Ordered Epistemic Logic: Semantics, Complexity and Applications

Hanne Vlaeminck Joost Vennekens Maurice Bruynooghe Marc Denecker

Department of Computer Science, KULeuven, Belgium

Abstract

Summary of a paper appearing in Principles of Knowledge Representation and Reasoning: Proceedings of the Thirteenth International Conference, KR 2012, Rome, Italy, June 10-14, 2012. AAAI Press 2012.

Ordered Epistemic Logic (OEL) was first defined under the name *Hierarchic Autoepistemic Theories* by Konolige [10]. He observed that, in non-monotonic reasoning, the notion of inference from a *specific* body of knowledge often plays an important role. Recently, OEL was independently reintroduced by [8] in order to merge the contributions of ASP [5, 1] on the level of Knowledge Representation into classical first order logic (FO). As many of the original motivating examples of ASP involve defaults and autoepistemic propositions [9], this was done by adding an epistemic operator to FO.

We observed that these motivating examples for ASP, as well as many examples of Default Logic (DL) [13], often have a simple stratified structure: the goal is to reason on an existing incomplete knowledge base, e.g., by adding default assumptions. More complex examples can have several levels of stratification. However, neither autoepistemic logic (AEL) [11], default logic, nor ASP preserve this inherent stratification. As a result, a theory in each of these logics is a theory that refers to its own information content through a reflexive epistemic operator (see [6] for a recent account). This is a source of complexity that complicates both their semantics and their reasoning procedures.

By contrast, OEL maintains a stratified representation where each level extends the knowledge of the lower levels. This simplifies the logic considerably, while still being able to handle a lot of useful applications from AEL or DL, as shown in the full paper. Contrary to AEL, DL or ASP, an OEL theory always defines a unique belief set, represented as a set of possible worlds. The full paper shows that OEL solves some well-known problems of ASP in the context of epistemic applications. Syntactically, OEL extends FO; the only difference with FO is that OEL is a *closed domain* version of FO: all possible worlds share the same domain and interpretation of terms. This is like in many first order modal logics. With exception of this feature, OEL is a conservative extension of FO; its epistemic operator stands orthogonal to many other extensions of FO (e.g., types, inductive definitions, aggregates,…), and hence seamlessly integrates with them. By combining them, a very rich KR language is obtained in which many of the motivating examples in DL, AEL and ASP, as well as other extensions of FO such as FO(ID) [7], have a natural expression.

We here extend the initial work in [10, 8], in several ways. First, we prove that, in a given finite domain, the data complexity of model checking, satisfiability checking and query answering for OEL theories is in Δ_2^P, which is indeed lower then for AEL and DL, where some instances of satisfiability checking problems can be proven to be Σ_2^P-complete. We also show how a model generator for OEL can be implemented.

Second, we illustrate the use of OEL and of model generation in the context of a scheduling problem with an epistemic component. Third, we extend OEL to a logic for distributed epistemic agents, which we call *distributed ordered epistemic logic* (d-OEL). Knowledge bases are still hierarchically ordered, but now theories at one level no longer automatically possess all the knowledge of lower levels.

Distributed ordered epistemic logic can cope with distributed knowledge, which makes it relevant for a number of new application areas. One example is the specification of access control policies. Formal specification languages used for this purpose (e.g., [2]) often include a construct allowing one policy manager to query the knowledge of another. Another potential application area is the Semantic Web, which can be seen as a huge network linking different sources of data and knowledge, often in the form of ontologies. Several proposals haven been made to combine data from such ontologies, for example, through the use of *bridge* *rules* [4]. Others address the need for expressing defaults such as “if source *x* does not specify the color of the car, we assume its color is black”. In yet other approaches, the web is seen as an open environment consisting of purely positive knowledge bases that do not contain negative information. This is to avoid potential inconsistencies between different sources [3]. Here, a limited form of “negation-as-failure” called *scoped negation* [12, 3] has been proposed: this is an epistemic operator to query whether a specific source (e.g., some ontology) does not know a proposition.

Common to all such applications is the presence of a number of knowledge sources that can query each other through some form of epistemic operator. We argue in the full paper that the logic d-OEL provides a simple formalism with a precise semantics to tackle some of these applications in a natural way.

The full paper starts with recalling some preliminaries. Next, syntax and semantics of both OEL and d-OEL are defined, examples are given and a number of properties about both languages are proved. The paper continues with a closer look at model generation for OEL and d-OEL and with proving our complexity results. After that, related work is discussed in detail, in particular the relationship with Answer Set programming, with Default Logic and with other hierarchical approaches but also with other formalisms for handling distributed (possibly contradictory) knowledge. The paper ends with a brief discussion.

References

[1] C. Baral. Knowledge representation, reasoning and declarative problem solving. 2003.

[2] S. Barker. The next 700 access control models or a unifying meta-model? In *Proceedings of the 14th ACM symposium on Access control models and technologies*, SACMAT ’09, pages 187–196. ACM, 2009.

[3] T. Berners-Lee, E. P. Dan Connolly, and Y. Scharf. Experience with N3 rules. In *W3C Workshop on Rule Languages for Interoperability*, 2005.

[4] P. Bouquet, F. Giunchiglia, F. V. Harmelen, L. Serafini, and H. Stuckenschmidt. C-OWL: Contextualizing ontologies. In *Journal Of Web Semantics*, pages 164–179. Springer Verlag, 2003.

[5] G. Brewka, T. Eiter, and M. Truszczyński. Answer set programming at a glance. *CACM*, 54(12):92–103, 2011.

[6] M. Denecker, V. W. Marek, and M. Truszczynski. Reiter’s default logic is a logic of autoepistemic reasoning and a good one, too. *CoRR*, abs/1108.3278, 2011.

[7] M. Denecker and E. Ternovska. A logic of nonmonotone inductive definitions. *ACM Transactions on Computational Logic (TOCL)*, 9(2):Article 14, 2008.

[8] M. Denecker, J. Vennekens, H. Vlaeminck, J. Wittocx, and M. Bruynooghe. Answer set programming’s contributions to classical logic. An analysis of ASP methodology. In M. Balduccini and S. Tran, editors, *MG-65: Symposium on Constructive Mathematics in Computer Science*. Lexington, October 26-27 2010.

[9] M. Gelfond and V. Lifschitz. Classical negation in logic programs and disjunctive databases. *New Generation Computing*, 9(3/4):365–386, 1991.

[10] K. Konolige. Hierarchic autoepistemic theories for nonmonotonic reasoning. In *AAAI*, pages 439–443, 1988.

[11] R. C. Moore. Possible-world semantics for autoepistemic logic. In *Proceedings of the Workshop on Non-Monotonic Reasoning*, pages 344–354, 1984. Reprinted in: M. Ginsberg, ed., *Readings on Nonmonotonic Reasoning*, pages 137–142, Morgan Kaufmann, 1990.

[12] A. Polleres, C. Feier, and A. Harth. Rules with contextually scoped negation. In *Proc. 3 rd European Semantic Web Conf. (ESWC2006*, pages 332–347. Springer, 2006.

[13] R. Reiter. A logic for default reasoning. *Artif. Intell.*, 13(1-2):81–132, 1980.