Study of magnetic reconnection in large-scale magnetic island coalescence via spatially coupled MHD and PIC simulations

Kirit D. Makwana,^{1, a)} Rony Keppens,¹ and Giovanni Lapenta¹

*Centre for mathematical Plasma Astrophysics, Department of Mathematics,
KU Leuven, Celestijnenlaan 200B, 3001 Heverlee, Belgium*

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We study the process of magnetic reconnection in a coalescing magnetic island setup by means of numerical simulation. This process mimics flux tube merging which can take place in the solar corona, laboratory, as well as astrophysical objects. The simulations are performed with magnetohydrodynamics (MHD), Hall-MHD, and a newly developed Coupled MHD and PIC model (CMAP). This model treats the global simulation domain with MHD while the region around the reconnection zone is treated with PIC. This CMAP code allows us to simulate larger-scale domains with lesser computing power compared to fully PIC simulations. CMAP reproduces the dynamics of fully kinetic simulations which Hall-MHD does not capture, as seen in the Hall magnetic field and the reconnecting current sheet structure. For large islands in kinetic simulations the current sheet does not form smoothly and shows chaotic behavior and the magnetic islands also bounce and slosh. The current sheet thickness, length, and aspect ratios are calculated. They show that in the CMAP model, the thickness remains close to ion skin depth while the length changes weakly with system size, giving a steady aspect ratio for the two largest system size simulations. The pressure tensor also shows large deviations from isotropy and gyrotropy near the current sheet. **The CMAP simulations for smaller system sizes are compared to fully kinetic simulations and we find that a minimum fraction of area has to be provided PIC feedback in the CMAP simulations in order to produce reconnection rates and dynamics similar to fully kinetic simulations.** The reconnection rate reduces with increasing island size. For the CMAP model this reduction is steeper compared to MHD and Hall-MHD initially, but for larger system sizes, **the reconnection rates in CMAP simulations show a steady behavior.**

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^{a)}Electronic mail: kirit.makwana@kuleuven.be

I. INTRODUCTION

Magnetic reconnection is observed or believed to occur in a wide range of systems from Earth-based laboratories to the heliosphere to even interstellar environments^{1,2}. Magnetic reconnection involves rearrangement or change in topology of magnetic field lines along with a conversion of magnetic energy into plasma kinetic or thermal energy. There are many aspects of this process which are mysterious and subject of active research. Some of the interesting questions are about the rate of reconnection, the geometry of reconnection in 2D and 3D, and the coupling of large and small scales.

The rate of reconnection is observed to depend critically on a variety of aspects. Numerical simulations show that the physics model used for simulations affect the rate of reconnection, with Hall physics playing an important role³ in speeding up reconnection compared to purely magnetohydrodynamic (MHD) physics given by the Sweet-Parker rate^{4,5} which itself is significantly faster than the diffusion rate. On the other hand, even in MHD physics the Lundquist number can affect the reconnection rate and dynamics. At large Lundquist numbers in large size systems, reconnection rate can become weakly dependent on resistivity⁶ as well as chaotic and bursty^{7,8} due to secondary island formation^{9,10}. Also, turbulence can affect the reconnection where it can speed up the rate of reconnection and make it independent of the resistivity^{11,12}. Moreover, the rate of reconnection is not constant but varies. In solar flares the magnetic flux has to first build up, implying slow reconnection, before it is released explosively in a flare, implying a fast reconnection. Reconnection can also be spontaneous like in the tearing mode instability¹³. Therefore, another aspect of reconnection is how does it depend on the driving environment. Reconnection can be driven in stable current sheets by driving fluctuations¹⁴.

The other confounding aspect of reconnection is that it is a multi-scale as well as multi-physics phenomena. It can be driven on large scales by global physics, whereas the reconnection zone is intimately affected by small-scale physics. For ex, in Earth's magnetosphere the solar wind driver operates at the scales of magnetosphere which is of $\sim 10^9 m$ while the plasma sheet which forms and reconnects is of the scale $\sim 10^7 m$. Furthermore, kinetic physics plays an important role in this reconnection where the ion skin depth scale is at $\sim 10^5 m$ and the electron skin depth is at $\sim 10^3 m$. Kinetic effects like anisotropic and agyrotropic pressure play an important role in governing this reconnection dynamics.

The formation of current sheets is also an important process which affects the reconnection. Here the question is how long current sheets can form in realistic plasmas? The aspect ratio of a current sheet controls the rate of reconnection. Kinetic numerical studies have indicated that the length of the reconnection region remains limited to around 10 ion skin depths independent of the global system scale⁵¹. Some other numerical studies in both kinetic and fluid models have shown that the reconnection rate is affected by the system size. Since then several studies have focused on the magnetic island coalescence problem as a test case for analyzing these different aspects of reconnection^{16–21}. In the coalescence of magnetic islands, the current sheet forms consistently as the islands come together. This is a test model for flux tube merging in tokamaks²³ as well as in the solar corona²⁴. These studies have investigated the effect of the physics used in simulations, with differences between MHD, Hall-MHD, hybrid, and fully kinetic models. **Previous studies have investigated dependence of reconnection rates on Lundquist number (S) in coalescence of magnetic islands and found that they become independent of S at low S ^{25,26}.** They have also shown dependence of the reconnection rate on system size. However, simulations of these problems are not easy because of the vastly different scales involved and also the different physics. While the system size is of the order of 100's of d_i , the current sheet that forms is barely one d_i and needs to be properly resolved with kinetic physics. Also, these simulations proceed at Alfvén time scales, so the simulation time increases with the system size.

To overcome multi-scale and multi-physics difficulties, we present simulations with a new model of coupled MHD and PIC domains (denoted as CMAP model henceforth)²⁸. In this model the global simulation domain is handled efficiently with MHD while the region around the reconnection zone is handled by fully kinetic PIC method. These two domains talk to each other both ways, with the MHD code giving initial and boundary conditions to PIC, whereas the PIC code constantly updates the MHD solution. This saves computing power compared to fully kinetic simulations as only a fraction of the domain is simulated with the computationally expensive PIC model. This allows modeling larger system sizes with limited computing power. We briefly describe some details of this two-way coupling in Sec. 2. It describes the problem setup of magnetic island coalescence along with the numerical details of the problem setup. We are able to reproduce several results that have been found in kinetic simulations before with this CMAP model. In Sec. 3, we show a comparison of the results produced by MHD, Hall-MHD, CMAP, and fully PIC simulations (for smaller

system sizes). We find that the coupled simulation saves computing time compared to a full PIC simulation while capturing the relevant kinetic physics effects which are not reproduced in MHD and Hall-MHD. We also show properties of the current sheet formation and kinetic signatures near the current sheet. In Sec. 4 the reconnection rates obtained in these simulations are shown and how we can get the correct reconnection rates from the coupled MHD-PIC approach. It is shown that in the CMAP model the reconnection rate drops as the system size increases, but reaches a steady value for island sizes larger than $20d_i$. Sec. 5 gives the conclusions and implications of this study.

II. NUMERICAL SETUP

We setup the problem of magnetic island coalescence given by the Fadeev equilibrium²⁹. This has the magnetic vector potential given as^{20,21}

$$A_z = B_0 \lambda \ln[\cosh(y/\lambda) + \epsilon \cos(x/\lambda)] \quad (1)$$

where $\epsilon = 0.4$. The density is given by

$$n = n_b + \frac{n_0(1 - \epsilon^2)}{[\cosh(y/\lambda) + \epsilon \cos(x/\lambda)]^2} \quad (2)$$

The island size λ is related to the system size in the x direction as $L_x = 4\pi\lambda$. The simulation domain in the y direction is taken to be same $L_y = L_x$. The asymptotic magnetic field is kept at $B_0 = 0.1$. The background density is taken as $n_b = 0.3n_0$ where the ion mass is normalized to unity and n_0 is also unity. The velocity is zero and the pressure is given by $p = nk_B(T_{i0} + T_{e0})$ where $T_{i0} + T_{e0} = B_0^2/(2\mu_0 n_0 k_B)$ to maintain the equilibrium. The initial condition is perturbed by a sinusoidal perturbation of the form given in Ref.²² to initiate the coalescence of the two islands.

This setup is implemented in the fluid code MPI-AMRVAC^{27,30}. MPI-AMRVAC solves a variety of hyperbolic advection problems, including MHD, with conservative, shock-capturing finite volume and finite difference schemes. It offers versatility in terms of geometry of the grid, dimensionality, the spatial and temporal discretization, and computer platform³⁰⁻³². It has the resistive MHD and Hall-MHD modules which are critical in coupling with a kinetic code. It also has a grid-block based adaptive mesh refinement scheme. In this work we have used the TVDLF spatial discretization with a CADA3 limiter

function³³, along with a three-step RK time stepping scheme. The $\nabla \cdot \mathbf{B} = 0$ condition is maintained by a generalized lagrangian multiplier (GLM) scheme³⁴. The MPI-AMRVAC simulations are done with the resistive MHD module as well as the Hall-MHD module. The resistive MHD solves the standard resistive MHD equations. The normalized resistivity is kept at $\eta = 0.003$. The Hall module solves the Ohm's law with an extra Hall term $\frac{\eta_h}{\hat{\rho}} \hat{\mathbf{J}} \times \hat{\mathbf{B}}$ as shown below (hatted quantities meaning they are normalized),

$$\hat{\mathbf{E}} = -\hat{\mathbf{v}} \times \hat{\mathbf{B}} + \frac{\eta_h}{\hat{\rho}} \hat{\mathbf{J}} \times \hat{\mathbf{B}} + \eta \hat{\mathbf{J}} \quad (3)$$

and this electric field is then substituted in the induction equation. The value of η_h depends on the normalization selected. If c_{ref} , B_{ref} , L_{ref} , ρ_{ref} , are the reference values for normalization of velocity, magnetic field, length, and density respectively, then $\eta_h = \frac{m_i}{e} \frac{B_{ref}}{c_{ref} \mu_0 L_{ref} \rho_{ref}}$. We use the Alfvénic units in the MHD simulations, i.e., a reference Alfvén speed $v_{A,ref}$ for c_{ref} , ion skin depth d_i for the reference length L_{ref} , and inverse ion cyclotron frequency $\omega_{c,i}^{-1}$ for time. These quantities are defined with respect to a reference magnetic field B_{ref} . Then η_h is simply unity. The asymptotic magnetic field B_0 is taken to be $0.1 B_{ref}$. This helps in conveniently making the units in MHD same as units in the PIC part of CMAP which normalizes time with plasma frequency $\omega_{p,i}^{-1}$ and velocity with the speed of light c . The resistivity in Hall-MHD simulations is again kept to 0.003. For CMAP simulations the resistivity in the MHD part of the model is kept zero to match with the collisionless nature of the PIC part. For the MHD and Hall-MHD simulations the boundary conditions are symmetric for the density, pressure, tangential velocity and normal magnetic field component, while antisymmetric for the normal velocity and tangential magnetic field along the x -boundaries. It is continuous for the density and pressure, symmetric for tangential velocity and tangential magnetic field, while antisymmetric for the normal velocity and magnetic field in the y -boundaries.

The island coalescence setup for smaller system sizes ($L_x \leq 120 d_i$) is also implemented in iPIC3D. iPIC3D is a semi-implicit moment method for PIC simulations of plasmas^{35,36}. It solves a linearized version of the implicit Ampère's law to advance the electric field. Then the magnetic field is obtained from the induction equation and the particles are also moved implicitly. As a result of this semi-implicit scheme, iPIC3D can take larger grid spacing ($10 \times -50 \times$) and larger time steps ($5 \times -10 \times$) compared to explicit PIC codes. The magnetic field is initialized as given by Eq. 1 plus the perturbation, while the electric field

is zero. The particles are initialized with a Maxwellian distribution in each cell, having a density given by Eq. 2 and the temperature ratio between the two species is $T_{i0}/T_{e0} = 5$. The mass ratio is taken to be reduced to $m_i/m_e = 25$.

This setup is also simulated with a coupled MHD and PIC model (CMAP). This model spatially couples MPI-AMRVAC with iPIC3D, with an iPIC3D region embedded within a global MPI-AMRVAC domain. This follows similar coupling between MHD and PIC codes^{37,38}. The details of this CMAP model can be found in Ref.^{28,40}. Briefly, in this coupling the global domain is simulated with MPI-AMRVAC. At some time step, a region within the global domain is selected by the user to be simulated with iPIC3D. In MHD to PIC coupling, information from MPI-AMRVAC is used to provide initial conditions to iPIC3D to start the simulation. The primitive variables of MHD namely density (ρ), pressure (p), velocity ($\mathbf{v}$), and magnetic field ($\mathbf{B}$) are supplied to iPIC3D. Using this the electromagnetic fields are initialized. The particles are initialized with a Maxwellian distribution using the thermal velocity derived from MHD pressure and drift velocity derived from MHD velocity and current density. After this MPI-AMRVAC is advanced by one time step and the time averaged state at the interface between MHD and PIC is supplied to iPIC3D as the boundary conditions for its time step. After iPIC3D advances its state to the next time step, moments of the distribution function are gathered to obtain the MHD primitive variables. In PIC to MHD coupling this information from iPIC3D is used to update the MPI-AMRVAC solution. The MHD solution is overwritten by a weighted average of PIC solution and the MHD solution in the PIC domain. The weight function varies smoothly across the interface region such that the updated MHD solution smoothly changes from purely MHD at the edge to purely PIC in the interior. This is the two-way coupling method.

These simulations are carried out with varying system size. For the larger system size simulations, we make use of the mesh refinement ability of MPI-AMRVAC, with increase in the mesh resolution as we get closer to the central reconnection zone. Table. I shows the box sizes for the different simulations performed going from $60d_i$ up to $480d_i$. All these 5 simulations are performed with MHD, Hall-MHD, and CMAP. The box is centered at the origin so the x -coordinates go from $-L_x/2$ to $L_x/2$, and the same for y -coordinates. The number of refinement levels are also shown, which are the same for MHD, Hall-MHD and CMAP simulations. In the case of CMAP simulations, the table also specifies the size of the domain where the PIC simulation operates and gives feedback to MPI-AMRVAC. This box

Run ID	$L_x/L_y(d_i)$	$\lambda(d_i)$	Ref. levels	Size of PIC region
R0	60	4.77	1	24 (240×240)
R1	120	9.55	2	48 (480×480)
R2	240	19.1	3	96 (960×960)
R3	360	28.65	3	144 (1440×1440)
R4	480	38.2	4	192 (1920×1920)

TABLE I. List of simulations parameters. The simulation domain is square with $L_x = L_y$ values given in units of ion skin depth d_i , which is translated into the island width λ in the third column. The number of refinement levels used in the AMRVAC simulation are also shown. The PIC domain is also square shaped so its edge length is given in the last column (also specifying the number of PIC cells in the brackets).

is also square and centered around the origin. For ex., in case of simulation R2 a PIC domain of size 96 means that iPIC3D is working in the domain $-48d_i \leq x \leq 48d_i$, $-48d_i \leq y \leq 48d_i$. The base resolution and the number of refinement levels are chosen such that the highest level of refinement always has a resolution of $\Delta x = \Delta y = 0.1d_i$. The embedded PIC domain always lies in the center around the central reconnecting zone and it is always covered by the highest resolution MHD grid. The time step in MHD and Hall-MHD simulations is limited by the CFL condition. It is found that this time step is always slightly larger than $\Delta t = 0.2\omega_{p,i}^{-1}$. Therefore, in the CMAP model, the time step is limited to this value and the iPIC3D time step is also chosen to match this value. The iPIC3D time step is constrained by the thermal velocity. The ion thermal velocity in the PIC domain here is $u_{th,i} \approx 0.066c$ and the electron thermal velocity is $u_{th,e} \approx 0.15c$. This satisfies the accuracy constraint of $u_{th} < dx/dt$ for both the species with $dx = 0.1d_i$ and $dt = 0.2\omega_{p,i}^{-1}$. It is also close to the finite grid stability criterion for ions: $u_{th} * dt/dx < 0.1$.

The simulations R0 and R1 are also performed with purely iPIC3D, with the full domain simulated as PIC. This is for comparison and verifying the CMAP results. Below we present this comparison and the savings in computing time due to CMAP compared to iPIC3D. The CMAP is expected to be computationally cheaper compared to PIC simulations of the entire domain. We found that the CMAP simulation of size R0 took 26% of the computing time as the full PIC simulation of size R0. Similarly, the CMAP simulation of size R1 took

FIG. 1. The 8 MHD quantities in the CMAP simulation R1 at time $0.6\tau_A$. Panels show the (a) B_x , (b) B_y , (c) B_z , (d) ρ , (e) v_x , (f) v_y , (g) v_z , (h) p_{th} . The black square box represents the region which has PIC feedback. We see that the transition from MHD to PIC domain is smooth.

22% of the computing time as the full PIC simulation of size R1. The PIC domain in CMAP simulations has an area of 16% compared to the MHD domain and so a speedup of 5 times is close to the idealized speedup of 6.25 which can be expected in the limit of MHD and data transfer computations taking infinitesimal time compared to PIC computations. This is also seen in the comparison of CMAP time with MHD simulations, where the CMAP simulation takes much longer than the MHD simulation. For size R1 the CMAP simulation was 20 times more expensive than MHD while for the R2 size CMAP was 50 times more expensive than MHD. This shows that PIC computations are much more expensive than MHD computations while the communication costs are causing some slowdown compared to the ideal speedup.

Fig. 1 shows an example of a CMAP simulation. The 8 different MHD quantities are plotted on the second (highest) refinement level of the R1 simulation. The black box shows the extent of the PIC region ($-24 \leq x \leq 24$, $-24 \leq y \leq 24$). The islands initially start

with a separation of $60d_i$ whereas in Fig. 1 they have now moved close to $40d_i$. They move smoothly across the MHD-PIC interface boundary, which can be prominently seen in the inflow velocity v_x , density, and pressure plots. More detailed analysis of the interface region shows smooth matching of the MHD-PIC solution at the interface⁴⁰.

III. CHARACTERISTICS OF THE RECONNECTION ZONE

FIG. 2. The out-of-plane magnetic field B_z in simulation R1 performed with the four different models: (a) MHD, (b) Hall-MHD, (c) CMAP, and (d) fully PIC. This is shown at time $t = 1.0\tau_A$. We see that Hall-MHD produces this field which MHD does not capture. However, the strength and shape of this field is quite different in Hall-MHD compared to fully PIC which agrees very well with the CMAP model.

We compare the characteristics of the reconnection zone across the four different simulation models for simulation R1. The out-of-plane magnetic field B_z is shown in Fig. 2. Resistive MHD does not produce an out-of-plane field as it does not have the Hall physics. The Hall-MHD produces a B_z field which is different from that produced by CMAP and a

FIG. 3. The out of plane current density J_z in simulation R1 performed with the 4 different models: (a) MHD, (b) Hall-MHD, (c) CMAP, and (d) fully PIC. These are shown at time $t = 1.0\tau_A$. The Hall-MHD produces a smaller and thinner current sheet structure compared to MHD. The CMAP and fully PIC simulations agree quite well, while producing weaker current density compared to Hall-MHD.

fully PIC simulation. It is slightly weaker in strength and different in its geometry. On the other hand, CMAP produces a result very similar to the full PIC simulation. They both show a stronger B_z field compared to the Hall-MHD value. Fig. 3 shows the out of plane current density J_z in the four different models for the R1 simulation. The MHD shows a current sheet structure that is like a sheet. The thickness of this sheet is resolved by the resistivity value of $\eta = 0.003$, lower resistivities do not resolve this current sheet. In the Hall-MHD run, the current sheet becomes thinner and shorter. **However, we find that the Hall-MHD current sheet is not very different from the resistive-MHD regime current sheet. This is seen further in the reconnection rates and island dynamics also. The transition from resistive MHD to Hall-MHD is determined by the Sweet-Parker current layer width, which when it becomes smaller than the ion skin depth transitions to the Hall-MHD regime. The**

theoretical Sweet-Parker current layer width is $l = \sqrt{\eta L / v_{A,i}}$, where L is the current layer length and $v_{A,i}$ is the Alfvén velocity of the plasma flowing into the reconnection zone. The peak reconnecting magnetic field at the inflow site is close to $B_0 \approx 0.07$ while the density at this point is close to $\rho \approx 2.0$. This gives an inflow Alfvén velocity of $v_{A,i} \approx 0.05$. Using this and the length of the current sheet measured in Hall-MHD simulations shown in Fig. 7(b) we find the theoretical Sweet-Parker current sheet thickness. The theoretical Sweet-Parker widths are $0.44d_i, 0.67d_i, 1.06d_i, 1.4d_i, 1.6d_i$ for the 5 different Hall-MHD simulations from R0 to R4 respectively. So the three largest simulations are having current layer thickness larger than ion skin depth, while the smaller two simulations are close to it. This could be the reason that the Hall-MHD simulations are still very similar to the resistive-MHD regime. We would need to go to higher resolutions to be able to resolve the Hall-regime properly with a low enough resistivity, but that would not be comparable to the resolution in the CMAP simulations. The peak current density is also similar in MHD and Hall-MHD, but it is lower in the CMAP and PIC simulations. However, the CMAP model reproduces the full PIC results accurately with both the current sheet geometry and strength.

Fig. 4 shows the formation of the current sheet in the 4 CMAP simulations R0, R1, R2, and R3. A slice is taken along the x -axis of the current density J_z . These slices at regular time intervals are stacked vertically. For R0 and R1 we see the formation of the current sheet and it remains steady in thickness after that. The thickness is close to one ion skin depth. In R2, the current sheet forms at around $t = 0.75\tau_A$ but soon after that is disrupted by formation of a plasmoid (this is seen more clearly in Fig. 5), then it forms again but is more diffuse. This is because as can be seen in the Fig. 5 the current sheet has moved away from the center and hence the x axis is not passing through the current sheet. Similarly in R3 the current sheet forms much later, because earlier it forms off-center and only later on passes through the x axis.

Fig. 5 shows the current sheet structure at different times in the R1, R2, and R4 simulations. We see the formation of the reconnecting current sheet in this figure. For the smaller system size R1, the current sheet forms smoothly at the center. For the medium system size we see formation of a plasmoid at the center and its ejection in the upper direction. This could be the result of a plasmoid instability^{9,10}. In Figs. 5(e) & (f) we see structures of enhanced current density downstream of the reconnection zone which look like the mirror instability seen in Ref.³⁹. For the largest system size R4, the current sheet does not form

FIG. 4. Slices along x axis of the current density J_z stacked vertically as a function of time. This is shown for the 4 simulations (a) R0, (b) R1, (c) R2, and (d) R3. R0 and R1 simulations show a smooth current sheet formation process. The thickness is close to one ion skin depth. R2 and R3 show a chaotic current sheet formation.

smoothly. We see twisting and bending of the current sheet at times $0.7\tau_A$ and $0.9\tau_A$. A steady sheet forms at $1.1\tau_A$ but it is off center. It moves in the downward direction at time $1.3\tau_A$. Thus, in the CMAP simulations, the current sheet formation process is not smooth for larger system sizes. On the other hand, for the MHD and Hall-MHD simulations, we do not see this behavior of the current sheets, we see smooth formation of current sheets even for the largest system size.

Following Ref.²⁰ we analyze the motion of the island center and their separation. The island centers are identified by the maxima of the flux function ψ . The separation of islands as a fraction of their initial separation is shown in Fig. 6 as a function of time. For the smallest system size R0, we find that the separation reduces monotonically, although it slows down after $t = 1.0\tau_A$. For R1 simulation the CMAP and MHD simulations show

FIG. 5. Solution of the out-of-plane current density J_z in the R1, R2, and R4 simulations with CMAP model shown in the three different rows from top to bottom. The columns show 4 different times at $0.7\tau_A$, $0.9\tau_A$, $1.1\tau_A$, and $1.3\tau_A$ from left to right. The plot titles specify these details. The current sheet formation is smooth for R1 but not for R2 and R4 simulations.

a little bounce (increase in separation) at around $1.5\tau_A$. For R2, R3, and R4, the island separation increases after $t = 1.0\tau_A$, indicating that the islands bounce. The results in this case are quite similar across the different models. This confirms the findings of Ref.²⁰ that larger magnetic islands bounce and slosh, and are unable to merge into one island in one

FIG. 6. The time evolution of island separation in the five simulations. The separation is normalized to the initial separation, so initial separation is always unity. (a) shows MHD, (b) Hall MHD, and (c) CMAP. The islands move closer monotonically for the smallest system size, but bounce for larger system sizes.

go. For the two largest system sizes R3 and R4, the CMAP results for the island separation lie almost on top of each other. This indicates that the motion of the islands in these two simulations of CMAP is very similar.

As seen in Fig. 5 it is difficult to identify a smooth current sheet in the CMAP simulation of size R4 at earlier times, but at $t = 1.3\tau_A$ we see a steady current sheet. At this time we calculate some geometrical characteristics of the central current sheet, which are shown in Fig. 7. To do this first the current density is smoothed to remove noise. Then the maximum current density and its location is computed. Starting from this point of maximum current density, we scan along the x and y direction till we reach the points where the current density drops to $1/e$ of the maximum value. This defines the extent of the current sheet in the x and y directions, and the thickness denotes the length along x direction, while length along the y direction denotes the length of the current sheet. There are many critical differences between the MHD and Hall-MHD on one hand compared with the CMAP simulations on the other. For the lowest system size all three models are quite close. However, as the system size increases, the thickness also increases in the MHD and Hall-MHD models. On

FIG. 7. The geometrical characteristics of the central current sheet. (a) shows the current sheet thickness as a function of island size λ/d_i . (b) shows the current sheet length and (c) shows the aspect ratio given by length/thickness. The legend applies to all three panels.

the other hand the CMAP model shows current sheet thickness consistent with one ion skin-depth irrespective of system sizes. The length also shows stronger increase with system size in the fluid models compared to CMAP. In the fluid models the length shows almost linear growth with the system size. Surprisingly, the aspect ratio of the three models match quite well up to system size of R3, despite the different thickness and lengths. But in the largest system size of R4, the aspect ratio of CMAP model flattens out compared to the fluid models. This we can see is due to the length of the current sheet flattening out to around $15\text{-}20 d_i$ in the CMAP simulations. Other kinetic simulations of magnetic reconnection in a Harris sheet have seen much larger current sheets in larger domains, supported by pressure anisotropy^{41,42}. We do not find similarly long current sheets in our CMAP simulations of island coalescence in large system sizes. We do see some pressure anisotropy along the separatrixes for long distance from the X-point although this anisotropy is not as strong as seen in Ref.⁴².

Fig. 8 shows the various measures of pressure tensor asymmetries. These are purely kinetic effects and hence have to be extracted from PIC simulations. These are taken from the PIC part of the CMAP R2 simulation. There are three different measures of agyrotropy, taken from Scudder⁴³, Aunai⁴⁴, and Swisdak⁴⁵. For the Scudder prescription, we follow the simplified eigenvalue analysis presented in Ref.⁴³ where the characteristic polynomial is

FIG. 8. Various measures of kinetic effects in the pressure tensor. (a) shows the agyrotropy given by Scudder, (b) Aunai nongyrotropy measure, (c) Swisdak agyrotropy measure, (d) pressure anisotropy $p_{\parallel}/p_{\perp}$. These are taken from the CMAP R2 simulation at time $t = 1.0\tau_A$. The nongyrotropy and anisotropy are concentrated close to the current sheet.

simplified by removing the eigenvalue of the pressure tensor corresponding to the parallel direction. Qualitatively, the three measures look very similar. They are peaking near the current sheet which is at the center of the simulation domain. The anisotropy in the pressure tensor $p_{\parallel}/p_{\perp}$ is also shown in Fig. 8(d), giving a peak of more than 1.7 and a minimum of less than 0.7. We see similar pressure tensor asymmetries in the R3 and R4 CMAP simulations, but there these kinetic effects are not located close to the center and are not smooth features. Instead, they move around as the current sheet moves and they are spread out unevenly

around the reconnection zone. This implies measurements and observations of these kinetic features of the pressure tensor need to be spread out around the reconnection region.

IV. RECONNECTION RATE CHARACTERISTICS

FIG. 9. The reconnection rate time evolution in R1 simulation with the 4 different models.

The reconnection rate of the merging islands is an important property of the underlying physics of coalescence. To calculate the reconnection rate, the magnetic flux along the x axis is defined as $\psi(x) = \int_0^x B_y dx$. This is calculated by integrating along the x -axis starting from the X-point which lies at the origin. The reconnection rate (E_R) is calculated as²⁰

$$E_R = \frac{1}{v_{Am} B_m} \left| \frac{\partial(\psi_X - \psi_O)}{\partial t} \right|, \quad (4)$$

where ψ_X and ψ_O are the fluxes at X and O points respectively. The O-point is identified as the center of the island where the flux is maximum. For comparison with Ref.²⁰ it is normalized to the Alfvén velocity v_{Am} and magnetic field B_m at the point between the islands where the magnetic field is maximum, which is around 0.035 in our setup. For the simulation R0, the island size is $\lambda = 4.77d_i$. The iPIC3D simulation shows a peak reconnection rate of $E_R = 0.475$. This is close to the peak reconnection rate observed in PIC simulation of island size $\lambda = 5d_i$ in Ref.²⁰ which was $E_R = 0.435$. This indicates that

our reconnection rate is identically normalized and the simulation is verified. Fig. 9 shows the time evolution of the reconnection rate in the R1 simulation for the 4 different models. The Hall-MHD shows a larger reconnection rate compared to MHD, which itself shows a larger reconnection rate than the CMAP and PIC simulations. This is consistent with the observations in Ref.²⁰. The peak Hall-MHD value is more than twice the peak reconnection rate in pure PIC simulations. On the other hand, the CMAP produces a peak reconnection rate which is closer to the pure PIC value, but still 28% higher than it. The MHD and Hall-MHD codes show a non-zero reconnection rate in the beginning, while the CMAP and PIC simulations start from zero. This is because the MHD and Hall-MHD simulations are done with a finite resistivity which leads to resistive reconnection from the start while the resistivity is kept zero in CMAP simulations in order to match the PIC simulations. If we reduce the value of explicit resistivity in MHD and Hall MHD simulations then the initial reconnection rate also reduces. The time behavior of the reconnection rate is also similar between CMAP and PIC, where it peaks and then remains steady up to $1.2\tau_A$, whereas for MHD and Hall-MHD the reconnection rate drops sharply after the peak. We have found that the CMAP reproduces the PIC dynamics properly if we do not take the PIC domain to be too small. We found that this 16% area coverage worked well for this problem, where if we take the PIC feedback smaller, then the reconnection rate behavior starts behaving more like MHD. With this much PIC area coverage, the average reconnection rate in the CMAP model is within 20% of the PIC values, whereas for smaller PIC-feedback domains, the reconnection rate diverges more⁴⁰.

Fig. 10(a) plots the peak reconnection rates observed across the 5 different island size simulations for the three models MHD, Hall-MHD, and CMAP. Fig. 10(b) plots the corresponding average reconnection rates to see the scaling of the reconnection rates with system size. The average reconnection rate is taken between $t = 0.5\tau_A$ and $t = 1.5\tau_A$. For the smallest size simulation R0, the difference between CMAP and PIC reconnection rates is somewhat large, but for R1 this difference reduces, indicating that CMAP might work well at larger systems sizes. We find that MHD and Hall-MHD show similar scaling of the reconnection rate and their reconnection rates become closer as the system size increases. On the other hand, the CMAP simulation shows a steeper fall in the reconnection rate with system size. This is similar to the scaling observed in Ref.²⁰ upto island sizes $\lambda = 25d_i$. In CMAP simulations R0, R1, and R2 going up to island size $\lambda = 19d_i$ the average reconnection rate

FIG. 10. (a) Scaling of peak reconnection rate with system size, (b) Scaling of average reconnection rate with system size. These are given for the MHD, Hall-MHD and CMAP simulations, with PIC results for the two smallest system sizes R0 and R1.

drops close to the $(\lambda/d_i)^{-0.8}$ scaling. However beyond that for simulations R3 and R4, it seems to flatten and stay at around 0.09. In Ref.¹⁹ the peak raw reconnection rate and the average reconnection rate are reported. There the peak reconnection rate decreases going from island sizes $5d_i$ to $15d_i$. From $15d_i$ to $100d_i$ the peak reconnection rate stays close to 0.2. This is similar to what we observe for island sizes from $19d_i$ to $38d_i$ where the peak reconnection rate in CMAP simulations are also close to 0.2. The average reconnection rate in our simulations are also staying close to 0.1 for $\lambda > 20d_i$ whereas they drop further in Ref.¹⁹ as the island grows above $20d_i$. One difference is that we take time average for $1\tau_A$ around the peak reconnection time while in Ref.¹⁹ it is taken for $2\tau_A$. We recalculated

the average reconnection rate taking the average over $1.5\tau_A$. This shows some drop in the average reconnection for the largest island size also. This might be due to the bouncing of magnetic islands which might be affecting the average reconnection rate, since islands bounce further apart in the larger system sizes.

These simulations show that the reconnection rate in large collisionless systems might become independent of system size. This has been seen in other simulations also⁴⁹. It has also been observed that reconnection rate in a variety of kinetic simulations shows a value of 0.1 in units of the inflowing plasma Alfvén speed, while the outflow is at Alfvén speed. This has been linked with a similar aspect ratio of the reconnecting current sheet across these different simulations. Although the outflowing plasma velocity is not close to Alfvénic in our coalescence simulations with no guide field³⁹, we see a hint of something similar in CMAP simulations where the current sheet aspect ratio (measured at the time of peak reconnection, Fig. 7(c)) tapers off at larger island sizes and so does the peak reconnection rate. This needs to be explored more in future work by going to larger system sizes.

V. CONCLUSIONS

We have presented simulations of the magnetic island coalescence problem with a novel CMAP model which spatially couples global MHD simulations with embedded PIC region. This coupling is shown to work well for this problem setup, with its main advantage being that it saves computing time significantly compared to simulating the whole domain with a kinetic model. For the cases shown here the CMAP simulation was typically 5 times cheaper than the kinetic model, the speedup being faster for smaller PIC domains. However, we found that the PIC feedback has to cover at least 16% of the total simulation domain in order to produce reconnection rates close to the fully kinetic results. This seems to suggest that kinetic physics is important in a much larger range beyond the immediate diffusion zone. Simulations with MHD and Hall-MHD modules of MPI-AMRVAC are also presented alongwith a couple of full PIC simulations.

We find similarities with earlier work: the kinetic simulation is quite different from the Hall-MHD results. The out-of-plane Hall magnetic field is stronger in the CMAP model compared to Hall-MHD simulations. The current sheet structure is much larger in the MHD model, while it becomes smaller in Hall-MHD. On the other hand, CMAP model

shows a different X-point like structure of the current sheet which matches very well with PIC results. Magnetic island sloshing is observed in the CMAP model for large system sizes, which is not seen in the fluid simulations. This also leads to a chaotic behavior of the reconnection zone, with the current sheet strengthening, then weakening, and in general moving around. The current sheet geometrical characteristics show that the current sheet thickness in MHD and Hall-MHD increases with system size. On the other hand, kinetic current sheets remain at ion skin depth scale independent of system size. This corroborates with other results of kinetic scale current sheets^{46,47} which have their thickness close to the skin depth scale. However this does not necessarily imply that the ion physics is setting the current sheet thickness. Other studies have identified the electron meandering length scale⁴⁸ as a possible physics setting current sheet thickness. We calculate the electron meandering length scale by calculating the distance from the X-point along the x axis where the electron thermal gyroradius becomes comparable to this distance. This meandering length scale is around $1d_e$, giving a current sheet thickness of $2d_e$. However due to the limited mass ratio and resolution, it is difficult to separate clearly this length scale from the ion-skin depth. Further scaling studies with higher mass ratios can clarify this issue. The MHD and Hall-MHD also show that the lengths of the current sheets grow almost linearly with the system size, whereas the lengths in the CMAP simulations grow weakly. As a result of this, the aspect ratio of the current sheet very closely matches between the three different simulation models. This aspect ratio increases as system size gets larger. However, for the largest system size, the CMAP model shows that the aspect ratio steadies to between 10 and 20.

The CMAP model allows us to calculate the reconnection rates for large system sizes. The reconnection rate is also calculated for the different simulations and their scalings are shown. Initially, the CMAP reconnection rate drops rapidly as system size gets larger, compared to the drop in the reconnection rate shown by the fluid models. However, for islands larger than $\lambda > 20d_i$, the peak reconnection rate steadies at a value close to 0.2. The average reconnection rate also shows a steady value, although this may depend on the length of time averaging. This work indicates that the aspect ratio of the reconnection zone in kinetic simulations becomes constant at large enough system sizes, giving a similar maximum reconnection rate for different system sizes. Future work with even larger simulations will be needed to answer this.

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