetohydrodynamics could not strictly be maintained. The logical order required a quite advanced derivation of the MHD equations from kinetic theory (Chapter 3) at an early stage, different sections on advanced topics interspersed throughout the book, and rather complicated analyses of the initial value problem (Chapter 10) and flow in toroidal systems (Chapter 18). These parts are marked by a star (*) and can be skipped on a first study of the book. The same applies to text put in small print, in between triangles ($\triangleright \cdots \triangleleft$), usually containing tedious derivations or advanced material. The serious student is advised though not to skip the Exercises, which are also put in small print for typographical reasons. Frequent use of the vector expressions and tables of the Appendices is encouraged. Magnetohydrodynamics can only be mastered through intense practice.

An overview of the subject matter of the different parts and chapters of this book may help the reader to find his way.

Part I (Plasma physics preliminaries)

- Chapter 1 gives an introduction to laboratory fusion and astrophysical plasmas, and formulates provisional microscopic and macroscopic definitions of the plasma state.
- Chapter 2 discusses the three complementary points of view of single particle motion, kinetic theory and fluid description. The corresponding theoretical models provide the opportunity to introduce some of the basic concepts of plasma physics.
- Chapter 3 gives the ‘derivation’ of the macroscopic equations from the kinetic equations. Quotation marks because a fully satisfactory derivation can not be given at present in view of the largely unknown contribution of turbulent transport processes. The presentation provides some impression of the limitations of the macroscopic model.

Part II (Basic magnetohydrodynamics)

- Chapter 4 defines the MHD model and introduces the concept of scale independence. The central importance of the conservation laws is discussed at length. Based on this, the similarities and differences of laboratory and astrophysical plasmas are articulated in terms of a set of generic boundary value problems.
- Chapter 5 derives the basic MHD waves and describes their properties, with an eye on their role in spectral analysis and computational MHD. The theory of characteristics is introduced as a way to describe the propagation of nonlinear disturbances.
- Chapter 6 treats the subject of waves and instabilities from the unifying point of view of spectral theory. The force operator formulation and the energy principle are extensively discussed. The analogy with quantum mechanics is pointed out and exploited.

Part III (Standard model applications)

- Chapter 7 applies the spectral analysis of Chapter 6 to inhomogeneous plasmas in a plane slab. The wave equation for gravito-MHD waves is derived and solved in various limits. Here, all the intricacies of the subject enter: continuous spectra, damping of Alfvén waves, local instabilities, etc. The topic of MHD spectroscopy is shown to hold great promise for the diagnostics of plasma dynamics.
- Chapter 8 introduces the enormous variety of magnetic phenomena in astrophysics, in particular for the solar system (dynamo, solar wind, space weather, etc.), and provides basic examples of plasma dynamics worked out in later chapters.
- Chapter 9 is the cylindrical counterpart of Chapter 7, with a wave equation describing the various waves and instabilities. It presents the stability analysis of diffuse cylindrical plasmas (classical pinches and present tokamaks) from the spectral perspective.
- Chapter 10 solves the initial value problem for one-dimensional inhomogeneous MHD and the associated damping due to the continuous spectrum.
- Chapter 11 discusses resonant absorption and phase mixing in the context of heating mechanisms of laboratory plasmas, and solar or stellar coronae. Sunspot seismology is introduced as another example of MHD spectroscopy.

Part IV (Flow and dissipation)

- Chapter 12 initiates the most urgent extension of the theory of Chapters 1–10, viz. waves and instabilities in plasmas with stationary background flows, a theme of great interest for laboratory fusion and astrophysical plasma research. The old problem of how to find the complex eigenvalues of stationary plasmas is solved by means of the new method of constructing the Spectral Web in the complex plane.
- Chapter 13 applies the new theory of the Spectral Web to the two classical topics of shear flow in plane plasma slabs, including the Kelvin–Helmholtz instability, and rotation in cylindrical plasmas, including the magneto-rotational instability.
- Chapter 14 treats the considerable modification of plasma dynamics when resistivity is introduced in the MHD description, both in the linear domain of spectral theory and in the nonlinear domain of reconnection.
- Chapter 15 introduces the basic techniques of computational MHD, the discretization techniques, the methods of time stepping, etc. It thus provides the modern techniques needed to solve for the dynamics of plasmas in complicated magnetic geometries.

Part V (Toroidal geometry)

- Chapter 16 presents the classical theory of static equilibrium of toroidal plasmas, a topic of central interest in fusion research of tokamaks.
- Chapter 17 concerns the spectral theory of waves and instabilities in toroidal equilibria, again a central topic in tokamak research. Because of this important application, this part of MHD spectral theory is the most developed one, also with respect to comparison with experimental data. This activity is called MHD spectroscopy.
- Chapter 18 introduces the theory of transonic equilibria and spectral theory of toroidal equilibria rotating in both directions, a subject of great interest but still in its infancy.

Part VI (Nonlinear dynamics)

- Chapter 19 introduces the topic of 2D turbulence in magneto-fluids by deriving the scaling laws for MHD turbulence and presenting the high performance computing efforts needed to resolve the structures that occur.
- Chapter 20 presents the counterpart of Chapter 15 by introducing the numerical methods for nonlinear MHD, in particular for plasmas with large background flows, applied in the last two chapters of this book.
- Chapter 21 discusses the MHD shock conditions from a new perspective, scale independence leading to time reversal duality, and it introduces some of the important areas of application of nonlinear MHD, viz. astrophysical winds and transonic flows.
- Chapter 22 introduces special relativistic MHD, in particular the linear waves and nonlinear shocks that occur at relativistic speeds. The books ends with applications to astrophysical phenomena, like relativistic jets, and thus completes the panorama of the tremendously exciting field of magnetohydrodynamics dominated by flows.

The Appendices provide the essentials of the two indispensable tools for MHD calculations, vector relations and orders of magnitude of the plasma parameters.

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