RH: The Metaphysics of the “Divided Line”

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The Metaphysics of the “Divided Line” in Proclus: A Sample of Pythagorean
Theology
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Abstract: The aim of this paper is to expose Proclus, a fifth-century Athenian Platonist, as a key figure in a tradition of interpretation that exploited Plato’s divided line as evidence for the attribution of a doctrine of mathematical intermediaries to Plato. I will claim that to fully appreciate the rationale behind Proclus’s exegesis of the passage, it is necessary to read it against the backdrop of his Neoplatonic metaphysics—especially his views on the procession of all reality from a single principle and his particular views on the ontology of mathematical objects. I will further argue that Proclus’s insistence on the harmony between Plato and Pythagoreanism is the main motive behind this interpretation. For Proclus, the divided line is a typical sample of what he regards as “Pythagorean theology,” that is, instruction about the first principles of reality through mathematical images.

Keywords: Proclus, mathematics, divided line, Plato, Neopythagoreanism

1. Introduction

The famous passage at the end of the sixth book of the *Republic* (509d–511e), in which Socrates compares our cognitive states to different sections of a single line, is often read as a “map” of Plato’s epistemology. Symbolizing the different levels of cognition, the divided line may be taken to present a forceful image of the various stages that the soul must traverse in its quest for true knowledge. Its reputation as a

snapshot of the epistemology of Plato's mature dialogues makes the divided line, despite, or perhaps because of, persisting disagreements over its interpretation, one of the most vexed passages of the Platonic corpus.

In the ancient readings of the divided line, many of the modern exegetical debates have been anticipated. The line is, of course, as ubiquitous in the later ancient Platonic discussions of our cognitive states and capacities as it is in contemporary discussions of Plato's epistemology. Yet only a handful of comprehensive discussions have come down to us from the later Platonists. Apart from Plutarch's *Platonic Questions* III 1 and Iamblichus's discussion in the eighth chapter of *On General Mathematics* (in which material from the pseudo-Pythagorean Archytas and Brotinus is preserved), we also possess a more extensive treatment by Proclus, a fifth-century Neoplatonist, in the twelfth essay of his *Commentary on the Republic*, entitled *On the Cave in the Seventh Book of the Republic (in Remp., I.287–96)*.¹

My focus in this paper will be on Proclus, and my starting point will be his *Commentary on the Republic*. If our aim were to understand the importance of the divided line for Proclus's epistemology, then this text certainly would have its limitations. First, Proclus here deals with the divided line in a text that has a rather different aim, namely, to clarify the cave allegory, and accordingly introduces the line primarily to expose the strict parallelism that he thinks obtains between the two images.² Second, set against this backdrop, Proclus's main concern is not so much with the epistemology of the divided line, but rather with its metaphysics and with the ontological map it provides for the ascent of the prisoner from the cave. Partly for this reason, I will be mainly concerned with the metaphysics of the divided line, leaving the epistemological side of the story for another occasion.³ To this end, it will be necessary to complete the picture provided by his twelfth essay of the *Republic* commentary by appealing to scattered references and allusions to the line found elsewhere in Proclus's writings—especially in his commentaries on the *Timaeus*, *Parmenides*, and the first book of Euclid's *Elements*.

My aim is to show that Proclus stands at the origin of a tradition of interpretation that exploited Plato's divided line in support of the attribution of a doctrine of mathematical intermediaries to Plato. Proclus was certainly not the first to propound this view, but his version is the best documented among the ancient sources that are extant, and it has been influential among later readers of Plato. I will claim, however, that to fully appreciate the rationale behind Proclus's exegesis of the passage, it is necessary to read it against the backdrop of his Neoplatonic metaphysics—especially his views on the procession of all reality from a single principle and on the ontology of mathematical objects. I will further identify Proclus's insistence on the harmony between Plato and Pythagoreanism as the main motive behind this interpretation. I will argue that, for Proclus, the divided line is a typical sample of what he regards as “Pythagorean theology,” that is, instruction about the first principles of reality by means of mathematical images.⁴

2. The Divided Line in the Twelfth Essay of Proclus's *Commentary on the Republic*

In the twelfth essay of his *Commentary on the Republic*, Proclus's comments mainly concern the metaphysical implications of the divided line. This metaphysical dimension is, of course, far from absent in Plato, but Proclus exploits it to its full potential and interprets the analogy along distinctively Neoplatonic lines. This discussion also allows Proclus to take a position on a number of exegetical debates about the divided line that had tantalized his predecessors. For Proclus, the line provides a division of “all things” (τὰ πάντα), that is, of reality as a whole,⁵ and provides an image for the procession of all reality from a single first principle. As such, it can be taken as a geometrical representation of the Neoplatonic view of the dependence of the phenomenal world upon higher principles. Right from the outset, Proclus is eager to interpret the relations between the various sections of the line in terms of the Neoplatonic rules of causation:

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[T1.] Thus, since he [Plato] wanted to expose the procession of [all] beings [τῶν ὄντων] from the One as continuous [συνεχῆ] and unified, he compared this continuous [procession] with a single line [γραμμῇ μιᾷ], as derivative realities always proceed from primary ones through likeness and continuity [δι' ὁμοιότητος καὶ ἀλληλουχίας], without any gap holding these realities apart. For [such a gap] would not have been allowed, since the Good brings forth all things and makes them revert back on itself. The creation must indeed resemble [ὁμοιοῦσθαι] the creating [principle], and since that principle is one, it is necessary that the creation be continuous, for continuity is akin to unity. The cause of this continuity is the likeness [ὁμοιότης] of the later sections to the preceding ones, and we all agree that it [likeness] is on the side of unity, for likeness is a kind of unity. (*in Remp.*, I.288.6–18)⁶

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Proclus first insists on the fact that Plato aptly chose to represent the various sections of the divided line as parts of a single, continuous line.⁷ He then makes metaphysical sense of this detail by interpreting it in terms of the Neoplatonic doctrine of procession, to the effect that the line is a suitable image of the continuous procession of all reality from the One. We may interpret the analogy by observing that, just as the geometrical line is a continuous, one-dimensional magnitude “proceeding” from an indivisible point, the continuous procession of all reality sets out from a unique first principle.⁸ The notion of procession may well have been suggested to Proclus in this context by the originally Pythagorean idea of the line as a “flowing” (ῥύσις) of a point.⁹ It cannot be a coincidence that Plato opted for a geometrical representation of reality (i.e. by means of a continuous quantity), rather than, for instance, for the discrete quantity of a number series or of arithmetic proportions. It is typical for Proclus to insist on the fact that the procession of reality indeed occurs without any

gaps. From the lines that follow, it emerges that it is really the principle that like is generated from like, or more precisely, that causes produce effects that resemble them as much as possible, that supports this idea.¹⁰ It is the cyclical process of procession from and reversion to the cause—where the former represents the productive aspect and the latter the perfective aspect of causation—which makes the effect remain attached to its cause and warrants the continuity of reality. The likeness of effects to causes seems to be at play here on two levels. First, this likeness applies to the relation between the One and the entire reality proceeding from it: since its principle is one, and since continuity is akin to, that is, resembles, unity, reality as a whole displays continuity as its analogue for the indivisible unity of the One. Second, the likeness of effects to causes regulates the relations among the realities proceeding from the One, in the sense that the likeness between the higher and lower orders of beings also warrants the continuity between all members of the causative chain. At any rate, if the likeness between the realities represented by the different sections of the divided line is the unifying principle of reality, then Plato has aptly opted for a single, continuous line. The divided line is an image of the continuous procession of reality, and as such—we may submit—already entails the promise that it may disclose to us the path of reversion through cognitive perfection.

From *Republic* VI, 509d6, Plato makes Socrates give instructions for the division of the line. According to the reading adopted in the OCT edition, the line should be divided in “unequal parts” (ἄνισα τμήματα). However, some ancient authors, including Ps.-Archytas,¹¹ seem to have read the text differently, for they interpret it to the effect that it is concerned with a division into “equal sections” (ἴσα τμήματα).¹² It is against this background that we can understand Proclus’s insistence on the inequality of the sections in his *Republic* commentary (*in Remp.*, I.288.18–20). Proclus attaches an ontological meaning to this inequality: for him, it is a sign of the inequality among beings. Proclus speaks about differing value (ἄξια [*in Remp.* I.288.24]), and this is no doubt his understanding of Plato’s contention that the different sections of the line represent different grades of truth and clarity in the

objects they are concerned with and in their corresponding cognitive states (*Republic* 509d8–9). This criterion applies not only to the two main sections of the line, but also to the analogous proportions between the subdivisions of each section.¹³

Yet, if the line is divided into unequal parts, it is legitimate to ask—as contemporary readers also continue to do¹⁴—which section is the larger and which the smaller one. Plato is notoriously silent on this point—the only thing that can be inferred from the text is that he has a vertical line in mind, rather than a horizontal one.¹⁵ The point was controversial among the ancient readers of the passage, as evidenced by Iamblichus’s *On General Mathematics* and Plutarch’s *Platonic Questions*.¹⁶ In a Neoplatonic universe where reality unfolds from unity to multiplicity and to ever increasing grades of complexity, and where likeness of effects to causes is one of the main regulating principles of procession, one might expect that the highest stretch, which would be more akin to unity and indivisibility, would be taken to be the smaller one. This is what, according to Iamblichus, the “Pythagorean” Brotinus had advocated. Iamblichus himself explains that the smallness of the upper section is to be understood “not in terms of its power, but in terms of multiplicity” (οὐ τῇ δυνάμει ἀλλὰ τῷ πλήθει [*De comm. math.*, 35.7]), that is, in that it contains fewer items.¹⁷ Proclus disagrees and puts the larger section on top. On reflection, this is perhaps not all that far-fetched, for we have seen that Plato arranged the sections of the line according to their greater truth and clarity—which is at the very least suggestive that the sections of the line should decrease in length accordingly. Proclus, however, is not satisfied with a merely epistemological explanation, but adds a metaphysical account, which adopts productive power as a criterion:

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[T2.] Since four sections of the single line were displayed to him, he takes the two that fill up the largest section of it [τὸ μείζον αὐτῆς τμημα] to belong to the intelligible genus, and the two that compose the shortest section [τὸ ἔλασσον] to the visible genus. For we must attribute the largest section to the intelligible, which is

more excellent and encompasses the other section [κρείττονί τε ὄντι καί περιέχοντι θάτερον], and the shortest to the visible, for it is causally encompassed [περιέχεται . . . κατ' αἰτίαν] in the former. Now, the encompassed is everywhere inferior [ἔλασσον] to the encompassing, no matter whether you understand the encompassment [τὴν περιοχὴν] in terms of essence, power, or activity, as one sees both with respect to continuous quantities and with respect to discrete ones. Accordingly, of the two original unequal sections of the line the larger represents the intelligible genus, the other, smaller one the visible genus. (*in Remp.*, I.289.6–18)

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Proclus discusses the relation between the different sections in terms of their being “larger” (μεῖζον) or “smaller” (ἔλασσον) than one another. These terms could have both a quantitative (“more/greater,” “fewer/less”) and an evaluative meaning (“better/more important,” “worse/less important”), and Proclus may certainly be taken as exploiting this ambiguity.¹⁸ His point is that the *largeness* of the highest section symbolizes its ontological superiority and therefore its *greater importance*. The larger section thus stands for the intelligible, which is superior to the sensible. The argument Proclus advances to support this claim is based on the idea of causal containment, where the cause encompasses the effect and must therefore be greater or superior to the latter. The same argument can already be found in Plutarch,¹⁹ but the idea that the effect is causally contained by the cause gives it a Neoplatonic flavor. The doctrine looming behind this is indeed the idea that the cause is greater than the effect, which in turn is “causally” (κατ' αἰτίαν) pre-contained by it at a higher level.²⁰ Proclus refers this idea of causal pre-containment back to his understanding of the construction of mathematical objects, thus stressing once again the relevance of Plato’s choice for a geometrical simile in the divided line analogy. Indeed, the claim that for both continuous and discrete quantities the encompassing is always superior to the encompassed would make no sense if we were to understand it in the non-technical sense as applying to relations between different magnitudes, which would arguably be

the most natural way to understand it.²¹ Instead, what Proclus must have in mind is the construction of complex geometrical objects from more simple and basic ones, or, in arithmetic, the derivation of numbers from units. In his *Commentary on Euclid*, Proclus in this sense talks about the “encompassment” (περιοχή) of the line by the point, or of the surface by the line.²² Proclus thus understands the Euclidean construction of complex geometrical objects from more simple ones in terms of causal priority and containment, which may well resonate with his (neo-) pythagoreanizing sources. In the passage quoted above from the *Republic* commentary, Proclus adds that the superiority of the encompassing with respect to the encompassed not only concerns their “essence” (οὐσία), but also their “power” (δύναμις) and “activity” (ἐνέργεια). I am not too sure what the reference to this well-known triad adds in this context, but perhaps not much more is to be made of this than that the ontological priority of the containing over the contained applies not only to the nature of geometric and arithmetic entities (as expressed in their definitions), but also to what can be derived from that nature.²³ At any rate, according to Proclus, the larger section of the line is the more encompassing, in that it causally anticipates the lower sections (or better, the objects represented by the larger section pre-contain the objects corresponding to all the lower sections). For this reason, it is suitable that the intelligible realm is represented by the larger section, the visible world by the smaller. Later, in his exegesis of the cave analogy, Proclus will find further evidence for his arrangement of the divided line in the fact that the world outside the cave, which is assumed to correspond to the upper section of the line and to the intelligible world, is much “larger” (μεῖζον) than the petty dwelling where the prisoners spend their lives (*in Remp.*, I.293.5–9).

3. The Divided Line and the Levels of Reality

Before we look at Proclus's identification of the contents of the various subsections, it is worthwhile to turn our attention first to another text of his where the divided line is taken to represent the procession of reality from the first principle. The passage can be found in the sixth book of Proclus's *Commentary on the Parmenides*. In a historical survey of the interpretations of the hypotheses in the second part of the *Parmenides*, Proclus, from *in Parm.*, 1057.5 onwards, introduces a group of exegetes who were the first to sharply distinguish the first five, positive hypotheses (postulating the existence of the One) from the ensuing negative hypotheses (taking their starting-point from the non-existence of the One), and relate the positive hypotheses to the five main levels of reality. The first of these exegetes was the so-called "philosopher from Rhodes," whom Henri-Dominique Saffrey famously identified with Theodorus of Asine.²⁴ According to this philosopher, the first five hypotheses are concerned with the One, Intellect and the intelligibles, discursive objects, bodily forms, and the receptacle respectively (*in Parm.*, 1057.7–58.2). That the divided line is explicitly referred to in support of the succession of discursive and bodily forms (*in Parm.* 1057.20–22) strongly suggests that the entire scheme derives from that source. This suspicion is confirmed a little later in the text when Proclus introduces Plutarch of Athens, the Neoplatonist who re-established the Academy in Athens in the late fourth or early fifth century (not to be confused with his more famous namesake from Chaeronea). Plutarch interpreted the five positive hypotheses along broadly similar lines, making them correspond with the One, Intellect, Soul, enmattered forms (or the sense-world), and matter.²⁵ Apart from the One beyond Being, which falls outside the scope of Plato's analogy, Proclus traces these principles back, first, to the general distinction between being and becoming in the *Timaeus* (27d6–28a4), and second, to the divided line in the *Republic*. Even though Proclus's reading of the hypotheses does not entirely coincide with Plutarch's—like his master Syrianus, he advocates a more "theological" reading, according to which the second hypothesis discloses all the divine classes—their interpretations cover enough common ground to suppose that Proclus too endorsed the underlying correspondence between sections of the line and

levels of reality which the “philosopher from Rhodes” and Plutarch of Athens had made (see, e.g. in *Parm.*, VI, 1063.11–64.10). The passage therefore deserves our close attention:

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[T3.] We have, then, according to this arrangement, the First Hypothesis as being about the One, which Plato in the *Republic* [509b] firmly places beyond existence and Being; and the four following about beings, of which the first two concern eternally existent beings, while the latter two concern things created, following the distinction made in the *Timaeus* [27d], between those things apprehensible by intellection through reason [τῶν νοήσει μετὰ λόγου περιληπτῶν], and those things known by opinion through sense-perception [τῶν δόξει μετ’ αἰσθήσεως γνωστῶν]—or, if you wish, the four follow the division of the line in the *Republic*, of which he apportioned one section to the intelligible realm, the other to the sensible realm, one to the henads, the other to the things here termed the Others²⁶; and of the larger section [τοῦ μείζονος], one part to the intelligibles, the other to the objects of discursive reasoning [τοῖς διανοητοῖς], even as of the two former hypotheses the one is about Intellect, the other about Soul; while of the smaller section [τοῦ ἐλάττονος], he apportioned one part to things of sense, the other to things imaged [τοῖς εἰκαστοῖς], even as here we have said that the fourth hypothesis is about forms-in-matter [τῶν ἐνύλων εἰδῶν], which are the proper objects of sense [κυρίως αἰσθητά], while the fifth is about matter, which is analogous to the things imaged by reason of the indefiniteness of the knowledge of it attainable by us [διὰ τὴν ἀοριστίαν τῆς περὶ αὐτῆς ἡμῖν ἐπιγιγνομένης γνώσεως]. (in *Parm.*, 1060.22–61.7)²⁷

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The connection of the divided line with the distinction between the accounts of being and of becoming in the *Timaeus* can be found in many passages—for the later Platonists, the *Timaeus*’s distinction basically recapitulates the more detailed

description of our cognitive states and their corresponding objects provided by the divided line.²⁸ I here want to draw attention only to the use that is made of the line in T3. First of all, Plutarch is much more straightforward than Plato in excluding the Good—which formed the object of Plato’s analogy of the sun and which is here identified with the Neoplatonic One—from the contents of the divided line altogether.²⁹ The One, being beyond Being, cannot be an object of proper cognition and thus falls out of Plato’s account of our cognitive states. From Plotinus onwards, Platonists have taken it to be the proper object of the first hypothesis of the *Parmenides*, in which all attributes are denied of the One. The four subsequent hypotheses treat, in Plutarch of Athens’s view, intellect, the soul, enmattered forms, and matter respectively. These are said to correspond one-to-one to the four sections of the line. Nothing is surprising in the connection of intellect with the intelligibles, or of soul with the objects of *διάνοια*. What may be less obvious, however, are the attributions in the lower section of the line. Plutarch equates what should correspond to the objects of *πίστις* not with sensible objects as such, but with “forms-in-matter” (*ἔνυλα εἶδη*), that is, the formal components of bodily compounds which determine their nature and represent what is cognizable about them—in this sense they may be considered the “proper objects of sense” (*κυρίως αἰσθητά*). Equally remarkable is the identification of the “objects of conjecture” (*εἰκαστά*) with matter. This identification is no doubt partly imposed by the strict correspondence between the subsections of the line and the levels of reality, but the idea is perhaps not too difficult to understand if we remember that Proclus, in his *Commentary on the Parmenides*, compares matter to a mirror in which the forms are reflected.³⁰

4. The Contents of the Divided Line

Let us turn then to what Proclus says about the contents of the divided line in the twelfth essay on the *Republic* (*in Remp.*, I.289.18–92.21). In his exegesis of the

relevant passage in Plato, Proclus first remarks that, in his discussion of the objects with which the various subsections are concerned, Plato starts from what is more known to us (i.e. from “below” [*in Remp.*, I.289.18–19]). This allusion to the Aristotelian order of proceeding makes a very sound point with respect to Plato’s actual line of reasoning. Plato indeed seems to use the distinction between our cognizing sensible objects either directly or through their reflections as a means to clarify the distinction between two methods of approaching the intelligible (possibly, but not necessarily corresponding to two sets of objects). Proclus then embarks on a rather lengthy discussion of the objects of εἰκασία, the main aim of which is to establish their ontological relevance.

Proclus first points out that while Plato identifies the contents of the lowest subsection as “images,” he immediately adds a clarification to disambiguate this term, saying that by “images” he means shadows and reflections in water and on other reflecting surfaces (*Republic* 509e1–10a3; cf. *in Remp.*, I.289.21–26). The reason is, according to Proclus, that ‘images’ can be of various types and that Plato wants to keep us from thinking, for instance, of statues or paintings. I take it that Proclus’s concern is with the distinction between artificial and natural images and that his insistence that only the latter are referred to by Plato serves Proclus’s point about the substantial nature of the objects of εἰκασία. After a few remarks on the requirements mentioned by Plato for the reflecting surfaces,³¹ Proclus concludes that, for Plato, reflections are indeed in some sense “realities” (ὑποστάσεις)—a claim that he backs up with a reference to the *Sophist*.³² Proclus provides two further arguments to support his statement that reflections are somehow real. The first is concerned with shadows in particular, and it establishes their reality by virtue of their “affinity” (συμπάθεια) with the objects they reflect (*in Remp.*, I.290.10–21). Proclus refers to magical practices where shadows and other reflections are exploited, and remarks that even irrational beasts such as hyenas may learn about the dogs they are about to catch by observing their shadows.³³ It is with his last argument, however, that Proclus finally puts his cards on the table. He argues that the substantial nature of the

reflections is also implied by the proportions that Plato himself provides about the relations between the different subsections of the line, which entail that the relation between the “objects of conjecture” (εἰκαστά) and the “visible objects” (τὰ ὁρατά) is analogous to that between the “objects of discursive reason” (διανοητά) and the “intelligibles” (νοητά).³⁴ Now, since discursive reason has as its proper objects “certain forms that really are” (εἶδη τινὰ καὶ ὄντα), the objects of εἰκασία must have a substantial nature, too (*in Remp.*, I.290.25–27).

This ingenious argument suggests that what is truly at stake in Proclus’s elaborate discussion on the substantiality of reflections is the existence of mathematical intermediaries. Contemporary commentators often use the objects of εἰκασία’s relative lack of ontological relevance as an argument that in the upper section of the line Plato did not mean to introduce mathematical intermediaries as a separate level of ontological items.³⁵ For if the analogy between the subdivisions of both sections holds, and if in the lower section there is only one class of ontologically substantial entities, then it is unlikely that Plato’s intention was to assign different objects to the two upper subsections. Rather, what Plato did was to provide an analogy for two methods of approaching the Forms—which can be grasped either directly or by means of their sensible representations—by referring to the familiar relation between sensible objects and their shadows or reflections. It appears to me that Proclus was well aware of the possible threat of this line of reasoning. He rendered it harmless by turning the argument upside down. Regardless of the fact that Proclus employs the analogy argument in defense of the substantial character of shadows and reflections, I think that his actual agenda is to be sought in the opposite direction, namely, in the defense of mathematical intermediaries. I can hardly see how else one could make sense of the great pains he has taken to argue for the ontological relevance of the objects of εἰκασία, which he is prepared to downgrade in other contexts.³⁶

Proclus’s views on the contents of the other subsections of the line are much more straightforward. Proclus discusses them in ascending order, starting from the

second lowest subsection. This is said to contain natural and artificial particulars (*Republic* 510a5–6), and Proclus takes them to correspond to the objects carried behind the wall in the Cave and to the animals and plants that surround the passers-by (*in Remp.*, I.290.29–291.12). It should by now be hardly surprising that Proclus believes that the two subsections composing the great upper section of the line, which represents “the intelligible genus” (τὸ νοούμενον γένος), are concerned with two different classes of objects: “discursive objects” (τὰ διανοητά) and “the intelligible” *stricto sensu* (τὸ νοητόν) respectively (see *in Remp.*, I.291.12–92.21). His account of the distinction between them stays reasonably close to the Platonic text, mainly emphasizing the fact that discursive reason proceeds from hypotheses, whereas “intellect” (νοῦς) leaves the hypotheses behind in its search for their ultimate principle. The other difference between discursive reason and intellect mentioned by Plato (that the former makes use of images, whereas the latter does not), is not dealt with by Proclus—or perhaps it would be more accurate to say that it is reinterpreted with a view to the model/image relation that Proclus thinks obtains between all the levels of the divided line, as we will see shortly. In this way, the objects of διάνοια become truly intermediate between the Forms and sensible particulars.

5. The “Pythagorean” Division of Reality

The discussion in the previous paragraph suggested already that what is at stake in Proclus’s interpretation of the divided line is the defense of mathematical intermediaries. The most remarkable metaphysical application of the line is indeed to be found in Proclus’s discussions of the ontological status of mathematical objects. The divided line has of course time and again been referred to by those who want to attribute a doctrine of mathematical intermediaries to Plato,³⁷ and Aristotle’s statements to that effect may well derive from a reading of our passage.³⁸ It is in Proclus’s *Commentary on the First Book of Euclid’s Elements* that we find the most

instructive discussions à propos. In the first prologue to this commentary, Proclus repeatedly raises the question regarding the status of mathematical objects. Since mathematical objects turn out to have an intermediate status, the main structure of reality is threefold—it is divided into the intelligible, the discursive, and the sensible or opinable.³⁹ It is with this threefold division in mind that Proclus opens his *Commentary on the First book of Euclid's Elements* with a programmatic statement on the ontology of mathematical objects. After comparing the properties of mathematical objects with those of the higher and lower orders of reality, he concludes that they cannot belong to either of them. He then continues,

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[T4.] It is for this reason, I think, that Plato too distinguished between our cognitions [γνώσεις] of things [depending on whether they concern] the highest, the intermediate, or the lowest grades of reality. To indivisible realities he assigned intelligible cognition [τὴν νοητὴν], which discerns the intelligible with simplicity and immediacy, and by its freedom from matter, its purity, and its uniform mode of coming in contact with being is superior to all other forms of cognition. To divisible things which have received the lowest nature, that is, to all objects of sense-perception, he assigned opinion [τὴν δόξαν], which lays hold of a faint truth, whereas to intermediates, such as the forms studied by mathematics [τὰ τῆς μαθηματικῆς εἶδη], which fall short of indivisible but are superior to divisible nature, he assigned understanding [τὴν διάνοιαν]. Though second in rank to intellect and the highest knowledge, understanding is more perfect, more exact, and purer than opinion. . . . As the forms of cognition differ from one another, so also their objects are distinguished in nature. (*in Eucl.*, 3.15–4.18)⁴⁰

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Proclus clearly has the divided line in mind, distinguishing between the different states of cognition and relating them to their corresponding objects. However, rather

than working with the four states of the line, he instead works here with three cognitive states—ἡ νοητὴ (γνῶσις), διάνοια, and δόξα—thus relating the entire lower section of the line, which comprehends πίστις and εἰκασία, to the realm of δόξα.⁴¹ One may find some irony in the fact that the objects of εἰκασία, once they have done their job supporting the existence of mathematical intermediaries, no longer seem to have such an ontologically substantial role. In the context of the Euclid commentary, Proclus's main concern is of course with the ontological status of mathematical objects, which are here identified as the ontologically intermediate objects of διάνοια. The objects of intelligible cognition (elsewhere called νοῦς or νόησις) are called indivisible, and if they can be grasped in a pure way, free from matter, it is because they are themselves pure and immaterial in the first place. By contrast, the objects of opinion are divisible, precisely because of their material nature. If mathematical objects are intermediate between the truly indivisible and the entirely divisible nature, they must presumably be divisible in some sense, but not in the straightforward way of sensible objects.

Proclus's most elaborate and detailed reference to the divided line in the Euclid commentary comes a few pages later (*in Eucl.*, 10.16–11.25), when he tackles the question regarding “the cognitive faculty involved in mathematical inquiry” (τὸ κριτήριον τῶν μαθημάτων). Proclus here proceeds somewhat differently, in that after mentioning Plato's *Republic* explicitly he sets out by introducing the fourfold distinction between cognitive states and their corresponding objects of the line, and only then tries to determine where the criterion of mathematics belongs in this scheme. Eventually (*in Eucl.*, 11.16–25), the fourfold scheme again collapses into a threefold taxonomy, in which the objects of our cognitive faculty of “understanding” (διάνοια) occupy the intermediate position, between the objects of “intellection” (νόησις) or “the first science” or science proper (ἡ πρώτη ἐπιστήμη), and those of “opinion” (δόξα) or sense-perception.

The main division of reality is thus threefold. Among the later Neoplatonists, we often find this threefold division invested with Pythagorean authority.⁴² As such, it

also appears in Proclus's master, Syrianus, to whom Proclus is much indebted for his own views on mathematics. In the introduction to his commentary on books M and N of the *Metaphysics*, Syrianus writes the following:

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[T5.] They [i.e. the “divine” Pythagoras, and “all those who have genuinely received his doctrines into the purest recesses of their own thought” (*in Met.*, 81.32)] declared that there were, broadly, three levels of being, the intelligible, the dianoetic, and the sensible, and that there were manifested at each of them all the forms, but in each case in a manner appropriate to the particular nature of their existence. (*in Met.*, 81.38–82.2)⁴³

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This passage forms part of an introduction to Syrianus's commentary on books M and N of the *Metaphysics*, which is, of course, another classical text for the discussion of the status of mathematical objects. Yet, contrary to the passages discussed from Proclus's Euclid commentary, nothing in Syrianus's formulation here restricts the scope of the dianoetic entities to mathematical objects—which were Plato's main concern in the second section of the divided line. Indeed, the reference to the line may be rather thin at first sight, for Syrianus refers the theory not to Plato, but to the Pythagoreans, and rather than talking of Forms, mathematical objects, and sensible particulars, he merely refers to three levels of being—the purely intelligible, the dianoetic, and the sensible—to which three distinct levels of forms correspond. Nothing is surprising about the attribution of Forms to the upper subsection of the line, but the distinction between levels of forms in Syrianus goes much further. We do, however, find similar ideas in Proclus. As we have already seen, the third subsection of the line was associated in T3 with forms-in-matter, that is, the formal components of sensible objects, and in T4 the objects of *διάνοια* were identified as the forms of mathematical objects. It is not difficult to connect this doctrine with the

Neoplatonic theory of procession, according to which the Forms, after their appearance in the demiurgic Intellect, proceed to the sensible realm, where they will manifest themselves as the formal principles of body, through the intermediate level(s) of soul (and nature). At each of these fundamental levels of reality, the entire range of forms is thought to be present.⁴⁴

The intermediate forms are what both Syrianus and Proclus often call the ‘psychic’ forms or *logoi*, that is, the reason-principles which constitute the essence of the soul and in virtue of which the soul is said to comprehend the totality of the forms in a way proper to its own nature. These psychic *logoi*, images of the intelligible Forms, are the soul’s proper objects of thought.⁴⁵ Mathematical forms only constitute a subclass of the entire realm of dianoetic forms. According to Proclus, it is by “projecting” or “bringing forth” (προβάλλουσα) and “unfolding” (ἐξελίπτουσα) these innate *logoi* of which the soul is essentially constituted that discursive reason generates the entire manifold of mathematical sciences and theorems.⁴⁶ According to the famous ontology of geometrical objects that Proclus provides in the second prologue to his commentary on Euclid, the psychic *logoi* are strictly speaking not themselves the objects that geometrical truths are about.⁴⁷ In the soul, the *logoi* or principles of these objects are unique, unextended, and indivisible, and they are contained as it were “wrapped up” (συνεπτυγμένως [*in Eucl.*, 55.1]). In order for them to become manageable concepts they need to be projected by our discursive reason onto the intelligible matter of the imagination (φαντασία),⁴⁸ which serves as a mirror in which the *logoi* are reflected.⁴⁹ It is only by thus unfolding or articulating its *logoi* that the soul can come to know its innate concepts and understand the knowledge that it carries within. As a result of their projection the *logoi* will acquire the conceptual extension and multiplicity that they need to function in mathematical practice,⁵⁰ where, for instance, the relation between various types of angles may be studied or theorems about the properties of circles may be derived. This is the way in which Proclus accounts for both the intermediate status that Plato attributes to mathematical objects in the divided line, and for the properties that Aristotle, in *Metaphysics* A (6,

987b14–18), connects with such mathematical intermediaries: unlike sensible particulars, mathematical objects are eternal and unchangeable, while they differ from Forms in being multiple. Proclus often defends this Platonist view in opposition with what he regards as an Aristotelian, abstractionist view on the generation of mathematical objects.⁵¹

6. The Nature of Mathematical Objects in the Twelfth Essay

It is worthy of note that in the twelfth essay of his *Commentary on the Republic*, Proclus alludes to the account of the nature of mathematical objects that he had expounded in more detail in the *Euclid Commentary*. The passage can be found in the second part of the essay, where Proclus has returned to the discussion of the Cave allegory. After having reminded us of the broad correspondence between the line and cave analogies, Proclus has evoked the general setting of the allegory and related the situation of the prisoners who are looking at the cave wall with the lowest subsection of the line. After a lacuna in the text (at *in Remp.*, I.293.22, probably comprising at least an entire folio), the text resumes with an interpretation of the objects seen by the liberated prisoner immediately upon his ascent from the cave to the world outside. Even though the immediately preceding lacuna also affects our interpretation of the ensuing lines, the passage deserves to be quoted in its entirety because of its striking affinity with the doctrine from the *Commentary on Euclid*:

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[T6.] After all these things [τούτων, namely, sensible particulars] were seen first by us in our ascent,⁵² he added in between them and the really existing intelligibles [μεταξὺ τούτων τε καὶ τῶν ὄντως νοητῶν]—introducing it somewhat surreptitiously—something else [ἄλλο τι], and “then the things themselves” [καὶ ὅσπερ αὐτά].⁵³ Now what could he be indicating by this to those who are listening not inadvertently [τοῖς

μὴ παρέργως ἀκούουσιν], if not the fact that the soul possesses the *logoi* of the discursive objects [τῶν διανοητῶν], which geometry and arithmetic examine [σκοπεῖται], so that the objects contemplated by these sciences are images [ὡς εἰκόνας εἶναι τὰ ὑπὸ τούτων θεωρούμενα] and that our discursive reason [τὴν διάνοιαν], by training itself in these sciences [ἐν τούτοις]⁵⁴ and purifying itself, must bring forth [προβάλλειν] the *logoi* themselves of these discursive objects that are in the soul [αὐτοὺς . . . τοὺς ἐν αὐτῇ τούτων λόγους] and [καὶ] must behold them not in their images [ἐν εἰκόσιν], but in itself [ἐν ἑαυτῇ], where they are wrapped up and indivisible [συνεπτυγμένως ὄντα καὶ ἀμερῶς]. And it is indeed by unfolding [ἀνελίσσουσα] these things that [discursive reason] has produced such a great number of theorems. (*in Remp.*, I.293.23–94.4)

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The vocabulary of this passage is clearly reminiscent of the Euclid commentary. Not only does it speak about the “bringing forth” (προβάλλειν) of the soul’s *logoi* by discursive reason, but it also claims that these *logoi*, which are in themselves “indivisible” (ἀμερῶς) and as it were “wrapped up” (συνεπτυγμένως), must be “unfolded” (ἀνελίσσουσα)⁵⁵ so as to produce the theorems studied by the mathematical sciences. These wordings are all familiar from the *Commentary on Euclid* and especially from Proclus’s discussion of the nature of geometrical objects. One might therefore be tempted to go even further and also read Proclus’s more technical view about the projection of the soul’s *logoi* in the imagination into this passage.

Whether or not this is plausible depends on how one is prepared to understand the “images” that Proclus mentions at 293.28 (εἰκόνας) and—assuming that they are identical—at 294.1–2 (ἐν εἰκόσιν). At first sight, there are at least three possible interpretations. A first reading that has been proposed simply identifies the discursive objects that are called images with the soul’s *logoi*, which are indeed sometimes called “images” of the intelligible Forms.⁵⁶ Yet this reading fails to account for the

point that Proclus is making in the passage under consideration, which precisely requires an opposition between the soul's *logoi* themselves and their images that are examined by the mathematical sciences.

Taking this contrast seriously one may be inclined to adopt a more technical reading of the passage, along the lines of the projection theory of the Euclid commentary. According to this second reading, Proclus is drawing a contrast between the soul's own *logoi* and the discursive objects with which the sciences are concerned. The latter would be identified with the projections of the *logoi* in the imagination, which because of their derivative nature could be considered “images” of the essential *logoi*.⁵⁷ The idea would be that the sciences, which operate discursively, are usually concerned with the unfolded or projected *logoi*, that is, the discursive objects. Yet the training and purification for which the sciences allow disclose the path to the discovery of the principles from which such projections derive, and therefore allow us to go beyond the images and contemplate the soul's proper *logoi* in their unified state. This reading is very tempting, as it provides the best explanation for the presence of the technical vocabulary of projection: the vocabulary is present simply because the doctrine behind it is too. And yet I am hesitant to argue along these lines for three reasons. First, neither the divided line nor the cave analogy gives any indication about the objects of discursive reason (or what can be taken to correspond to it in the cave) that could—even by a reader as biased as Proclus sometimes is—be interpreted in terms of such a technical distinction. Second, Proclus's agenda in this commentary does not require such a distinction—for the main thrust of his exegesis is simply to establish the existence of mathematical intermediaries. Finally, upon consideration it proves difficult to take *προβάλλειν* (293.30–94.1) in a very technical sense here. For the projection of *logoi* in the imagination by discursive reason can hardly be contrasted with the examination of projected *logoi* by the sciences, and yet this would be a consequence of the more technical reading. On the same note, if the *καὶ* of 294.1 is epexegetic—as I think it is—we may take the clear contrast in 294.1–3 between considering the discursive objects in their images and beholding them as indivisible

and wrapped up in discursive reason itself as an explanation of the preceding distinction between the sciences' usual way of considering their objects and their "bringing forth" of the *logoi* themselves.

The only alternative that I can see is to take εἰκόνας at 293.28 to refer to something rather obvious, namely to the sensible representations of mathematical objects, such as the circle drawn in sand.⁵⁸ After all, Plato himself has told us quite emphatically in the divided line that mathematicians use perceptible objects as images of what they are really concerned with (*Republic* 510b4–5; 510e3–11a1; cf. *in Remp.*, I.291.12–27). According to this reading there is no sharp contrast between the truly discursive objects examined by the sciences and the principles in the soul. The opposition is rather between the proper discursive objects and their sensible images, which is compatible with Proclus's view that sensible particulars are images of discursive forms. The idea is that mathematicians at first need sensible representations of their objects, but through practice and purification they can learn to "bring forth" the soul's *logoi* themselves and thus contemplate pure mathematical objects at the ontological level where they belong: in the soul, which is intermediate between the sensible and the intelligible. Once these *logoi* are properly grasped by discursive reason, one is able to unfold them and thus properly derive the entire host of mathematical theorems. Even though this reading is not devoid of difficulties either,⁵⁹ it appears to me as the least problematic of the three. If this interpretation is correct, it means that the technical vocabulary used in the Euclid commentary's *second* prologue to explain the projection theory of geometrical objects is employed in the *Republic* commentary's interpretation of the cave to express the less technical doctrine—familiar from the Euclid commentary's *first* prologue—of the derivation of mathematical sciences and theorems by "bringing forth" their principles. Both the vocabulary and the doctrine of our passage from the twelfth essay of the *Republic* commentary still have to be read against the backdrop of the *Commentary on Euclid's Elements*, which is precisely the point that I wanted to make.

7. Model and Image Relations in the Line

In our discussion of the “Pythagorean” tripartition of reality in section 6, we have seen that, for Syrianus and Proclus, mathematical objects—or, more accurately, the principles of mathematical objects—constitute only a subclass of the intermediate forms. Yet there is also a second consequence of their acceptance of the “Pythagorean” division of reality that is worth considering. Plato’s assertions about the relation between the objects of *διάνοια* and those of intellection and opinion were open to at least two very different interpretations. It is not crystal clear whether Plato wants us to understand the sensible objects represented by the third subsection of the line merely as images of the Forms—which the mathematician then employs as images of the mathematical objects he contemplates, in the sense that he can, for example, use the trunk of a tree as a sensible illustration of a circle without the circle being causally active in the production of trees—or also as copies of mathematical intermediaries—and therefore only indirectly as images of the Forms.⁶⁰ The entire question hinges on the ontological status that one is prepared to attribute to the objects of *διάνοια*. The later Neoplatonists’s theory of procession and their doctrine of mathematical intermediaries allow them to resolve this ambiguity about the model/image relations in the divided line. Proclus and his fellow commentators opt for the second, ontologically more committed interpretation, according to which the objects of *διάνοια* are genuine ontological intermediaries, implying that they are images of the higher Forms in Intellect, yet models for the forms immanent in sensible particulars.⁶¹ Thus the three main levels of reality are all linked by model/image relations—which was of course also implied in Proclus’s insistence in T1 on the importance of likeness as a ruling principle in the procession of reality. These elaborations are by no means Proclus’s own, but can be traced back at least to Syrianus, if not Iamblichus.⁶²

8. The Pythagorean Provenance of the Line

T4 and T5 stemmed from very similar contexts, but only in Proclus was the reference to Plato explicit. It is well known that the first prologue to the Euclid commentary, in which Proclus treats mathematics in general, closely follows the structure of Iamblichus's *On General Mathematics*. It is indeed in the corresponding chapter of the latter work—chapter 8, in which the *criterion* of mathematics is discussed—that Iamblichus had introduced an elaborate discussion of the divided line, drawing on material that he found in (Ps.-)Brotinus and (Ps.-)Archytas, which he takes to have been Plato's sources. It is remarkable that, while Brotinus's treatise *On Intellect and Discursive Reasoning* and Archytas's treatise *On Intellect and Sense-Perception* are quoted at length, Plato himself is not even mentioned in the entire chapter. Moreover, in the second chapter of *On General Mathematics*, on which Proclus seems to rely in T4, Iamblichus has inserted a reference to “the division of the cognitive line” (ἐν τῇ τῆς γνωριστικῆς γραμμῆς τομῇ) made by the so-called Archytas, on which he says he relies for his views on the intermediary status of mathematical objects. It is against the backdrop of these texts that Syrianus credits the “Pythagoreans” with the elaboration of the threefold division of reality. This is confirmed by another text from his *Commentary on the Metaphysics*. In commenting on Aristotle's conclusion to his discussion of mathematical objects in M 4 (1078b7–9), Syrianus has inserted a short survey of insights to be gained from the “genuine doctrines” of the Pythagoreans on this subject. It has been noted that this list reads as a summary of the chapter headings of Iamblichus's *On General Mathematics*.⁶³ Concerning the *criterion* of mathematical objects (a topic discussed, as was just said, in chapter 8 of *De comm. math.*), Syrianus makes the following remarks:

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[T7.] [We can also learn from the doctrines of the Pythagoreans] what is the common *criterion* of all the mathematical sciences, and how it can be discovered from the division of the line transmitted by Archytas. (*in Met.*, 102.3–4)

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Syrianus thus follows Iamblichus in crediting Archytas with the invention of the divided line, and this is no doubt one of the reasons why he considers the threefold division of reality as Pythagorean. In the corresponding chapters of his *Commentary on Euclid*, Proclus reclaims the line for Plato. As Dominic O’Meara has argued,⁶⁴ the moral to be drawn from this is not that Proclus is in any way skeptical about the Pythagorean provenance of the doctrine concerning the tripartite structure of reality—which he readily accepts elsewhere⁶⁵—but rather that he finds Plato’s version of it far more accurate and scientific than anything we find in the extant texts of his alleged Pythagorean predecessors. For Proclus, no less than for Iamblichus and Syrianus, Plato is a prominent member of that ‘golden chain of wisdom’ that claimed Pythagoras as one of its charismatic founding fathers, and which assembled everything worthwhile in pagan thought and religion. This is the agenda that motivates many of the post-Iamblichean Platonists to scrutinize Plato’s dialogues for presumed parallels and correspondences with Pythagorean doctrines. The pseudo-Pythagorean writings of the early imperial age, such as the treatises by the so-called Archytas and Brotinus quoted by Iamblichus in *On General Mathematics*, could fruitfully be used to establish such connections. Proclus, for his part, finds no fault in giving credit to the pseudo-Pythagorean Timaeus of Locri for the entire subject-matter of Plato’s *Timaeus*.⁶⁶ Even though no comparable Pythagorean attribution can be found in Proclus for the divided line,⁶⁷ his understanding of both the form and the contents of the analogy allows for no other conclusion than that the divided line is for him a sample of Plato’s appropriation of Pythagorean theology.

9. The Line as a Sample of Pythagorean Theology

The picture that has started to emerge from our discussion of Proclus's interpretation of the divided line is indeed quite consistent. If we abstract from the particular context of the twelfth essay of the *Commentary on the Republic* (with its particular concern for the parallelism between the line and the cave analogies), a case can be made that the divided line, for Proclus, as for his Neoplatonic predecessors, first and foremost serves to introduce mathematical intermediaries. As such, it is, for him, Plato's clearest expression of allegiance to the "Pythagorean," threefold division of reality. For both Iamblichus and Syrianus, however, not only the doctrine behind the divided line, but also the very image used by Plato to represent it had a Pythagorean provenance. We find no explicit statement to that effect in Proclus, but it is striking how, in the twelfth essay of his *Commentary on the Republic*, he insists on the features that expose the line as a geometrical object: it represents the procession of reality by means of continuous quantity, it uses geometrical relations and proportions to illustrate ontological and epistemological hierarchies, and it makes use of analogy as a mathematical counterpart of metaphysical unification. Moreover, many details of Proclus's interpretation can, as we have had ample occasion to see, only be fully appreciated against the backdrop of the mathematical and geometrical doctrines discussed in Proclus's Euclid commentary.

From all these observations, it can be safely inferred that what Plato does in the divided line according to Proclus is provide instruction about the first principles of reality by means of a mathematical (in particular a geometrical) analogy. Such a procedure nicely matches the characterization that Proclus, in the opening chapters of his *Platonic Theology*, provides of the kind of theology that he identifies as Pythagorean. In this magnum opus, which provides an outline of what he considers as the systematic theological project behind Plato's dialogues, Proclus actually distinguishes between four modes of theological discourse that can be found in Plato.⁶⁸ One of these four ways in which Plato instructs about divine principles is,

according to Proclus, “through images” (ὑπὸ τῶν εἰκόνων, *Theol. Plat.*, I 4, 17.22). Even though such images can be ethical or physical representations as well (e.g. in the *Politicus*, where political leadership is, according to Proclus, actually conceived as an image of divine rulership and thus instructs us about the powers of divine principles), Plato often employs mathematical images to teach about the divine (e.g. in the construction of the soul and in the geometrical atomism of the *Timaeus*).⁶⁹ Teaching through mathematical analogies actually turns out to be the main procedure of this mode of divine instruction, when Proclus later tells us that it is typical for Pythagoras and his followers, who invented the mathematical sciences and used them to gain access to the divine (*Theol. Plat.*, I 4, 20.8–12).⁷⁰ Such teachings usually advance “by means of likeness of things here with divine principles” and provide images of the “processions, hierarchies and demiurgic powers” of such principles (*Theol. Plat.*, I 4, 19.20–22). This nicely matches Proclus’s understanding of the divided line, which indeed provides a geometrical analogy for the procession of all (levels of) reality from the One and which adopts proportion as a likeness of the first principle.

A passage from the third book of Proclus’s *Commentary on the Timaeus* (II, 245.23–46.9) confirms this qualification of the divided line as a specimen of “Pythagorean” theology through mathematical imagery. Proclus is here commenting on another crucial source-text for the Platonico-Pythagorean doctrine of the intermediate forms in the soul, namely, the passage from the *Timaeus* relating the construction of the World Soul out of geometric and arithmetic proportions (*Tim.* 35b–36b). Timaeus describes how two lines (or lengthwise strips) are first divided proportionally and then bended to form the circles of the Same and the Different, which make up the essence of Soul. This text is often cited by the later Platonists in support of the view that the principles of mathematics essentially belong to the soul and that mathematical objects are therefore ontologically superior to sensible particulars.⁷¹ In the passage under consideration here, however, Proclus tackles the question as to why Plato chose to make the soul out of a line, that is, out of a geometrical magnitude. According to Proclus, the line may well represent the soul’s

transitivity and discursiveness,⁷² but one should not infer from Plato's mode of presentation that the soul is also a magnitude. Plato actually makes use of a mathematical line to represent something that has an indivisible essence. His mode of exposition illustrates what Proclus in the *Platonic Theology* had identified as the mode of instruction through images:

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[T8.] Yet Plato secretly used mathematical terms like a disguise for the truth of things [οἷον παραπετάσμασιν ἐχρήσατο τῆς τῶν πραγμάτων ἀληθείας], just as the [Orphic] Theologians used myths and the Pythagoreans used symbols. For it is possible to see the paradigms in the images and to make the transition through the latter to the former. (*in Tim.*, II.246.4–8)⁷³

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Proclus is not very firm in his terminology, for while in the *Platonic Theology* he identified mythological discourse (especially Orphic myths) with the symbolic way of teaching, here he takes them as two alternatives, and while the Pythagoreans were there regarded as the founders of the mode of divine instruction through mathematical images that Plato often adopts, here they are rather said to provide something analogous but not identical to Plato's mathematical teaching. Furthermore, in the *Timaeus*, the line provides an analogy for the soul, and not for the entire procession of reality as in our passage from the *Republic*. Nonetheless, both the (presumably Pythagorean) context and the doctrinal content of the two source-texts (concerning the intermediary status of the soul and its objects) are sufficiently close to allow us to understand the divided line too as a sample of what in the *Platonic Theology* was identified as “Pythagorean” teaching about the divine through mathematical images.

Against the backdrop of all this evidence, the conclusion is inevitable that Proclus understands the divided line as a delicate sample of genuine Pythagorean theology. This mode of teaching has as its distinctive feature that it represents reality

through mathematical images that appeal to our imagination. Since our soul is bound to think in temporal succession, the projection of higher truths within our imagination is a way to disclose divine truths to our discursive reason. By providing a powerful geometrical image of the procession of all beings from the One, the divided line thus helps anyone who is able to go beyond the analogy to grasp the fundamental law of reality.⁷⁴

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¹ References to Proclus’s and other ancient works are to the page and line numbers adopted in the standard editions mentioned in the bibliography. Where applicable, I use Roman numbers to refer to ancient book divisions (except for Proclus’s *in Remp.* and *in Tim.*, where I follow common practice in using Roman numbers to refer to the volume numbers of Kroll’s and Diehl’s editions) and Arabic numbers for chapters. For a survey and discussion of the most relevant texts, see Heinrich Dörrie and Matthias Baltes, *Platonismus* IV, 84–97 and 332–55. This paper owes much to Dörrie and Baltes’s almost encyclopedic study of the reception of Plato’s line, especially for the abundant factual information that it provides. Among the many allusions and references to the divided line by the later Platonists, those by Proclus’s master, Syrianus, are discussed in Antoine Coté, “Syrianus’ Use of the Divided Line.”

² See *in Remp.*, I.288.5–6 and I.292.22–24. That the main aim of the twelfth essay is to clarify Plato’s cave allegory is already apparent from its title. Yet, while the

discussion of the divided line in this context serves to clarify the cave analogy, its interpretation actually occupies the largest part of the extant text (in the second part of the essay, there is a longer lacuna, the extent of which is hard to determine).

³ I will deal with the place of the divided line in Proclus's epistemology in a paper forthcoming in the proceedings of a conference organized in Gargnano in June 2015, devoted to epistemology in post-Hellenistic philosophy.

⁴ By pointing to Proclus's "pythagoreanizing" agenda, I by no means intend to take a position in the all too vexed debate about the authenticity of the doctrines that Proclus and other later Platonists attribute to Pythagoras or the Pythagoreans. I will rather look at the material from the perspective of Proclus's own claim to Pythagorean authority and of the role it plays in the justification of his exegesis of the divided line, regardless of the authenticity of his presumed Pythagorean sources—which is often highly problematic, to say the least—and of his own appropriation of them.

⁵ *In Remp.*, I.287.23 and I.288.4–5.

⁶ All English translations from Proclus's *Commentary on the Republic* quoted in this paper are mine.

⁷ Proclus emphasizes this point again in *in Remp.*, I.288.18 and I.289.6. For the same insistence, see also Iamblichus, *De comm. math.*, 38.15–16. Cf. Plutarch, *Quaest. Plat.*, III 1, 1001C, and Asclepius, *in Met.*, 142.8.

⁸ In his *Commentary on the Parmenides* (1141.29–30), Proclus compares the One as the first principle of all reality with the point as the first principle of magnitude. Cf. *in Remp.*, I.292.8–9, where the point is called the "origin of all geometrical entities" (ἀρχὴ τῶν ἐν γεωμετρίας πάντων).

⁹ In the Euclid commentary, Proclus often refers to the definition of the line as the "flowing of a point" (σημείου ῥύσις: see *in Eucl.*, 97.9; 179.24; 185.10; cf. *in Tim.*, III.137.2), while he stipulates that "'flowing' stands for 'procession'" (ἡ δὲ ῥύσις τὴν πρόοδον ἐνδείκνυται [*in Eucl.*, 97.13–14]). For this interpretation of the originally Pythagorean notion of ῥύσις, see also Nicolas Vitell, "La *rhysis* mathématique." For

the line as a procession (πρόοδος) from the point, see, e.g. in *Eucl.*, 101.13–23, and in *Tim.*, II.246.25–6, where Proclus talks about a continuous line as something that “proceeds” (πρόεισιν) from a point.

¹⁰ For this fundamental principle of Proclean metaphysics, see esp. *El. theol.*, § 28, which E. R. Dodds in his commentary *ad locum* (216) has aptly called the “law of continuity.” For the relation between continuity and likeness, see, e.g. *El. theol.*, §§ 132 and 147.

¹¹ As well as Iamblichus, who is reporting Ps.-Archytas’s views. See *De comm. math.*, 36.15–20 and 38.19–22.

¹² As we will further discuss below, later Platonists took Ps.-Archytas’s paraphrase of the divided line in his lost treatise *On Intellect and Perception* (quoted by Iamblichus) to be the *source* of Plato’s line analogy. Because of the pseudepigraphic nature of this work, it is difficult to tell whether or not its author relies on a different version of our Plato text. Ps.-Archytas’s alternative reading a priori allows for three possible explanations. (1) He simply had a different text before him (reading ΙΣΑ instead of ΑΝΙΣΑ), which he faithfully paraphrases. Yet there is no trace of such an alternative reading in the direct textual tradition, nor do the credentials of an author who fraudulently adopts the identity of Archytas and pretends to be Plato’s source urge us to assume that he is a reliable witness for Plato’s text. While (1) cannot be ruled out, I am therefore inclined to think that Ps.-Archytas probably relies on the same version of the text as in the ancient ancestors of our manuscripts, which presumably read ΑΝΙΣΑΤΜΗΜΑΤΑ (in *scriptio continua*), spelled out in A and D as ἄνισα τμήματα (the reading adopted by John Burnet and Siem Slings in their editions, and supported by Plutarch and Proclus), and as ἄν, ἴσα τμήματα in our manuscript F (for these *sigla*, see Slings’s edition, xxii). Yet this again allows for two hypotheses. (2) Ps.-Archytas may simply have adopted a different word division from that of the OCT edition—he could have *constructed* the text as ἄν’ ἴσα τμήματα, as Richter conjectured later, which gives a grammatically possible, if somewhat convoluted, reading (by contrast,

F's variant reading ἄν, ἴσα τμήματα, seems odd without further interventions in the text). This hypothesis, which was first suggested to me by an anonymous reader, is not implausible. Yet, nothing prevents us from thinking that, given his own philosophical agenda, (3) Ps.-Archytas may as well have deliberately *adapted* Plato's text to his own needs. This was suggested by Heinrich Dörrie (in "Formula analogiae," 39–40), who thinks that Ps.-Archytas's reading stems from a neo-Pythagoreanizing revision of the text, conforming it to a Pythagorean, non-transcendent dualism. At any rate, it may be useful to remark that, despite appearance to the contrary, the anonymous *scholion in Remp.* 509d (published by Karl Friedrich Hermann, *Appendix platonica*, 350–51, and discussed in Dörrie and Baltes, *Platonismus* IV, 344–53) cannot be used in support of (1). Even though the *scholion* opposes Iamblichus's and Ps.-Archytas's reading (ἴσα) with that of "some manuscripts of Plato" (ὥς ἐν τισι τῶν ἀντιγράφων τοῦ Πλάτωνος) that made a division into unequal parts, this need not mean more than that the scholiast, who clearly depends on Iamblichus, compared the interpretation that he found there with the received version of the text in the (recent) manuscripts he had access to. The *scholion*, which appears in *Venetianus Marcianus graecus* 184 (see the description by Gerard Boter, *Textual Tradition of Plato's Republic*, 56–57), must date from the fifteenth century (Siem Slings attributed it to John Rhosus: see his *Critical Notes*, 112–13). Dörrie's remark, in "Formula analogiae," 47–48n20, that the scribe of F "evidently . . . has taken account of the Scholiast's recommendation" can therefore not be correct. I thank an anonymous reader for urging me to elaborate on this point and Gerard Boter for generously providing clarification on a number of text-critical issues.

¹³ In the geometrical proportions between the different subsections of the divided line, Proclus seems to find yet another argument for his view that the line represents the unity and continuity of reality, for at *in Remp.*, I.288.29–89.2, he quotes *Tim.* 31c, where proportion is called the "most beautiful of all bonds." In his *Commentary on*

the Timaeus (in *Tim.*, II.13.15–56.11), Proclus makes much of this statement, interpreting the harmony and unification resulting from this bond as the second of ten gifts of the divine Demiurge to the cosmos. For the idea that proportion is an image of divine unification, see esp. II.13.19–21; 16.15–21; 23.5–8; 27.13–16 and 51.4–5.

¹⁴ See, however, Nicholas Denyer, who claims that the choice is arbitrary and that the point of Plato’s silence on this issue is precisely to show that one has to ignore such “irrelevant” questions as a kind of practice in abstract thinking. “The wrangle between Proclus and Plutarch,” he continues, “shows how much we need that practice” (“Sun and Line,” 292–94).

¹⁵ See the frequent use of words composed with ἀνω-/ἀν- and κατω-/κατα- in *Republic* 511a6–7; b8; d1; and d8. For this observation, see Dörrie and Baltes, *Platonismus* IV, 333n6.

¹⁶ See *De comm. math.*, 34.20–35.26, where Iamblichus quotes from and discusses Brotinus’s views. Plutarch’s *Platonic Questions* III 1 actually starts from the question, “What, then, did he have in mind when he divided the sum of things into unequal segments? And which of the segments is larger, the intelligible, or the perceptible?” (τί οὖν διανοηθεῖς εἰς ἄνισα τὰ τμήματα τὸ πᾶν ἔτεμε; καὶ πότερον τῶν τμημάτων, τὸ νοητὸν ἢ τὸ αἰσθητόν, μείζον ἐστίν [1001D4–6, trans. Cherniss]) and the remainder of the *quaestio* consists in a set of arguments for making either the sensible or the intelligible section the larger. Plutarch himself sympathizes with the latter view. For a more elaborate discussion of Plutarch’s interpretation of the divided line, see Franco Ferrari, “Trascendenza e immanenza.”

¹⁷ For a similar view, see, e.g. Benjamin Jowett and Lewis Campbell, *The Republic of Plato*, 3:307–8.

¹⁸ However, I do not think it is legitimate to render μείζον simply as “more important,” as André-Jean Festugière has done in his French translation of the Commentary (“plus importante” [*Commentaire sur la République*, 2:97]). For in such a translation the connection with the line as a geometrical object and with the

discussion that is found in Plutarch and Broctonius is lost (or at the very least no longer obvious). See already the criticism of Michele Abbate, *Commento alla Repubblica*, 404n13.

¹⁹ See Plutarch, *Quaest. Plat.*, III 2, 1002B12–C12, esp. B12–C1: “What is encompassed is in all cases surely less than that which encompasses [τὸ περιεχόμενον ἑλαττόν ἐστι τοῦ περιεχόντος]; and the nature of the sum of things [ἡ τοῦ παντὸς φύσις] encompasses the perceptible with the intelligible [τῷ νοητῷ περιέχει τὸ αἰσθητόν]” (trans. Cherniss).

²⁰ See, e.g. *El. theol.*, § 65, 62.16–17, where Proclus explains what it means to exist κατ’αἰτίαν by pointing out that “every cause pre-contains its effect within itself” (πάν τὸ αἴτιον ἐν ἑαυτῷ τὸ αἰτιατὸν προεἶληφε).

²¹ Think, e.g. of the way in which one could say that the number seven, being larger than the numbers three and four, contains them, or that a large circle contains a smaller, complex polygon circumscribed by it. It would be difficult to see how the larger number, which encompasses the smaller one, could be ontologically superior to the latter.

²² See, e.g. *in Eucl.*, 101.13–25, where the idea seems to be that the point, being the “boundary” or “limit” (πέρας) of the line, is the (formal/generative?) cause of the latter in the sense that (the *logos* of) the point causally pre-contains the line.

²³ For the triad of essence, power, and activity, see, e.g. *in Alc.*, 83.20–84.11, with the literature and passages quoted in Alain-Philippe Segonds’s note *ad locum* (169–70n8). Despite its frequent use in other contexts, I could not find any place where this triad is as such applied to the generation of mathematical objects, but Proclus does, e.g. make a distinction between the essence of figure in general, as expressed in its definition, and the species of figure, which are its δυνάμεις. See *in Eucl.*, 143.21–44.5 (at 144.3, it is probably better, with Morrow, to put the comma after instead of before οἱ παλαιοί). Proclus is replying here to an objection that some have raised against Euclid’s fourteenth definition, in which he defines figure as that which is contained

by either one or more boundaries, the objection being that Euclid seems to define the genus on the basis of its species (rectangular figures and circular ones). Proclus replies that it is a common procedure to define (the essence of) the genus on the basis of its δυνάμεις. This alludes to the (Aristotelian) procedure of defining the οὐσία of soul on the basis of its δυνάμεις, and its faculties on the basis of its ἐνέργειαι, from which the Neoplatonic triad took its inspiration. See the passages from *De anima* referred to in Carlos Steel, *The Changing Self*, 59–60. The distinction between δύναμις and ἐνέργεια (in the sense of potency and actuality) of course plays a role in discussions about infinite divisibility—see, e.g. in *Eucl.*, 158.2–10, where Proclus comments on the possibility of drawing an infinite number of diameters that divide a circle into two equal parts, thus generating a double infinity.

²⁴ See Henri-Dominique Saffrey, “Le Philosophe de Rhodes.” This identification was accepted by Carlos Steel, who in his recent OCT edition proposed to adapt the manuscript reading of *in Parm.*, 1057.6, accordingly.

²⁵ The differences between Plutarch’s and the “philosopher from Rhodes’s” readings, which mainly concern the number and interpretation of the negative hypotheses, need not concern us here.

²⁶ For this general use of the term “henads” for incorporeal being and of “the others” for the corporeal and visible, cf. *in Parm.*, 1048.1–7.

²⁷ Trans. Morrow and Dillon, slightly modified.

²⁸ See, e.g. Proclus, *in Tim.*, I.228.4–7 and 254.19–55.26; and *in Alc.*, 21.12–22.13.

²⁹ This is perhaps less obvious in Plato, for even though he says that the Good is beyond being and knowing, the “non-hypothetical principle” (ἀνυπόθετος ἀρχή) may be reached in dialectics and the liberated prisoner of the Cave will eventually see the sun outside the cave. For Proclus’s attempt to keep the two perspectives together, see, e.g. *in Remp.*, I.295.4–18. It is noteworthy that Proclus, at *El. theol.*, § 123, 108.32–10.3, makes the point that the henads are also unknowable by denying all types of cognition represented by the different sections of the line from them.

³⁰ See in *Parm.*, 839.16–21 and 839.29–40.1, where this is one of a set of (admittedly traditional and not entirely satisfying) comparisons that Proclus employs to clarify the notion of participation.

³¹ See *Republic* 510a1–2, where Plato talks about “reflections in water and in all close-packed, smooth, and shiny materials” (trans. Grube and Reeve). Proclus makes much of the three qualities mentioned (πυκνά τε καὶ λεῖα καὶ φανὰ): see in *Remp.*, I.289.26–90.7.

³² See in *Remp.*, I.290.7–10: ὅτι κατὰ Πλάτωνα αἱ ἐμφάσεις ὑποστάσεις εἰσὶν εἰδῶλων τινῶν δαιμονία μηχανῇ δημιουργούμεναι, καθάπερ αὐτὸς ἐν Σοφιστῇ διδάσκει. The reference is to *Sophist* 266a–c, where it is said that not only natural things such as living beings, fire and water are products of divine craftsmanship, but even εἰδῶλα such as dream appearances and shadows are (see esp. 266b6–7): “the εἰδῶλα of all these things . . . which have come about by divine craftsmanship as well” (τούτων δέ γε ἐκάστων εἰδῶλα . . . δαιμονία καὶ ταῦτα μηχανῇ γεγονότα). Proclus only infers from this passage that, since shadows are products of demonic craftsmanship, they must somehow be “real.” It is difficult to see how he would fit dream appearances into this picture. I fail to see, moreover, why both Festugière and Abbate, in their translations, connect εἰδῶλων τινῶν with ὑποστάσεις: “les réalités de certains corps spectraux” (*Commentaire sur la République*, 98); “fondamenti sostanziali di certi tipi di immagini riflessi” (*Commento alla Repubblica*, 271 and 404–5n18 and n23). It seems more natural to me to understand it as a *genitivus obiecti* with μηχανῇ, which is well attested (see LSJ II.1 s.v. μηχανή). The phrase could then be rendered as “that according to Plato reflections are realities produced by the demonic skill to procure certain εἰδῶλα, as he himself teaches in the *Sophist*.” There is no need to qualify the claim that εἰδῶλα are ὑποστάσεις the way Festugière and Abbate do, since Proclus repeats it again without such qualification at 290.22.

³³ For the example of the hyena, see also the source texts referred to by Kroll from Aelianus (*De nat. an.*, VI 14), Ps.-Aristotle (*Mirab. ausc.*, 145, 845a24–7), and

Plinius (*Nat. hist.*, VIII 106). Cf. also Festugière's remarks, *Commentaire sur la République*, 98–99n3. Proclus also refers to Aristotle's allegation, in *De insomn.*, 2, 459b27–30, that when women in their menstrual period look into a mirror, the latter will become bloodlike. It is interesting to observe, as Abbate, *Commento alla Repubblica*, 409n22 has done, that at *in Alc.*, 219.2–20.1, Olympiodorus uses the same two examples of the hyena and the menstrual blood to establish the substantial reality of shadows.

³⁴ *In Remp.*, I.290.22–24; for Plato's analogies, see *Republic* 534a2–5.

³⁵ See, e.g. Julia Annas, *Introduction*, 248–52.

³⁶ See, e.g. *in Tim.*, I.343.18–44.5, where Proclus talks about the value of εἰκαστά in a discussion of the *Timaeus*'s εἰκὼς λόγος. For a depreciation of the εἰκαστά, compare the judgement of Syrianus: “For as to the dimness and worthlessness and total unknowability of the objects of conjecture (εἰκαστά), even if we learn from nowhere else, certainly we learn with accuracy from the division of the line in the *Republic*” (*in Met.*, 101.3–5, trans. Dillon and O'Meara).

³⁷ A reference to (not an endorsement of) such an interpretation is to be found, e.g. in Annas, *Introduction*, 251. David Ross, *Theory of Ideas*, 59–65 (esp. 59n1), refers this interpretation, which he himself declines, back to Proclus, but as we will see Proclus could hardly have been the first to propound this view.

³⁸ Recently the case has been made for this by Carlos Steel, “Plato as Seen by Aristotle,” 184–85.

³⁹ For this division, see also, e.g. *El. theol.*, § 123, 108.33–10.1.

⁴⁰ Trans. Morrow, modified.

⁴¹ Cf. already Plato, who at the end of his discussion of the divided line calls understanding “intermediate between opinion and intellection” (ὥς μεταξύ τι δόξης τε καὶ νοῦ τὴν διάνοιαν οὖσαν [*Republic* 511d4–5]).

⁴² For Proclus, see, e.g. *in Tim.*, I.8.14–21, and II.23.16–20. In their translations of these passages, André-Jean Festugière, *Commentaire sur le Timée*, 3:49n1, Harold

Tarrant, “Proclus on the Socratic State,” 101n39, and Dirk Baltzly, “Proclus on the World’s Body,” 69n96, all seem to be at a loss to explain Proclus’s attribution of this threefold division of reality to the Pythagoreans, as is Diehl in the *apparatus* to his edition (he twice refers to Iamblichus, *De vit. pyth.*, 15/16, 64–70, where nothing relevant is said). It is true, as Festugière remarks, that Aristotle, in the *Metaphysics*, writes that for the Pythagoreans numbers are the primary realities (*Met.*, A 5, 986a1 and 15–16) and not intermediates (as in the doctrine Aristotle attributes to Plato in *Met.*, A 6, 987b14–18). However, for the later Neoplatonists it was common to think that the divided line made a division between three fundamental levels of reality, and since for at least Iamblichus and Syrianus the line was—as we will see—a Pythagorean analogy, the most obvious explanation is that Proclus is relying on a Neopythagorean interpretation of the divided line.

⁴³ Trans. Dillon and O’Meara.

⁴⁴ See, e.g. Proclus, *in Parm.*, 795.25–96.9. For the doctrine, see also my “Platonic Forms,” 109–10 (and the passages quoted there).

⁴⁵ For this doctrine, see, e.g. Christoph Helmig, *Forms and Concepts*, 263–72. According to Syrianus (*in Met.*, 82.20–29), the doctrine relies on at least three Platonic source-texts. Apart from the divided line, Syrianus also mentions the construction of the World Soul according to mathematical ratios in the *Timaeus* (35a–36b) and the argument from recollection from the *Phaedo* (72e–78b). According to the later Platonists, these two latter dialogues of course largely relied on Pythagorean teachings as well. Cf. *in Met.*, 4.5–20.

⁴⁶ See *in Eucl.*, 17.22–18.4; cf. 16.27–17.6.

⁴⁷ For Proclus’s most elaborate discussion of the status of geometrical objects, see *in Eucl.*, 49.4–56.22. For a discussion, see, e.g. Alain Lenoir, “Le statut ontologique.”

⁴⁸ See esp. *in Eucl.*, 52.20–53.5. At 56.15–18, Proclus clearly says that the discursive forms are projected by discursive reason *onto* the passive intellect (i.e. for Proclus, the imagination).

⁴⁹ The metaphor of imagination as a mirror in which the soul projects its own *logoi* and through which the soul gets to know itself is beautifully developed in *in Eucl.*, 141.2–42.7. For an interpretation of the metaphor, see, e.g. Guy Claessens, “Imagination as a Symptom.”

⁵⁰ See, e.g. *in Eucl.*, 54.18–22.

⁵¹ See esp. *in Eucl.*, 12.2–18.4. For Proclus’s criticism of Aristotelian abstraction or induction, see also Helmig, *Forms and Concepts*, 208–21.

⁵² In this phrase, transmitted by our manuscript as τούτων δὲ πρώτων ἀπάντων ἡμῖν ἀνιοῦσιν θεάν εἰπὼν, Wilhelm Kroll has conjectured φανεισῶν for the nonsensical θεάν εἰπὼν, presumably because of the sequence at *Republic* 516a6–8, where the prisoner, after ascending from the cave, habituates himself to seeing real objects by looking at the following things in succession: “At first, he’d see shadows most easily, then images of men and other things in water, then the things themselves” (καὶ πρῶτον μὲν τὰς σκιὰς ἂν ῥᾶστα καθορῶ, καὶ μετὰ τοῦτο ἐν τοῖς ὕδασι τὰ τε τῶν ἀνθρώπων καὶ τὰ τῶν ἄλλων εἶδωλα, ὕστερον δὲ αὐτά [trans. Grube and Reeve]). In Kroll’s reading, φανεισῶν would refer to an unexpressed σκιῶν. I do not think, however, that this conjecture makes good sense, for then the τούτων of 293.24 must refer to the same σκιῶν, implying that Proclus puts the emphasis on the existence of something intermediate between the shadows outside the cave and the real things found there—i.e. the reflections in water that the liberated prisoner sees—and on the fact that these intermediates are the *logoi* of mathematical objects in the soul. However, besides making one wonder what the shadows outside the cave could then stand for (as distinct from the reflections), it would be much more natural to think that Proclus is making the already familiar point that mathematical objects are intermediate between sensible particulars and Forms and that the shadows and reflections are to be taken together as the reference of the ἄλλο τι at 293.25. Both Festugière (*Commentaire sur la République*, 102n1) and Abbate (*Commento alla Repubblica*, 407n42) have (not without hesitation) followed Kroll in adopting his conjecture, but

since they refer ἄλλο τι to both the shadows and the reflections outside the cave (cf. Festugière, *Commentaire sur la République*, 102n2: “les σκιαί [vues cette fois dans le monde extérieur] et les reflets dans l’eau, cf. 516 A 5–7”), I take it that they refer φανεισῶν to the shadows that the prisoners see in the cave (which could have been mentioned in the lacuna at 293.22). Yet this seems even more problematic, since these shadows are not seen during the ascent, but actually before the prisoners are liberated in the first place, and it would be pointless to say that Plato introduces the shadows and reflections outside the cave as intermediate between the shadows in the cave and the real things outside of the cave without mentioning the more substantial items in the cave (the statues and other artefacts whose shadows are cast on the cave wall). I take it that τούτων at 293.24 actually refers to the artefacts carried behind the wall, which represent sensible particulars and are the first things seen during the ascent (see 293.13 and 293.18, where Proclus calls the progression towards seeing the artefacts already an ἄνοδος). In that case, we would expect a neuter participle at 293.23. My conjecture is to read something like θεαθέντων here, with ἡμῖν ἀνιοῦσιν as the agent of the passive perfect participle. The passive perfect of θεάομαι is frequent only in late ancient and Byzantine Christian authors, but while it is not attested in Proclus, it is used as early as Philo (*De gigant.*, 15.6) and occurs also in Simplicius, *in Cat.*, 186.27, and Philoponus, *in De an.*, 455.28.

⁵³ Cf. *Republic* 516a8: ὅσπερ δὲ αὐτά.

⁵⁴ One could perhaps expect ταύταις rather than τούτοις for the reference to the sciences (ἐπιστήμαι), but the neuter may be a slight confusion resulting from the occurrence of τούτων in the preceding line in reference to the sciences. At any rate, this remark about the soul training and purifying itself in the sciences is of course an allusion to the function of the mathematical sciences in the educational program of *Republic* VII.

⁵⁵ For this term, see, e.g. *in Eucl.*, 55.3.

⁵⁶ This is how I understand Abbate's note (*Commento alla Repubblica*, 407n45), explaining the εἰκόνας at *in Remp.*, I.293.28: "gli oggetti del pensiero discorsivo sono immagini e copie degli enti intelligibili, come si evince sulla base del sezionamento della linea." Proclus calls the soul's inner *logoi* "images" (εἰκόνας) of the Forms at, e.g. *in Eucl.*, 140.16, and this original/copy relation is indeed implied by the intermediate status of the dianoetic forms within the tripartite structure of reality.

⁵⁷ I am not aware of any passage where Proclus calls the projections of the *logoi* "images," but the idea is at least suggested by the mirror metaphor of *in Eucl.*, 141.2–42.7. Note that Proclus in that context does repeatedly call the projections of the soul's *logoi* εἰδωλα. See, e.g. *in Eucl.*, 141.6, 9, and 18.

⁵⁸ I thank Guy Claessens for first suggesting this interpretation to me in a private discussion of the text.

⁵⁹ One difficulty is that Proclus, at *in Remp.*, I.293.27, would say that geometry and arithmetic "examine" (σκοπεῖται) the discursive objects, the principles of which are possessed by the soul, and hardly one line later that these sciences are actually "contemplating" (θεωρούμενα) sensible particulars. Furthermore, it seems inelegant to use θεωρούμενα, which usually has an intellectual connotation, for the mere observation of sensible particulars, all the more so since Proclus, in *in Eucl.*, 54.24–26, says that the mathematician "sees" (ὁρᾷν) one circle (i.e. in the imagination), while he actually "contemplates" (θεωρεῖν) another, namely, the circle in discursive reason (i.e. the *logos* of the circle). Finally, upon this construction of the argument, I fail to see the point of insisting on the indivisible and clenched character of the soul's *logoi*, as Proclus does at 294.2–3, since that is not required for the opposition between mathematical objects and their sensible representations (and a more common understanding of mathematical objects certainly takes them to be divisible and articulated).

⁶⁰ The only model/image relations that Plato explicitly mentions in the divided line are the following: shadows and reflections in water are "images" (εἰκόνες) of sensible

particulars (*Republic* 510a5; cf. 510e3); the “object of (proper) cognition” (τὸ γνωστόν) and “the object of opinion” (τὸ δοξαστόν) are related as “the likeness to what it is a likeness of” (τὸ ὁμοιωθὲν πρὸς τὸ ᾧ ὁμοιώθη); and finally, διάνοια employs sensible particulars as images of the things that it is concerned with (*Republic* 510b4; cf. 510b8; 510d7; and 511a6), such as the tetragon itself or the diameter itself (*Republic* 510d7–8). Since it seems most natural to take these last expressions as references to the Forms of tetragon and diameter, sensible particulars seem to be regarded as images of the Forms themselves, and not of mathematical intermediaries. Yet someone who wants to find room for a doctrine of mathematical intermediaries could argue that they are precisely what such expressions denote, which would make sensible particulars images of the intermediaries. And since the relation between the objects of πίστις and εἰκασία is implied to be analogous to that between the objects of νοῦς and διάνοια (as results from the analogies provided at *Republic* 534a2–5), one could then argue that mathematical intermediaries are also images of Forms. This would make them both images of Forms and models of sensible particulars. An additional difficulty for this last interpretation, however, is that it can hardly account for the equality of the second and third subsections of the line, which, as many readers have noticed, is a necessary consequence of the ratios between the sections of the line. It may be interesting to note that I am not aware of any passage where Proclus alludes to the equality of the two middle segments of the line. Because of his strong ontological commitments to mathematical intermediaries, I think Proclus should be more worried about this consequence than Plato.

⁶¹ See in *Remp.*, I.291.21–27; cf., e.g. *El. theol.*, § 195; in *Tim.*, I.349.26–28; and in *Eucl.*, 62.22–26.

⁶² According to Carlos Steel, the doctrine results from Iamblichus’s particular views on the intermediate nature of the soul as both an image of intellect and a model of sensible particulars. See Steel, “Modèle et image,” 19–21. For Syrianus’s endorsement of this view, with particular reference to the divided line, see in *Met.*,

82.22–25. A very clear statement to this effect can also be found in *Anon. Proleg.*, § 20.13–18, and 20–21.

⁶³ See Dominic O’Meara, *Pythagoras Revived*, 157.

⁶⁴ O’Meara, *Pythagoras Revived*, 165.

⁶⁵ See above, n. 39.

⁶⁶ See *in Tim.*, I.1.1–16; 7.18–21; cf. Iamblichus, *in Nicom.*, 105.10–17; and *Anon. Proleg.*, § 5.32–39.

⁶⁷ To my knowledge, Proclus mentions Archytas only once, in a historical survey of the development of geometry in *in Eucl.*, 66.15, where he is mentioned as a contemporary of Plato’s who expanded the body of geometrical theorems and had a more scientific approach to them than his predecessors. Brotinus is never mentioned.

⁶⁸ See *Theol. Plat.*, I 4, with the parallel text in *in Parm.*, 646.2–47.18. For a discussion of these four “modes” of theological discourse, see, e.g. Jean Pépin, “Modes de l’enseignement”; and Stephen Gersh, “Theological Methods.”

⁶⁹ See *Theol. Plat.*, I 4, 19.6–17. The geometric atomism example is probably taken to exemplify both physical and mathematical imagery of the divine.

⁷⁰ *Theol. Plat.*, I 4, 20.8–12; cf. *in Eucl.*, 22.1–16; *in Parm.*, 646.22–47.4, where especially Philolaus is given credit as the founder of this kind of theology.

⁷¹ See, e.g. *in Eucl.*, 16.16–17.14, and *in Parm.*, 924.2–7. Cf. Syrianus, *in Met.*, 4.5–20 and 82.20–29.

⁷² See, e.g. *in Tim.*, II.243.17–28.

⁷³ Trans. Baltzly, slightly modified.

⁷⁴ The research for this paper was conducted as part of a project on epistemic authority in the later Neoplatonic commentators on Plato, funded by the KU Leuven Research Fund. The present paper includes material that I presented at a conference on epistemology in post-Hellenistic philosophy in Gargnano (June 2015), and I thank the audience (especially Jan Opsomer, Franco Trabattini, and Christoph Helmig) for useful comments and suggestions. For the final redaction of my paper, I was able to

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