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HIGHLIGHTS

- The mechanical behaviour under combined compression-shear loading of polymer foams is poorly understood due to the difficulty of performing such tests. We have designed, developed and used a novel test rig to perform such tests under quasi-static and high strain rate dynamic conditions for different densities of EPS foam.
- The compressive and shear stresses at yield of the EPS foam are density and strain-rate dependent; increasing the impact loading angle tends to decrease the compressive stress and increase the shear stress at yield of the EPS foam.

Deformation response of EPS foam under combined compression-shear loading. Part II: High strain rate dynamic tests

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ABSTRACT

In Part I, we performed quasi-static compression-shear tests on expanded polystyrene (EPS) foam. In the present study, dynamic combined compression-shear loading tests were performed on different densities of EPS foam by integrating the previously developed loading rig with a lab-built drop weight tower. Two impact loading wedge angles (45° and 60°) and three different drop heights (50, 130 and 175 cm) were tested in the experiments. It was found that both the compressive and shear stresses at yield of the EPS foam are density and strain-rate dependent. In addition, increasing the impact loading angle tends to decrease the compressive stress and increase the shear stress at yield of the EPS foam.

Keywords: EPS foam, combined compression-shear loading, high strain rate, drop tower dynamic tests

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1. Introduction

Understanding the dynamic mechanical behaviour of EPS foam under combined compression-shear loading is important for many applications that involve such combined loadings, including structural core material [1, 2], geofoam products [3, 4, 5], personal protective equipment and helmet liners [6-10]. Related experiments and data, however, are rarely found in the open literature due to the difficulties of performing such experiments.

Among the few studies available, several experimental techniques have been developed to study the deformation of materials and structures under dynamic combined compression-shear loading based on the Split Hopkinson Pressure Bar (SHPB) [11-13]: Hou et al. [11] developed a combined compression-shear set-up using the classical SHPB, but instead of using bars with flat ends, they used bevel-ended input and output bars to produce mixed compression-shear loading. Zhao et al. [13] also developed a test rig to create mixed compression-shear loading by modifying the SHPB into what they call the Split Hopkinson Pressure Shear Bar 'SHPSB' test, in which a 90° wedge-shaped bar was used instead of a flat surface incident bar, and two transmission bars were orientated vertically to the incident bar to create a dynamic combined compression-shear loading condition. Chung et al. [14] studied the compressive behaviour of polycarbonate honeycombs under in-plane biaxial dynamic loading using a drop weight testing system and found that the magnitude of the collapse load and stress-strain response varied as a function of load biaxiality.

In this study, we present experimental results of dynamic combined compression-shear tests for four different densities of EPS foam based on the test rig previously developed in our companion Part I paper [15]. The experimental setup is described in detail in Sec. 2. The principles of force and strain measurement are described in Sec. 3. Experimental results are presented in Sec. 4 and Conclusions are summarised in Sec. 5.

2. Experimental setup

Dynamic combined compression-shear loading on EPS foam was performed by integrating a lab-built drop weight tower testing system (shown in Fig. 1) with the test rig described in Part I [15]. The drop weight tower testing system is purpose built apparatus for which the drop height ranges from 0.2 to 2 m and the falling mass ranges from 2 to 5 kg.

Fig. 1. The homemade drop weight tower testing system

In addition to the existing 45° impact wedge loading test rig (Fig. 2 (a)), two add-on 15° wedges were mounted onto the two 45° wedges separately to create a 60° impact loading test rig, as shown in Fig. 2 (b).

(a)

(b)

Fig. 2. The two fixtures designed for dynamic combined compression-shear loading on EPS foam with (a) 45° impact wedge loading (b) 60° impact wedge loading

The details of the fixture were presented in Part I when we discussed the quasi-static testing of EPS foam on an Instron universal testing machine. Here, for the dynamic combined compression-shear loading tests, the fixture was bolted to the base of the drop weight tower (see Fig. 3).

Fig. 3. The setup of the test rig integrated with the drop-weight tower

Four wedge blocks were also used at the front and back sides of the setup; they were wedged against the side panel plate of the drop tower to increase the stability and robustness of the testing system during the impact tests. During the tests, the top impact wedge was initially in contact with the top surface of the foam sample; the impact wedge was fixed with a vertical frictionless guide on the left side, which allowed for movement in only the vertical direction; a drop mass of 3.83 kg was lifted to three different heights (50, 130 and 175 cm), from where the mass was dropped onto the stationary loading wedge, transferring the

impact energy; the wedge would then impart combined compression and shear loading on the foam sample beneath.

High-speed camera

Light

(a)

(b)

Fig. 4. (a) Setup of the high-speed camera (note the camera was rotated to either 45° or 60° to the horizontal during the tests); (b) Determination of the initial impact velocity from the snapshots taken by high speed camera

A high-speed camera (Photron Mini AX 100, <https://photron.com/mini-ax/>) was used during the tests, the camera was rotated through 45° or 60° to the horizontal during the tests, corresponding to the wedge impact loading angle of a specific test. This was performed to ensure that the bottom surface of the foam sample would appear horizontal in the pictures and provide a rectangular field of view, which would aid in the subsequent strain calculation using Digital Image Correlation (VIC-2D software). A wedge-shape of mm-scale grid paper (Fig. 4 (b) shows the smallest grid dimensions $2 \times 2 \text{ mm}^2$) was attached on the top wedge to calculate the initial impact velocities for the different drop heights.¹

3. Measurement principle and analytical model

3.1. Force measurement

The measurement of the force during the dynamic tests shares the same principle with the quasi-static tests as described previously in detail in Part I [15]. Fig. 5 illustrates how the compression and shear forces are calculated using Kistler force sensors (Type 9041A):

¹ By measuring the time taken for the top wedge to travel one step in the small grid (i.e., 2 mm) at the very beginning of the impact, we can calculate the initial impact velocities (dimension of the small grid divided by the travelling time taken). This assumes the impact velocity is constant, which is reasonable given that the time in which it happens is less than 1 ms.

Fig. 5. The scheme of the force measurement (The test rig includes a frictionless hinge joint on the bottom platen, whose rotation was restrained by the horizontal force sensor attached on the right side of the bottom wedge; the vertical force sensor was set in the bottom wedge, from which the corresponding vertical force was recorded during the impact test)

Depending on the test wedge angle, the shear and compressive forces acting on the foam sample are obtained directly via:

$$F_S = F_V \sin \theta - F_H \cos \theta , \quad (1)$$

$$F_C = F_H \sin \theta + F_V \cos \theta , \quad (2)$$

where F_S and F_C are the shear force and the compressive force on the foam material, F_V and F_H are the vertical and horizontal forces recorded by the Kistler force sensors and θ is the angle between the vertical direction and the normal direction of the contact surface.

3.2. Strain measurement

The surface strain fields of the EPS foam under dynamic loading were obtained using the Digital Image Correlation (VIC-2D software [16]), which is the same setup used for the quasi-static tests described in Part I [15]. The

foam surfaces were marked with a grid pattern using a CNC machine, which proved to be successful for the VIC-2D software to track and calculate the strain from the pictures obtained by the camera. However, being different from the quasi-static tests, a high-speed camera was used to capture the pictures of the deformation of EPS foam during the dynamic tests. Fig. 6 shows two pictures of a 64 kg/m^3 EPS foam taken by the high-speed camera at the beginning and the end of the impact from a 50 cm drop height, respectively. The camera captures the pictures of the EPS foam during the impact at a sampling rate (4000 fps) pre-set before the test. These pictures are subsequently analysed by the VIC-2D image correlation software, from which the strain of the foam can be obtained with the change of time.

Fig. 6. The pictures of a 64 kg/m^3 EPS foam taken by the high-speed camera (at a pre-set sampling rate 4000 fps) at (a) the beginning and (b) the end of the impact from the 50 cm drop height

4. Testing methods and results

4.1. Material and specimens

In this study, four different densities of EPS foam (43, 64, 80 and 120 kg/m^3 respectively) were used. The foam material was cut into blocks with dimensions of $50 \times 50 \times 25 \text{ mm}^3$ and loaded in the direction of the 25 mm

dimension. A black marking pen² was mounted in a CNC machine, to mark the grids (with dimensions $2.5 \times 2.5 \text{ mm}^2$) on the side surface of the foam samples.

4.2. Experimental design

The tests were performed at several loading conditions with two different loading wedge angles, 45° and 60° , and three drop heights, 50, 130 and 175 cm, where the dropped mass was 3.83 kg for all the tests. A high-speed camera was rotated to either 45° or 60° with the horizontal line, ensuring that the bottom surface of the foam was horizontal in the recorded pictures, for ease of subsequent strain processing with the VIC-2D software. Black tape was attached to a few areas of the test rig, as shown in Fig. 7, to reduce the level of light reflection from those areas: this helped the high-speed camera to provide pictures of the EPS foam with best resolution during tests.

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²Due to the black colour of the 120 kg/m^3 foam, a white marking pen instead was used for the grid marking of 120 kg/m^3 foam.

Fig. 7. The test setup of the impact test of EPS foam at 60° impact loading angle

4.2.1. Determination of the camera sampling rate and initial impact velocities

The high-speed camera provides several different choices of sampling rates, from 4000 fps to 540000 fps; increasing the sampling rate reduces the picture resolution. For a typical impact loading in our tests, the duration of the impact is approximately 10-20 ms, depending on the drop height. Considering the balance of the picture resolution and the amount of the pictures that can be recorded by the high-speed camera during the impact for strain analysis, 4000 fps was chosen as the sampling rate, which means the camera takes a picture of the deformed foam every 1/4000 second. This sampling rate provides the best combination of picture resolution

(1024×1024 pixels) and number of images (40-80) for the VIC-2D strain analysis software.

Once the sampling rate of the camera was set, the initial impact velocities from the three different drop heights can be determined, based on the method described in Sec. 2. For the three different drop heights (50, 130 and 175 cm), at the very beginning of the impact, the time taken to travel through a grid step on the impact wedge with a distance of 2 mm were 1, 0.6 and 0.5 ms respectively, from which we calculated that the initial impact velocities for these three drop heights would be 2, 3.3 and 4 m/s respectively. The corresponding average compressive and shear strain rates can thus be calculated based on the initial impact velocities and the dimensions of the foam sample³. Table 1 shows the list of the average compressive and shear strain rates for the two impact wedge loading angles from the three drop heights.

Table 1. The list of the average compressive and shear strain rate at 45° and 60° impact wedge loading angle

Impact wedge loading angle	Drop heights (cm)	Initial impact speed (m/s)	compressive strain rate (s^{-1})	shear strain rate (s^{-1})
45°	50	2	57	28
	130	3.3	93	47
	175	4	113	57
60°	50	2	40	35
	130	3.3	66	57

³ Note the strain-rates calculated here are based on the initial impact speed and initial foam height; however, both the velocity and height vary during the impact, resulting in a non-constant strain-rate.

	175	4	80	69

4.3. Experimental repeatability

Three repeat tests for the same density foam material were performed to check the repeatability of the experiments. As shown in Fig. 8, the agreement of the three curves, the vertical and horizontal forces, which were measured from the 43 kg/m^3 EPS foam at 45° impact loading angle from 50 cm drop height, was very good. This shows the test rig to be robust and the test results to be highly repeatable. Thus, for each subsequent loading case, only two repeat experiments were performed and the average was taken for the corresponding forces.

Fig. 8. The force curves of the three tests of 43 kg/m^3 EPS foam at 45° impact loading angle from 50 cm drop height

4.4. The experimental results

The experimental results for the 45° and 60° impact loading are presented and discussed in this section. For both impact loading angles, three impact heights and four densities of EPS foam were tested. The compressive

and shear forces were calculated from the vertical and horizontal force recorded by the force sensors according to Equations (1) and (2). The force curves presented in this paper were all filtered with a low-pass frequency of 300 Hz. The strain information was obtained through the VIC-2D software. The force vs. time results are shown below in Figures 9 and 10.

Fig. 9. The compressive and shear force curves for different densities of EPS foam (120, 80, 64, 43 kg/m³) at 45° loading angle at different impact heights (a) Compressive force, 50 cm drop height; (b) Shear force, 50 cm drop height;

(c) Compressive force, 130 cm drop height; (d) Shear force, 130 cm drop height;
 (e) Compressive force, 175 cm drop height; (f) Shear force, 175 cm drop height.

Fig. 10. The force curves for different densities EPS foam (120, 80, 64, 43 kg/m³) at 60° impact loading angle from different impact heights

- (a) Compressive force, 50 cm drop height; (b) Shear force, 50 cm drop height;
(c) Compressive force, 130 cm drop height; (d) Shear force, 130 cm drop height;

(e) Compressive force, 175 cm drop height; (f) Shear force, 175 cm drop height;

4.4.1. Compressive and shear force of EPS foam at 45° and 60° impact wedge loading

A strong density dependency of both the peak compressive and shear force of EPS foam is shown in both of these figures. For the 50 cm drop height (Fig. 9 (a)(b) and Fig. 10 (a)(b)), both the peak compressive and shear force decrease with decreasing foam density. For the 130 and 175 cm drop heights, however, the situation is different: as the density of the foam decreases from 120 kg/m³ to 64 kg/m³, the peak compression force decreases similarly, while the peak compression force of the 43 kg/m³ foam exceeds that of the 64 and 80 kg/m³ foam, both for the 130 cm drop height at 45° loading, and for the 175 cm drop height at 60° loading. Obviously, the impact energy increases with the elevation of the drop heights; a typical compressive stress-strain curve of foam material usually consists of a very short elastic regime, followed by a relatively long and flat stress plateau phase, due to the bending and buckling of the cellular structure, and ends up with the densification phase of the foam, during which the stress increases dramatically, even at very small changes of strain. If the foam is unable to dissipate this increased impact energy in the stress plateau region, the stress within the foam will increase dramatically as it enters into the densification phase, which will result in a higher reaction force and increased resistance against the shear deformation. This phenomenon is widely known as the ‘bottom out’ effect of the foam material in engineering applications. As shown for the 43 kg/m³ foam in our tests, a dramatic increase of the compression force can be seen followed by a relatively flat force plateau phase after 15 ms from both Fig. 9 (c) and Fig. 10 (c) due to this ‘bottom out’ effect. This effect can obviously be noted from both the compressive and

shear force curves, especially for the loading conditions of the low density foam from higher drop heights.

The shapes of the curves for the same density of foam also show differences at different drop heights: for the 120 kg/m^3 foam, the curve shapes for both compression and shear force show a ‘bell-like’ form at the 45° impact loading angle. However, for the other foam densities, the situation is different: at the 50 cm drop height, the curves have a relatively long and flat force plateau, as evident in both Figs 9 (a)(b) and Figs 10 (a)(b). For the impact loading at the 130 and 175 cm drop heights, the ‘force plateau’ section becomes more inclined and shorter compared with the loading case under the 50 cm drop height, as can be seen in Figs 9 and 10. Recall the typical stress-strain curve of EPS foam mentioned above: this explains the long and flat force plateau shown in both Figs 9 (a)(b) and Figs 10 (a)(b). Studies have also shown that the strain-rate effect tends to incline the stress-strain curve at the stress plateau phase of the foam material under dynamic loading cases [17, 18]. In our study, the strain-rate effect is not that obvious for the 50 cm drop height, due to the relatively low strain rate. Thus, a long and relatively flat force plateau was found for the low density foams. With an increase of drop height, the strain-rate effect becomes more obvious. Thus, the force plateau section becomes inclined due to this effect, as shown from Figs 9 (c)(d) and Figs 10 (c)(d) for the 130 cm drop height and Fig. 9 (e)(f) and Fig. 10 (e)(f) for the 175 cm drop height. It is also notable that the inclination rate of this force plateau section also increases, as seen in both Figs 9 and 10 (c)(d) to (e)(f), due to the increase of the strain-rate, as a result of the increasing drop height.

Similar to the 45° impact wedge loading, for the 120 kg/m^3 foam at 60° impact loading angle, it shows a ‘bell-like’ curve for both compressive and shear forces at the 50 cm drop height. However, the force curves behave

differently at both the 130 and 175 cm drop heights. As shown from the dotted ellipse marked in Figs 10 (c)-(f), the force curve reaches its peak first, then gradually decreases before steeply dropping back to zero. This phenomenon can be explained by the fact that, during the test, local cracks were observed within the foam samples due to localised shear deformation of the foam (as shown in Fig. 11). This cracking phenomenon was only observed for both the 120 kg/m^3 density foam at the 60° impact wedge loading from 130 and 175 cm drop heights, but no such crack phenomenon was observed in the other loading conditions. This indicates that the 120 kg/m^3 high density foam tends to behave in a more brittle manner compared with the other low density foams, especially for the loading of the 60° impact wedge when the extent of the foams' shear deformation under such a steep loading angle is larger compared with the 45° impact wedge loading.

Fig. 11. The deformation of 120 kg/m^3 foam at the beginning, middle and end of the impact at 60° impact wedge from 130 cm drop height

It is also interesting to note that the crack within the 120 kg/m^3 foam, as shown in the middle picture of Fig. 11, is in a localized diagonal direction through the foam sample. The foam fails (cracks) in a brittle manner when the principal stress exceeds the tensile strength of the foam, as demonstrated both from experiments and simulations. The diagonal direction shape of the crack of the 120 kg/m^3 foam sample in our test suggests that a localized high stress area formed along the diagonal line of the foam sample.

Tables 2 and 3 summarise the test results at the 45° and 60° impact wedge loading for different drop heights. The peak forces are obtained

directly from the curves in Figs 9 and 10, while the average compressive and shear stresses at yield for the different loading cases can be obtained by the method described in Appendix A. The quasi-static compressive and shear stresses to cause yielding of the foam at the 45° wedge loading, determined in Part I of this work [15], are also included in Table 2 for reference purposes. Figs 12 and 13 show the variation of compressive and shear stresses at yield for the 45° and 60° loading with the change of the foam densities and drop heights: it is evident that both the compressive and shear stresses at yield are dependent on both density and strain-rate. The compressive and shear stresses at yield of the EPS foam are lowest at quasi-static compression and increase with increasing foam density and strain rates. However, one exception is the shear stress at yield for the 43 kg/m³ foam at 45° loading, where the shear stress at yield from quasi-static is between that of the dynamic tests from the 130 cm and 175 cm drop heights. Nevertheless, the values of the shear stress at yield from these three loading conditions are very close and this might be due to test deviations.

Table 2. Summary of the test results of the EPS foam at 45° impact wedge loading from different drop heights⁴

Foam density (kg/m ³)	43			64			80			120		
Drop height (cm)	50	130	175	50	130	175	50	130	175	50	130	175
compressive strain rate (s ⁻¹)	57	93	113	57	93	113	57	93	113	57	93	113
shear strain rate (s ⁻¹)	28	47	57	28	47	57	28	47	57	28	47	57
Peak compression force (N)	1353	2706	3893	1761	2368	3018	2310	2606	3123	3252	3908	4017
Peak shear force (N)	270	673	1100	390	656	863	500	666	889	646	818	813

⁴ The forces listed in this table and Table 3 were averaged from two repeat tests for a specific loading condition.

Compression force at yield (N)*		1132	1300	1573	1609	1843	2267	2179	2300	2387	/	/	/
Shear force at yield (N)*		200	220	268	363	410	440	473	490	509	/	/	/
Compressive stress at yield (MPa)*	Dynamic tests	0.453	0.52	0.63	0.64	0.74	0.91	0.87	0.92	0.955	/	/	/
	Quasi-static tests**	0.22			0.32			0.47			0.92		
Shear stress at yield (MPa)*	Dynamic tests	0.08	0.09	0.11	0.15	0.16	0.18	0.19	0.2	0.204	/	/	/
	Quasi-static tests**	0.10			0.13			0.16			0.27		
Von Mises stress at yield (MPa)***		0.47	0.54	0.66	0.69	0.79	0.96	0.93	0.98	1.02	/		

*The method to calculate the forces and stresses at yield can be found in Appendix A. (Note: the value of the stresses at yield for the 120 kg/m³ foam at different drop heights are not included in the table as the foam did not yield under such loading conditions)

** The data is taken from Part I of this work.

***The method to calculate the von Mises stress can be found in Appendix B. (Note: the von Mises yielding stresses here are for dynamic tests only)

Fig. 12. (a) Compressive and (b) shear stresses at yield with the change of (b) foam densities and drop heights at 45° impact wedge loading (Note: The value of the stresses at yield for the 120 kg/m³ foam from different drop heights are not included in this figure as the foam did not yield under such loading conditions)

Table 3 Summary of the test results of the EPS foam at 60° impact wedge loading from different drop heights

Foam density (kg/m ³)	43			64			80			120		
Drop height (cm)	50	130	175	50	130	175	50	130	175	50	130	175
compressive strain rate (s ⁻¹)	40	66	80	40	66	80	40	66	80	40	66	80
shear strain rate (s ⁻¹)	35	57	69	35	57	69	35	57	69	35	57	69
Peak compression force (N)	1203	1830	2383	1684	1974	2156	1994	2114	2338	3211	2734	3295
Peak shear force (N)	399	626	883	680	828	931	729	908	1005	1170	1080	1039
Compression force at yield (N)*	1203	1234	1275	1674	1772	1877	1857	2088	2222	/	/	/

Shear force at yield (N)*	399	433	468	678	763	821	708	819	933	/	/	/
Compressive stress at yield (MPa)*	0.48	0.49	0.51	0.67	0.71	0.75	0.74	0.84	0.89	/	/	/
Shear stress at yield (MPa)*	0.16	0.17	0.19	0.27	0.31	0.33	0.28	0.33	0.37	/	/	/
Von Mises stress at yield (MPa)**	0.55	0.57	0.61	0.82	0.89	0.94	0.88	1.02	1.10	/		

* The method to calculate the forces and stresses at yield can be found in Appendix A. (Note: the value of the stresses at yield for the 120 kg/m³ foam at different drop heights are not included in the table as the foam did not yield under such loading conditions)

** The method to calculate the von Mises yielding stress can be found in Appendix B.

Fig. 13. (a) Compressive and (b) shear stresses at yield with the change of the foam densities and drop heights at 60° impact wedge loading

Fig. 14. Comparison of the (a) compressive, (b) shear and (c) von Mises stresses at yield of EPS foam between 45° and 60° impact wedge loading

Figures 14 (a) and (b) compare the average compressive and shear stresses at yield for the 45° and 60° loading, respectively. It is clear from Figs 14 (a) and (b) that the 60° loading causes a smaller compressive stress at yield and a higher shear stress at yield compared with the 45° loading at the same drop height.

Figure 14 (c) plots the comparison of the average von Mises yielding stresses for the 45° and 60° loading: it shows that the von Mises yielding stress of EPS foam under dynamic combined compression and shear loading is dependent on both foam density and strain rate, with higher foam densities and higher strain rates leading to a higher average von Mises yielding stress.

4.4.2. Strains of different density EPS foam from different drop heights

The average compression and shear strains for different densities of EPS foam at different drop heights were obtained via the VIC-2D software by processing the pictures taken from the high-speed camera. The method to calculate the average strains is the same as described in detail in Part I.

It is evident from Figs 15 and 16 that both the compressive and shear strain values converge when the inspection area becomes progressively smaller and more focussed on the central region of the foam samples, i.e., away from high gradient edge effects. Thus in this work, it was inspection area 4 that was used for all the foam samples under both the 45° and 60° degree loading angles.

Fig. 15. (a) The inspection areas of a 64 kg/m^3 foam under 130 cm drop height at 45 degree loading angle; (b) The compressive and shear strain of the four inspection areas, Areas 1 to 4 (subscripts 1 to 4 for the strain contours).

Fig. 16. (a) The inspection areas of a 64 kg/m^3 foam under 130 cm drop height at 60 degree loading angle; (b) The compressive and shear strain of the four inspection areas, Areas 1 to 4 (subscripts 1 to 4 for the strain contours).

4.4.2.1. The compressive strains

Fig. 17. The comparison of the compression strains
(a) 130 and 50 cm drop height at 45° impact wedge loading

- (b) 130 and 50 cm drop height at 60° impact wedge loading
- (c) 45° and 60° impact wedge loading at 130 cm drop height
- (d) 45° and 60° impact wedge loading at 50 cm drop height

Fig. 17 compares the compression strains at various loading conditions for the different foam densities. Typically, the strain curves go from zero to their peak values (negative values indicating compressive strains), after which there is an increase of the strain value; some of the curves show a relatively constant strain value at the end of the impact time period. This is consistent with the typical impact scenario of the foam tests in this study: the foam was compressed by the wedge during the loading phase, where the strain goes from zero to its peak. During the unloading phase, when the wedge bounced up after the impact, the elastic strain of the foam was recovered, resulting in an increase in the strain; the relatively constant strain values at the end of the impact period were due to permanent plastic strain. Fig. 18 shows four snapshots of the 64 kg/m³ foam at 60° loading from the 130 cm drop height taken by the high-speed camera during the impact. Note that Fig. 18 also clearly shows the non-uniform deformation of the foam sample. This is mainly due to the boundary conditions of the foam sample during the test: the bottom surface of the sample was attached directly to the bottom wedge with strong double-sided adhesive tape while the top surface was constrained by friction at the top wedge, which ensured nearly zero ε_{xx} strain. The left and right surfaces of the foam sample were unconstrained, resulting in zero σ_{xx} and σ_{xy} stresses at these free surfaces. These edge effects are assumed to be small, and thus have been neglected in the analysis.

Fig. 18. The snapshots of the 64 kg/m^3 foam at the 60° impact wedge loading from the 130 cm drop height taken by the high-speed camera: (a) the very beginning of the impact; (b) the middle of the impact; (c) the peak strain moment of the foam when the foam compressed and sheared to its most; (d) the recovery of the elastic strain of the foam after the unloading.

Fig. 17 clearly shows that, under the same loading conditions (impact loading angle, drop height), the peak strain of the foam is density dependent, with higher density foams reaching smaller magnitudes of compressive strain. From Figs 17 (a) and (b), it is obvious to note that for both 45° and 60° , the magnitude of peak compressive strain increases with increasing drop height. From Figs 17 (c) and (d), it shows that the peak compressive strain is smaller at 45° loading compared with 60° , under the same drop heights⁵. This indicates that the foam samples were compressed to a greater extent at 45° loading compared with 60° .

⁵ The only exception is in Fig. 17 (c), where the peak compression strain of the 120 kg/m^3 foam at the 60° impact wedge loading from the 130 cm drop height is higher than for the 45° impact wedge loading. This is due to the fact that, during the test, the 120 kg/m^3 foam cracked along the diagonal direction of the foam sample under the 60° impact wedge loading from the 130 cm drop height, resulting higher peak compression strain.

4.4.2.2. The shear strains

Fig. 19. The comparison of the shear strains

(a) 130 and 50 cm drop height at 45° impact wedge loading

- (b) 130 and 50 cm drop height at 60° impact wedge loading
- (c) 45° and 60° impact wedge loading at 130 cm drop height
- (d) 45° and 60° impact wedge loading at 50 cm drop height

Fig. 19 shows the shear strain of EPS foam under different loading conditions. It is shown clearly in Figs 19 (a) and (b) that, under the same impact loading angle, the shear strains increase with increasing drop height, which indicates that the higher drop height causes more shear deformation of EPS foam and results in higher shear strain. Figs 19 (c) and (d) compare the influence of impact loading angle on the shear strain from the 50 and 130 cm drop heights, respectively. For most foams, the peak shear strains from the 45° impact wedge loading are higher than those at the 60° impact wedge loading, although for a few foam densities, especially for the 43 kg/m³ foam, as seen in Figs 19 (c) and (d), the peak shear strains are similar under these two loading angles. One factor that could explain this is as follows: the average shear strain of the foam is dependent on the total shear deformation of the EPS foam material under loading; for the same drop heights impact, as shown in Figs (c) and (d), the total impact energy that needs to be dissipated by the foam is the same. The foam dissipates the energy by deformation in compression and shear of the foam material, however, for the case of low density foam under high impact height loading, the energy cannot be totally dissipated during the stress plateau phase. Hence, the foam will enter its densification phase during which the shear resistance of the foam becomes very high: it becomes difficult for the shear stress to cause any further shear deformation, thus the peak shear strains are getting closer for such loading conditions, as shown by the 64 and 43 kg/m³ foam from Fig. 19 (c), and the extra impact energy is dissipated by further compressive deformation. Fig. 20 shows the impact snapshots of the 43 kg/m³ foam from the 130 cm drop height at 45° and 60° impact wedge loading. It shows clearly that the impact wedge goes

further at 45° loading compared with 60° loading, as shown from (b) and (d), however, the shear deformation is similar from these two loadings.

Fig. 20. The beginning and peak strain moment of the impact of 43 kg/m^3 foam from 130 cm drop height: (a)(b) 60° impact wedge loading; (c)(d) 45° impact wedge loading.

5. Conclusions

In this study, dynamic combined compression and shear tests were performed on different densities of EPS foam by integrating a newly developed test rig with a drop weight tower testing system. Several loading conditions with various impact loading angles and drop heights were tested in this study. Due to the limitations of the maximum drop height and mass of the drop tower system, and the manufacture and assembly difficulties for the wedges with other loading angles, limited strain rates and loading angles are investigated in this work. It will be worthwhile in future to investigate the deformation of EPS foam under combined compression and shear loading, at higher strain rates and other various loading angles.

The compressive and shear stresses at yield were obtained from the force curves recorded by Kistler load cells. The average compressive and shear strains of the EPS foam were obtained using the Digital Image Correlation (VIC-2D software) method, where snapshots of the deformed EPS foam were taken during testing using a high-speed camera.

We note that it was not possible to obtain the local distribution of stresses directly from our setup since the forces recorded by the load cells are actually the average forces measured during the loading process. Because of edge effects, which result in a non-uniform deformation of the foam sample, we have chosen the inner inspection area to calculate the strain of the foam during the loading. The size and location of the inspection area remained the same for all samples, to ensure that the results were consistent. Separate work that will be presented elsewhere will describe the use of 3D finite element analysis to estimate the full-field distribution of stresses.

In this paper, we have compared the compressive and shear stresses at yield of EPS foam under different loading cases, where the main conclusions that can be drawn are as follows:

- Both the compressive and shear stresses at yield of EPS foam are dependent on foam density and strain-rate; the average von Mises stress at yield shows density and strain-rate dependency.
- The shear stress at yield of EPS foam is much smaller compared with the compressive stress at yield under the same loading conditions, both at quasi-static and dynamic strain rates.
- Higher impact loading angles tend to decrease the compressive stress and increase the shear stress at yield for the EPS foam.

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Appendix A. Method to determine the force and stress at yield of the EPS foam

The compressive and shear forces and stresses at yield, listed in Tables 2 and 3, were obtained from the force curves from Figs 9 and 10. The method will be explained by considering Fig. 10 (e) as an illustrative example.

Fig. A-1. The determination of the forces at yield

In the tests, the force curve of the EPS foam varies with the loading condition and foam density. However, it can be divided generally into four categories, as presented typically in Fig. A-1. The first category is for the highest density foam (120 kg/m^3), where the curve is always ‘bell-like’ except for those loading cases when cracking of the foam happened, as shown in Fig. A-1. However, as the 120 kg/m^3 foam did not reach its yielding point in our experiments, we cannot obtain the force at yield directly from the curve. The second category of force curve has a relatively long and flat force plateau, as for the 80 kg/m^3 foam shown in Fig. A-1. In this case, the force at yield can be

determined by averaging the highest and lowest value of the plateau, as shown by label ①. The third category of force curve shows an inclined force plateau due to the strain-rate effect, as for the 64 kg/m^3 foam shown in Fig. A-1. This curve shows a clear start point of the force plateau phase, as marked with the dotted circle in this figure. Thus, the force at yield in this instance can be determined by drawing a horizontal line through the start point of the inclined force plateau, as shown by label ②. The last category of force curve has a relatively small section of force plateau at the beginning, with the force increasing dramatically later due to densification of the foam, as for the 43 kg/m^3 foam shown in this figure. In this case, a line passing through the early small force plateau section can be made to determine the force at yield, as illustrated by the label ③.

Once the force at yield has been determined, the corresponding stress at yield can then be obtained by dividing the force by the foam compression and shear area, which is $20 \times 20 \text{ mm}^2$ in our case.

Appendix B. Method to calculate the von Mises yielding stresses of the EPS foam

The general equation to calculate the von Mises stress is:

$$\sigma_v = \sqrt{1/2 [(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)]} \quad (\text{B.1})$$

where σ_{11} , σ_{22} , σ_{33} , σ_{12} , σ_{23} and σ_{31} are the stress vectors for the stress at a point in 3D space. For our experimental tests on EPS foam, the problem can be simplified as a plane strain 2D problem, and we use σ_c and σ_s to represent the compressive and shear stresses from our experiments, i.e.:

$$\sigma_{11} = \sigma_c; \sigma_{12} = \sigma_s; \sigma_{33} \approx \sigma_{31} \approx \sigma_{23} \approx \sigma_{22} \approx 0.$$

Eq. (B.1) can then be simplified as:

$$\sigma_v = \sqrt{\sigma_c^2 + 3\sigma_s^2} \quad (\text{B.2})$$

Note that when we simplify Eq. (B.1) into Eq. (B.2), we assumed that $\sigma_{33} \approx \sigma_{31} \approx \sigma_{23} \approx \sigma_{22} \approx 0$, as the edge effect results in an inhomogeneity of stresses near the specimen edges. However, it decays rapidly away from the edge, and can be neglected close to the centre of the specimen. Therefore, the authors believe that the simplification to reduce Eq. (B1) to Eq. (B2) results only a small error and is therefore justified. This can be shown easily using numerical techniques such as three dimensional finite element analysis.

The von Mises yielding stresses at 45° and 60° impact wedge loading angle can then be calculated by substituting the compressive and shear stresses at yield into Eq. (B.2), as listed in Tables 2 and 3.

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Graphical Abstract