Properties of decaying plasma turbulence at sub-proton scales

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ABSTRACT

Using fully kinetic electromagnetic three-dimensional simulations, with different initial conditions, we study the properties of plasma turbulence at sub-proton scales. Particle-in-cell modeling of the Taylor-Green vortex have been performed, starting from three different magnetic field configurations. All simulations expose very similar energy evolution in which the large-scale ion flows and magnetic structures deteriorate and transfer their energy into particle heating. **Heating is more intense for electrons, decreasing the initial temperature ratio and guiding the two species to have same temperatures.** A full turbulent cascade, with a well-defined power-law shape at sub-proton scales, establishes much faster than the characteristic turnover time in the domain. Spectral indices for magnetic field fluctuations in two simulations are close to $\alpha_B \approx 2.9$, and are steeper in the remaining case with $\alpha_B \approx 3.1$. Energy is dissipated by a complex mixture of plasma instabilities and magnetic reconnection, and is milder in the latter simulation. The number of magnetic nulls, and the dissipation pattern observed in this case, differ from two others. Spectral indices for the kinetic energy deviate from magnetic spectra by ≈ 1 in the first simulation, and by ≈ 0.75 in two other runs. The difference between magnetic and electric slopes confirm previously observed value of $\alpha_B - \alpha_E \approx 2$.

Keywords: particle-in-cell simulations — kinetic turbulence

1. INTRODUCTION

Plasma turbulence observed in the solar wind and in the Earth's magnetosphere is usually characterized by the presence of an inertial range, with a power spectrum of the type $P(k) \sim k^{-\alpha}$ (Bruno & Carbone 2013). At smaller scales, on the order of the ion skin depth and the Larmor radius, a spectral break is evident and a sub-inertial (secondary) range is commonly observed (Alexandrova et al. 2012). This small scale regime is characterized by a magnetic power spectrum with α close either to 7/3 or to 8/3 (Sahraoui et al. 2009; Alexandrova et al. 2009; Sahraoui et al. 2013; Matteini

et al. 2017). Electric field fluctuations are less steep, with $P(k) \sim k^{-0.8}$ (Bale et al. 2005; Eastwood et al. 2009; Matteini et al. 2017). However, deviations from a universal behavior are regularly observed (Tessein et al. 2009; Sahraoui et al. 2010; Šafránková et al. 2013; Huang et al. 2017) and the nature of these anomalies is far from being understood.

Great effort has been devoted to the numerical modeling of small scale plasma turbulence. Kinetic simulations of turbulence driven by the shear flow (Wan et al. 2012), produced by the decaying Orszag-Tang vortex (Parashar et al. 2015), initiated with a long wavelength isotropic spectrum of fluctuations (Wan et al. 2015), or generated by magnetic reconnection (Pucci et al. 2017), have found magnetic and electric spectra consistent with the observations. Little attention, how-

ever, has been dedicated to the dependence of small scale turbulence on the initial conditions and the global properties of turbulence. At large scales, the non-universality of three-dimensional magnetohydrodynamic (fluid) turbulence has been described in Lee et al. (2010), using a modification of the Taylor-Green (TG) flow with various magnetic field combinations. Here we follow the above approach in order to understand the termination of the cascade in collisionless plasmas, at sub-proton scales.

2. MODEL

2.1. Taylor-Green problem

The TG problem represents a fully periodic adaptation of the vortex flow between a pair of counter-rotating disks

$$\mathbf{u} = u_0 \begin{pmatrix} \sin x \cos y \cos z \\ -\cos x \sin y \cos z \\ 0 \end{pmatrix} \quad (1)$$

The z component of velocity is initially null, but the flow quickly develops fluctuations in u_z , producing small-scale eddies, thus establishing a full turbulent cascade (Taylor & Green 1937). The flow is defined in a fully periodic three-dimensional box extending by 2π in each dimension, and in total 8 symmetric vortices are present in the initial state. In fluid modeling, the flow preserves its symmetries relative to the $n\pi$ surfaces as described in (Lee et al. 2008). However, due to particle noise the symmetries can be broken in a particle-in-cell (PIC) simulation.

Following Lee et al. (2010), we initiate our simulations with a magnetized TG flow. Three magnetic field distributions have been studied, denoted as cases 1, 2, and 3. In case 1, defined by

$$\mathbf{B}^{(1)} = B_0^{(1)} \begin{pmatrix} \cos x \sin y \sin z \\ \sin x \cos y \sin z \\ -2 \sin x \sin y \cos z \end{pmatrix}, \quad (2)$$

the magnetic field is perpendicular to the “insulating” symmetry planes. Hereafter, $B_0^{(j)}$ are magnetic field amplitudes for the three cases. In case 2 the magnetic field is parallel to the “conducting” symmetry planes

$$\mathbf{B}^{(2)} = B_0^{(2)} \begin{pmatrix} \sin 2x \cos 2y \cos 2z \\ \cos 2x \sin 2y \cos 2z \\ -2 \cos 2x \cos 2y \sin 2z \end{pmatrix}. \quad (3)$$

Finally, case 3 is also “insulating” but z component of magnetic field is zero

$$\mathbf{B}^{(3)} = B_0^{(3)} \begin{pmatrix} \cos 2x \sin 2y \sin 2z \\ -\sin 2x \cos 2y \sin 2z \\ 0 \end{pmatrix}. \quad (4)$$

These configurations are designed to have zero magnetic cross-helicity $H_c = \langle \mathbf{u} \cdot \mathbf{B} \rangle = 0$. However, the local distribution of $\mathbf{u} \cdot \mathbf{B}$ is initially different: in case 1 the alignment is strictly zero in all points of the domain, while in cases 2 and 3 it gives a zero integral because of annihilation of positive and negative contributions. Phenomenological theories of strong incompressible MHD turbulence, described for example in Grappin et al. (1983); Goldreich & Sridhar (1995), would give the same spectral prediction for the above 3 cases, since H_c is null. However, unlike MHD, our fully kinetic model does not preserve the frozen-in condition, and H_c is not conserved. New theories are needed to describe turbulence in such systems.

2.2. Simulation details

We performed direct numerical simulations of a 3D plasma using the fully kinetic electromagnetic particle-in-cell code iPic3D Markidis et al. (2010). The time has been normalized using the ion (proton) plasma frequency ω_{pi} and scales using the ion inertial length d_i , ion mass m_i , and elementary charge e .

The simulation domain was scaled to $30d_i$ and divided into 384 grid cells in each dimension to fully describe the TG flow, and well resolve the largest proton Larmor radius in the initial field. Each grid cell initially contained 125 uniformly distributed particles of two species, ions and electrons, with the reduced mass ratio of $m_i/m_e = 256$, and a temperature ratio of $T_i/T_e = 5$. Taylor-Green flow was assigned to both species with $u_0 = 0.049c$ (where c is the speed of light), much larger than the thermal speed of ions $u_{th,i} = 2.8 \cdot 10^{-3}c$. Time step $\Delta t = 0.1\omega_{pi}^{-1}$ resolves well the shortest electron gyration time in the initial magnetic field. The magnetic field amplitudes have been chosen as $B_0^{(1)} = B_0^{(2)} = 0.03$, and $B_0^{(3)} = \sqrt{3}B_0^{(1)}$ to have the same initial kinetic and magnetic energies in all cases.

3. RESULTS

3.1. Energy evolution.

Major contributors to the total energy in our kinetic simulations are the magnetic field energy, the bulk kinetic energy of ions (energy of electron bulk flows is negligible), and, at later stages, the thermal energy of particles. No numerical resistivity or viscosity is used, hence the changes in the particle energy are balanced by the dissipated electromagnetic energy, $\partial W_{kin}/\partial t = \mathbf{E} \cdot \mathbf{J} = -\partial W_{mag}/\partial t$. The evolution of different energy components in the three runs is shown in Figure 1. At $t = 0$ energetics is dominated by the ion bulk flows

Figure 1. Evolution of the domain-averaged magnetic (green), electron thermal (red dashed), ion thermal (blue dashed), and ion bulk (blue solid) energy components in 3 cases (from top to bottom).

$W_{bulk,i}$, and the magnetic field energy W_{mag} embedded in the large scale structures.

Abrupt evolution begins with relaxation of the initial pressure imbalance. Tension of the closed magnetic field lines compensates for the imbalance pushing plasma to the low-field regions at the symmetry planes. Similar dynamics was observed already in simulations initiated from non-equilibrium configuration by Olshevsky et al. (2013). At $t\omega_{pi} \approx 90$ in case 1, and somewhat earlier in cases 2 and 3, the large-scale ion flows start to break down, redistributing energy to the smaller scales and increasing the dissipation rate and vorticity in the domain. The turbulent cascade in plasma flow is being established in this phase, with an increase of the thermal energy contributions. The cascade effectively transfers energy of the fluid (bulk) flows and large-scale magnetic structures into particle heating. By the end of the simulation, the ion/electron temperature ratio decreases to $T_i/T_e \approx 3$, hence electrons are heated more efficiently.

Except for a minor hump at $t\omega_{pi} \approx 100$, magnetic energy steadily decreases in cases 2 and 3. In case 1, W_{mag} is stagnant at $t\omega_{pi} > 100$, and even increases by the end of the simulation. It reflects the relaxation of plasma turbulence which drives electromagnetic and kinetic energies to equipartition. Although I have to say, the energies are not going to equipartition. As far as I observed here and in other relaxing simulations with iPic3D, magnetic energy ALWAYS decay, and energy ALWAYS go to particle heating. This might be a feature of the particular code/method, of course. The behavior of magnetic energy in all three PIC simulations is different from the MHD simulations of three-dimensional decaying turbulence (Politano et al. 1995; Lee et al. 2010), where a well-defined initial increase of magnetic energy is typically observed. Interestingly, the three configurations of the magnetized TG flow modeled by (Lee et al. 2010) expose different energy evolutions.

3.2. Dissipation of energy

Different measures of energy dissipation are shown in Figure 2: the domain-averaged squared magnetic vorticity $[\nabla \times \mathbf{B}]^2$, the domain-averaged energy dissipation in the center-of-mass frame $D_{cm} = \mathbf{J} \cdot (\mathbf{E} + [\mathbf{u} \times \mathbf{B}])$, and

Figure 2. Dissipation measures in the three simulations (red - green - blue, correspondingly). (a) Domain-averaged $[\nabla \times \mathbf{B}]^2$ (squared magnetic enstrophy). (b) Domain-averaged energy dissipation in the frame of bulk flow $\mathbf{J} \cdot (\mathbf{E} + [\mathbf{u} \times \mathbf{B}])$. (c) Maximum value of the fluid-frame energy dissipation.

the extrema of D_{cm} . In the first simulation, the peak of energy dissipation at $T^*\omega_{pi} \approx 135$ is delayed in comparison to cases 2 and 3 where $T^*\omega_{pi} \approx 110$. This delay could be attributed to the initial condition. Indeed, in the cases 2 and 3 shorter wavelengths are present in the initial magnetic field configuration Eq. 3, 4. The maximum values of D_{cm} are 4 orders of magnitude larger than its domain-averaged values, hence the dissipation is very intermittent.

The regions of intense energy exchange are adjacent to the symmetry planes where the shear layers form. They are highlighted with red and blue fog-like volume rendering in Figure 3. As the highest values of D_{cm} are detected only in a few cells, the color scale is saturated by ~ 100 times in comparison to the peak values in Figure 2c. The probability distribution of D_{cm} (not pre-

Figure 3. Simulation snapshots in three cases at peak dissipation, from left to right: case 1, $t\omega_{pi} = 135$; case 2 and 3, $t\omega_{pi} = 110$. Symmetry planes are outlined with black lines. Magnetic topology is depicted by field lines traced from random locations (grey), and magnetic nulls (cyan spheres). Energy exchange in the center-of-mass frame $\mathbf{J} \cdot (\mathbf{E} + [\mathbf{u} \times \mathbf{B}])$ is shown with blue and red volume rendering. Note, the colors are saturated to seize wider regions of intense dissipation.

Figure 4. The evolution of the total number of magnetic nulls in the three simulations (solid lines), and the percentage of the spiral nulls (dashed lines).

sented here) is very far from a bell curve. In many locations of intense energy exchange, the regions of positive and negative D_e interchange and form wave-like patterns signalling of an instability worth to be addressed in more details elsewhere.

Intense energy exchange regions in cases 1 and 2 are also attributed with a large number of magnetic null points, believed to be the preferred locations for magnetic reconnection. We used the method of topological degree to locate magnetic nulls in our simulations, as described in Olshevsky et al. (2016). The latter study suggests that energy release does not always accompany the locations of magnetic nulls in PIC simulations. Indeed, snapshots of cases 2 and 3 don't expose such a clear correlation. In case 1, however, nulls clearly align with the current sheets at the symmetry planes. The total number of magnetic nulls in this simulation increases just before the moment of peak dissipation, and relative number of spiral nulls drops to 50%. This is caused by breeding and decay (reconnection) of non-spiral (radial) magnetic nulls at the low-field regions adjacent to the symmetry plane current sheets. Later on, the percent-

age of spiral nulls increases but is still lower than in cases 2 and 3, resonating with lower magnetic energy decay rate in case 1 (Fig. 1a). Correlation of intensified energy dissipation with the locations of spiral magnetic nulls in the magnetic reconnection diffusion region was recently observed by Fu et al. (2017). Small-scale localized magnetic reconnection events may play an essential role in the process of energy dissipation in our simulations. The number of nulls detected during the peak of dissipation in case 2 is almost three times lower than in two other simulations, which hints to the very high intermittency of such dissipation, and the failure to find very high values of D_{cm} in this simulation (see Fig. 2c).

3.3. Spectral evolution

At $t = 0$ most energy is contained in large scale vortices and magnetic structures. Particle noise, common in PIC simulations, is higher at the smallest scales, and produces a plato in the right-hand wing of the kinetic energy (center-of-mass velocity fluctuations) spectrum shown in Figure 5. This noise is not present in the spectra of magnetic or electric field oscillations due to the dedicated smoothing procedure. The smoothing is needed to stabilize the numerical solution, and consists in weighted averaging of the electric field and current densities on a five-point stencil as described in, e.g., Birdsall & Langdon (1991). The anisotropic magnetic field and kinetic energy spectra shown in Figure 5 were computed by integration over spherical shells in three-dimensional Fourier space. The oscillations between even and odd wavenumbers, salient in cases 1 and 3, are possibly caused by interferences of the shear layers adjacent to insulating walls. Lee et al. (2008) applied two-point running average to filter these oscillations out. However, such filtering affects the estimated spectral slopes, therefore we don't use it here. As the simula-

tion evolves, the energy is drained from large scales to feed the small scales by nonlinear transfer, until the full cascade is established, and the spectra of B and u obtain their ‘conventional’ monotonic forms (except for the aforementioned plateau in the kinetic energy). This process may be triggering observed plasma instabilities and magnetic reconnection events in the high-dissipation regions of the computational domain.

The time needed to establish the full cascade $T_*\omega_{pi} \sim 100$ is much shorter than the fluid turnover time in the domain, $\pi L/u_0 \approx 2000$. **This indicates the important difference between the fluid turbulence and plasma turbulence at kinetic scales.** In case 1, the initial magnetic configuration varies on the box scale, while in two other cases it varies at twice shorter wavelengths, which delays the establishment of the full cascade in the first simulation to $T_*\omega_{pi} \approx 135$. When the cascade is established, the relaxing turbulence becomes, for a short time, representative of a full-scale plasma system where energy is constantly transferred from large (fluid) scales to small (kinetic) scales and dissipates into heat. Energy dissipation during this time reaches its maximum as shown in Fig 2. Our simulations fully resolve the dynamics between the two well defined scales, ion and electron inertial lengths $d_i = 16d_e$. In the remainder of this paper we investigate the spectral slopes within this interval of wavenumbers, henceforth referred to as sub-proton scale. Note, our models do not include a strong background (guide) magnetic field, therefore gyro-scales may only be estimated locally and can’t be used to characterize the spectral ranges.

Observations in the solar wind and the Earth’s magnetosheath suggest that magnetic field fluctuations spectrum exposes a well defined steepening (break) at proton scales (Sahraoui et al. 2009), with the second break at electron scales (Sahraoui et al. 2013), indicating the changes of dominant energy dissipation mechanisms. Both breaks are present in our simulations and are visible in Fig. 5 as well. Power-law fit is the traditional, simplest way of interpretation of the inertial range of fluid turbulence spectrum $W(k) = Ck^{-\alpha}$. Observations suggest that power-law fit is also eligible to represent a sub-proton range of plasma turbulence, giving $\alpha \approx 8/3 \div 9/3$ (Sahraoui et al. 2010; Alexandrova et al. 2012).

We focus on the spectra of velocity, magnetic and electric field fluctuations, averaged over several simulation snapshots at the period of peak dissipation, shown in Figure 5. To find out, whether a power-law fit is a good representative of the spectrum, we compute the ‘running slope’, i.e., the centered finite-difference derivative of the spectrum in logarithmic scale, at each discrete $\log(k)$. Inspection of the running slope plots in Figure 5 suggests

Table 1. Spectral best-fit parameters.

<i>case</i>	$T_*\omega_{pi}^a$	α_u	α_B	α_E
1	135	1.87	2.86	1.00
2	110	2.30	3.05	0.96
3	110	2.11	2.86	0.74

^aTime moment of peak dissipation.

that the interval $3 < kd_i < 12$, could be well represented by a horizontal line in the corresponding plots, despite the noise caused by the aforementioned resonances at the symmetry planes. This range of wavenumbers was chosen to compute the nonlinear least-squares best-fits to the spectra, shown in Figure 5. The spectral slopes α are summarized in Table 1.

The magnetic field spectra are similar in cases 1 and 3, with $\alpha_B \approx 2.9$. In the second case, the spectrum is steeper, with $\alpha_{B,3} \approx 3.1$, in accordance with different dissipation patterns, and the presumable suppression of magnetic reconnection in this simulation. Interestingly, the difference $\alpha_B - \alpha_E \approx 2$ (with account for a large fitting error in $\alpha_{3,E}$) has universal value in all three simulations, confirming the simulations of Pucci et al. (2017) and observations of Eastwood et al. (2009). A discrepancy between the slopes of magnetic and kinetic energies is found in all three simulations. The difference $\alpha_B - \alpha_u \approx 1$ in the first simulation, and is somewhat smaller (≈ 0.75) in cases 2 and 3. Note, that similarly to the magnetic field fluctuations, the kinetic energy spectrum is also steeper in case 2.

Summary. We have performed three-dimensional kinetic electromagnetic simulations of decaying turbulence at sub-proton scales initiated with Taylor-Green flow. Three different initial magnetic field configurations with zero magnetic cross-helicity were studied. The flows that develop at sub-proton scales in the three simulations, do not exhibit such prominent differences as in the previously reported MHD simulations

Energy evolution in all three simulations is very similar, driven by the fast compensation of the initial pressure imbalance, followed by (also fast) decay of ion bulk flows, and a more gradual decay of magnetic energy. The dissipated energy is transferred into particle heating, with electrons being heated more efficiently, so that ion/electron temperature ratio decreases with time.

Energy is gradually fed from large (fluid) scales to the small (kinetic) scales, until the full turbulent cascade establishes and a peak dissipation is observed. At this moment of peak dissipation the flows are representative of the real-world plasma turbulence. The time needed to reach this moment is twenty times shorter

Figure 5. For this draft, I combined 6 figures: u (top) and B (bottom) spectra evolution for the three cases. In each figure, three panels show: (a) evolution of the spectrum where color corresponds to the time. Vertical lines indicate the inertial lengths of ions (solid) and electrons (dashed). (b) average spectrum at peak dissipation and its power-law fit. (c) the running slope computed over this average spectrum.

Figure 6. Same as Fig. 5 but for E .

than estimated turnover time in the simulation domain. Magnetic field and velocity spectra exhibit two spectral breaks, at the inertial scales of ions and electrons. The spectra have a well-defined power-law shape at sub-proton scales, while simulations with much higher resolution are needed to study the electron-scale range.

The energy exchange during peak dissipation period is very intermittent, with the most active regions adjacent to the TG flow symmetry planes. The maximum values of the energy exchange rate in the center-of-mass frame D_{cm} exceed the domain-averaged values by several orders of magnitude. Regions of positive and negative D_{cm} form oscillatory patterns and are often adjacent to the locations of magnetic nulls. Hence, a complex mixture of plasma micro-instabilities and magnetic reconnection is responsible for turbulent energy dissipation in plasma. Important difference is observed in the second simulation where initial magnetic configuration symmetries are different, the number of magnetic nulls is smaller, the dissipation pattern is different, and the overall decay of magnetic field is slower.

The power-law fits to the sub-proton range of the spectrum reflect this difference, the spectral slopes are $\alpha_B \approx 2.9$ in cases 1 and 3, while a steeper spectrum $\alpha_{B,3} \approx 3.1$ is found in the second simulation. A commonly observed difference between the slopes of magnetic and electric fluctuations spectra $\alpha_B - \alpha_E \approx 2$ is found in all three runs considered here. The slopes of kinetic energy spectra deviate from the B spectra. This deviation is larger in the first simulation, $\alpha_B - \alpha_{u,u} \approx 1$, and milder in two other runs ≈ 0.75 .

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