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Physics Letters A 323 (2004) 439–444

PHYSICS LETTERS A

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The image charge effects on plasma waves in the presence of neutral dust grains

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Received 30 November 2003; accepted 10 February 2004

Communicated by F. Porcelli

Abstract

A quantitative analysis of the image charge effects in dusty plasmas is presented. It is emphasized that the electrostatic interaction caused by the induced charge on grains can play an important role in dusty plasmas even in the case when in the equilibrium dust grains are not charged. As an example, the behavior of the ion electrostatic perturbations propagating in a plasma containing static and neutral (i.e., uncharged) dust grains is examined. A dispersion equation describing a dispersive and always unstable ion wave is obtained. Both the wave dispersion and the instability are a direct consequence of the image charge effect.

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PACS: 52.27.Lw; 52.35.Fp; 95.30.Wi

Keywords: Dusty plasma; Image charge

1. The conventional picture of the electrostatic interaction in a dusty plasma is based on the Coulomb-type interaction which implies that the potential around a grain can be approximated by the Coulomb potential, i.e., proportional to $1/r$. The additional Debye–Hückel screening, which one takes into account, changes to some extent the range of the electrostatic interaction, while the general picture in principle does not change too much. Yet, contrary to that approach, charged and uncharged dust grains should behave like isolated conductive or semi-conductive ob-

jects, which then results in the fact that the charge on grains is polarized in the presence of some external field. In other words a charge at some distance from the grain will induce an image charge on the grain, resulting in an additional image charge force which is always attractive, regardless of the sign of the outside charge and the equilibrium charge on the grain. The physics and the analytical expressions describing that effect are known from classical literature [1]. In the domain of the dusty plasma, however, apart from a comprehensive study [2] performed about 15 years ago little has been done so far [3]. To describe the physical background of the phenomenon and its possible influence on the waves in dusty plasmas, in the present Letter we shall highlight some effects of the

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image charge that can be deduced from the existing classical literature [1,2]. A quantitative description of the phenomenon will be given, and, finally, the obtained results will be applied to the simple case of the ion acoustic wave.

2. As one can find in classical books on electrodynamics theory [1], in the simple case of an isolated conducting spherical object with the charge Q , placed at the distance r from a point charge q (ion or electron), the force acting on the point charge q is given by $F = F_c + F_{ic}$ where

$$F_c = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2},$$

$$F_{ic} = -\frac{1}{4\pi\epsilon_0} \frac{q^2 a^3}{r^3} \frac{2r^2 - a^2}{(r^2 - a^2)^2}. \quad (1)$$

Here a denotes the radius of the spherical object (dust grain), and the subscript ic denotes the force due to the image charge effect. For $r \gg a$ the image charge term in the above expression is negligible, as it behaves like $\sim 1/r^5$, while it becomes dominant for $r \rightarrow a$. Intuitively it is clear that this will result in significantly modified (i.e., larger) impact parameters, as well as in larger effective cross-sections for the capturing of particles. One should note that the image charge force exists, and it is always attractive, even for uncharged, i.e., neutral dust grains, which could be of great importance in the application to astrophysical environments. The numerical calculations for collision cross sections [2] show considerable changes when the image charge force is taken into account, compared to the classical Coulomb-type interaction. The calculations have been done for conducting and very small grains. Yet the story turns out to be applicable for dielectrics as well, and in that case there appears a constant of proportionality only [2] of the form $(\epsilon - 1)/(\epsilon + 2)$, where ϵ is the dielectric constant of the material. The modified collision cross-sections for plasma particles will further influence any process in a dusty plasma that involves collisions, like the charge fluctuations and shadow force effects. For equal sign of charges the standard Coulomb interaction is repulsive, while the image charge effect yields an attractive force acting at short distances. The shadow force, which can be understood from the mechanical point of view as an effective attraction and agglomeration of dust grains caused by

the bombardment by ions, will be influenced as well, i.e., it is expected to be more efficient due to the image charge effects. In laboratory conditions dust grains are usually heavily charged, which appears as a necessity to keep them levitating (by means of some applied electric field) in the field of the external gravity force. So, in such an environment the image charge effect should not be expected to play a decisive role in the behavior of dust grains. The situation can be very different for the interstellar grains, composed of graphite, silicate and metallic compounds. Due to the low ionization rate these grains are as a rule embedded in a weakly ionized plasma background, acquiring $\pm 1, \pm 2$ electronic charges only, or remaining neutral [4,5]. The signs $\pm$, in principle, indicate some possibly different charging mechanisms that can take place in such environments. The electrically charged grains are rare in dense molecular clouds as these clouds are shielded from UV radiation.

Using Eq. (1) one can write the following expression for the potential energy

$$E_p = \frac{Q}{r} - \frac{1}{2r^2(r^2 - 1)}. \quad (2)$$

Here the energy is normalized as $E_p \rightarrow E_p/(\kappa q^2/a)$, $\kappa = 1/4\pi\epsilon_0$, the distance is expressed in units of the grain radius $r \rightarrow r/a$, and the charge of the spherical object (grain) is expressed in units of the external charge $Q \rightarrow Q/q$.

The profile of the potential energy for the case of the charges of the same sign ($Qq > 0$), for several values of Q/q is given in Fig. 1. Contrary to the standard description, which involves the Coulomb interaction only and results in a permanently growing potential, here obviously there is a maximum value of the potential, for some $r = r_m$, which is given by

$$\frac{dE_p}{dr} = \frac{2r^2 - 1 - Qr(r^2 - 1)^2}{r^3(r^2 - 1)^2} = 0. \quad (3)$$

The potential at distances close to the grain is always attractive. The maxima of the potential energy grow with the amount of charge on the grain. The distance r_m of the maxima of the potential energy, in terms of Q , is given in Fig. 2. As one see from Fig. 1, there exist a distance of the zero force acting on the external charge q , i.e., at which the image charge force and the Coulomb force balance each other. It formally follows from the equality

Fig. 1. The profile of the potential energy for the grain with the charge Q and a charge of the same sign q in terms of the distance between the charges, in the presence of the image charge effect. The distance is in units of the grain radius.

Fig. 2. The locus of maxima of potential energy as a function of the charge on the grain.

$F_c/F_{ic} = 1 \equiv -Qr(r^2 - 1)^2/(2r^2 - 1)$ and has the shape similar to Fig. 2. In general, the smaller Q the relatively stronger the image charge effect.

The collisional charging is, by its nature, the attachment of lighter plasma particles (electrons and ions) onto the grains. It is determined by the electrostatic interaction and, consequently, must be influenced by the image charge effect. To demonstrate that fact we shall apply the standard derivation of the collision cross-section with the difference due to the image charge force only. The model implies an isolated dust grain, with other dust grains being relatively far etc. It should be said that this is another weak side of the presently well accepted theory of dust charging, developed from the probe theory, which therefore neglects the collective effects and the presence of other dust grains.

From the momentum and energy conservation laws

$$m_q v_{q0} b = m_q v_{qc} r_c, \quad (4)$$

$$\frac{1}{2} m_q v_{q0}^2 = \frac{1}{2} m_q v_{qc}^2 + E_{pc}, \quad (5)$$

one calculates the closest approaching distance r_c of a particle with the charge q for the given impact parameter b .

Here v_{q0} is the velocity of the charge q far from the grain (i.e., from the charge Q) and E_{pc} is the electrostatic potential energy (the Coulomb energy plus the image-charge energy, Eq. (2)) at $r = r_c$. When the impact parameter decreases for the given velocity, the particle will hit the grain. We note that for the standard (Coulomb) case the closest distance is just the grain radius a . However, in general, $r_c > a$. In the presence of the attractive image-charge force the particles at distances less than r_c will perform a spiral motion towards the grain surface.

Introducing $E_0 = m_q v_{q0}^2/2$, $E_c = m_q v_{qc}^2/2$, normalizing the energy to $\kappa q^2/a$, and expressing r_c and b in units of a we arrive at the following equation:

$$-E_0 r_c^2 + b^2 E_0 + Q r_c = \frac{1}{2(r_c^2 - 1)}. \quad (6)$$

Eq. (6) determines r_c in terms of b , E_0 , and Q . Eq. (6) can be solved graphically. The smallest value of b , i.e., b_c is obtained when the left-hand side (LHS) and the right-hand side (RHS) are tangent. Mathematically this reads $d(\text{LHS})/dr = d(\text{RHS})/dr$, and yields the following condition for r_c :

$$(r_c^2 - 1)^2 (2E_0 r_c - Q) - r_c = 0. \quad (7)$$

From Eq. (6) we have

$$b_c^2 = r_c^2 + \frac{1}{E_0} \left[\frac{1}{2(r_c^2 - 1)} - Q r_c \right], \quad (8)$$

so the collision cross-section $\sigma_c = \sigma_c(v_0, r_c, Q)$, for $E_0 > E_{p,\max}$, is

$$\sigma_c = \pi b_c^2 a^2 = \pi a^2 \left[r_c^2 + \frac{1}{E_0} \left(\frac{1}{2(r_c^2 - 1)} - Q r_c \right) \right], \quad (9)$$

where r_c is to be determined from Eq. (7), and $E_{p,\max}$ is the maximum potential energy, at the distance r_m determined from Eq. (3) (see Figs. 1 and 2). In the classical case $E_{p,\max}$ is the potential on the grain

Fig. 3. The solution of $-Er_c^2 + b^2 E + Q r_c = 1/(2r_c^2 - 2)$ for $E = 8$, $Q = 10$.

surface, and it is generally larger than in the present case. As an illustration, the graphical solution of Eq. (6) for $E_0 = 8$, and $Q = 10$ is given in Fig. 3. Here $r_c \approx 1.169$ and $b_c = 0.275$.

We note that after neglecting the image-charge effect, and setting $r_c = 1$, instead of Eq. (9) we have:

$$\sigma_c = \pi a^2 \left(1 - \frac{qQ}{2\pi \epsilon_0 m_q v_0^2 a} \right), \quad (10)$$

which is the standard expression for the collision cross-section in the presence of the Coulomb force only [6].

On the other hand, to emphasize the effect of the image charge force only, i.e., for the *electrically neutral dust grains*, we take $Q = 0$, $E_{p,\max} = 0$, and from (7) we have $r_{cn}^2 = 1 + 1/\sqrt{2E_0}$. From Eq. (9) we have the collision cross-section for the uncharged grains:

$$\sigma_{cn}(Q = 0) = \pi a^2 \left(1 + \sqrt{\frac{2}{E_0}} \right). \quad (11)$$

Here the subscript n denotes the case of the *neutral*, i.e., uncharged grains. Consequently, due to the image charge, the electromagnetic effects exist even in the case of an uncharged dust.

In the general case the collision cross-section σ_c can be calculated numerically from Eqs. (7)–(9), or found approximately; one can see details in Ref. [2]. The charging currents should be further calculated

from:

$$I_j = n_j q_j \int_{v_{\min}}^{\infty} v_{\perp} \sigma_c f_j(\vec{v}) d^3 \vec{v}. \quad (12)$$

Here $v_{\perp} = \vec{v} \cdot \vec{n}$, is the perpendicular component of the velocity of the plasma species with the charge q_j , and $\vec{n}$ is the unit vector perpendicular to the dust grain surface. The analytical expression can be obtained for the case of uncharged grains, otherwise a numerical procedure should be applied, or some approximate expressions can be used [2].

3. We are now interested in examining the effects of the image charge on plasma waves. As a simple example we shall take the case of the electrostatic ion perturbations in the limit of massless Boltzmann distributed electrons, in the presence of immobile dust particles. The results may look unusual from the conventional point of view, i.e., when the Coulomb-type interaction is present only, however, in accordance with the argument stated above they are certainly expected in the presence of the image charge force.

We assume that dust particles are neutral at equilibrium, but become charged by the effect of the image charge in the perturbed state of the plasma. Thus, in the equilibrium the quasineutrality condition reads $n_{i0} = n_{e0} \equiv n_0$. The dynamics of one-dimensional ion-acoustic waves, whose phase speed is much smaller (larger) than the electron (ion) thermal speed, is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \quad (13)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{e}{m_i} \frac{\partial \phi}{\partial x}, \quad (14)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\epsilon_0} (n_e - n_i - n_d Z_d), \quad (15)$$

where n_i is the ion number density, u_i is the ion fluid velocity, ϕ is the electrostatic wave potential, Z_d is the dust charge number normalized by e , and n_d is the constant dust number density. We note that Z_d is not constant but varies with space and time because of the image charge effect [2]. Thus, the equations are completed by the dust charging equation

$$\frac{\partial Z_d}{\partial t} = -\alpha_n (n_e - n_i). \quad (16)$$

Eq. (16) is obtained by using the general expression for the charging of the neutral grains $\partial Q/\partial t = \sum_j I_j$, where in the equilibrium $\sum_j I_{j0} = 0$, and the charging currents are given in the form [2] $I_j = \text{const} \times n_j \tilde{I}_j$. In view of Eq. (11) here $\tilde{I}_j$ in the case of neutral grains can be calculated exactly [2] yielding $\tilde{I}_{jn} = 1 + [\pi \kappa e^2 / (2ak_B T_j)]^{1/2}$. Here $\alpha_n = s_e \pi a^2 (8k_B T_e / \pi m_e)^{1/2} (1 + \sqrt{\pi \kappa e^2 / 2ak_B T_e})$, s_e is the electron sticking coefficient, $Q_0 = 0$, and the subscript n stays for the neutral grains. To study the effects of the image charge on ion-acoustic waves, we shall carry out a normal mode analysis. We first express our dependent variables n_i , u_i , ϕ , and Z_d in terms of their equilibrium and perturbed parts as $n_i = n_0 + \tilde{n}_i$, $u_i = 0 + \tilde{u}_i$, $\phi = 0 + \tilde{\phi}$, and $Z_d = 0 + \tilde{Z}_d$. We then substitute these into Eqs. (13)–(16), and express the latter in linearized form as

$$\frac{\partial \tilde{n}_i}{\partial t} + n_0 \frac{\partial \tilde{u}_i}{\partial x} = 0, \quad (17)$$

$$\frac{\partial \tilde{u}_i}{\partial t} = -\frac{e}{m_i} \frac{\partial \tilde{\phi}}{\partial x}, \quad (18)$$

$$\frac{\partial^2 \tilde{\phi}}{\partial x^2} = \frac{e}{\varepsilon_0} \left[\left(\frac{n_0 e}{k_B T_e} \right) \tilde{\phi} - \tilde{n}_i - n_d \tilde{Z}_d \right], \quad (19)$$

$$\frac{\partial \tilde{Z}_d}{\partial t} = -\alpha_n \left[\left(\frac{n_0 e}{k_B T_e} \right) \tilde{\phi} - \tilde{n}_i \right]. \quad (20)$$

We now assume that all perturbed quantities (viz. $\tilde{n}_i$, $\tilde{u}_i$, $\tilde{\phi}$, and $\tilde{Z}_d$) are proportional to $\exp(-i\omega t + ikx)$. Thus, using Eqs. (17)–(20), we obtain the dispersion relation

$$1 + k^2 \lambda_{De}^2 - \frac{k^2 c_s^2}{\omega^2} + i \frac{\gamma}{\omega} \left(1 - \frac{k^2 c_s^2}{\omega^2} \right) = 0, \quad (21)$$

where $c_s = (k_B T_e / m_i)^{1/2}$, $\lambda_{De} = (\varepsilon_0 k_B T_e / n_0 e^2)^{1/2}$, and $\gamma = n_d \alpha_n$. Neglecting the second term, $k^2 \lambda_{De}^2$, in Eq. (21) one readily obtains the dispersion relation for the ion-acoustic wave $\omega = kc_s$, and a purely damped perturbation, due to the image charge, described by $\omega = -i\gamma$. However, for any finite value of $k^2 \lambda_{De}^2$ the dispersion equation (21) has an unstable solution. Setting $\omega = \omega_r + i\omega_i$, where $|\omega_r| \gg |\omega_i|$, from (21) one finds approximately

$$\omega_r^2 = \frac{k^2 c_s^2 + 2\gamma \omega_i}{1 + k^2 \lambda_{De}^2}, \quad (22)$$

$$\omega_i = \frac{\gamma}{2} \frac{k^2 \lambda_{De}^2}{1 + k^2 \lambda_{De}^2}. \quad (23)$$

Consequently, the mode is always unstable regardless of how small the departure from the quasineutrality condition is. At the same time the real frequency of the mode, Eq. (22), represents a dispersive electrostatic ion wave, where the dispersion is a direct consequence of the image charge effects. In the limit $\gamma \rightarrow 0$ it converts into the frequency of the standard ion acoustic wave $\omega_r = kc_s / (1 + k^2 \lambda_{De}^2)^{1/2}$.

4. To conclude, in the present Letter we have given a physical background of the phenomenon known in the literature as the image charge effect, for the case of a plasma containing dust grains. According to the general electrodynamics theory the grains should behave as conductive or semi-conductive objects, resulting in the induced charge on the grains in the presence of some outside charge or electric field. As we have demonstrated in the Letter, the collision cross-section and all physical effects related to the inelastic collisions of plasma particles and grains, should consequently be modified due to the image charge. To emphasize the phenomenon we have applied the results to the case of the electrostatic ion waves in the presence of uncharged dust grains. The grains acquire some charge in the perturbed plasma case due to the image charge effect, which further influences the plasma waves. The analysis performed here shows the existence of a new plasma mode, i.e., the dispersive ion plasma oscillation that propagates as an unstable wave. Both the dispersion and instability are caused by the image charge effect. Similar effects one can expect for some other plasma waves. Consequently, we believe that the effects presented here should be used in the appropriate studies of wave phenomena in astrophysical dusty plasmas.

Acknowledgements

This work has been partly done under the auspices of the Belgian Office for Scientific, Technical and Cultural Affairs. Useful discussions with A.A. Mamun are gratefully acknowledged.

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