Temporal relations between magnetic bright points and the solar sunspot cycle

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Abstract

The Sun shows a global magnetic field cycle traditionally best visible in the photosphere as a changing sunspot cycle featuring roughly an 11-year period. In addition we know that our host star also harbours small-scale magnetic fields often seen as strong concentrations of magnetic flux reaching kG field strengths. These features are situated in inter-granular lanes, where they show up bright as so-called magnetic bright points (MBPs). In this short paper we wish to analyse an homogenous, nearly 10-year-long synoptic Hinode image data set recorded from 2006 November up to 2016 February in the *G*-band to inspect the relationship between the number of MBPs at the solar disc centre and the relative sunspot number. Our findings suggest that the number of MBPs at the solar disc centre is indeed correlated to the relative sunspot number, but with the particular feature of showing two different temporal shifts between the decreasing phase of cycle 23 including the minimum and the increasing phase of cycle 24 including the maximum. While the former is shifted by about 22 months, the latter is only shifted by less than 12 months. Moreover, we introduce and discuss an analytical model to predict the number of MBPs at the solar disc centre purely depending on the evolution of the relative sunspot number as well as the temporal change of the relative sunspot number and two background parameters describing a possibly acting surface dynamo as well as the strength of the magnetic field diffusion. Finally, we are able to confirm the plausibility of the temporal shifts by a simplistic random walk model. The main conclusion to be drawn from this work is that the injection of magnetic flux, coming from active regions as represented by sunspots, happens on faster time scales than the removal of small-scale magnetic flux elements later on.

Key words: dynamo — Sun: activity — Sun: magnetic fields — sunspots — techniques: high angular resolution

1 Introduction

An important feature of our host star—the Sun—is its magnetic cycle, visible as changing activity cycle throughout the whole solar atmosphere (see, e.g., Hathaway 2010). While the varying global magnetic fields cause a lot of turbulent energy releases, such as flares and coronal mass ejections, in the higher solar atmosphere (e.g., Webb & Howard 1994) the most evident feature of the solar magnetic field cycle is the changing sunspot patterns. The variation in number as well as in configuration of the sunspots must have been known for quite a long time, but it was Schwabe (1844) who first realised that the variation in number follows certain patterns and especially a period of about 11 years. Hale (1908) realized that sunspots are magnetic and thus it was established that there is a global magnetic field dynamo acting on the Sun (e.g., Babcock 1961). Recent progress in dynamo theories is now often driven by helioseismology on the one hand (see, e.g., Broomhall et al. 2014), which is practically the only method to look into the subsurface configuration of the Sun, and by simulations on the other (e.g., a recent review by Karak et al. 2014).

In addition to this large-scale dynamics of strong and extended magnetic fields, we know of the existence of plenty of small-scale magnetic features (see Zwaan 1987). Among them are so-called magnetic bright points (MBPs; e.g., Berger & Title 2001; Abramenko et al. 2010). Most often these are investigated by filtergram observations in the *G* band (see Schüssler et al. 2003; Feng et al. 2013), but they are also covered by spectropolarimetric means (e.g., Romano et al. 2012) and, recently, by numerical approaches (among others Riethmüller et al. 2014). The interest in them arises due to their importance, as they do not only represent a significant part of the small-scale magnetic flux budget (e.g., Schrijver et al. 1997; Gošić et al. 2016) due to their large magnetic field strength in the range of a kG (Utz et al. 2013), but additionally they contribute to the coronal heating problem via the creation and propagation of magnetohydrodynamic waves into the higher solar atmosphere (e.g., Choudhuri et al. 1993; Muller et al. 1994; Jess et al. 2009; Mathioudakis et al. 2013).

In this particular contribution we investigate the link between the large-scale magnetic cycle visible via sunspots and the small-scale magnetic field cycle by investigating the number of MBPs at the solar disc centre. While there has been plenty of information gathered about the sunspot cycle over the years, much less is known about the variation of small-scale magnetic fields. This is especially true for MBPs (see, e.g., Muller & Roudier 1984). A good overview of earlier attempts of establishing relations between small-scale solar flux elements and the sunspot cycle is given within

the opening chapter of the work of Jin et al. (2011). In an earlier paper by the current authors (Utz et al. 2016), an in-phase correlation was found between the number of MBPs and the sunspot cycle, which was shifted by about 2.5 years with the sunspots being ahead of the corresponding MBP number. In the current work we wish to re-investigate this correlation, but using an updated, data set (described in section 2) which is now more complete and extended by several months and thus also covers the solar maximum of the current cycle 24 (for full analysis see section 3); the older work relied on only the minimum of cycle 23 and the early rise phase of cycle 24. Moreover, we also wish to investigate in much more detail the contribution of the sunspots to the number of MBPs detected at the disc centre and to model this influence (section 4). In section 5 we show the usefulness of the model for predicting the MBP activity close to the equator, before we finish this small paper with a discussion of what can be learned from the modeling efforts in section 6 and the presentation of our conclusions and outlook in section 7.

2 Data and basic analysis

In this study we used the Hinode/SOT (Solar Optical Telescope) synoptic filtergram data taken at the solar disc centre. In the early phase of the spacecraft mission the data was obtained on a daily basis while the amount of data taken was reduced after problems occurred with the downlink antenna. Within this programme, images are taken in principle with all six broadband filters of the Hinode/SOT/BFI (Broadband Filter Imager; Kosugi et al. 2007; Tsuneta et al. 2008) instrument, which observes the blue, green, and red continua as well as the Ca II H line by a corresponding filter. Two more filters are available for molecular lines, namely the *G*-band centred around 4305 Å, and the CN band-head centred around 3883.5 Å. In the beginning the data were taken with the highest possible sampling resolution of 0.054 arcsec pixel⁻¹, with a full field-of-view (FOV) of 4048 by 2024 pixels corresponding to about 220 by 110 arcsec². After the failure of the main downlink antenna the operators decided to perform an onboard binning of the data to reduce the amount of data to be downloaded to the ground control station. Thus the data taken after 2008 February 17 were already reduced in pixel sampling on-board, to a value of 0.108 arcsec pixel⁻¹. To have a homogenous data set, we decided to also implement a down-sampling of the earlier data already recorded to-date.

For the analysis and investigation of MBPs, a preferential spectral region is the so-called *G*-band which covers

Fig. 1. Raw results; the number of detected MBPs in all the analysed Hinode G-band images is shown versus time. This is an updated and extended representation of figure 2 from Utz et al. (2016).

molecular lines. These molecular absorption lines happen to be weakened in small-scale strong magnetic field agglomerations, and thus such field concentrations can be observed with a higher contrast compared to the normal continuum (see, e.g., Schüssler et al. 2003). Therefore, we selected all 6343 available G-band images covering the period from 2006 November up to 2016 February. Unfortunately, a problem occurred in the middle of 2016 February which lead to a malfunctioning CCD and finally to the loss of the BFI and NFI (Narrowband Filter Imager) instruments onboard the Hinode spacecraft, as both of those instruments use the same two CCD chips. Thus the synoptic programme cannot be prolonged any further, and we therefore have the longest available and most stable data set ever recorded from space for such a purpose at hand.

The principle data calibration follows the idea already outlined in Utz et al. (2016). First we calibrate the data (flat-fielding, dark current calibration) with the SSWIDL routine `fg_prep`. In the next step we apply an automated image segmentation and MBP identification algorithm as described in detail in Utz et al. (2009, 2010). After applying the algorithm to all 6343 images, we obtain the graph shown in figure 1.

On average, the number of detected MBPs in the disc centre of the Sun is around 1000, corresponding to roughly 4 per 100 arcsec² (which would correspond to typical meso-granular diameters). Besides, it is visible that there are strong peaks reaching up to values of two to three thousand, i.e., more than doubling the usually visible bright point activity. These outliers are related to images in which sunspots are visible. These sunspots lead to an enhanced magnetic network and thus to a higher detection rate of MBPs. Furthermore, there are measurement points clearly below the 800 to 1000 level. These points are related to images out of focus and/or broken images, e.g., only a part of the image was successfully downloaded from the

Fig. 2. The evolution of the median MBP number per month is plotted together with the evolution of the relative sunspot number for the decreasing solar cycle 23 and the increasing phase of the solar cycle 24, including both the minimum between the cycles and the maximum of the new ongoing cycle. This is an updated plot of the upper panel of figure 3 from Utz et al. (2016). Sunspot data for this and the following figures were obtained from the World Data Center SILSO, Royal Observatory of Belgium, Brussels.

satellite.¹ To dispose of most of these active regions and broken images, and thus to obtain a clearer insight into the overall behaviour, we applied a two-step selection and smoothing criterion. In the first step we applied a contrast criterion and selected only those images which have a contrast close to the optimum focal position, which we defined as 10.8% to 11.8% (standard deviation of the image intensity divided by the mean image intensity times 100). After thus selecting only the best-focused images, which already helps to discard sunspot images and images that are out of focus, we calculated a median MBP number per month for the selected images of each monthly period. The result is depicted in figure 2. For completeness, we wish to mention that on average 73% of the images taken in a month survive our stringent selection method, and that on average the median MBP number is then calculated over 41 images (for most months the number ranges between 25 and 60). In the least statistically robust month, at least four images were still considered for the calculation of the median monthly MBP number. This low-confidence month occurred in the period spanning from roughly 2008 to 2010, when Hinode was not taking as many images within the synoptic programme as usual (see also figure 1).

3 Results

The solid line in figure 2 illustrates the number of MBPs detected in the Hinode synoptic G-band images at the disc centre after calculating a corresponding median number for each month. The dashed line shows the corresponding relative sunspot number obtained from the SIDC (Solar

¹ For a detailed analysis and discussion of the data stability (which was excellent) we wish to refer the interested reader to Utz et al. (2016) at this point.

Fig. 3. The cross-correlation between the sunspot cycle as given by the relative sunspot number and the number of MBPs measured at the disc centre. The indicated monthly shifts correspond to the number of months the sunspot cycle would be ahead of the MBP cycle for the calculated and corresponding correlations.

Influence Data Centre).² A similar result, but for a limited data set (data only available up to 2014 August), was shown already in Utz et al. (2016). In that earlier paper we found that the MBP cycle is shifted by about 2.5 years from the sunspot cycle. We wish to repeat this analysis with the extended data set now at hand, which also covers the maximum of the cycle 24 which was missing in the earlier work.

Thus we wish to have a detailed look at the correlation between the relative sunspot number and the detected monthly median number of MBPs. The resulting plot for the cross-correlation coefficient versus temporal shift between both quantities is depicted in figure 3.

Here we can identify roughly three local maxima, with the largest being situated at a temporal shift of 35 months between the sunspots and the number of MBPs. This would mean that the sunspot cycle is nearly three years ahead of the MBP activity at the disc centre. The other two maxima would correspond to 28 months (the previously found 2.5-year lag) and only 11 months.

Plotting the sunspots shifted for the nearly 3-year period (35 months) yields figure 4. It becomes clear that the 3-year period is most likely an artifact caused by the large variation around the solar cycle maximum at the end of the observational period. On the other hand, it becomes clear that while there is an acceptably good agreement between the rising phase and maximum of sunspot cycle 24 with the rising activity of MBPs at the disc centre, the decreasing phase of the previous cycle, and especially the minimum between both cycles, shows a clear mismatch when the shifting period would be around 3 years. Thus we wish now to investigate the shifts for the decreasing cycle 23 and the rising phase of cycle 24 separately.

Fig. 4. The number of detected MBPs at the disc centre (solid line) together with the evolution of the relative number of sunspots. This plot shows the sunspot axis shifted by 35 months (dashed line) which corresponds to the general maximum of the cross-correlation (0.51) between both time series.

Fig. 5. Cross-correlations between the number of MBPs during the rising phase of the sunspot cycle with the rising phase of sunspot cycle 24 (stars) and number of MBPs during the decreasing phase of cycle 23 with the corresponding part of the relative sunspot number during the decreasing phase (crosses). The selected periods as well as the time-series plots with temporal shifts featuring the maximum correlations are shown in figure 6 (for the rising phase) and figure 7 (decreasing phase), respectively.

This approach yields figure 5. Immediately it becomes clear that the rising phase of the sunspot cycle (correlation shown by stars) is nearly co-aligned with an increase in MBP activity. There is at most a shift of a few months, e.g. the correlation coefficient starts with 0.34, reaches a first local maxima of 0.48 after 6 months, and the absolute maximum of the correlation is reached after 11 months, or roughly a year, with a value of 0.58. Thus the injection of new magnetic field into the network region follows the sunspot cycle strongly as well as temporally very closely within a year. On the other hand, the picture is quite different for the decreasing phase of solar cycle 23. The correlation for this part of the period is shown by crosses and one can see that the time lag is much more pronounced. Here we reach the maximum correlation of 0.7 after a shift of 22

² (<http://sidc.oma.be/>).

Fig. 6. Number of MBPs at the disc centre (solid line) together with the relative sunspot number (dashed line) during the rising phase of cycle 24. The shift between both curves is 12 months and the correlation coefficient amounts to 0.58.

Fig. 7. Similar to figure 6, but for the decreasing phase of cycle 23. The shift between both plotted curves is 22 months and the correlation coefficient reaches a maximum value of 0.7.

months, which means that the network takes quite some time to discard the magnetic field injected from the previous sunspot cycle.

The next two figures, figures 6 and 7, depict the used periods for the rising and decreasing phases of the solar cycle with the activity of MBPs at the disc centre shifted according to the previously found best correlation values.

4 Modeling of the temporal behaviour of MBPs

In the previous section we have seen that the injection of new flux into the network due to the rising phase of the new solar cycle 24 occurs on a much faster time scale, with a shift of only a few months up to a year, while the disposal of the flux within the magnetic network takes a longer, about 2 years (22 months). Thus one may wonder if it is possible to construct a model to predict the change of magnetic network activity as seen by MBPs in the solar disc centre due to the sunspot activity as measured by the relative sunspot number.

Input

Output

Fig. 8. Electronic circuit schematic representing the governing differential equation which controls the change of the number of MBPs at the disc centre. (Color online)

After some reflection, we propose the following governing equation for the change of the number of MBPs $N(t)$ with time at the solar disc centre:

$$\frac{dN(t)}{dt} \propto a n_s(t - \delta t_1) + b \frac{dn_s(t - \delta t_2)}{dt} + c [N(t) - N_0]. \quad (1)$$

The idea behind this conceptual equation is that the change in the number of MBPs will be proportional to the number of sunspots n_s expressed by the proportionality parameter a , to the monthly derivative of the number of sunspots (the rate of sunspot change) dn_s/dt expressed by parameter b ,³ and naturally to the amount of available MBPs related to the coefficient c (which can be interpreted similar to a radioactive decay constant).⁴ Having in mind that there might be a background solar surface dynamo working on, e.g., the granulation scale, we should reduce the number of affected MBPs in the equation above by a background permanent “rest-flux” number, N_0 .⁵ This equation can be graphically illustrated as a flowchart, such as in figure 8. Another aspect which should be mentioned here is that the influence of the sunspot number as well as its change might be shifted in time by a certain amount, which is expressed as δt_1 and δt_2 in the equation above. This is a reasonable assumption, as we have learned in the previous section that the number of detected MBPs generally lags behind the number of sunspots as well as that the decay of

³ This parameter is justified by the idea that when a sunspot emerges there will be an immediate transport of new magnetic flux to the network.

⁴ The “decay”—flux dispersion—will depend (be proportional) on the amount of magnetic flux in place and can be caused by, e.g., the meridional flows transporting flux away from the disc centre to the poles.

⁵ While the meridional flows will sweep magnetic fields away from the disc centre (constant c), the background surface dynamo will “replenish” the network with new magnetic fields, leading ultimately to some balanced number of MBPs—the constant N_0 .

Fig. 9. Principle input components/curves for the differential equation [see, equation (1)] governing the change of the MBPs at the disc centre (see also schematic figure 8). From top left to bottom right: relative sunspot number (solid line depicts a 12-month running average), the monthly derivative of the relative sunspot number calculated from the 12-month running averaged relative sunspot number as shown before, the number of detected MBPs at the disc centre (solid line again depicts a 12-month running average), and finally the to-be-modeled output curve, the change of MBPs at the disc centre calculated via averaging the number of MBPs over a yearly mean and then calculating the derivative of this curve and applying another running 12-month average. Originally measured data points are shown as crosses.

flux seems to happen more slowly than the injection of new flux.

Thus our approach is based on the idea of constructing the profile and evolution of the dN/dt temporal evolution as depicted in the bottom right-hand panel of figure 9 by three input variables; the sunspot number, the monthly derivative of the sunspot number (sunspot change rate), and the number of bright points itself. The temporal evolutions of these input variables are shown from the top left, relative sunspot number, via the top right, the monthly derivative of the relative sunspot number, to the bottom left, the actual detected number of MBPs. The four given quantities are depicted as running yearly average curves (solid line) and the actual measured values for the number of sunspots and also the number of MBPs are depicted by crosses.

After thorough testing of our approach we learned that an improved fit can actually be achieved when one uses the square root of the number of sunspots as an input parameter rather than the sunspot number itself. This might be understood in the way that a small number of sunspots can already contribute significantly to the number of MBPs at the disc centre, while a higher number of sunspots start to saturate the magnetic network, or, at least, lead to a

decreasing impact on the number of detected MBPs. Thus, the finally applied equation is (already taking into account the running yearly average,⁶ i.e., summation and averaging of quantities from 6 months before, $t - 6$, to 5 months after, $t + 5$, the month of interest, where t represents time in months):

$$\begin{aligned} \frac{dN(t)}{dt} = & a \sqrt{\frac{1}{12} \sum_{t-6}^{t+5} n_s(t - \delta t_1)} \\ & + \frac{b}{12} \sum_{t-6}^{t+5} \frac{d \sum_{t-6}^{t+5} n_s(t - \delta t_2)/12}{dt} \\ & + c \left[\frac{1}{12} \sum_{t-6}^{t+5} N(t) - N_0 \right]. \end{aligned} \quad (2)$$

Due to the temporal averaging we cut off the first and last months of our time series and fitted the period from 2007 May to 2015 August. The obtained fitting parameters are

⁶ A 12-index smoothing in IDL leads to considering six items before, the item itself, and the five following items, giving rise to the rather strange-looking asymmetry in the equation.

Table 1. Fitting parameters.*

Modeling period	a [month ⁻¹]	b	c [month ⁻¹]	N_0	δt_1 [month]	δt_2 [month]
2007 May/2015 August	2.728	0.962	-0.018	255	22	8
2008 July/2014 May	2.575	1.516	-0.038	448	23	10

*Gives the best-fitting coefficients found for the general fitting of the dN/dt behavior for the temporal change of the detected MBPs measured at the disc centre for running yearly averages of all parameters.

Fig. 10. Actual obtained best-fitting candidates for the input parameters of the predictive model. From top left to bottom right: the square root of the relative sunspot number for fitting the change in the number of MBPs (shifted by 22 months) and the monthly derivative of the square root of the relative sunspot number (shifted by eight months). Both curves are shown as running yearly averages and thus slightly different from the ones depicted in the figure 9. For the obtained fitting parameters (weights of the different curves) see table 1.

Fig. 11. Detected change of the number of MBPs after applying an annual running mean filtering shown with crosses and the fitted change of MBPs according to equation (2) for the longest possible data set period from 2007 May to 2015 August.

listed in table 1. To check the stability of the model parameters we have redone the fitting for a more limited data set where we only used the period from 2008 July until 2014 May. While the model parameters do change quite considerably, the overall possible predictions due to the model are still acceptable and by far better than by a simple polynomial fit (see below). The shifted input profiles for the square root of the relative sunspot number, $\sqrt{N_s}$, and the temporal derivative of the sunspot number are depicted in figure 10. The result of the modeling can be seen in figure 11. Here the detected change in number of MBPs (running yearly

average) is shown with crosses and the obtained best fit by a solid line.

5 Predictive capabilities of the model

After obtaining a well-fitted dN/dt curve we can use the obtained information to try to predict the variation of the number of MBPs at the disc centre by a simple recursive equation:

$$N(t) = N(t-1) + \Delta N(t-1), \quad (3)$$

where $N(t)$ is the number of MBPs at the disc centre and $\Delta N(t-1)$ can be calculated via equation (2) and is in principle only dependent on $N_s(t)$ (the number of sunspots), its derivative, and the fitting parameters. Using this equation and a chosen starting value we calculated the predicted number of MBPs due to the given sunspot number as well as its change with time (running annual mean values).⁷ The result is shown in figure 12. Here the predicted number of MBPs is shown by a solid line. For comparison, the actual number of MBPs (running annual mean) is shown by a dashed line and the single real measurements are depicted as stars. The correlation between the predicted number of

⁷ We used as a starting value the median MBP number of 2006 November, the first month of available Hinode data.

Fig. 12. Actual measured number of MBPs at the disc centre (crosses) are depicted together with an annual running mean of the actual data (dashed line) and the predicted number of MBPs at the disc centre (solid line) obtained via a single starting number of MBPs and the fitting parameters listed in table 1 and equation (2).

Fig. 13. Truly measured evolution of MBPs at the disc centre after application of a yearly running average (solid line) together with two predictive models according to equation (2) and two simple fifth-order polynomial fits. The vertical dashed lines give the used data range for the full fitting and the limited fitting. (Color online)

MBPs and the actual measured MBPs (running 12-month average) reaches a value of 0.959.

To prove the usefulness of our model we want to compare the predictive capability of our empirical model with a standard polynomial fit of 5 order (six free fitting parameters). The result can be seen in figure 13. Here we show the running yearly average number of measured MBPs by a solid line, two different predictive models as dashed lines (the darker one represents the model using the maximum data range, and the lighter one represents the model that used the limited data range from 2008 July to 2014 May), and two fifth-order polynomial fits by dash-dotted lines (again, one model created using the full data range, and the other one using the limited data set). While the polynomials in general correlate as well with the measured data as the

Table 2. Correlation coefficients between the measured evolution of MBPs, the predicted MBP evolutions, and via polynomial modeled MBP evolutions.

Correlation type	Value
Predictive model, full data range	0.9586
Predictive model, small data range	0.9427
Predictive model, small data range extended to full period	0.9028
Polynomial model, full data range	0.9515
Polynomial model, small data range	0.8986
Polynomial model, small data range extended to full period	−0.2588

predictive model (see also table 2), when we come to predictions, they completely start to fail, while our empirical models can still follow the true evolution quite closely.

6 Discussion

In section 3 we have seen that there is in general a good correlation between the number of sunspots and the number of MBPs detected at the disc centre. However, this correlation only shows up when one shifts the two time series to each other. A question arising is, is it possible that this established correlation is just an artifact (e.g., due to instrumental aging)?

We have discussed this possibility at length already in our previous work Utz et al. (2016). In that work we found that a decrease of the instrument quality happens in the beginning, which is largely offset by permanent refocusing attempts of the Hinode operation team.⁸ Moreover, we saw that after several years the quality of the instrument starts to stabilise and that there should be no influence of the instrument on the quality of the data any more. Our conclusion in the previous work, which we would like to uphold, was that the overall behaviour of the measured evolution of the number of MBPs at the disc centre is due to physical reasons and created by physical processes in the Sun. However, the exact numerical values can be slightly altered due to the changing instrument quality. Besides that, it is totally reasonable from the physical point of view that the number of sunspots will influence the activity and strength of the magnetic network. When a sunspot starts to decay, it will supplement and feed magnetic flux via the so-called moving magnetic features (see, e.g., Harvey & Harvey 1973) into the magnetic network. Thus it is highly plausible that there is indeed a temporally shifted correlation between the number of sunspots and the number

⁸ On a regular basis, about once a month, the Hinode team is measuring the optimum focus position and thus correcting for long term changes of the focal position due to ageing.

of MBPs at the disc centre (which represent strong magnetic flux elements of about 1 kG). Of course, at the end a correlation between two quantities can never prove that there is a true link between two quantities or a real process relating them. This could be only proven to a higher extent by direct and detailed investigations of the transport of flux elements, e.g., by investigating in full detail the flow maps in and around active regions (e.g., J. I. Campos Rozo et al. in preparation). However, it is a first strong indication for such processes. Moreover, from the viewpoint of physics, such a linkage between magnetic network activity and sunspot activity is very plausible and also used in theoretical solar cycle modeling (e.g., Thibault et al. 2012, 2014).

Coming back to the correlation between the sunspot number and the number of MBPs at the disc centre, we would like to update our earlier work in the sense that we are actually now able to establish that there is not one single good correlation between the number of MBPs at the solar disc centre and the number of sunspots, but that indeed the MBP cycle is broken in two parts. The decreasing phase of solar cycle 23 is shifted in time compared to the MBP decreasing phase by nearly two years (exactly 22 months) while the increase follows the rising phase of cycle 24 nearly co-temporally (as the correlation is already high for small time shifts but reaches a local maximum after 11 months).

How can these temporal shifts plus the ones from the modeling now be explained or justified (see also discussions in Thibault et al. 2012, 2014; Utz et al. 2016)?

We have to keep in mind that we observe the number of MBPs at the disc centre, which means very close to the equator,⁹ while sunspots are emerging more or less in a latitude band between $\pm 30^\circ$ in the beginning of the cycle. Later on, these zones of emergence migrate closer and closer to the equator. Finally, at the end of the cycle, they can appear practically at the solar equator itself. Thus, the magnetic elements, created during the decay of the sunspots, need to be transported by distances as far reaching as 30° from their emergence locations towards the equator. Only then can they be detected and visible in the analysed data set. This already explains, from a qualitative viewpoint, the time shifts seen in the rising phase as well as in the fitting parameters. For a more quantitative, but still basic, first principle analysis, we would now need to consider the available physical processes for such a magnetic flux transport. Among them are meridional flows, which would, however, rather transport the magnetic field away from the equator and to the poles with speeds of up to 10 m s^{-1} (e.g., Hathaway & Rightmire 2010). Besides such meridional flows,

we can also imagine that evolving supergranular cells distribute and diffuse magnetic fields. For our first principle analysis we would like to consider evolving supergranular cells as pushing magnetic elements on the solar surface in a simplified one-dimensional random walk manner.^{10,11} Looking at characteristic scales of supergranular cells, we find the following values: sizes from 20 to 63 Mm with an average size of 36 Mm and turn-over, or lifetimes, of about 1.7 d (see, e.g., the review by Rieutord & Rincon 2010).

Having a look now at the theory of a simple, one-dimensional random walk process, one can obtain the equation for the expectation value of travelled distance (in normalized characteristic distances) after a certain number of normalized (time) steps (approximated for large n) as $E(|n|) = \sqrt{2n/\pi}$ (E. W. Weisstein 2017)¹². In our case, the normalization of the step size is the typical lifetime and thus we can replace the step counting number n on the right-hand side by t_i/t_c , where t_c is the characteristic lifetime of the supergranulation and t_i is the necessary transportation time. Similarly, the necessary expectation value $E(|n|)$ is equivalent to the number of steps necessary to transport the magnetic fields over the necessary distance, d_i , and thus is given in multiples of the characteristic travel distance per time, d_c . Replacing these mentioned parts of the equation we end up with

$$d_i/d_c = \sqrt{\frac{2t_i}{\pi t_c}}, \quad (4)$$

or, resolved for the necessary transportation time,

$$t_i = (d_i/d_c)^2 \pi / 2 t_c; \quad (5)$$

but what are the characteristic times and distances now?

For the correlation between the decreasing phase of the sunspot cycle and the number of MBPs, the delay happens most likely due to the fact that time is needed for the magnetic field to be transported out of the observable range. Thus, to estimate the time needed to clean the region close to the equator from magnetic elements originating from the sunspots, we can start by assuming that the last sunspots of the cycle were formed at the equator and that the magnetic field needs to be transported as far away as we could detect MBPs. Thus we can argue that these magnetic fields need to be transported across the observable position furthest away from the equator, which would be the top boundary of the FOV at maximum solar tilt ($3.3 + 7.5 \approx 11^\circ$). The second

¹⁰ We are only interested in the latitudinal component.

¹¹ That this is a reasonable approach can be seen by the fact that the behaviour of small-scale magnetic fields also follows granular scales random walks (e.g., Utz et al. 2012).

¹² Weisstein, E. W. 2017, Random Walk–1-Dimensional, MathWorld–A Wolfram Web Resource (<http://mathworld.wolfram.com/RandomWalk1-Dimensional.html>).

⁹ Due to the solar tilt angle to Earth's ecliptic plane, the solar latitude that is actually observed, by taking images at the apparent solar disc centre, varies slightly ($\pm 7^\circ$) during the year.

Table 3. Expected time shifts on a first principle random walk analysis (for details see text).

Parameter	Distance	Characteristic displacement scale	Time
	[°]; [Mm]	[Mm]	[months]
Rising phase, max elongated FOV position	20; 240	31.5	5.1
Rising phase, centre FOV position	27.5; 350	31.5	10.8
Decreasing phase	11; 135	9	19.8
Travel time fastest elements for mean distance	15; 185	31.5	3.2
End of bulk travel time for mean distance	15; 185	9	38.9
Mean delay time for mean distances (δt_1)	—	—	21
Travel time for fast elements mean distances (δt_2)	15; 185	18	8.8

necessary parameter is now the characteristic distance travelled per turn-over time of a supergranule. For that we can assume that the elements are transported with a characteristic scale of around 9 Mm (half the typical radius of the supergranulation).¹³ Inserting these values in the above equation we end up with 19.8 months, which fits quite well with the observed 22 months of time lag. The used values for the needed parameters to perform this calculation, as well as the ones for similar calculations for the other time shifts, can be found in table 3.

For the rising phase, we assume in one case that the FOV is situated closer to the onset of the sunspot belt due to a preferable solar tilt situation reducing the distance to be covered (called in table 3: Rising phase, max elongated FOV position) and in another we considered a more realistic set-up where we would assume that newly arrived elements (30° latitude) would be required to move inside the FOV of Hinode while the satellite is pointing at the equator. For the characteristic displacement we assumed the radius of the maximum size of the supergranules. In this way we made sure that these are really minimum time-lags; under normal conditions the true time-lag should be rather larger than estimated here. In the first case we end up with times of 5.1 months, while the second one gives 10.8 months, which agrees very well with the observed 11-month time-lag. Also, we have seen in the first part of the paper that the rising phase is already highly correlated when smaller time shifts are applied, which can be due to the FOV of the satellite being more favorably positioned (as we have seen above). Moreover, the correlation coefficients are of course also influenced by the later cases of sunspot emergence, which will happen closer to the solar equator.

To validate the model time-shifts with this first principle analysis we assume that the flux needs to be transported from an average position (15°) to the equator. The average position was chosen as the model should be valid for the whole solar cycle and thus the sunspots would be on average

at a latitude position of the mentioned 15°. We assume fast movements for the δt_2 parameter as it relates to the immediate change in number of MBPs due to new emerging flux. Thus we assumed that the elements get pushed by a full average supergranular radius each time. The result amounts to 8.8 months, which again agrees quite well with the obtained 8 months. The last parameter, δt_1 , measures the time lag to the sunspot number itself. As we have seen, and also know from theory, a movement by random walk is quite dispersive and thus we would see first elements reaching a certain position long before the bulk of features would arrive at a similar distance. Thus we estimated, for obtaining δt_1 , the principle time when first elements could arrive, if they would be so luckily as to be pushed all the time by the largest possible supergranules. This principle time amounts to 3.2 months. Moreover, we estimated the time it would take for the last of the bulk of the elements to arrive, by assuming that the elements would always only be pushed by half of the radius of a mean-sized supergranular cell. That time would amount to 38.9 months. Thus the first elements should arrive after 3.2 months while after 38.9 months the last elements should arrive and therefore the influence from a certain sunspot should be over. Therefore the average time-lag between a sunspot arriving on the solar surface and its influence in our model should be the average of these two numbers, 21 months, which is in good agreement with the 22 months obtained in the model.

Thus we can conclude from the observational correlation analysis, the modelling, and the interpretational random walk model findings that the injection of new flux from the sunspot cycle into the magnetic network happens on a faster temporal scale than the reduction of magnetic flux. Thus the diffusion mechanism which weakens and finally disperses these magnetic fields at the solar disc centre, before the magnetic flux ultimately gets transported to the poles, is acting more slowly than the injection mechanism.

In section 4 we followed up with a more quantitative investigation into the formation and disintegration of MBPs and their correlation with the sunspot cycle. We introduced a model which can reproduce the temporal change of the

¹³ The factor of half was chosen because not every supergranule will always transport all elements by a full radius, but rather, on average, by about half of its radius.

number of MBPs. The fitting parameters of this model, as depicted in table 1, are very interesting, as they are related to physical processes and mechanisms.

A plausible interpretation of the N_0 parameter is that it represents a possible acting small-scale solar surface dynamo which permanently induces new magnetic fields and thus also some magnetic field enhancements independently of the sunspot activity. Hence there should be some permanent and constant number of MBPs visible, even when there is no global field acting as in the case of during the extended minimum of cycle 23 (e.g., Muller et al. 2011). These constantly created background MBPs would be exactly described by N_0 . Similarly, the parameter c could be seen as related to the large-scale flow pattern (meridional flows) as it governs the speed of reduction of magnetic flux (number of MBPs). A possible enhanced flow field could destroy flux concentrations more rapidly and in addition move magnetic flux out of the FOV faster (dispersing the field in the direction of the poles). Then there are the parameters a and b , which together with δt_1 and δt_2 describe the coupling of the sunspots to the magnetic network as seen by MBPs. There are two striking features with these parameters. First of all we recover the temporal shift from the previous data analysis with 22 months for the sunspot coupling (the temporal shift in the decreasing phase of cycle 23) and 8 months (which is close to the one year for the increasing phase of cycle 24) for the monthly derivative of the sunspot number. As the rate of change in the sunspot number is directly related to fresh magnetic flux injection, we get another indication that the flux injection from sunspots works much faster than the removal of magnetic flux from the network. The other striking feature is that the coupling parameter a and the temporal shifts are very stable parameters in regards of changing the data range for creating the predictive model, while the other parameters, b , c , and N_0 seem to vary to a higher degree and are less well determined. This has also to do with the kind of fitting as, e.g., c and N_0 are counteracting parameters. A larger background flux density N_0 could be compensated by a larger meridional flow and magnetic field dispersion related to c . Thus, as these parameters couple, they cannot be estimated that easily or with high accuracy.

At this point we would like to make clear that the developed model creates a good representative and prediction capability for the observed roughly 10-year period, however, it is (a) just a toy model based on phenomenological terms (for an older, in some way similar, approach, see, e.g., Schrijver & Harvey 1994), and (b) the parameters have to be taken extremely carefully as they represent only a particular (albeit useful) local minimum solution to the multitude of possible fitting solutions for a six-parameter model. In the future it would be necessary to compare the findings with other, more direct methods, like flow field analysis (e.g., the

local correlation technique, see November & Simon 1988), which could also yield insights into the magnetic flux transport by plasma motions by comparing plasma flows with changes of the magnetic field and its position (see, e.g., Vargas Domínguez et al. 2008).

A main reason for the variation of the parameters b , c , N_0 within the covered period (see table 2) will be the non-separation of the hemispheric sunspot numbers in this study. When the Hinode satellite is observing the disc centre within its campaign, for half a year it is covering, to a larger extent, the equator and the region slightly north of it (so the northern hemispheric sunspots will have a larger influence on the observed network) and for the other half of that year the equator plus the region slightly south of it (so the influence will be to a higher degree coming from the southern hemisphere; see also Utz et al. 2016). This is due to the fact that the apparent solar tilt angle varies between roughly $\pm 7.5^\circ$ while the total FOV of Hinode in latitude is about 6.6° . Thus for a more accurate model these effects should be considered in future models and might help to determine the parameters more uniquely, while neglecting this effect will lead to a small systematic error (likely increasing the observed time-lags) and more likely to larger errors in the estimated time-lags. The theoretical interpretation of the parameters is outstanding, but should be quite interesting as the decay constant should be coupled to the meridional flows dispersing and transporting the magnetic field from the centre to the poles, and the background number of MBPs will be related to an acting surface dynamo and thus could, in principle, tell us about the strength of it. Thus a more thorough analysis could yield interesting new and additional insight in the future.

7 Conclusion

In this work we had a detailed look at the evolution of MBPs close to the solar disc centre by investigating the Hinode synoptic data set which was recorded over a period of roughly 10 years and is the most suitable data set available in the world for such a purpose, as it is stable and seeing-free due to the space conditions. The evolution of the number of MBPs at the disc centre can tell us about the important coupling mechanism between the small-scale magnetic fields and the large-scale solar cycle induced sunspot cycle as well as about magnetoconvection processes, especially the magnetic flux transport. We have shown that there is not a simple unique coupling between the variation of MBPs but that actually the temporal shift between the minimum in the sunspot cycle and the minimum in the number of MBPs is different from the one obtained through the investigation of the shift between the corresponding maximum values. This means that the injection of new magnetic flux from the global field dynamo works on a faster time-scale

than the reduction of the magnetic flux from the magnetic network, which is probably due to plasma flows dispersing the magnetic field and transporting it away from the disc centre. In the second part of this work we developed a coupling model between the temporal change of the number of MBPs at the disc centre and the sunspot number. Detailed investigations of this model have shown that characteristics like the decay constant (in this work denoted as c) and the background magnetic field constant (N_0) cannot easily nor with full stability obtained by the enormous yet limited data set. However, the predictive capability of the introduced empirical model still gives correlations better than 0.9 for periods extended by roughly 2 years anterior and posterior to the used data set. Thus, future work on such a model seems promising and probably a huge step forward can be achieved by separating the hemispheric sunspot numbers. In addition, we should keep in mind that ephemeral regions are also a main source for the injection of magnetic flux into the network, and therefore it would be good to include them in the modeling efforts. Finally, we remark that the observed temporal lags between the sunspot cycle and the detected MBPs close to the solar equator might be explained by a simple random walk model where supergranules push and move the magnetic elements.

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