A Boundary Integral Equation Scheme for Simulating the Nonlocal Hydrodynamic Response of Metallic Antennas at Deep-Nanometer Scales

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Abstract—Modeling the interaction between light and a plasmonic nanoantenna, whose critical dimension is of a few nanometers, is complex owing to the “hydrodynamic” motion of free electrons in a metal. Such a hydrodynamic effect inevitably leads to a nonlocal material response, which enables the propagation of longitudinal electromagnetic waves in the material. In this paper, within the framework of a boundary integral equation and a method of moments algorithm, a computational scheme is developed for predicting the interaction of light with 3-D nonlocal hydrodynamic metallic nanoparticles of arbitrary shape. The numerical implementation is first demonstrated for the test example of a canonical spherical structure. The calculated results are shown to be in the excellent agreement with the theoretical results obtained with the generalized Mie theory. In addition, the capability of treating 3-D structures of general shapes is demonstrated by ellipsoids and dimers.

Index Terms—Computational electromagnetics (EMs), deep-nanometer nanoantennas, nonlocal material response, plasmonics.

I. INTRODUCTION

THE material characteristics of metals vary with a frequency over the entire electromagnetic (EM) spectrum, and this variation must be accounted for when modeling the EM response of an antenna. For example, it is well known that, at microwave frequencies, free electrons in a metal are highly mobile. Hence, they respond to an external EM perturbation almost immediately, which leads to a very large conductivity and a vanishingly small skin depth (compared to the thickness of the antenna under consideration). As a result, metal-based microwave antennas can be treated by

using surface currents [1], as opposed to volume currents that are needed to handle.

As the frequency increases to the optical regime, the free electrons are too inertial to follow the external field and there is a considerable temporal delay between the material response and the field [2]. This effect can be captured by using the Drude model, and its extensions which account for the material frequency dispersion in the response, by utilizing a frequency-dependent relative permittivity $\varepsilon(\mathbf{r}, \omega)$. This is in contrast to the microwave case where the conductivity of the metal is almost frequency independent. Also, at optical frequencies, the field can penetrate into the antenna, since the skin depth is comparable to the dimensions of a nanoantenna (around tens of nanometers). Consequently, the current flowing in the antenna must be modeled as a volume current [3]. Hence, the modeling of a plasmonic nanoantenna is not fundamentally different from the modeling of a dielectric antenna [4], [5].

If we move to even higher frequencies (e.g., ultraviolet or above) or if we consider a nanoantenna whose characteristic length is on the order of a few nanometers or less, we need to trigger new physical considerations beyond the local Drude model. Specifically, a crucial observation is that the metal responds to an external field nonlocally in the spatial domain and its permittivity must be expressed as $\varepsilon(\mathbf{r}, \mathbf{r}', \omega)$, which not only depends on the field point $\mathbf{r}$ but also on the source point $\mathbf{r}'$, in contrast to the local case where the permittivity $\varepsilon(\mathbf{r}, \omega)$ is only a function of the field point $\mathbf{r}$. In the Fourier-transformed space, this is equivalent to saying that there is a spatial dispersion in the response of the material. Physically, the nonlocality is attributable to the fact that the free electrons begin to exhibit a quantum nature [6] as the critical size of the nanoantenna enters the deep-nanometer (i.e., a few nanometers) regime. The effects of the nonlocality have been demonstrated to be very relevant for the emerging research domain of quantum plasmonics [7] and its related applications, for example, chemistry [8], biology [9], and energy [10].

Rather than employing a full quantum mechanical treatment (e.g., the time domain density functional theory approach, i.e., time-dependent density functional theory [11]), the complex dynamics of the free-electron gas could be captured to the first order by a semiclassical nonlocal hydrodynamic model (HDM) [12]. The HDM has been successfully applied

Manuscript received October 16, 2017; revised March 12, 2018; accepted April 4, 2018. Date of publication June 28, 2018; date of current version August 31, 2018. The work of X. Zheng and M. Kupresak was supported by Katholieke Universiteit Leuven under Contract 3E140402 and Contract 3E151025. The work of X. Zheng, M. Kupresak, and G. A. E. Vandenbosch was supported by the C2 Project (C24/15/015) of KU Leuven and Research Foundation–Flanders (FWO). (*Corresponding author: Xuezhi Zheng.*)

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Digital Object Identifier 10.1109/TAP.2018.2851290

Fig. 1. Illustration of (a) interaction of EM waves with matter and (b) main equations which control the physics of the interaction.

to explain, for example, the frequency shifts and broadening in the plasmonic resonances [13], the modification of the field enhancement [14]–[16], and the extra peaks above the plasma frequency [17]. In addition, the HDM serves as a starting point for the study of the nonlinear effects in metal-based nanoantennas [18]–[20].

Specifically, in the context of the HDM, two physical phenomena should be considered (see Fig. 1): 1) the dynamics of EM fields and 2) the mechanical motion of free electrons in a metal. The former is described by Maxwell's equations, while a hydrodynamic equation is used to model the latter. These two physical phenomena are coupled through the fact that the EM fields drive the motion of free electrons (to induce free-electron polarization currents), while the induced free-electron motion reradiates and affects the total EM fields.

Much effort has been devoted to solve the coupled system in a self-consistent manner. On the one hand, analytical solutions could be derived for some canonical geometries, e.g., spheres and cylinders, by using the generalized Mie theory [21]–[23] and, for dimer, by using transformation optics [24]. On the other hand, for structures with arbitrary geometries, the coupled system can be solved numerically via differential equation (DE)-based methods, e.g., the finite-element method (FEM) (the use of COMSOL in [25] and [26]), the finite-difference time-domain method [27], and the discontinuous Galerkin method [28]. However, as with any DE-based algorithm, all the above mentioned methods involve the step of discretizing a certain region in the spatial domain, which demands extensive computational resources, and this can be very challenging for generic 3-D geometries. In contrast to this, it is well known that integral equation (IE)-based methods focus on the surface or volume representation of the induced current of a scatterer and they provide an efficient alternative to the DE methods. In the past, a volume IE simulation has been pursued in [29], instead of focusing on the volume of a scatterer in [30], while a boundary IE (BIE) formalism has been presented for the case of general 2-D nanowire systems by applying the surface equivalence principle [31].

In this paper, we develop a general modeling scheme to solve the coupled system for generic scatterers with arbitrary 3-D geometries. In contrast to some recent work involving quasi-static limits ([32], as well as to the nonretarded BIE model in the supplementary information of [33]), the full

dynamics of the EM waves is considered in this paper. The scatterer is assumed to be located in a homogeneous medium. However, this limit will be removed in our future publications on this topic, based on our “work in progress.”

This paper is organized as follows. We begin by discussing the principle that controls the dynamics of EM fields and free electrons in a homogeneous space. In common with the treatment in [30], the discussion first emphasizes on the separation of the transverse and longitudinal field components and the vector wave equations of EM fields. However, instead of focusing solely on this commonly used field-based formalism, we develop a potential-based representation (comprising vector potential and scalar potential), which forms the theoretical basis of the proposed computational scheme. In addition, we pay a close attention to the behavior of the EM waves at the interface between the local and nonlocal media. Next, the BIE formalism is derived for the governing DEs by using the Huygens principle and adopting the concept of fictitious boundary sources [34], [35]. Finally, the derived BIEs are solved numerically by using the method of moments (MoM) algorithm [36]. The numerical results are first compared with the analytical ones for both the near- and far-field quantities for a canonical structure, namely, a sphere. Next, the scattering (or extinction) cross sections of ellipsoids and dimers are calculated to demonstrate the capability of the modeling scheme to treat objects with arbitrary 3-D shapes, and not just canonical ones which lend themselves easily to analytical treatments.

It is worth noticing that although this paper focuses on a nonlocal HDM; the proposed modeling scheme can be readily extended to a much finer model, e.g., the generalized nonlocal optical response (GNOR) model proposed in [25], since the GNOR shares the same mathematical structure with the HDM. Moreover, it is interesting to see that although the current work is developed in the context of quantum plasmonics, the BIE formalism may also be extended and applied to model the interaction of EM waves with materials that behave similar to plasma waves, e.g., microwaves with warm plasma [37], microwaves with spatially dispersive metamaterials, and isotropic connected wire media [38], [39].

II. GOVERNING EQUATIONS

In this section, we present the theoretical background for the dynamics of EM waves propagating in a space, which is homogeneously filled with a nonlocal hydrodynamic metal. Before going into the details, we enunciate several conventions that we use in this paper. We conduct our discussion in the frequency domain, adopt time convention $e^{-i\omega t}$ where ω is the angular frequency, and we suppose that the variable ω unless otherwise mentioned is systematically suppressed. Furthermore, instead of the International System of Units system, we choose the Gaussian unit system to eliminate the two basic quantities, viz., the vacuum permittivity ϵ_0 and the vacuum permeability μ_0 . A conversion between the two unit systems can be found online [40]. Note that since a metal exhibits only very small magnetic effects at optical frequencies, we assume that its relative permeability is one.

A. Helmholtz Decomposition of a Vector Field

The Helmholtz theorem can be stated as follows: the divergence and curl of a field quantity $\mathbf{F}$, together with the boundary condition, define $\mathbf{F}$ uniquely. For instance, the field $\mathbf{F}$ can be uniquely decomposed into a transverse component $\mathbf{F}^T(\mathbf{r})$ and a longitudinal component $\mathbf{F}^L(\mathbf{r})$, as follows:

$$\mathbf{F}(\mathbf{r}) = \mathbf{F}^T(\mathbf{r}) + \mathbf{F}^L(\mathbf{r}). \quad (1)$$

In (1), the superscripts T and L denote the transverse part whose divergence is zero and the longitudinal part whose curl is zero. Mathematically, $\mathbf{F}^T$ and $\mathbf{F}^L$ satisfy the following two relations, respectively:

$$\nabla \cdot \mathbf{F}^T(\mathbf{r}) = 0, \quad \nabla \times \mathbf{F}^T(\mathbf{r}) \neq 0 \quad (2)$$

$$\nabla \cdot \mathbf{F}^L \neq 0, \quad \nabla \times \mathbf{F}^L = 0. \quad (3)$$

B. Dynamics of Electromagnetic Fields

The dynamics of the EM fields are controlled by Maxwell's equations

$$\nabla \cdot \mathbf{D}(\mathbf{r}) = 4\pi\rho_{\text{ext}}, \quad \nabla \cdot \mathbf{B}(\mathbf{r}) = 0$$

$$\nabla \times \mathbf{E}(\mathbf{r}) - ik\mathbf{B}(\mathbf{r}) = 0, \quad \nabla \times \mathbf{H}(\mathbf{r}) + ik\mathbf{D}(\mathbf{r}) = \frac{4\pi}{c}\mathbf{j}_{\text{ext}}(\mathbf{r}). \quad (4)$$

In (4), $\mathbf{E}$ and $\mathbf{D}$ are the electric field and the electric displacement field, respectively, $\mathbf{B}$ and $\mathbf{H}$ are the magnetic flux and the magnetic field, respectively, ρ_{ext} and $\mathbf{j}_{\text{ext}}$ are the impressed external charge and external current, respectively, and k and c are the free space wavenumber and the speed of light, respectively. The application of the Helmholtz decomposition to (4) gives two separate systems. For the transverse field component, the governing equations are

$$\begin{aligned} \nabla \cdot \mathbf{D}^T(\mathbf{r}) &= 0, \quad \nabla \cdot \mathbf{B}^T(\mathbf{r}) = 0 \\ \nabla \times \mathbf{E}^T(\mathbf{r}) - ik\mathbf{B}^T(\mathbf{r}) &= 0, \quad \nabla \times \mathbf{H}^T(\mathbf{r}) + ik\mathbf{D}^T(\mathbf{r}) \\ &= \frac{4\pi}{c}\mathbf{j}_{\text{ext}}^T(\mathbf{r}). \end{aligned} \quad (5)$$

While for the longitudinal component, the corresponding equations are

$$\begin{aligned} \nabla \cdot \mathbf{D}^L(\mathbf{r}) &= 4\pi\rho_{\text{ext}}, \quad \mathbf{B}^L(\mathbf{r}) = 0 \\ \nabla \times \mathbf{E}^L(\mathbf{r}) &= 0, \quad ik\mathbf{D}^L(\mathbf{r}) - \frac{4\pi}{c}\mathbf{j}_{\text{ext}}^L(\mathbf{r}) = 0. \end{aligned} \quad (6)$$

C. Dynamics of Bound Electrons and Free Electrons in Metals

In metals, two types of electrons may be distinguished: the bound electrons and the free electrons. Roughly speaking (more rigorous discussions can be found in [2]), the bound electrons are assumed to be more confined to the nuclei of atoms and, hence, can respond to external EM fields only locally. In contrast to this, the free electrons that are less bounded to the nuclei can move around the entire nanoantenna and can, therefore, respond to the external EM fields nonlocally.

In response to the external EM perturbation, we can define two types of currents, namely, the bound electron and free-electron polarization currents. They follow the constitutive relations, which can be found in [30] for instance, and which reads

$$\mathbf{P}_{bd}(\mathbf{r}) = \chi_{bd} \cdot \mathbf{E}(\mathbf{r}) \quad (7)$$

$$\frac{\beta^2}{\omega^2 + i\omega\gamma} \nabla(\nabla \cdot \mathbf{P}_f(\mathbf{r})) + \mathbf{P}_f(\mathbf{r}) = -\frac{1}{4\pi} \frac{\omega_p^2}{\omega^2 + i\omega\gamma} \mathbf{E}(\mathbf{r}). \quad (8)$$

In (7) and (8), the subscripts bd and f denote the origin of the polarization currents, respectively, the parameters γ and ω_p are the damping and plasma frequencies, respectively, as in the local Drude model, and β is equal to $\sqrt{3/5} v_F$, where v_F denotes the Fermi velocity.

It can be readily seen that due to the assumption of local motion, the susceptibility of the bound electrons in (7) only depends on one single field point $\mathbf{r}$. In contrast to this, the presence of the differential operator $\nabla(\nabla \cdot)$ in (8) makes the free-electron polarization current dependent on all the spatial points in the immediate neighborhood of a spatial point $\mathbf{r}$. This, in turn, mathematically reassures that the nonlocal effects are taken into consideration.

Again, the Helmholtz decomposition is applied to (7) and (8). As a result, the transverse component of the bound electron and free-electron currents read

$$\mathbf{P}_{bd}^T(\mathbf{r}) = \chi_{bd} \cdot \mathbf{E}^T(\mathbf{r}) \quad (9)$$

$$\mathbf{P}_f^T(\mathbf{r}) = \chi_m^T \cdot \mathbf{E}^T(\mathbf{r}) = -\frac{1}{4\pi} \frac{\omega_p^2}{\omega^2 + i\omega\gamma} \mathbf{E}^T(\mathbf{r}). \quad (10)$$

The longitudinal component of the bound electron and free-electron polarization currents are

$$\mathbf{P}_{bd}^L(\mathbf{r}) = \chi_{bd} \cdot \mathbf{E}^L(\mathbf{r}) \quad (11)$$

$$\frac{\beta^2}{\omega^2 + i\omega\gamma} \nabla^2 \mathbf{P}_f^L(\mathbf{r}) + \mathbf{P}_f^L(\mathbf{r}) = -\frac{1}{4\pi} \frac{\omega_p^2}{\omega^2 + i\omega\gamma} \mathbf{E}^L(\mathbf{r}). \quad (12)$$

It should be noticed that (9)–(12) form the governing equations that describe the dynamics of free and bound electrons in metals.

D. Coupling Electrodynamics With Hydrodynamics

The physical systems are then coupled via the electric displacement field, which can be written as

$$\mathbf{D}(\mathbf{r}) = \mathbf{E}(\mathbf{r}) + 4\pi\mathbf{P}_f(\mathbf{r}) + 4\pi\mathbf{P}_{bd}(\mathbf{r}). \quad (13)$$

For the transverse component, a transverse wave permittivity can be defined as

$$\mathbf{D}^T(\mathbf{r}) = \varepsilon^T \cdot \mathbf{E}^T(\mathbf{r}). \quad (14)$$

In (14), the transverse permittivity ε^T is given by

$$\varepsilon^T = \varepsilon_{bd} - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}. \quad (15)$$

For future use, we can also define the permittivity for the bound electron polarization current as

$$\varepsilon_{bd} = 1 + 4\pi\chi_{bd}. \quad (16)$$

After some manipulations, Maxwell's equations for the transverse field in (5) and the constitutive relation in (14) can be combined to yield

$$\nabla \times \nabla \times \mathbf{E}^T(\mathbf{r}) - k^2 \varepsilon^T \mathbf{E}^T(\mathbf{r}) = ik \frac{4\pi}{c} \mathbf{j}_{\text{ext}}^T(\mathbf{r}) \quad (17)$$

$$\nabla \times \nabla \times \mathbf{H}^T(\mathbf{r}) - k^2 \varepsilon^T \mathbf{H}^T(\mathbf{r}) = \frac{4\pi}{c} \nabla \times \mathbf{j}_{\text{ext}}^T(\mathbf{r}). \quad (18)$$

Equations (17) and (18) are the governing wave equations for the transverse EM waves. In a source-free region, the well-known dispersion relation $c^2 k_T^2 = \omega^2 \varepsilon_T$ can be readily recovered, by going to the Fourier-transformed domain.

The governing vector wave equation for the longitudinal waves is

$$ik \mathbf{D}^L(\mathbf{r}) = \frac{4\pi}{c} \mathbf{j}_{\text{ext}}^L(\mathbf{r}). \quad (19)$$

In (19), the electric displacement field is expressed as

$$\mathbf{D}^L(\mathbf{r}) = \varepsilon_{bd} \cdot \mathbf{E}^L(\mathbf{r}) + 4\pi \mathbf{P}_f^L(\mathbf{r}). \quad (20)$$

Note that the electric field in (20) can be expressed in terms of the free-electron polarization current through (12). Then, by combining (12) with (19) and (20) leads to the following equation:

$$\nabla^2 \mathbf{P}_f^L(\mathbf{r}) + (k^L)^2 \mathbf{P}_f^L(\mathbf{r}) = -\frac{\omega_p^2}{\beta^2} \frac{\mathbf{j}_{\text{ext}}^L}{i\omega \varepsilon_{bd}}. \quad (21)$$

In (21), a longitudinal wavenumber k_L is defined as

$$(k^L)^2 = \frac{1}{\beta^2} \left[\omega^2 + i\omega\gamma - \frac{\omega_p^2}{\varepsilon_{bd}} \right]. \quad (22)$$

Equation (22) defines the dispersion relation for the longitudinal EM waves in a source-free region.

It is interesting to notice that the longitudinal electric displacement field vanishes in the absence of the impressed external source, while the longitudinal electric field continues to exist and is related to the longitudinal free-electron polarization current through the following equation:

$$\frac{\varepsilon_{bd}}{4\pi} \mathbf{E}^L(\mathbf{r}) = -\mathbf{P}_f^L(\mathbf{r}). \quad (23)$$

If we regard the longitudinal free-electron polarization current as a "source" term, we readily conclude that the electric field and the polarization current support each other, and they enable the propagation of the longitudinal electric field in a nonlocal hydrodynamic metal. This is very different from the case of the transverse wave where the electric and magnetic fields are coupled and they propagate in a self-sustained manner.

Finally, it is worth mentioning that the longitudinal EM waves do exist in the case of local medium. This has been demonstrated by the vector wave functions in an unbounded medium (see [31, pp. 393–397]) which supports, if we express them in the Cartesian coordinate system, three types of waves: the M waves (TE waves), the N waves (TM waves), and the L waves (longitudinal waves). Especially, the L waves are closely related to the singular behavior of the electric dyadic Green's Function in the source region ([31, pp. 398]). This fact further implies that the longitudinal waves are very relevant

in the near-field region of an antenna. However, they decay quickly as going to the far-field region and, hence, they do not propagate in a source-free local medium environment.

E. Potential Formalism for (17), (18), and (21)

The main field equations, namely, (17), (18), and (21) can be rewritten in terms of potentials. On the one hand, since the transverse magnetic field is divergence-free [as in (5)], the concept of vector potential $\mathbf{A}^T$ and scalar potential φ^T can be introduced similar to the local medium case

$$\mathbf{E}^T(\mathbf{r}) = ik \mathbf{A}^T(\mathbf{r}) - \nabla \varphi^T(\mathbf{r}), \quad \mathbf{B}^T(\mathbf{r}) = \nabla \times \mathbf{A}^T(\mathbf{r}). \quad (24)$$

In (24), the vector and scalar potentials are fixed by imposing the Lorentz gauge

$$\nabla \cdot \mathbf{A}^T(\mathbf{r}) = ik \varepsilon^T \varphi^T(\mathbf{r}). \quad (25)$$

Using (24) and (25) in (5), the vector wave equations [i.e., (17) and (18)] can be recast as a set of the Helmholtz equations as follows:

$$\nabla^2 \varphi^T(\mathbf{r}) + k^2 \varepsilon^T \varphi^T(\mathbf{r}) = 0 \quad (26)$$

$$\nabla^2 \mathbf{A}^T(\mathbf{r}) + k^2 \varepsilon^T \mathbf{A}^T(\mathbf{r}) = \frac{4\pi}{c} \mathbf{j}_{\text{ext}}^T(\mathbf{r}). \quad (27)$$

On the other hand, since the longitudinal field component is in general free of curl, in common with the electrostatic case, scalar potentials can be defined for the longitudinal electric field $\mathbf{E}^L$, the longitudinal free-electron polarization current $\mathbf{P}_f^L$, and the longitudinal component of the impressed external current $\mathbf{j}_{\text{ext}}^L$ and expressed as

$$\begin{aligned} \mathbf{E}^L(\mathbf{r}) &= -\nabla \varphi^L(\mathbf{r}) \\ \mathbf{P}_f^L(\mathbf{r}) &= -\nabla \psi(\mathbf{r}) \\ \mathbf{j}_{\text{ext}}^L(\mathbf{r}) &= -\nabla \tau(\mathbf{r}). \end{aligned} \quad (28)$$

Recognizing the longitudinal nature of the above quantities and directly substituting the definitions in (28) into (21), we get

$$\nabla^2 \psi(\mathbf{r}) + (k^L)^2 \psi(\mathbf{r}) = -\frac{\omega_p^2}{\beta^2} \frac{\tau(\mathbf{r})}{i\omega \varepsilon_{bd}}. \quad (29)$$

From (19) and (20) in a source-free region, the potential of the longitudinal free-electron polarization current can be directly related to the potential of the longitudinal electric field as follows:

$$\psi(\mathbf{r}) = -\frac{\varepsilon_{bd}}{4\pi} \varphi^L(\mathbf{r}). \quad (30)$$

Therefore, in a source-free region, (29) can be rewritten in terms of the scalar potential φ^L

$$\nabla^2 \varphi^L(\mathbf{r}) + (k^L)^2 \varphi^L(\mathbf{r}) = 0. \quad (31)$$

The greatest advantage of the potential formalism consisting of (26), (27), and (29) [or (31) for the source-free case] over the conventional field formalism comprising (17), (18), and (21) is that the DEs in the potential formalism all follow the same mathematical structure, specifically, the Helmholtz equation and, hence, this enables us to develop a unified treatment of the corresponding integral representations, as will be more apparent in Section IV.

Fig. 2. Illustration of the boundary S in between a local medium and a nonlocal hydrodynamic metal. This boundary has a surface normal $\mathbf{n}$ pointing from the internal region (region 1) to the external region (region 2). The EM property of the external region is characterized by a relative permittivity ϵ_2 , while the internal region is by a relative permittivity ϵ_1^T for the transverse wave component and a longitudinal wavenumber k_1^L . Fictitious surface charge and surface current sources may be imposed on the boundary: for the outer side of the boundary, σ_2 and $\mathbf{j}_2$, and for the inner side of the boundary, σ_1^T , σ_1^L , and $\mathbf{j}_1^T$. The imposed sources can be linked with some observation surfaces in the space via Green's functions: for S_0^+ and $G_2(\mathbf{r}_0, \mathbf{r})$ and for S_0^- , $G_1^T(\mathbf{r}_0, \mathbf{r})$, and $G_1^L(\mathbf{r}_0, \mathbf{r})$.

III. TWO-REGION PROBLEM

In this section, the behavior of EM waves is discussed at a boundary interface S , which separates an external region V_2 filled with a local nonmagnetic medium (a dielectric, for instance), from an internal region V_1 filled with a nonlocal hydrodynamic metal (see Fig. 2). It is assumed that the impressed source only exists in the external region.

A. Differential Equations and Interface Conditions

The interface problem in Fig. 2 can be formally handled by using the potential formalism introduced in Section II-E for both the external and internal regions. In the external region, the scalar potential and the vector potential satisfy the Helmholtz equation in a conventional local medium

$$\nabla^2 \varphi_2(\mathbf{r}) + k_2^2 \varphi_2(\mathbf{r}) = -\frac{4\pi \rho_{\text{ext}}(\mathbf{r})}{\epsilon_2} \quad (32)$$

$$\nabla^2 \mathbf{A}_2(\mathbf{r}) + k_2^2 \mathbf{A}_2(\mathbf{r}) = -\frac{4\pi}{c} \mathbf{j}_{\text{ext}}(\mathbf{r}). \quad (33)$$

In the internal region, three DEs have to be satisfied: the following two equations are for the transverse wave:

$$\nabla^2 \varphi_1^T(\mathbf{r}) + (k_1^T)^2 \varphi_1^T(\mathbf{r}) = 0 \quad (34)$$

$$\nabla^2 \mathbf{A}_1^T(\mathbf{r}) + (k_1^T)^2 \mathbf{A}_1^T(\mathbf{r}) = 0 \quad (35)$$

the following equation is for the longitudinal wave:

$$\nabla^2 \varphi_1^L(\mathbf{r}) + (k_1^L)^2 \varphi_1^L(\mathbf{r}) = 0. \quad (36)$$

In (32)–(35), the vector potential and the scalar potential are related through the Lorentz gauge condition

$$\nabla \cdot \mathbf{A}_2(\mathbf{r}) = ik\epsilon_2 \varphi_2(\mathbf{r}) \quad (37)$$

$$\nabla \cdot \mathbf{A}_1^T(\mathbf{r}) = ik\epsilon_1^T \varphi_1^T(\mathbf{r}). \quad (38)$$

In order to connect the vector potentials and the scalar potentials in different regions, a set of interface conditions must be established based on Maxwell's equations in (4)

(see the proofs in the Appendix). Since both the internal and external media are assumed to be nonmagnetic, the magnetic field is continuous across the boundary. The continuity of the normal component of the magnetic field together with the Lorentz gauge condition in (38) forces the vector potential to be continuous. Hence, we have

$$\mathbf{A}_2 = \mathbf{A}_1^T = \mathbf{A}. \quad (39)$$

In (39), the vector potential is assumed to be $\mathbf{A}$.

Next, the continuity of the tangential component of the electric field and that of the vector potential lead to the continuity of the scalar potential. Thus, we have

$$\varphi_2 = \varphi_1 = \varphi_1^T + \varphi_1^L. \quad (40)$$

Turning now to the continuity of the tangential component of the magnetic field, we can invoke the Lorentz gauge condition in (38) to get

$$\frac{\partial \mathbf{A}_2}{\partial n} - \frac{\partial \mathbf{A}_1^T}{\partial n} = ik(\epsilon_2 \varphi_2 - \epsilon_1^T \varphi_1^T) \mathbf{n}. \quad (41)$$

Finally, we assume that the normal component of the free-electron polarization current totally vanishes at the boundary

$$\mathbf{n} \cdot \mathbf{P}_f = 0 \Leftrightarrow \chi_m^T \frac{\partial \varphi_1^T}{\partial n} - \frac{\epsilon_{1,bd}}{4\pi} \frac{\partial \varphi_1^L}{\partial n} = ik\chi_m^T \mathbf{n} \cdot \mathbf{A}. \quad (42)$$

If we utilize the definition for the electric displacement field in (13) and the vanishing free-electron polarization current assumption in (42), the interface condition for the normal component of the electric displacement field becomes

$$\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \mathbf{n} \cdot (\epsilon_2 \mathbf{E}_2 - \epsilon_{1,bd} \mathbf{E}_1) = 0 \quad (43)$$

in terms of the potentials which reads

$$\epsilon_2 \frac{\partial \varphi_2}{\partial n} - \epsilon_{1,bd} \frac{\partial \varphi_1}{\partial n} = ik(\epsilon_2 - \epsilon_{1,bd}) \mathbf{n} \cdot \mathbf{A}. \quad (44)$$

It is evident that, in contrast to the conventional local medium case where the behavior of EM waves is uniquely defined with four interface conditions, the nonlocal medium case requires five boundary conditions instead. This is because of the appearance of the longitudinal wave in the metal, which inevitably results in an extra variable, namely, the scalar potential φ_1^L , as well as an additional wave equation viz., (36). Specifically, (42) is known as the additional boundary condition (ABC) [5]. This interface condition is based on the physical fact that no free electrons can escape from the metal, and this in turns, causes the charge density at the boundary to vanish and, hence, the normal component of the corresponding current is zero. This is very different than the conventional local medium case, for instance, in the sense that for a dielectric–dielectric boundary, a surface charge accumulation always appears, meaning that the “surface charge” concept fails at the deep-nanometer scale. Actually, the concept of “surface charge” implies the accumulation of charges of the same polarity in an infinitely thin region. This is not physical due to quantum pressure effects and induced charge diffusion kinetics [25]. Thus, the charges in reality cannot stay on a single interface but must spread within a certain region close to the surface of the metal (see [25, Fig. 1(c)]).

For large structures, e.g., an antenna operating at microwave frequencies, this region is so thin that the region can be considered as a “surface” and, hence, the use of the concept of “surface charge.” However, for very small structures, the scale of this region is comparable to the size of the structure, which introduces interesting nonlocal effects. We hasten to point out that the ABC is not unique since it is based on pure conjecture, which is merely a model that approximates what happens at the surface of a metal. Various options and even finer descriptions, for instance, the ABC is not needed at all in the density functional theory. The ABC in (42) is adopted here only because it has been employed in many occasions and it yields good results, in general, when compared with experiments at measurements. Other possible ABCs, as well as improved models of this phenomenon will be postponed until a future work.

B. Boundary Integral Equation Formalism

The DEs in (32)–(38) together with the interface conditions given in (39)–(44) uniquely define the vector potential and scalar potentials at all points in space. Furthermore, in this context, we can invoke the well-known Huygens principle for the problem at hand. Here, similar to the case of the local medium [34], [35], a set of fictitious surface charges (σ) and surface currents ($\mathbf{j}$) are introduced at the boundary for both the external and internal regions. Then, solving the system of equations for the surface charges and currents yields the devised field distributions in the external and internal regions in a self-consistent manner.

Next, the potentials are expressed in terms of the fictitious sources. We begin with the scalar potentials in the internal and external regions given by

$$\varphi_1^T = G_1^T \sigma_1^T, \quad \varphi_1^L = G_1^L \sigma_1^L, \quad \varphi_2 = G_2 \sigma_2 + \varphi_2^e. \quad (45)$$

Our next step is to express the directional derivatives of the scalar potentials along the normal of an observation surface S_0 for the internal and external regions as follows:

$$\frac{\partial \varphi_1^T}{\partial n} = H_1^T \sigma_1^T, \quad \frac{\partial \varphi_1^L}{\partial n} = H_1^L \sigma_1^L, \quad \frac{\partial \varphi_2}{\partial n} = H_2 \sigma_2 + \frac{\partial \varphi_2^e}{\partial n}. \quad (46)$$

Following this, the vector potentials in the internal and the external regions are expressed as:

$$\mathbf{A}_1^T = G_1^T \mathbf{j}_1^T, \quad \mathbf{A}_2 = G_2 \mathbf{j}_2 + \mathbf{A}_2^e. \quad (47)$$

Finally, the directional derivatives of the vector potentials along the normal of an observation surface S_0 for the internal and external regions are written as

$$\frac{\partial \mathbf{A}_1^T}{\partial n} = H_1^T \mathbf{j}_1^T, \quad \frac{\partial \mathbf{A}_2}{\partial n} = H_2 \mathbf{j}_2 + \frac{\partial \mathbf{A}_2^e}{\partial n}. \quad (48)$$

In the above equations, a set of linear operators that link source points $\mathbf{r}$ on the boundary S to the observation points $\mathbf{r}_0$ on the observation surface S_0 are defined in terms of the following

equations:

$$G_1^T = \int_S g_1^T(\mathbf{r}_0, \mathbf{r}) d\mathbf{s}(\mathbf{r}), \quad H_1^T = \int_S \mathbf{n}(\mathbf{r}_0) \cdot \nabla_{\mathbf{r}_0} g_1^T(\mathbf{r}_0, \mathbf{r}) d\mathbf{s}(\mathbf{r}) \quad (49)$$

$$G_1^L = \int_S g_1^L(\mathbf{r}_0, \mathbf{r}) d\mathbf{s}(\mathbf{r}), \quad H_1^L = \int_S \mathbf{n}(\mathbf{r}_0) \cdot \nabla_{\mathbf{r}_0} g_1^L(\mathbf{r}_0, \mathbf{r}) d\mathbf{s}(\mathbf{r}) \quad (50)$$

$$G_2 = \int_S g_2(\mathbf{r}_0, \mathbf{r}) d\mathbf{s}(\mathbf{r}), \quad H_2 = \int_S \mathbf{n}(\mathbf{r}_0) \cdot \nabla_{\mathbf{r}_0} g_2(\mathbf{r}_0, \mathbf{r}) d\mathbf{s}(\mathbf{r}). \quad (51)$$

In (49)–(51), the scalar Green’s functions g_2 , g_1^T , and g_1^L share the same functional form, namely,

$$\frac{e^{i\kappa r}}{r}. \quad (52)$$

In (52), κ can accordingly be k_2 , k_1^T , or k_1^L , and r is the distance between an observation point and a source point, i.e., $|\mathbf{r}_0 - \mathbf{r}|$, while $\nabla_{\mathbf{r}_0}$ indicates that the gradient is taken with respect to the observation point $\mathbf{r}_0$. In (45)–(48), φ_2^e and $\mathbf{A}_2^e$ are the external scalar (vector) potentials generated by the impressed sources spanning a volume V in the external region, as follows:

$$\varphi_2^e(\mathbf{r}_0) = \frac{1}{\varepsilon_2} \int_V g_2(\mathbf{r}_0, \mathbf{r}) \rho_2^{\text{ext}}(\mathbf{r}) d\mathbf{v}(\mathbf{r}) \quad (53)$$

$$\mathbf{A}_2^e(\mathbf{r}_0) = \frac{1}{c} \int_V g_2(\mathbf{r}_0, \mathbf{r}) \mathbf{j}_2^{\text{ext}}(\mathbf{r}) d\mathbf{v}(\mathbf{r}). \quad (54)$$

Furthermore, if we move the observation surface S_0 to the boundary S and impose the boundary conditions expressed in (39)–(44), we obtain the following linear system:

$$G_1^T \sigma_1^T + G_1^L \sigma_1^L - G_2 \sigma_2 = \varphi_2^e \quad (55)$$

$$G_1^T \mathbf{j}_1^T - G_2 \mathbf{j}_2 = \mathbf{A}_2^e \quad (56)$$

$$H_1^T \mathbf{j}_1^T - H_2 \mathbf{j}_2 - i\mathbf{k}\mathbf{n}(\varepsilon_1^T G_1^T \sigma_1^T - \varepsilon_2 G_2 \sigma_2) = \frac{\partial \mathbf{A}_2^e}{\partial n} - i\mathbf{k}\mathbf{n}\varepsilon_2 \varphi_2^e \quad (57)$$

$$\begin{aligned} \varepsilon_1^T H_1^T \sigma_1^T - \varepsilon_2 H_2 \sigma_2 - i\mathbf{k}\mathbf{n} \cdot (\varepsilon_1^T G_1^T \mathbf{j}_1^T - \varepsilon_2 G_2 \mathbf{j}_2) \\ = -i\mathbf{k}\varepsilon_2 \mathbf{n} \cdot \mathbf{A}_2^e + \varepsilon_2 \frac{\partial \varphi_2^e}{\partial n} \end{aligned} \quad (58)$$

$$\varepsilon_{1,bd} H_1^L \sigma_1^L - \tilde{\chi}_m^T H_1^T \sigma_1^T + i\mathbf{k}\mathbf{n} \cdot \tilde{\chi}_m^T G_1^T \mathbf{j}_1^T = 0. \quad (59)$$

In (59), $\tilde{\chi}_m^T$ is defined as $4\pi\chi_m$. It can be readily seen that in (55)–(59), there are five equations with five unknowns. Hence, the system of equations is consistent.

IV. NUMERICAL IMPLEMENTATION AND RESULTS

In this section, we consider, as an illustrative example, the interaction of a plane wave with a spherical structure. The linear system of BIEs in (55)–(59) is first numerically solved by using the MoM algorithm. Next, the numerical results for the near field and extinction (or scattering) spectra of the sphere are compared with the analytical solutions from the generalized Mie theory. Following this, we consider a set of ellipsoidal structures and a dimer structure to demonstrate the capability of the solver to treat generic 3-D structures that, unlike the sphere, do not lend themselves to convenient analytical solutions.

A. Numerical Implementation

In the implementation, a linear system of equations whose form is slightly different than (55)–(59) is used to solve for the inner surface charges σ_1^T and σ_1^L . After some basic algebraic manipulations, we can derive the following equations:

$$M_{11}\sigma_1^T + M_{12}\sigma_1^L = K_1 \quad (60)$$

$$M_{21}\sigma_1^T + M_{22}\sigma_1^L = K_2 \quad (61)$$

where we have used the definitions

$$M_{11} = (\varepsilon_1^T \Sigma_1^T - \varepsilon_2 \Sigma_2 + k^2(\varepsilon_1^T - \varepsilon_2)^2 \mathbf{n} \cdot \Delta^{-1} \mathbf{n}) G_1^T \quad (62)$$

$$M_{12} = -(\varepsilon_2 \Sigma_2 + k^2 \varepsilon_2 (\varepsilon_1^T - \varepsilon_2) \mathbf{n} \cdot \Delta^{-1} \mathbf{n}) G_1^L \quad (63)$$

$$M_{21} = (\tilde{\chi}_m^T \Sigma_1^T + k^2 \tilde{\chi}_m^T (\varepsilon_1^T - \varepsilon_2) \mathbf{n} \cdot \Delta^{-1} \mathbf{n}) G_1^T \quad (64)$$

$$M_{22} = -(\varepsilon_{1,bd} \Sigma_1^L + k^2 \tilde{\chi}_m^T \varepsilon_2 \mathbf{n} \cdot \Delta^{-1} \mathbf{n}) G_1^L \quad (65)$$

$$K_1 = -ik\varepsilon_2 \mathbf{n} \cdot \mathbf{A}_2^e + ik\mathbf{n} \cdot \Delta^{-1} \left((\varepsilon_1^T - \varepsilon_2) \frac{\partial \mathbf{A}_2^e}{\partial n} - (\varepsilon_1^T \Sigma_2 - \varepsilon_2 \Sigma_1^T) \mathbf{A}_2^e \right) \quad (66)$$

$$K_2 = ik\mathbf{n} \cdot \Delta^{-1} \tilde{\chi}_m^T \left(\frac{\partial \mathbf{A}_2^e}{\partial n} - \Sigma_2 \mathbf{A}_2^e \right) \quad (67)$$

where

$$\Sigma_1^T = H_1^T (G_1^T)^{-1}, \quad \Sigma_1^L = H_1^L (G_1^L)^{-1}, \quad \Sigma_2 = H_2 G_2^{-1} \\ \Delta = \Sigma_1^T - \Sigma_2. \quad (68)$$

Other fictitious sources can be solved by substituting the solution to (60) and (61) into the following equations:

$$\sigma_2 = G_2^{-1} G_1^T \sigma_1^T + G_2^{-1} G_1^L \sigma_1^L. \quad (69)$$

$$\mathbf{j}_1^T = (G_1^T)^{-1} \Delta^{-1} \left[\frac{\partial \mathbf{A}_2^e}{\partial n} - \Sigma_2 \mathbf{A}_2^e + ik\mathbf{n} \left(\varepsilon_1^T G_1^T \sigma_1^T - \varepsilon_2 G_2 \sigma_2 \right) \right] \quad (70)$$

$$\mathbf{j}_2 = G_2^{-1} \Delta^{-1} \left[\frac{\partial \mathbf{A}_2^e}{\partial n} - \Sigma_1^T \mathbf{A}_2^e + ik\mathbf{n} \left(\varepsilon_1^T G_1^T \sigma_1^T - \varepsilon_2 G_2 \sigma_2 \right) \right]. \quad (71)$$

Since a plane wave source in the external region is purely transverse, the incident scalar potential in (53) is set to be zero for the sake of simplicity and is no longer present in the excitation, as may be seen from (66) and (67).

To solve the system in (60) and (61), the linear operators G 's and H 's defined in (49)–(51) are evaluated by following the conventional MoM procedure. This is done by first discretizing the boundary of the scatterer, for instance, a sphere [see Fig. 3(a)], into N small triangular patches [see Fig. 3(b)]. Due to the fact that the size of the targeted scatterer (usually around a few nanometers) is very small compared to the wavelengths of interests (usually around hundreds of nanometers), we choose pulses for the basis function.

Fig. 3. Illustration of (a) topology under consideration and (b) surface mesh that is used in the simulation.

Next, we consider the coupling between an observation patch (i th patch) and a source patch (j th patch) to get

$$G_{ij} = \int_{\Delta S_i} \int_{\Delta S_j} \frac{e^{ik|\mathbf{r}_0 - \mathbf{r}|}}{|\mathbf{r}_0 - \mathbf{r}|} ds(\mathbf{r}) ds(\mathbf{r}_0) \quad (72)$$

$$H_{ij} = \int_{\Delta S_i} \int_{\Delta S_j} \mathbf{n}(\mathbf{r}_0) \cdot \nabla_{\mathbf{r}_0} \left(\frac{e^{ik|\mathbf{r}_0 - \mathbf{r}|}}{|\mathbf{r}_0 - \mathbf{r}|} \right) ds(\mathbf{r}) ds(\mathbf{r}_0). \quad (73)$$

Extra care must be exercised in evaluating the above expressions when the observation surface approaches the source surface, i.e., when $|\mathbf{r}_0 - \mathbf{r}| \rightarrow 0$. This spatial singular behavior is handled in the same way, as in [41], by writing

$$g(\mathbf{r}_0, \mathbf{r}) = \left(\frac{e^{ikr}}{r} - \frac{1}{r} + \frac{k^2}{2} r \right) + \frac{1}{r} - \frac{k^2}{2} r \quad (74)$$

$$\nabla_{\mathbf{r}_0} g(\mathbf{r}_0, \mathbf{r}) = \nabla_{\mathbf{r}_0} \left(\frac{e^{ikr}}{r} - \frac{1}{r} + \frac{k^2}{2} r \right) + \nabla_{\mathbf{r}_0} \frac{1}{r} - \frac{k^2}{2} \nabla_{\mathbf{r}_0} r. \quad (75)$$

The first term in (74) and in (75) is numerically evaluated by using the Gaussian quadrature, while the rest are calculated by using the integral tables provided in [41].

At this point, the computational advantage of the potential formalism becomes obvious. On the one hand, since the targeted linear operators G 's and H 's share the same mathematical structure, the codes evaluating the main matrices in (72) and (73) have to be implemented only once. Reusing the same code with different parameters, e.g., different wave numbers, i.e., k_1^T , k_1^L , and k_2 can readily generate the required main matrices, which significantly shortens the programming time. On the other hand, the potential formalism is at most strongly singular, while the field formalism contains the $\nabla \times \nabla \times$ operator which introduces a hypersingular behavior in the integrand. Finally, we obtain the fictitious sources and the resultant field quantities by inverting the M matrices in (60) and (61) whose sizes are $2N \times 2N$.

B. Sphere

Three different cases are considered in this section: 1) gold (Au) nanoparticle in the vacuum; 2) silver (Ag) nanoparticle in water; and 3) aluminum (Al) nanoparticle in the vacuum. All the nanospheres have a radius of 1 nm and are discretized by using 648 triangular patches. In all the calculations, the incident field is assumed to be polarized along the x -axis and is an incident direction along the negative

z-axis. The derived physical quantities are all normalized to 1 and are plotted using a common logarithmic scale.

Especially, the material properties of gold (Au) and silver (Ag) are described by the nonlocal Drude model, while the material properties of aluminum (Al) are described by the nonlocal Drude–Lorentz model. The following parameters are employed in the calculation. The plasma frequency ω_p and the damping factor γ in (8) for Au, Ag, and Al are taken to be the same as [42]. The Lorentz oscillators used to fit the bound electron susceptibility in (9) for Al are obtained from [42]. The Fermi velocity v_F for β in (8) is set to 1.40×10^6 m/s for Au, 1.39×10^6 m/s for Ag, and 2.03×10^6 m/s for Al. The applied frequencies are normalized with respect to the plasma frequencies of the materials in these calculations. The relative permittivity of the vacuum and water is, respectively, taken as 1 and 1.7956.

We begin with the evaluation of the extinction cross sections for cases 1) and 3). The extinction cross section is defined in the same manner in [43, eq. (10.139)] as follows:

$$\sigma_{\text{ext}} = \frac{4\pi}{k_2} \text{Im}[\mathbf{e}_0^* \cdot \mathbf{f}(\mathbf{k}_2)]. \quad (76)$$

In (76), $\mathbf{k}_2$ is the wavenumber in the external region, while $\mathbf{e}_2$ and $\mathbf{k}_2$ (in bold) denote, respectively, the unit vector pointing along the polarization of the incident wave, and its wave vector. Also, $\mathbf{f}$ is the amplitude of the scattered electric field, which can be expressed in terms of the external fictitious current source as follows:

$$\mathbf{f}(\mathbf{k}) = ik_0 \cdot \int_S e^{-i\mathbf{k} \cdot \mathbf{r}} \cdot \mathbf{j}_2(\mathbf{r}) d\mathbf{r}. \quad (77)$$

In (77), the integral yields the forward scattered vector potential [43] and $\mathbf{k}_0$ is the wavenumber in the vacuum. Here, we ignore the contribution from the fictitious surface charges because they are only significant in the near-field region of the particle. Fig. 4 shows that the numerical results in both cases agree very well with the theoretical results derived from the generalized Mie theory [21], and the normalized rms error is 5.74% for Au while it is 2.98% for Al.

Next, we emphasize the scattering cross sections for case i2), which is defined as the ratio of the scattered power to the intensity of the incident field

$$\sigma_{\text{sca}} = \frac{\frac{1}{2} \oint_{S_\infty} \mathbf{E}(\mathbf{r}_0) \times \mathbf{H}^*(\mathbf{r}_0) \cdot d\mathbf{s}(\mathbf{r}_0)}{\frac{1}{2}}. \quad (78)$$

In (78), it is assumed that the incident wave carries a unit amplitude. Again, the results from the BEM solver agree well with the theoretical ones derived from the generalized Mie theory [21], with the normalized rms error being 2.54%.

Furthermore, we examine the electric field in the near-field region for cases 1) and 2). For case 1), the electric field at the spatial position (0, 0, 2) nm is evaluated for all frequencies, while for case 2), the same field is evaluated at 150 spatial points that are linearly distributed in between the spatial positions (0, 0, 2) and (0, 0, 10) nm at a single-frequency point, which corresponds to the first resonance in the scattering spectra shown in Fig. 5. For case 1) [see Fig. 6(a)], despite the fact that there are some differences, the normalized

Fig. 4. Comparison between the extinction cross sections between the BEM results (blue solid lines) and the theoretical results (red dashed lines) for (a) Au nanoparticle and (b) Al nanoparticle. The results calculated within the local response material model are plotted in green solid lines as reference.

Fig. 5. Comparison between the scattering cross sections between the BEM results (blue solid lines) and the theoretical results (red dashed lines) for the Ag nanoparticle. The results calculated within the local response material model are plotted in green solid lines as reference.

rms error in the electric field with respect to the theoretical ones derived from the generalized Mie theory [21] is still less than 3%. For case 2) [see Fig. 6(b)], the agreement is excellent with virtually no differences between the two points.

It should be noted that for the sake of completeness, the extinction cross sections, scattering cross sections, and near fields are also evaluated against frequencies within the local response model (green solid lines in Figs. 4–6). There, two main features are observed. First, the main resonances from the local response model are in general redshifted with respect to the ones from the nonlocal response model. It should be noted that in Fig. 5, since the silver particle is positioned in water, the main peak in its local response is so redshifted that it is not visible in the frequency range of interests anymore. Second, due to the spatial dispersion [i.e., the gradient term in (8)], the results from the nonlocal response model demonstrate additional resonances above the plasma frequency.

Finally, in order to demonstrate the performance of the proposed modeling scheme, a weak form formulation of

Fig. 6. Comparison between the near fields from the BEM algorithm (blue solid lines) and the generalized Mie theory (red dashed lines) for (a) Au nanoparticle and (b) Ag nanoparticle. In (a), the results calculated within the local response material model are plotted in green solid lines as reference.

Fig. 7. Comparison between scattering cross sections from the generalized Mie theory (red dashed lines), the BEM solver (blue solid lines), and COMSOL Multiphysics (green solid lines).

the nonlocal HDM was implemented in COMSOL Multiphysics [44]. Therefore, the scattering cross section of a gold sphere (described by the nonlocal Drude model) with a radius of 2 nm in the vacuum is simulated and contrasted with the results from: 1) the generalized Mie theory and 2) the proposed IE algorithm. It can be immediately seen from Fig. 7 that the results from the generalized Mie theory and the BEM algorithm are almost indistinguishable. However, although being able to catch the general trend qualitatively, the results from COMSOL have a significant deviation especially in the high-frequency range (above the plasma frequency).

In terms of computational cost, both the BEM and FEM simulations were conducted on a desktop computer with a CPU, Core i7 4790 (3.6GHz), and a RAM size of 32 GB. COMSOL takes around 28 h and almost all the memory.

Fig. 8. Illustration of extinction cross sections of spheroidal structures. The length of the major axis varies from 1 to 2.5 nm. A step of 0.1 nm is taken. The intensity is in the common logarithmic scale and marked by the yellow and the blue to, respectively, denote the highest and lowest values.

Fig. 9. Illustration of scattering cross sections of spheroidal structures. The length of the major axis varies from 1 to 2.5 nm. A step of 0.1 nm is taken. The intensity is in the common logarithmic scale and marked by the yellow and the blue to, respectively, denote the highest and lowest values.

In the run, the gold particle and its environment (vacuum) are discretized with a tetrahedral mesh (0.25 nm for the particle and 11 nm for the vacuum). Moreover, a five-layer perfect matching layer is added to truncate the space. The simulation includes 500 sampling points in wavelengths/frequencies. However, the IE solver only takes less than 10 min for 150 sampling points and less than 5 GB memory.

C. Ellipsoids

By elongating the radius of the sphere along the z -axis from 1 to 2.5 nm, we can morph the sphere into an ellipsoid with an increasingly longer profile. The ellipsoids are assumed to be filled with the Drude silver (Ag) and immersed in the vacuum. We illuminate the spheroid with an x -polarized plane wave incident from the top, compute the scattering and extinction cross sections, and present them in Figs. 8 and 9.

For a step size of 0.1 nm, the results in Figs. 8 and 9 are seen to be extrapolations of the scattering from a sphere. Two main observations on these results are when we compare them with the spherical cases: 1) the four main resonances continue to be present in both scattering and extinction cross sections and 2) additional spectral features (e.g., extra peaks and valleys)

Fig. 10. Comparison between the scattering cross sections from the BEM algorithm (blue solid lines) and the generalized Mie theory (red solid lines). (a) $\theta = 0^\circ$ and $d = 0.2$ nm. (b) $\theta = 30^\circ$ and $d = 0.5$ nm.

are seen to be present, as expected, since increasing the height of the ellipsoids accommodates more resonant modes. Since the current work focuses on the modeling scheme, the detailed discussions on the physical interpretations of the additional resonances will be deferred to our future work.

D. Dimers

Last but not least, the proposed algorithm is applied to evaluate the scattering cross section of a dimer topology. Here, the dimer is made up of two gold (described by the nonlocal Drude model) spheres which are separated by a gap size of d [see Fig. 10(a) (inset)]. Each sphere has a radius of 1 nm and is sufficiently discretized by 324 triangular patches [see Fig. 10(b) (inset)]. The surrounding of the dimer is assumed to be the vacuum. The incident field is assumed to be a plane wave with an in-plane polarization [see the inset of Fig. 10(a)].

In Fig. 10, the scattering cross sections from the BEM solver are contrasted with the ones calculated from a T-matrix method [45]. On the one hand, it can be readily observed from Fig. 10(a) and (b) that for different gap sizes, although there are some small shifts in the low-frequency range, the results from the BEM algorithm and the T-matrix method almost coincide with each other. On the other hand, with an oblique incident field in Fig. 10(b), the main resonance in Fig. 10(a) splits to two, which physically corresponds to the bonding and antibonding modes in a plasmonic dimer [5].

V. CONCLUSION

In conclusion, the modeling scheme based on the BIE has been proposed in this paper to study the interaction of light with 3-D arbitrarily-shaped nanoantennas at deep nanometer. By distinguishing the transverse from the longitudinal field components, the governing DEs which control the dynamics of the light-matter interaction are presented. These DEs are formulated in terms of: 1) the vector wave equations (field based) and 2) Helmholtz equations (potential based). In the potential formalism, based on Huygens' principle, fictitious boundary sources are introduced to formulate a set of BIEs. Next, the IEs are solved by applying the MoM algorithm. Excellent agreement between numerical and analytical results is realized for canonical-type structures, e.g., a sphere. The scheme's capability of dealing with generic 3-D structures is demonstrated for the ellipsoidal structures and the dimer structure. We believe that the proposed modeling scheme provides

Fig. 11. Illustration of auxiliary geometries in the derivation of the interface conditions. (a) Boundary loop. (b) Pillbox. (c) Boundary line segment pointing from a point P_1 in region 1 to a point P_2 in region 2. (d) Boundary line segments used in the proof of (A9) and (A10).

an efficient and effective approach for modeling the light-matter interaction in the research domain of nanophotonics and nanoplasmonics and, hence, can become an essential element in paving a path for engineering the propagation of light at deep-nanometer scale.

APPENDIX

In this appendix, the interface conditions presented in (39)–(44) are derived. Under the sole assumption that all the physical observables, namely, fields, potentials, and so on, must be finite.

First of all, the continuity conditions for the vector and scalar potentials are developed based on the textbook “boundary loop” and “pillbox” methods [see Fig. 11(a) and (b)]. For the vector potential $\mathbf{A}$, we can write

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \text{and} \quad \int_S \mathbf{B} \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\mathbf{l}. \quad (\text{A.1})$$

In (A1), the first integral is carried out, over a small surface area S whose circumference is a loop C , and the Stokes's theorem is applied. By applying the integral form in (A1) to an interface, we establish the continuity of the tangential component of the vector potential, as follows:

$$A_{1t} - A_{2t} = 0. \quad (\text{A.2})$$

Then, integrating both sides of the Lorentz condition with respect to a small volume V , whose boundary is ∂V , and then employing Gauss's theorem leads to

$$\nabla \cdot \mathbf{A} = i k \epsilon \phi \Leftrightarrow \int_{\partial V} \mathbf{A} \cdot d\mathbf{s} = i k \epsilon \int_V \phi dv. \quad (\text{A.3})$$

Under the assumption that the medium is nonmagnetic, we can clearly see that, if we consider a vanishingly small volume and the finiteness of the scalar potential ϕ , (A3) assures the continuity of the normal component of the vector potential at the interface, that is,

$$A_{1n} - A_{2n} = 0. \quad (\text{A.4})$$

We can now combine (A2) with (A4), to establish the continuity of the vector potential in (39).

Using the definition of the scalar potential in conjunction with the gradient theorem, we can get

$$\int_l \mathbf{E} \cdot d\mathbf{l} = ik \int_l \mathbf{A} \cdot d\mathbf{l} + (\varphi_2 - \varphi_1). \quad (\text{A.5})$$

In (A5), l is a line segment which connects two points, say point “1” and point “2.” When the line segment is across an interface [see Fig. 11(c)], (A5) enforces the continuity of the scalar potential in (40), that is,

$$\varphi_2 - \varphi_1 = 0 \quad (\text{A.6})$$

for the vanishing line segment, in view of the finiteness of the electric field (which is piecewise continuous up to the interface), and the finiteness of the vector potential (which is continuous). For the region filled with a nonlocal hydrodynamic metal, the scalar potential is to be taken as the sum of the transverse and longitudinal potentials.

Next, we derive the interface condition for the Lorentz gauge, i.e., for (41). We assume that the $\mathbf{x}y$ plane of the local coordinate system is chosen such that it is aligned with the tangential plane of the interface, and we take the partial derivatives of the vector potentials with respect to x as an example [see Fig. 11(d)]. On both sides of the interface, we have

$$\begin{aligned} \left. \frac{\partial \mathbf{A}_1}{\partial x} \right|_{P_1} &= \lim_{\Delta x \rightarrow 0} \frac{\mathbf{A}_1(P_1 + \Delta x) - \mathbf{A}_1(P_1)}{\Delta x} \\ \left. \frac{\partial \mathbf{A}_2}{\partial x} \right|_{P_2} &= \lim_{\Delta x \rightarrow 0} \frac{\mathbf{A}_2(P_2 + \Delta x) - \mathbf{A}_2(P_2)}{\Delta x}. \end{aligned} \quad (\text{A.7})$$

Furthermore, by enforcing the continuity of the vector potential, we get

$$\begin{aligned} \mathbf{A}_1(P_1 + \Delta x) &= \mathbf{A}_2(P_2 + \Delta x) \\ \mathbf{A}_1(P_1) &= \mathbf{A}_2(P_2). \end{aligned} \quad (\text{A.8})$$

Equation (A8) leads to

$$\frac{\partial \mathbf{A}_1}{\partial x} = \frac{\partial \mathbf{A}_2}{\partial x}. \quad (\text{A.9})$$

Equation (A9) can be viewed as the continuity candidate for the derivatives of the vector potential. A similar proof can be derived for the continuity of the partial derivative with respect to y as well, that is,

$$\frac{\partial \mathbf{A}_1}{\partial y} = \frac{\partial \mathbf{A}_2}{\partial y}. \quad (\text{A.10})$$

We point out that the magnetic field is continuous across the interface, because we have assumed that the material is nonmagnetic. Thus, we have

$$\mathbf{B}_1 - \mathbf{B}_2 = 0 \Leftrightarrow \nabla \times \mathbf{A}_1 - \nabla \times \mathbf{A}_2 = 0. \quad (\text{A.11})$$

Next, we expand the curl in the second equation of (A11) as follows:

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \mathbf{i} - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) \mathbf{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \mathbf{k}. \quad (\text{A.12})$$

By enforcing the continuity of the partial derivatives of the vector potential in (A9) and (A10), and the continuity of the magnetic field, we get

$$\frac{\partial A_{x1}}{\partial z} = \frac{\partial A_{x2}}{\partial z}, \quad \frac{\partial A_{y1}}{\partial z} = \frac{\partial A_{y2}}{\partial z}. \quad (\text{A.13})$$

Note that neither the continuity relations in (A9), (A10), and (A13) nor the Lorentz condition is changed as we cross an interface; hence, we have

$$\left(\frac{\partial \mathbf{A}_1}{\partial z} - \frac{\partial \mathbf{A}_2}{\partial z} \right) - ik(\varepsilon_1 \varphi_1 - \varepsilon_2 \varphi_2) \hat{\mathbf{z}} = 0. \quad (\text{A.14})$$

In (A14), since the local coordinate system is chosen such that it is aligned with the tangential plane of the interface, the unit vector $\hat{\mathbf{z}}$ and the partial derivative with respect to z should be understood, respectively, as the surface normal and the directional derivative along the normal to the interface. For the case where region 1 is filled with a nonlocal hydrodynamic metal. The permittivity ε_1 should be taken as the transverse permittivity ε_1^T since the Lorentz condition only links the transverse vector potential and the transverse scalar potential.

The interface condition for the normal component of the electric displacement field should be derived in conjunction with the ABC, which assumes that the free-electron polarization current vanishes at the boundary. According to the ABC, the electric displacement field for region 1 is only related to the bound electron polarization current. As a result, we have

$$\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = 0 \Leftrightarrow \mathbf{n} \cdot (\varepsilon_2 \mathbf{E}_2 - \varepsilon_{1,bd} \mathbf{E}_1) = 0 \quad (\text{A.15})$$

which can be written as in terms of the potentials

$$\varepsilon_2 \frac{\partial \varphi_2}{\partial n} - \varepsilon_{1,bd} \frac{\partial \varphi_1}{\partial n} = ik(\varepsilon_2 - \varepsilon_{1,bd}) \mathbf{n} \cdot \mathbf{A}. \quad (\text{A.16})$$

Finally, the ABC can be expressed in terms of the potentials as

$$\mathbf{n} \cdot \mathbf{P}_f = 0 \Leftrightarrow \chi_m^T \frac{\partial \varphi_1^T}{\partial n} - \frac{\varepsilon_{1,bd}}{4\pi} \frac{\partial \varphi_1^L}{\partial n} = ik \chi_m^T \mathbf{n} \cdot \mathbf{A}. \quad (\text{A.17})$$

Equation (A17) is used in (55)–(59) to complete the system of equations.

REFERENCES

- [1] C. A. Balanis, *Antenna Theory: Analysis and Design*, 3rd ed. Hoboken, NJ, USA: Wiley, 2006.
- [2] S. A. Maier, *Plasmonics: Fundamental and Applications*. New York, NY, USA: Springer, 2007.
- [3] X. Zheng *et al.*, “Volumetric method of moments and conceptual multilevel building blocks for nanotopologies,” *IEEE Photon. J.*, vol. 4, no. 1, pp. 267–282, 2012.
- [4] X. Zheng, G. A. E. Vandenbosch, and V. V. Moshchalkov, “Understanding the physical behavior of plasmonic antennas through computational electromagnetics,” in *Nanoplasmonics—Fundamentals and Applications*, G. Barbillon, Ed. Rijeka, Croatia: InTech, 2017.
- [5] X. Zheng *et al.*, “Interacting plasmonic nanostructures beyond the quasi-static limit: A ‘circuit’ model,” *Opt. Express*, vol. 21, no. 25, pp. 31105–31118, 2013.
- [6] N. A. Mortensen, “Nonlocal formalism for nanoplasmonics: Phenomenological and semi-classical considerations,” *Photon. Nanostruct.-Fundam. Appl.*, vol. 11, no. 4, pp. 303–309, 2013.
- [7] W. Zhu *et al.*, “Quantum mechanical effects in plasmonic structures with subnanometre gaps,” *Nature Commun.*, vol. 7, Jun. 2016, Art. no. 11495.
- [8] M. E. Stewart *et al.*, “Nanostructured plasmonic sensors,” *Chem. Rev.*, vol. 108, no. 2, pp. 494–521, Feb. 2008.
- [9] J. N. Anker, W. P. Hall, O. Lyandres, N. C. Shah, J. Zhao, and R. P. Van Duyne, “Biosensing with plasmonic nanosensors,” *Nature Mater.*, vol. 7, no. 6, pp. 442–453, Jun. 2008.
- [10] A. Polman and H. A. Atwater, “Photonic design principles for ultrahigh-efficiency photovoltaics,” *Nature Mater.*, vol. 11, pp. 174–177, Feb. 2012.
- [11] M. A. L. Marques, C. Ullrich, F. Nogueira, A. Rubio, K. Burke, and E. K. U. Gross, Eds., *Time-Dependent Density Functional Theory*. Berlin, Germany: Springer, 2006.

- [12] F. Forstmann and R. R. Gerhardt, *Metal Optics Near the Plasma Frequency*. Berlin, Germany: Springer, 2006.
- [13] C. David and F. J. G. de Abajo, "Spatial nonlocality in the optical response of metal nanoparticles," *J. Phys. Chem. C*, vol. 115, no. 40, pp. 19470–19475, 2011.
- [14] G. Toscano, S. Raza, A. P. Jauho, N. A. Mortensen, and M. Wubs, "Modified field enhancement and extinction by plasmonic nanowire dimers due to nonlocal response," *Opt. Express*, vol. 20, no. 4, pp. 4176–4188, 2012.
- [15] G. Toscano *et al.*, "Surface-enhanced Raman spectroscopy: Nonlocal limitations," *Opt. Lett.*, vol. 7, no. 13, pp. 2538–2540, 2012.
- [16] A. Wiener, A. I. Fernández-Domínguez, A. P. Horsfield, J. B. Pendry, and S. A. Maier, "Nonlocal effects in the nanofocusing performance of plasmonic tips," *Nano Lett.*, vol. 12, no. 6, pp. 3308–3314, 2012.
- [17] S. Raza, G. Toscano, A.-P. Jauho, M. Wubs, and N. A. Mortensen, "Unusual resonances in nanoplasmonic structures due to nonlocal response," *Phys. Rev. B, Condens. Matter*, vol. 84, no. 12, p. 121412(R), 2011.
- [18] J. E. Sipe, V. C. Y. So, M. Fukui, and G. I. Stegeman, "Analysis of second-harmonic generation at metal surfaces," *Phys. Rev. B, Condens. Matter*, vol. 21, no. 10, p. 4389, 1980.
- [19] C. Ciraci, E. Poutrina, M. Scalora, and D. R. Smith, "Origin of second-harmonic generation enhancement in optical split-ring resonators," *Phys. Rev. B, Condens. Matter*, vol. 85, no. 20, p. 201403(R), 2012.
- [20] A. Hille *et al.*, "Second harmonic generation from metal nano-particle resonators: Numerical analysis on the basis of the hydrodynamic drude model," *J. Phys. Chem. C*, vol. 120, no. 20, pp. 1163–1169, 2016.
- [21] R. Ruppin, "Optical properties of a plasma sphere," *Phys. Rev. Lett.*, vol. 31, no. 24, p. 1434, 1973.
- [22] R. Fuchs and F. Claro, "Multipolar response of small metallic spheres: Nonlocal theory," *Phys. Rev. B, Condens. Matter*, vol. 35, no. 8, p. 3722, 1987.
- [23] R. Ruppin, "Optical absorption by a small sphere above a substrate with inclusion of nonlocal effects," *Phys. Rev. B, Condens. Matter*, vol. 45, no. 19, p. 11209, 1992.
- [24] A. I. Fernández-Domínguez, A. Wiener, F. J. García-Vidal, S. A. Maier, and J. B. Pendry, "Transformation-optics description of nonlocal effects in plasmonic nanostructures," *Phys. Rev. Lett.*, vol. 108, no. 10, p. 106802, 2012.
- [25] N. A. Mortensen, S. Raza, M. Wubs, T. Søndergaard, and S. I. Bozhevolnyi, "A generalized non-local optical response theory for plasmonic nanostructures," *Nature Commun.*, vol. 5, May 2014, Art. no. 3809.
- [26] K. R. Hiremath, L. Zschiedrich, and F. Schmidt, "Numerical solution of nonlocal hydrodynamic Drude model for arbitrary shaped nanoplasmonic structures using Nédélec finite elements," *J. Comput. Phys.*, vol. 231, no. 17, pp. 5890–5896, 2012.
- [27] M. Fang, Z.-X. Huang, W. E. I. Sha, X. Y. Z. Xiong, and X.-L. Wu, "Full hydrodynamic model of nonlinear electromagnetic response in metallic metamaterials," *Prog. Electromagn. Res.*, vol. 157, pp. 63–78, 2016.
- [28] L. Li, S. Lanteri, N. A. Mortensen, and M. Wubs, "A hybridizable discontinuous Galerkin method for solving nonlocal optical response models," *Comput. Phys. Commun.*, vol. 219, pp. 99–107, Oct. 2017.
- [29] G. Miano, G. Rubinacci, A. Tamburrino, and F. Villone, "Linearized fluid model for plasmon oscillations in metallic nanoparticles," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 822–825, Jun. 2008.
- [30] W. Yan, N. A. Mortensen, and M. Wubs, "Green's function surface-integral method for nonlocal response of plasmonic nanowires in arbitrary dielectric environments," *Phys. Rev. B, Condens. Matter*, vol. 88, no. 15, p. 155414, 2013.
- [31] W. C. Chew, *Waves and Fields in Inhomogeneous Media*. Piscataway, NJ, USA: IEEE Press, 1995.
- [32] A. Trügler, U. Hohenester, and F. J. G. de Abajo, "Plasmonics simulations including nonlocal effects using a boundary element method approach," *Int. J. Mod. Phys. B*, vol. 31, no. 24, pp. 1740007, 2017.
- [33] T. Christensen, W. Yan, A.-P. Jauho, M. Solijačić, and N. A. Mortensen, "Quantum corrections in nanoplasmonics: Shape, scale, and material," *Phys. Rev. Lett.*, vol. 118, no. 15, p. 157402, 2017.
- [34] F. J. G. de Abajo and A. Howie, "Retarded field calculation of electron energy loss in inhomogeneous dielectrics," *Phys. Rev. B, Condens. Matter*, vol. 65, no. 11, p. 115418, 2002.
- [35] U. Hohenester and A. Trügler, "MNPBEM—A Matlab toolbox for the simulation of plasmonic nanoparticles," *Comput. Phys. Commun.*, vol. 183, no. 2, pp. 370–381, 2012.
- [36] R. F. Harrington and J. L. Harrington, *Field Computation by Moment Methods*. London, U.K.: Oxford Univ. Press, 1996.
- [37] K. Balmin, "Impedance of a short dipole in a compressible plasma," *Radio Sci.*, vol. 69, pp. 559–566, Apr. 1965.
- [38] G. W. Hanson, E. Forati, and M. G. Silveirinha, "Modeling of spatially-dispersive wire media: Transport representation, comparison with natural materials, and additional boundary conditions," *IEEE Trans. Antennas Propag.*, vol. 60, no. 9, pp. 4219–4232, Sep. 2012.
- [39] E. Forati and G. W. Hanson, "Scattering from isotropic connected wire medium metamaterials: Three-, two-, and one-dimensional cases," *IEEE Trans. Antennas Propag.*, vol. 61, no. 7, pp. 3564–3574, Jul. 2013.
- [40] *Gaussian Units*. [Online]. Available: https://en.wikipedia.org/wiki/Gaussian_units
- [41] P. Ylä-Oijala and M. Taskinen, "Calculation of CFIE impedance matrix elements with RWG and $n \times$ RWG functions," *IEEE Trans. Antennas Propag.*, vol. 51, no. 8, pp. 1837–1846, Aug. 2003.
- [42] A. D. Rakić, A. B. Djurišić, J. M. Elazar, and M. L. Majewski, "Optical properties of metallic films for vertical-cavity optoelectronic devices," *Appl. Opt.*, vol. 37, no. 22, pp. 5271–5283, 1998.
- [43] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. Hoboken, NJ, USA: Wiley, 1999.
- [44] *COMSOL Product Page*. [Online]. Available: <https://www.comsol.com/>
- [45] J. Bruning and Y. Lo, "Multiple scattering of EM waves by spheres Part I—Multipole expansion and ray-optical solutions," *IEEE Trans. Antennas Propag.*, vol. AP-19, no. 3, pp. 378–390, May 1971.

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