The power of interactive whiteboards for secondary mathematics teaching: two case studies

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Abstract

Interactive Whiteboard (IWB) offers a high-potential innovative tool in mathematical educational environments, in which teachers' modelling processes and students' exploring activities can easily be executed. Nevertheless, these affordances are not self-evident. There is a gap between the potential claims of the tool and its actual use in the classrooms. This study investigated, through two parallel case studies, the IWB potential in view of optimising the exploitation of the IWB, improving conceptual understanding and fostering interactivity in secondary mathematics classrooms. Two main patterns for a productive IWB use emerged: i) a ***problem solving*** *pattern, consisting in IWB supporting problem solving activities; ii) an* ***organize****r pattern, consisting in using IWB as a kind of advanced organizer, stimulating students' reflection and mathematical contributions. The two patterns were used to design and teach lessons that were analysed through the Instrumental Orchestration framework elaborated by Drijvers et al. (2013). Analysis showed how the IWB acted as a useful instrument for students' discussion and collective construction of mathematical knowledge.*

Keywords: Interactive Whiteboards, Mathematics Education, Learning Environments, Teaching/learning Strategies, Instrumental Orchestration

1. Introduction

The capability of technological resources for mathematics education has been widely recognized. Research in learning with technological tools has shown that technology can help produce learning environments in which students have ampler opportunities to construct mathematical meanings, to explore and experiment with mathematical ideas and to express these using a wide range of representations (Ruthven, 2007). In their contribution to the *17th ICMI Study on Mathematics Education and Technology*, Olive et al. (2010) stress how technology provides access to new understandings of relations, processes, and purposes. They state that "the use of technologies in schools has encouraged a closer relationship between mathematical knowledge and mathematical practice, providing learners with opportunities to experiment, visualize, and test emerging mathematical understandings. From the use of digital technologies, a new model of interaction between the student, the mathematical knowledge and the instrument emerges." (Olive et al., 2010, p. 153). NCTM's position statement claims that ‘‘content-specific mathematics technologies support students in exploring and identifying mathematical concepts and relationships. Effectively applied technologies increase students' access to information and ideas and enhance student–student and student–teacher interactions to support and enrich sense making.'' (NCTM, 2015, p. 1).

Nonetheless, the use of technology for instructional purposes has a rather disappointing history. As Reiser (2001) notes in his *History of instructional design and technology*: "as a new medium enters the educational scene, there is a great deal of initial interest and much enthusiasm about the effects it is likely to have on instructional practices. However, enthusiasm and interest eventually fade, and an examination reveals that the medium has had a minimal impact on such practices" (p. 62). Salomon and Ben-Zvi (2006) remark how there are exemplary cases of extraordinary integrations of technology into education, but they constitute rather islands in an ocean of wasted good intentions and mediocre usages of ICT. According to Kozma (1991), the ability to take advantage of the power of emerging technologies will depend on the creativity of designers and teachers, their ability to exploit the capabilities of the media, and their understanding of the relationship between these capabilities and learning. Also for the Interactive Whiteboard (IWB), currently one of the most popular educational technologies all over the world, many studies (e.g. Moss et al., 2007; Somekh et al., 2007) report a gap between claims and actual contribution to learning and teaching.

IWB prevalence is rapidly increasing in many countries (Hennessy & London, 2013). Further rapid growth is expected in the next few years. IWB systems provide a multimodality environment wherein images, texts, insertions from other software programmes (e.g. mathematical software) can be combined and manipulated directly on the screen by teachers and students. IWBs are equipped with their own specific software but they can also be considered a digital hub that allows teachers and students to integrate Internet or other hardware resources into lessons. Objects from other technologies, for instance geometric dynamic software, can easily be displayed on the IWB and can directly be manipulated by teachers and students to create an interactive experience accessible to all during the lessons. Results from these manipulations can also be stored and retrieved for use in future lessons (Mercer, Hennessy & Warwick, 2010).

These affordances make IWB a high-potential innovative tool in mathematical educational environments in which teachers' modelling processes, students' exploring activities and other instructional strategies can easily be executed. Glover, Miller, Averis and Door (2007) state that the IWB has the potential to transform mathematics teaching, and in many cases it clearly has done so. As teachers become more confident in using the IWB, research has also highlighted its potential to develop productive classroom dialogue (Mercer et al., 2010).

Nevertheless, these affordances are not self-evident; many teachers seem to use the IWB solely as a large-scale visual blackboard or a simple presentation tool. The studies by Moss et al. (2007) and Higgins et al. (2005) did not reveal a significant improvement in students' attainments. Teachers often fail to exploit the above-mentioned innovative pedagogical advantages of IWBs (Moss et al., 2007; Somekh et al., 2007). Moreover, if teachers merely use IWBs in a presentation mode, they can be induced to teach in an expository way, reducing rather than stimulating students' activity. In those cases, the IWB will thus lead to a more teacher-centred instructional approach (Glover et al., 2007).

Ruthven (2009) highlights that integrating new technologies in mathematics teaching, such as dynamic geometry and computer algebra, is a complex challenge. Introducing new mathematical technologies confronts teachers with contexts that challenge their routines and require the development of new teaching practices for technology-rich environments (Lagrange & Ozdemir Erdogan, 2009; Ruthven, 2007).

The point in question is, therefore, what kind of practices teachers can adopt to exploit and maximize the potential of IWBs in building an instructional environment in which students are more cognitively engaged in the domain of mathematics. The purpose of the present study is investigating, through two parallel case studies, the IWB potential in view of coming up with useful guidelines for optimising the exploitation of the IWB affordances, improving conceptual understanding and fostering interactivity in secondary mathematics teaching and learning.

2. **Theoretical and empirical background**

Most mathematics educators have argued that mathematics consists of more than knowledge of mathematical concepts, principles, techniques, procedures (e.g., Collins, Brown & Newman, 1989; Schoenfeld, 1992). For them, mathematics does not consist only in applying standard procedures generally explained in school textbooks, but also in engaging in the processes of mathematical thinking, in reasoning about key mathematical concepts and in solving and managing mathematical problems. The importance of mathematical tasks is highlighted by Doyle (1988) and Hiebert and Wearne (1993), who observe that students' learning is mainly defined by the tasks they are given. Doyle (1988) argues that "the focus for tasks involving higher cognitive processes is on comprehension, interpretation, flexible application of knowledge and skills, and assembly of information from several different sources to accomplish work" (p. 170). Henningsen and Stein (1997) observe that "not only must the teacher select and appropriately set up worthwhile mathematical tasks, but the teacher must also proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demands of the task" (p. 546). Hiebert and Grouws (2007) highlight the concept of *teaching for conceptual understanding*, i.e. "the mental connection among mathematical facts, procedures and ideas" (Hiebert & Grouws, 2007, p. 380). They individuate two critical approaches in the pattern of teaching for conceptual understanding: (1) teachers and students attending explicitly to concepts, i.e. treating mathematical connections in an explicit way, discussing the mathematical meaning underlying procedures, attending to the relationships between mathematical ideas, etc.; and (2) students struggling with important mathematics, i.e. expending efforts to make sense of mathematics, figuring something out that is not immediately apparent, solving problems "that are within reach and grappling with key mathematical ideas that are comprehensible but not yet well formed" (Hiebert & Grouws, 2007, p.387).

In addition to the level of the mathematical tasks, the quality of discourse interaction between teacher and students and amongst the students themselves is another essential element of an effective instructional set. Instruction consists of interactions involving teachers, students, and content, i.e. how teachers and students interpret and interact with one another and with the task at hand. Effective instruction is dynamic, including teachers' and students' interaction, the mathematical tasks in which they engage, and the environment in which they act (Ball & Forzani, 2009; Cohen & Ball, 2001).
In their review of researches in mathematics classrooms, Walshaw and Anthony (2008) highlight how "the opportunity for learning is influenced by what students are helped to coproduce through dialogue. The effective use of classroom discourse makes students' mathematical reasoning visible and open for reflection. In an environment where ideas are shared, students' own ideas become resources for their own learning" (p. 539). According to Walshaw and Anthony (2008), "a context that supports the growth of students' mathematical identities and competencies builds on students' responses, shapes the reasoning and thinking to an appropriate level, and moves ideas and solutions toward a satisfactory conclusion" (p. 539).

What kind of support may the IWB provide to the previous two aspects: mathematical tasks and quality of discourse interaction? In the last years, several studies attempted to explore teachers' and students' perceptions of IWB actual use in mathematics teaching.
Heemskerk, Kuiper and Meijer (2014) report the results of a study focusing on the combined use of the IWB and a Virtual Learning Environment in mathematics education in a Dutch secondary school. For three years, teachers developed digital teaching materials for the IWB they used in their

mathematics teaching. They also made the lessons they developed for the IWB available on the Virtual Learning Environment used by the school. Students' mathematics performance was monitored during one school year and their motivation for mathematics was followed for 3 years. There was no relation between frequency of being taught with an IWB and mathematics performance. Results from the study showed no evidence that the use of an IWB in mathematics lessons is associated with better learning results. However, the authors found a positive relation between students' motivation for mathematics and the combination of lessons made for the IWB and availability of these lessons on the Virtual Learning Environment.

Van Laer, Beauchamp and Colpaert (2014) conducted a study about IWB use in Flemish secondary schools to assess, through an online survey, how IWBs are used and how teachers develop their IWB skills in the classroom. To classify teachers' IWB use, they applied a detailed IWB use framework, elaborated by Beauchamp (2004), that identifies different stages, from *Black/Whiteboard Substitute* through *Apprentice User, Initiate* (sic) *User, Advanced User*, till to *Synergistic User*. The results show that, in terms of the level of IWB use, teachers classified themselves predominantly in the first two stages of the framework (*Black/Whiteboard Substitute* and *Apprentice user*). This would suggest that teachers have been initiated (in a technological sense) in using the IWB and are beginning to initiate (in a pedagogic sense) wider usage, incorporating students' use of the IWB. In this process of initiation, however, teachers appeared to be more confident in the technical use of the ICT skills, but less confident in developing new pedagogical approaches which may exploit the full potential of the IWB.

Drijvers (2014) refers to three review studies (Cheung & Slavin, 2011; Li & Ma, 2010; Rakes, Valentine, McGatha & Ronau, 2010) that report small but positive effects of the use of digital technologies (not only referring to IWB) in mathematics education. According to Drijvers (2014), there is modest support to the claim that technology may have positive effects on students' achievement, but "little is known about successful approaches in teaching that may optimize the possible benefits" (p. 24).

An extensive literature review about the effects of IWBs on students' learning (not only concerning mathematics) by Di Gregorio and Sobel-Lojeski (2010) led to the conclusion that "in order for IWBs to have their greatest positive influence on student learning and achievement, an interactive school culture is needed" (p. 269). Following a classification introduced by Glover et al. (2007), Di Gregorio and Sobel-Lojeski (2010) state that when teacher reach the *enhanced interactivity* stage, they can look to integrate conceptual understanding and cognitive processes in a way that exploits the interactive capacity of the IWB, this way maintaining student motivation over time.

An OECD report (2015) states that "positive effects were achieved in interventions that followed the same principles of learning that apply for traditional teaching as well: ICT is particularly effective when used to extend study time and practice, when used to allow students to assume control over the learning situation (e.g. by individualising the pace with which new material is introduced), and when used to support collaborative learning." (p.162)

These studies, and others (e.g. Beauchamp & Kennewell, 2013; Pepin, Gueudet, & Trouche, 2017), show positive and negative aspects about the actual IWB use and the need to implement an interactive pedagogy. Often, the studies refer to a stage of *enhanced* (or *synergistic*) *interactivity* that teachers should reach to fully exploit the IWB affordances, but the analysis of this stage is complicated by the lack of a shared framework that could provide an integrated view of the different observations and perspectives.

Looking for guidelines for improving IWB exploitation aiming at conceptual mathematics understanding and fostering interactivity between students and teacher, two previous studies (De Vita, Verschaffel & Elen 2014; De Vita, Verschaffel & Elen, 2017) analysed in different constellations various attempts of teachers to design and enact high-quality IWB based math lessons aimed at conceptual understanding.

Two main patterns for a fruitful and productive IWB use emerged. Both patterns are characterized by high level mathematical tasks, in the form of students attending explicitly to mathematical concepts or in the form of problem solving activities. In both patterns, what seemed important was developing a strong synergy between IWB affordances and interaction with it by the students.

- The first pattern, named *problem solving pattern*, consists in IWB supporting problem solving activities achieved through a large use of geometrical or other mathematical software. IWB allows a dynamic visualization of the tasks, and engages students' participation at whole classroom level. Students are stimulated by the teacher in performing the tasks. The teacher guides an 'at the moment' activity, scaffolding and addressing students' conjectures and explorations, leaving students free to find their own solutions;

- The second pattern, named *organizer pattern*, consists in using IWB as a notepad, a kind of advanced organizer, that the teacher, in collaboration with the students, 'tailors' following a thread, including links to external sources, mathematical and geometrical constructions, problems or activities proposals. The teacher presents the topic to the classroom and stimulates the discussion between students and leads the students' reflection and mathematical contributions.

The two patterns base on the three important aspects introduced above, i.e. mathematical tasks, interactivity between students and teacher, and IWB support, for an integrated and effective IWB interactive environment for mathematics teaching and learning:

- the *level of the mathematical content*, i.e. the cognitively demanding mathematical tasks in which students are engaged;
- the *quality of the discourse interaction* between the teacher and the students, and amongst the students themselves. An important result of this type of classroom discourse is a clear articulation of students' thinking, supported by careful listening and responsive scaffolding by the teacher;
- *the support that IWB can lend to the previous two elements*, i.e. mathematical content and discourse interaction.

An articulation of these features in the two patterns is summarized in Table 1.

Table 1. Differences between the two patterns of IWB exploitation.

	Problem solving pattern	*Organizer pattern*
Mathematical tasks	Students doing mathematical problem solving tasks	Students attending explicitly to mathematical concepts
Interactivity	Students working in groups, presenting work at IWB, and discussing results with the teacher and the whole classroom	Single students participating in the elaboration at the IWB, whole classroom discussion about the processes and the results

IWB use	Use of mathematical software for checking mathematical hypotheses, modelling, comparing and transforming	Use of prepared teacher's outlines, retrieving materials from different sources (Internet, textbooks, etc.), using mathematical software for multiple visualisation

Despite the usefulness of this framework consisting of three analytic dimensions, the two cited studies (De Vita et al., 2014; De Vita et al., 2017) clearly revealed the restrictions of the framework, because the three dimensions remained rather separated and the framework did not allow an integrated picture of the classroom activities. Therefore, a more 'holistic' analytic framework was looked for that could do justice to the interconnectedness of the three separate analytic dimensions, i.e. mathematical tasks, interactivity and IWB use. Such a framework was found in the work of Drijvers, Trouche and others. Particularly, the notion of *instrumental orchestration*, elaborated by Trouche (2004) and Drijvers, Doorman, Boon, Reed and Gravemeijer (2010) revealed to be appropriate for the analysis, supporting an integrated description of the different activities carried out in the classroom. *Instrumental orchestration* focuses on the didactic management of the available artefacts by the teacher (Drijvers & Trouche, 2008; Trouche, 2004). It is defined by Drijvers et al. (2010) "as the teacher's intentional and systematic organisation and use of the various artefacts available in a learning environment - in this case computerised - in a given task situation" (p. 2).

In different studies concerning the use of IWB in mathematical classrooms Drijvers and others (Drijvers et al., 2010; Drijvers, 2012; Drijvers, Tacoma, Besamusca, Doorman & Boon, 2013) elaborated and improved an *instrumental orchestration* framework that classifies classroom types of orchestration (though they do not consider this list as exhaustive).

Eight kinds of orchestration were defined:

- the *Technical-demo* orchestration concerns the demonstration of tool techniques by the teacher;
- in the *Guide-and-explain* orchestration, on the one hand the teacher provides a closed explanation based on what is on the screen. On the other hand, there are some, often closed, questions for students, but interaction with students is so limited and guided that it cannot be considered as an open discussion;
- in the *Link-screen-board*, the teacher stresses the relationship between what happens in the technological environment and how this is represented in the conventional mathematics of paper, book and board;
- the *Explain-the-screen* orchestration concerns whole-class explanation by the teacher, guided by what happens on the computer screen;
- the *Discuss-the-screen* type concerns a whole-class discussion about what happens on the computer screen and requires a classroom setting favourable for discussion;
- in the *Spot-and-show* orchestration, student reasoning is brought to the fore through the identification of interesting student work during preparation, by the teacher or by the students, of the lesson, and its deliberate use in a classroom discussion. In both these two last types, *Discuss-*

the-screen and *Spot-and-show*, starting from student work, or a task, or a problem, a teacher or a student can reason at the IWB and discuss with other students, exploiting suggestions for different representations that can be easily and quickly tried out;

- in the *Sherpa-at-work* mode, a so-called Sherpa student (Trouche, 2004) uses the technology to present his or her work, or to carry out actions the teacher requests, and the other students follow and discuss the actions at the IWB;
- the *Board-instruction* orchestration is the traditional one of a teacher in whole-class teaching in front of the board, with different degrees of student involvement and interaction. The board can be a chalk board, a whiteboard or an interactive whiteboard, but in any case it is just used for writing, with no use of digital technology.

Using this framework may help to reveal differences in teacher-students interactions. In four orchestrations (*Technical-demo*, *Guide-and-explain*, *Link-screen-board* and *Explain-the-screen*) the teacher manages the communication and leads the interaction, while students' interventions are limited. Therefore, these four orchestrations may be considered as teacher-centred. Three orchestrations (*Discuss-the-screen, Spot-and-show* and *Sherpa-at-work*) allow more interaction for students, they have more voice then in the first four types of orchestrations. These three types can be considered as student-centred orchestrations. Finally, *Board-instruction* does not involve the IWB exploitation as a technological tool and its connotation, i.e. whether it is teacher or student centred, depends on the use made in the classroom activities.
Drijvers et al. (2013) used the orchestrations framework in a study concerning how 'mid-adopting' teachers orchestrate technology-rich activities in mathematics classrooms. 'Mid-adopting' teachers are teachers with a limited experience in the field of digital resources in mathematics. The study involved twelve teachers, who taught three modules concerning geometry, linear equations and quadratic equations. The lessons were video recorded and coded following the above framework.
Results show a large prevalence of *Board-instruction*, suggesting that teachers did not feel the need to drastically transform their whole-class teaching. Student-centred orchestrations like *Discuss-the-screen* or *Sherpa-at-work* are rarely exploited. Teacher-centred orchestrations are prevalent in the lessons. Table 2 shows the number of instances for each orchestration in the three topics.

Table 2. Instrumental orchestration instances (Drijvers et al., 2013)

	Geometry (%)	Linear equations (%)	Quadratic equations (%)
Teacher-centred orchestrations			
Guide and explain	3	11	8
Link screen-board	18	10	8
Explain the screen	36	2	0
Total	57	23	16
Student-centred orchestrations			
Discuss the screen	3	0	0
Spot and show	0	7	17
Sherpa at work	5	2	0
Total	8	9	17

Board instruction	23	66	67
Technical demo	12	2	0

These results raise the question whether the IWB necessarily favours a way of teaching focused on the teacher rather than on the students. In fact, if IWBs are merely used as a support to teachers' activities, they may induce teachers to teach in a teacher-centred way, reducing rather than stimulating students' activity and interactivity and exploiting the affordances of a media-rich content only for surface aims (Moss et al., 2007, Somekh et al., 2007).

Orchestrations do not refer to isolated activities but are part of orchestration sequences. In the same lesson the teacher can sequentially exploit different kinds of orchestration, dealing with didactical schemes and technological aspects to achieve the tasks. Analysing orchestration sequences may help to identify powerful patterns and hence add another layer to the analyses made by Drijvers et al. (2013).

In view of investigating the IWB potential and to come up with practically useful guidelines for optimising the exploitation of the IWB affordances, improving conceptual understanding and fostering interactivity in teaching and learning, two case studies -one for each pattern- were initiated to reveal the potential of IWB use in secondary mathematics classrooms. To maximize the chance that the implementation of the two patterns would be done reliably, these case studies were planned and realized by the researcher/teacher himself.

To come up with useful guidelines, the intent was checking the effectiveness of both patterns in fostering mathematical conceptual understanding, highlighting the differences between them, and promoting a student-centred environment that supports the growth of students' mathematical identities and competencies, shaping the reasoning and thinking to an appropriate level. More specifically, we aimed at answering the following research questions:

- Are the two patterns, *Problem solving* and *Organizer*, effective in promoting a synergic interactivity in the IWB exploitation?
- In what way are the two patterns different with respect to their use in classroom?
- To what extent does the IWB favour a more teacher or more student-centred approach? More particularly, are there indications that IWB use intrinsically results in a teacher-centred approach?

Aiming at answering these research questions, the Drijvers orchestrations classification was used in this study (as Drijvers et al., 2013, used it in their study, at least partially) to describe in depth the activity-patterns *at the class level*, allowing more integrated understandings of the dynamic relationship between teacher and students. In view of extending the framework an analysis of orchestration sequences is done as well.

3. Method

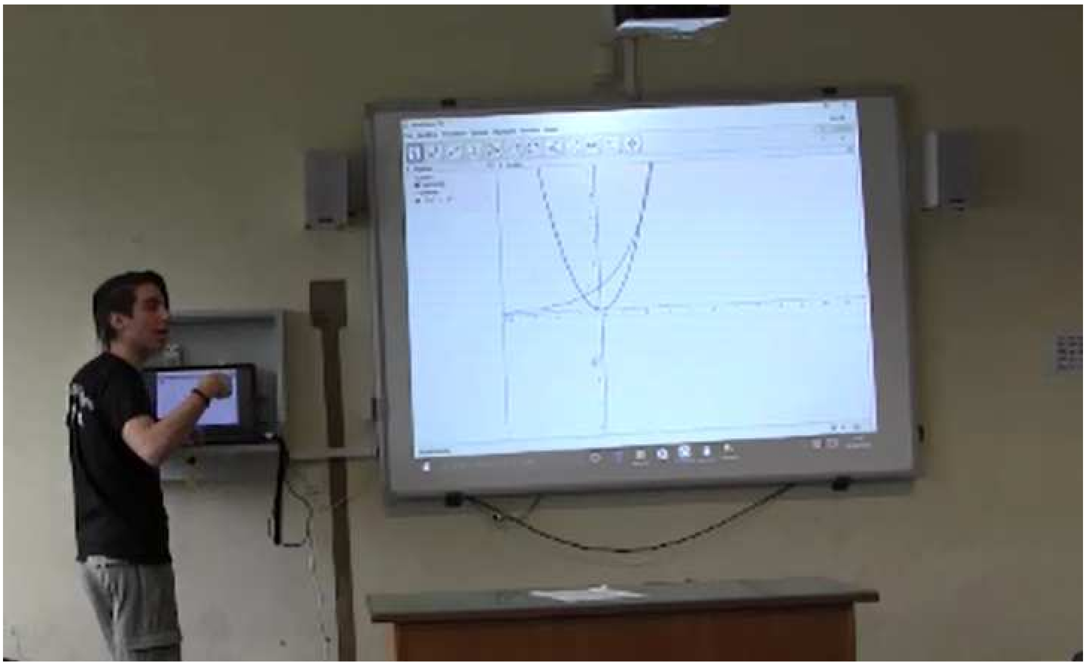
The present study uses a classroom action research method, a qualitative approach similar to design-based research with a number of lessons designed and enacted by the teacher. Action research typically involves the use of qualitative interpretive modes of inquiry and data collection by a teacher. Primacy is given to teachers' self-understandings and judgments. The emphasis is 'practical', that is, on what teacher and students are making and acting on in the situation (Kemmis & McTaggart, 2005). In the study, the teacher acted also as researcher, planning the lessons, teaching and video recording them and performing the analysis. By this choice the teacher was allowed to have a direct access to the enactment of the lessons and could immediately control the process of realizing the intended plans in classroom activities, in order to ensure in a more reliable way the tension between acting and monitoring moment by moment the course of the lessons, and evaluating and sustaining students' efforts (Anderson, 2002; Mills, 2006).

The research has been carried out in two 3rd grade classrooms of a Scientific Lyceum (age of students ~16) in Italy. One classroom accounted for 23 students (9 females and 14 males), the other one accounted for 25 students (13 females and 12 males). The two classrooms had the same performances at the mathematical level, and were both accustomed to IWB use in daily classroom activities. The lessons were taught and videotaped from April to June 2016. The mathematical topic was the introduction of exponential and logarithmic functions and equations. The choice of the mathematical topic was driven by the Italian regular curriculum. Exponential functions are very useful in real world situations. They are used to model populations, carbon date artefacts, help coroners determine time of death, compute investments, as well as many other applications. The topic was chosen because it is an important theme in mathematics, it is possible to set high level mathematical tasks and to propose problem solving activities, and it pertains to the 3rd grade curriculum. The classroom in which the *Problem solving* pattern (PS classroom) was applied, had six one period lessons, while the classroom in which the Organizer pattern (OR classroom) was applied, had only five one period lessons, due to curriculum constraints. The main features of the topic were anyway covered in both classrooms.

In the preparatory phase, the teacher elaborated a detailed lesson plan for each classroom.

In the PS classroom, the topic has been explored using the first pattern, namely IWB used as support for problem solving, achieved through a large use of geometrical or other mathematical software. Following the distinction between *teachers and students attending explicitly to concepts* and *students struggling with important mathematics*, introduced by Hiebert and Grows (2007) the mathematical tasks were settled as *students struggling with important mathematics,* i.e. expending efforts to make sense of mathematics, figuring something out that is not immediately apparent. In the first lesson the exponential functions were introduced through problem solving activities, with small student groups working at a first draft solution of the problem they were assigned, and then presenting at the IWB, elaborating and discussing it with the entire classroom. In the following lessons, exponential equations and inequations were presented through a graphical method, still using a problem solving approach and leaving students practice them with the help of mathematical software. For instance, in Figure 1 a student graphed an exponential function and a parabola, and was trying to individuate their intersections trough the graph.

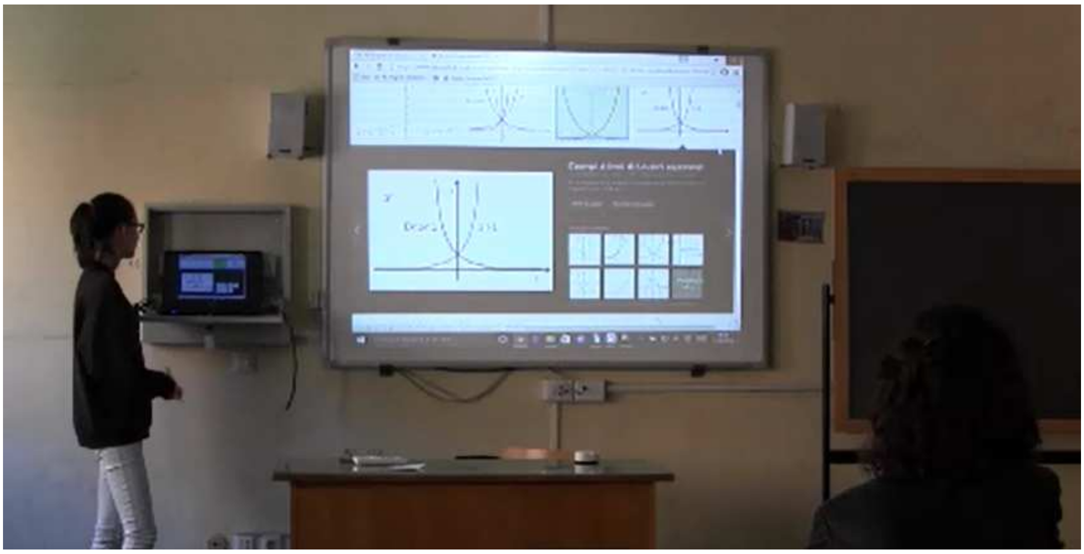
Figure 1. Graphing and solving equations graphically using the software Geogebra (managed by the student)



The logarithmic function was introduced through the concept of inverse of exponential function, starting from the achievements attained in the first phase, and following the same route: problem solving activities and consolidation of the results.

In the OR classroom the topic has been pursued according to the second pattern, namely IWB used as an advanced organizer. The mathematical tasks were focused on *teachers and students attending explicitly to concepts* (Hiebert & Grows, 2007), i.e. treating mathematical connections in an explicit way, discussing the mathematical meaning underlying procedures, attending to the relationships between mathematical ideas. The teacher, in collaboration with the students, followed a prepared instructional itinerary, including links to external sources, mathematical and geometrical constructions, problems or activities proposals. The first lessons introduced the topic through guided worksheets, at the beginning the teacher proposed applications of the exponential functions and then introduced theoretical concepts and examples. During the following lessons, students conducted Internet searches about applications of the exponential functions and used spreadsheets and geometrical constructions to simulate different models (exponential growth, exponential decrease).

Figure 2. Students discussing function representations retrieved from the Internet



Then the teacher proposed some issues leaving space for a whole classroom discussion, elaborating it at the IWB. For instance, in Figure 2 a student was showing different representations of exponential functions, retrieved from the Internet, and discussing with the whole classroom the link between graphs and algebraic forms of the functions. In the same way, the teacher introduced the logarithmic function (guided worksheets about logarithmic as inverse of exponential function, students' Internet research about).

All the lessons were video recorded. To analyse the two patterns we used the framework elaborated by Drijvers et al. (2013) presented above. Each lesson was analysed considering the instances of each instrumental orchestration and their sequence. For each minute, the prevailing instrumental orchestration was coded. For each classroom, we accounted for the percentages of instrumental orchestration instances that occurred in the total of the lessons. An independent observer coded randomly chosen fragments of each lesson, approximately for half an hour in each classroom. The achieved intercoder reliability was calculated using the criterion that the observer's coded responses were assumed as equivalent only if they are identical (Lombard, Snyder-Duch & Bracken, 2002). Reliability of instrumental orchestrations coding averaged 95%.
As the orchestrations are not isolated, but part of sequences, and the sequences could add important information about classroom activities, recurrent sequences of orchestrations were looked for in the two patterns. A sequence was considered recurrent when it occurred at least six times in one of the two patterns. Examining the orchestrations coding, three recurrent sequences were individuated, *Sherpa at work – Discuss the screen, Board instruction – Sherpa at work, Board instruction – Sherpa at work.* No other sequences revealed recurrent occurrences.

4. Results

In this section results from the analysis are reported. First, results concerning the occurrences of the different instrumental orchestrations in each classroom are reported; second, results are reported about the recurrent sequences of instrumental orchestrations.

4.1 Orchestrations

In both classrooms the *Technical Demo* and the *Link-screen-board* orchestrations were not used at all. *Technical Demo* was not used because both teacher and students were already accustomed to the use of the IWB and there was no need for technical demonstrations. *Link-screen-board* was not used because the teacher did not use a mathematical book, preferring to exploit his own prepared resources or resources retrieved from the Internet.

The teacher used the orchestrations *Guide-and-explain* (3%) and *Explain-the-screen* (10%) to introduce the topic and to address students' elaborations. In *Spot-and show* (6%) students presented their group works at the IWB. *Sherpa-at-work* was largely used (37%), fostering students' autonomous elaboration, involving them in the elaboration of the function graphs and in transforming and comparing them. In both these orchestrations students extensively exploited mathematical software (Excel and Geogebra). *Discuss-the-screen* occurs often (25%). This orchestration was used mainly when students presented their groups work at the IWB using mathematical software and when students discussed tables summarizing function properties. Last, *Board-instruction* (19%) was intended by the teacher as instances in which students make practice in algebraically solving equations and inequations.

Generally, the two student-centred orchestrations *Spot and show* and *Sherpa at work* are predominant and account for 43% of the total instances (n=229), showing a strong emphasis on the role of the students in the development of the lessons. Teacher's interventions are limited to short instances (*Guide and explain* and *Explain the screen*, 13% in total), used for introducing, clarifying, and addressing some tasks.

In the OR classroom, two types of orchestration, *Link-screen-board* and *Spot-and-show* were not used at all. In the case of *Link-screen-board,* the teacher did not use a mathematical book, preferring to exploit his own prepared resources or resources retrieved from the Internet. *Spot-and-show* was not used because in this pattern students did not make any preparatory work and the teacher did not use students' work.

The teacher used shortly *Guide-and -explain* (1%) and more extensively *Explain-the-screen* (14%) to introduce the topic and to explain some basic rules during the lessons. *Discuss-the-screen* occurs very frequently (34%). This orchestration is used mainly when students used mathematical software at the IWB and when students discussed tables summarizing function properties on the board. *Sherpa-at-work* was extensively used (23%), involving students in the elaboration of the function graphs and in the construction of tables representing the main characteristics of the functions. Last, the large use of the *Board-instruction* (29%), reflects the teacher's intention to make students practice in solving equations and inequations algebraically, and not only graphically. In all these orchestrations, the teacher had a minor role (about 14 % of the instances), mainly the work was carried on by the students (about 86% of the instances).

Also in the *Organizer* pattern classroom, the orchestrations alternated in each lesson. Most lessons show a prevalence of *Discuss the screen* (lesson 1, 2, 3, 5). As in this pattern the teacher did not envisage work groups, activities at the IWB performed by single students (*Sherpa at work*) were discussed by the whole classroom and by the teacher. Discussion was also intertwined with *Guide and explain* and with *Board instruction*. Students actively participated and contributed to the development of the subject.

Comparing orchestrations in the two classrooms (*Discuss the screen*, *Spot and show* and *Sherpa at work*), student-centred orchestrations are predominant: in *PS* classroom about 68% of the instances and in the *OR* classroom about 59%. Teacher-centred orchestrations account for 13% in the PS

classroom and for 11% in the OR classroom. *Board instruction* accounts for 19% in the PS classroom and for 30% in the OR classroom.
Table 3 shows the comparison between the orchestration instances in the PS classroom and the OR classroom, specifying teacher-centred and student-centred orchestrations.

Table 3. Instrumental orchestration instances (%) in the PS and OR Classroom

	PS Classroom	**OR Classroom**
Teacher-centred orchestrations		
Technical Demo	0	0
Guide and explain	3	1
Link screen-board	0	0
Explain the screen	10	10
Total	13	11
Student-centred orchestrations		
Discuss the screen	25	37
Spot and show	6	0
Sherpa at work	37	22
Total	68	59
Board instruction	19	30

4.2 Orchestration sequences

For a more detailed representation of the classroom activities, the orchestration sequences have been considered for each lesson. As observed by Drijvers (2013), orchestrations are not isolated, but part of orchestration sequences, and play particular roles in such a sequence. Drijvers et al. (2010) observe also how it is not always easy to separate the different orchestrations. Sometimes one orchestration can gradually shift into another one. For instance, a *Spot-and-show* or *Sherpa-at-work* orchestration can turn into a *Discuss-the-screen* orchestration, or vice versa.
From the analysis of the orchestration sequence it emerges that in each lesson different kinds of orchestration are used, alternating moments in which students work directly at the IWB (*Spot and show* and *Sherpa at work*) with moments of collective discussion (*Discuss the screen*).
The alternation allows the shift between different IWB exploitation registers, i.e. use of mathematical software or tables construction, and maintains high the students' concentration. For instance, in the first lesson in the PS classroom, after a short teacher's introduction, students showed their elaboration about a problem (*Spot and show*) and then further elaborated it at the IWB using mathematical

software (*Sherpa at work*). Successively, students built a table with the data (*Board instruction*), and there was a general discussion about the meaning of the table. The lesson proceeded alternating *Sherpa at work, Board instruction, Discuss the screen* and *Spot and show* instances.
From the orchestrations coding, three main sequences recurred, *Sherpa at work – Discuss the screen, Board instruction – Sherpa at work, Board instruction – Sherpa at work.*
In the PS classroom, the two most frequently occurring sequences are the sequence *Sherpa at work – Discuss the screen* and the sequence *Board instruction – Sherpa at work.* The first, *Sherpa at work – Discuss the screen,* appears in all the lessons. The students' elaborations carried out at the IWB, generally through mathematical software, are then discussed by their classmates and by the teacher, for better understanding, deepening and for further elaboration. The second sequence, *Board instruction – Sherpa at work,* arises frequently. In many lessons (not all) in the PS classroom, after a topic is elaborated as *Board instruction* orchestration, i.e. as the traditional 'paper-and-pencil' work, then it is represented by a student at the IWB using mathematical software.
Also in the OR classroom, the sequence *Sherpa at work – Discuss the screen* occurs in all the lessons. As told above, it is expected (and required) that after an elaboration by a student at the IWB the whole classroom collectively discusses this elaboration. Another significative sequence in the OR classroom is the *Board instruction – Discuss the screen*. After an elaboration at the IWB without using mathematical software, the classroom and the teacher feel necessary a discussion about the concerned arguments.
Table 4 shows the recurrent sequence occurrences in the two classrooms.

Table 4. Sequence occurrences -PS and OR Classrooms

Sequences	PS classroom	OR classroom
Sherpa at work – Discuss the screen	*14*	*10*
Board instruction – Sherpa at work	*9*	*2*
Board instruction – Discuss the screen	*4*	*8*

As the table shows, there are clear differences between the two patterns. In the PS classroom, two of the sequences involve the *Sherpa at work* orchestration, once followed by the discussion of what happened at the screen, and the other one preceded by *Board instruction,* i.e. students elaborated through a software at the IWB a topic introduced as whole- class teaching, by the teacher or by the students. This confirms that direct elaboration at the IWB by students is central in this model. In the OR classroom, both recurrent sequences include the *Discuss the screen* orchestration, confirming that the whole-class discussion, preceded by an elaboration by students or a whole-class teaching, was one of the main activity in this pattern.

5. Discussion and conclusions

The analysis of the two patterns through the instrumental orchestration framework leads to a positive overview of their effectiveness in promoting a collaborative interaction between teacher and students, with the significant support of the IWB

In this section, first we will answer the three research questions posed above. Second, we will discuss some important features in building a teaching and learning environment for optimising the exploitation of the IWB affordances, improving conceptual understanding and fostering interactivity in teaching and learning.

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5.1 *Research questions*

The three research questions set out were the following

- Are the two patterns, *Problem solving* and *Organizer*, effective in promoting a synergic interactivity in the IWB exploitation?
- In what meaning are the two patterns different with respect to their use in classroom?
- To what extent does the IWB favour a more teacher or more student-centred approach? More particularly, are there indications that IWB use intrinsically results in a teacher-centred approach?

The first research question concerns the effectivity of the two patterns, *Problem solving* pattern and *Organizer* pattern, in promoting a synergic interactivity in the IWB exploitation.

The two patterns show similarities and differences. The design of the former pattern, *Problem solving,* aimed at making students build the concept of the exponential function through real world problems. This construction implied a preceding group work and the implementation of the results by the students through mathematical software. The teacher based his lessons on activities of problem solving that called for a more direct students' elaboration, and these activities were carried on by the students directly at the IWB. The instrumental orchestrations used reflect the stress on these autonomous students' elaborations. The two orchestrations *Discuss-the-screen* and *Sherpa-at-work* were student-centred and were the most used in the pattern. The teacher used them to enhance the interaction between the students and himself. In these orchestrations students had the opportunity to state their ideas, to articulate their observations, to express their doubts, and to discuss with the teacher and with their peers, this way learning through interaction. In *Spot-and-show* the preparatory work conducted by the students' work groups was presented at the IWB and used in classroom discussion. Starting from students' work, students and teacher reasoned at the IWB, exploiting suggestions for different representations that were easily and quickly tried out. In the latter pattern, *Organizer*, the aim was to lead students through a guided path in the discovery of the topic. In this pattern, the role of the teacher's design was prevailing, though students actively participated in the elaboration. Students were guided in exploring the functions through a prepared route, and this lead to a decrease of the direct students' interventions and to an increase of general classroom discussion involving teacher and students. The prevalent instrumental orchestration was *Discuss the screen*, which could be an elaboration by a student, a graph proposed by the teacher or a picture retrieved by the Internet. It is developed as a broad and deep discussion by the whole classroom of the elaborations at the IWB.

The PS pattern makes students confront with problems and mathematical concepts that are within their range, but are not yet completely shaped, configuring the features that Hiebert and Grows (2007) call 'students struggling with important mathematics'. Furthermore, the structure of the lessons (small student groups working at the solution of the problem) favours the cooperation and the active involvement of the students, building a dynamic environment (Ball & Forzani, 2009) supported by the mathematical software exploited by the IWB.

In the OR pattern teacher and students treat explicitly mathematical connections, discuss the mathematical meaning underlying procedures, and attended carefully to the relationships between the mathematical concepts, this way approaching the teaching for conceptual understanding through what Hiebert and Grows (2007) call 'teachers and students attending explicitly to concepts'. Paying attention to and discussing with the teacher students' mathematical contributions allows students to see mathematics as a collective construction; it sustains students' learning by involving them in the formulation and verification of concepts, and helps students conceptualize mathematical activities (Walshaw & Anthony, 2008),

In sum, both patterns foster an active participation aimed at conceptual understanding by the students, integrating mathematical concepts and investigation through the IWB affordances.

The second research question considers how the two patterns, *Problem solving* and *Organizer,* differ with respect to their use in classroom.

Both patterns use the same instrumental orchestrations, *Guide and explain, Discuss the screen, Explain the screen, Sherpa at work* and *Board instruction*. *Spot and show* is used only in *Problem solving* pattern.

Only one kind of orchestration was not exploited in the lessons, *Link-screen board*. This orchestration refers to the relationship between the technological environment and the textbook. As highlighted by Monaghan (2004), teachers often prefer to not use textbooks in lessons in which technological tools are exploited. They often set open tasks that involve extended inquiries, without a close answer, and these contrast with closed textbooks tasks with single correct answer. For this reason, the teacher decided not to use the textbook giving more spaces to open students' investigations, and *Link-screen board* is not used.

Though the same instrumental orchestrations were used, their sequence and frequency were different in the two patterns. *Sherpa-at-work* is the main orchestration in the PS classroom (37%), while in the OR classroom the main orchestration is *Discuss the screen* (37%). The *Problem solving* pattern stimulates creative students' resources, using orchestrations that promote autonomous thinking, and foster difficult problems. The large use of the *Sherpa-at-work* orchestration is indicative of the teacher's intention to leave room to the students as far as possible. This orchestration is an opportunity for students to use their collective resources, but is also a challenge for the teacher because it requires a clear articulation of students' thinking, supported by careful listening and responsive scaffolding by the teacher. It is not always easy to lead students' reasoning without interfering excessively in their elaborations.

Also in the *Organize* pattern the teacher used the *Sherpa at work* orchestration, but its use aimed to carrying out single student's elaboration, with the teacher posing questions to the student and asking him/her to accomplish specific actions, this way stimulating whole classroom discussion.

Spot and show was only used in the P*roblem solving* pattern, as this orchestration starts from previous students' work, and students had to elaborate in small groups a first draft solution of the problem they were assigned to. No previous work was assigned in the *Organizer* pattern.

Discuss the screen was the main activity in the *Organizer* pattern. It arises often, in the sequences *Sherpa at work – Discuss the screen* and *Board instruction – Discuss the screen*. In this pattern, which is more whole classroom oriented, *Discuss the screen* is the main orchestration used by the teacher to promote and address students' autonomous elaborations. By soliciting students with open questions and suggestions, the teacher can shape the reasoning and thinking to a clear articulation. Nevertheless, teacher's solicitations have to be accurately calibrated, and can not completely substitute students' reasoning. This is a complex challenge, and the risk is to fall into a 'path smoothing' scenario

(Anthony, 1996), where the teacher leads students through a prepared itinerary and students just contribute with low level interventions, precluding an effective conceptual evolution.

Also the *Board instruction* orchestration, in which the IWB has been used as a ‘paper and pencil’ tool, differs with respect to the use in the two patterns. In the *Problem solving* pattern there was less need for whole class instruction, because many issues were directly managed and solved by the students while elaborating at the IWB. In the *Organizer* pattern, some concepts required a specific attention, and could be explained only through a ‘paper and pencil’ exposition. Nevertheless, also in this pattern students were actively involved in the activities, both directly elaborating and discussing what was happening at the board.

The *Problem solving* pattern is more challenging (both for teacher and students) than the *Organizer* pattern. It implies students’ workgroups that require a special attention by the teacher when exposing and discussing their works, and a skilful use of the mathematical software. The *Organizer* pattern is less challenging. Students are guided through the topic, explicitly attending to mathematical connections and to the underlying meaning. It could be said that the *Problem solving* model is more suitable in classrooms in which students are more clever and smart and accustomed to a ‘laboratorial’ way of teaching and learning, while the *Organizer* model is more appropriate in a situation where the students are less active and need an instructional activity that follows a straighter path.

The third research question concerns the extent to which the IWB favours a more teacher or a more student centred approach.

From the results, it seems clear that IWB does not necessarily favour a teacher-centred approach. In the two classrooms, student-centred orchestrations (*Discuss the screen, Spot and show* and *Sherpa at work*), are predominant (respectively PS 68% and OR 59%), while teacher-centred orchestrations have a minor role (PS 13% and OR 11%). It depends on the teacher’s intentions and experiences and on the quality of the lessons’ design and on the use to support collaborative learning.

Comparing the results of the study by Drijvers et al. (2013) with the results of this study (see Table 2 and Table 3), orchestrations used in the lessons observed in Drijvers study appear more teacher-centred than the lessons observed in this study. Furthermore, in Drijvers lessons *Board instruction* has a preponderant role, at least in two topics, Linear and Quadratic equations. This suggests that teachers were not prone to changing the traditional whole class teaching and left little space to students’ contributions and initiatives.

Of course, the two studies can not directly be compared, as they have different aims and different premises. The participants in the Drijvers study were teachers with limited experience in mathematics instruction through technologies, and the study was designed as an exploratory study, aiming at observing which kind of orchestrations they developed to fit the use of digital resources. In this study the teacher (and researcher) had a consolidate experience in teaching mathematics through technologies, and he deliberately planned and monitored the lessons aiming at optimising the exploitation of the IWB, through an accurate use of the instrumental orchestrations. Nevertheless, a comparison can be indicative of the differences in exploiting digital resources. In these two case studies, there was a constant and accurate attention to promote students’ participation, both through discussion and direct interventions by the students, leading to a student-centred learning approach. As highlighted by Cohen and Ball (2001), instruction consists of interactions involving teachers, students, and content. The interactions occur in varied settings, as small groups in classrooms, informal groups, tutorials, whole classroom discussion. This interaction can be achieved through an adequate lesson design and through the teacher's ambition to favour the growth of students' mathematical competencies based on autonomous thinking.

5.2 *Guidelines for optimising IWB exploitation.*

The goal of the present study was coming up with useful guidelines for optimising the exploitation of the IWB affordances, improving conceptual understanding and fostering interactivity in teaching and learning.

Useful guidelines concern different factors, that seem relevant in the design and in the enactment of the two patterns: the educational environment, the design of the lessons, the role of the teacher in orchestrating, and the mathematical software employed.

First, the relevance of the educational environment. It is important to embed the technology in daily practice, so that teachers and students get accustomed to its use. In the study by Drijvers et al. (2013) the *Technical Demo* orchestration took a large part in the first sequence. This orchestration decreased in the two following sequences. In the present study, there was no need for *Technological demo*, because students and teacher had already experience in using the IWB during the normal educational practice. Thus, it was possible to concentrate on the mathematical tasks and on the better ways to accomplish them through the technology.

Second, the decisive function of the design. The lessons should be accurately planned, in terms of tasks involved, teaching and learning activities, their sequence, software concerned, and timing, more than non IWB lessons, particularly because the teacher manages different tools (as mathematical software, paper-and pencil activities, etc.). In this study, the teacher accurately designed the lessons before their enactment in terms of timing, instrumental orchestrations, their sequence, IWB actions, and the motivations for each activity. The design was more detailed for the *Organizer* pattern as it required a precise sequence of instructional activities, while in the *Problem solving* pattern more room was left to the autonomous elaboration by the students, leaving to the teacher the role of 'director'. As highlighted by Drijvers (2013), "the criterion for appropriate design is that it enhances the co-emergence of technical mastery to use the digital technology for solving mathematical tasks, and the genesis of mental schemes that include the conceptual understanding of the mathematics at stake." (p. 15).

Third, the crucial role of the teacher in orchestrating the lessons. Both the teacher and the student work *together* to attain the goal, provide feedback, and ascertain whether the student has attained the goal (Hattie, 2012). The teacher balances different instrumental orchestrations, the sequences in which orchestrations are used, decides when and how to use mathematical software or resources retrieved form the Internet or other technological resources, and when to introduce different activities, e.g. paper-and pencil activities. He leads the whole classroom discussion, and promotes and values students' mathematical contributions allowing students to see mathematics as a collective construction. The teacher assumes the role of 'adaptive expert' who knows what students are learning, how to adapt strategies, resources, and even the classroom climate in order to meet learning goals, and is skilled at monitoring the current status of student understanding. Effective planning involves deciding on appropriately challenging goals and *then* structuring learning situations so students can reach those goals (Hattie, 2012). In the PS pattern, besides the design, the teacher is required to carry out an 'at the moment' didactical performance, exploiting all the resources (technological and human) of the environment.

Fourth, the exploitation of mathematical software. Mathematical software (spreadsheet, Geogebra, etc.) represents the main potentiality of the IWB in mathematics lessons, favouring a multiple dynamic visualisation of the mathematical objects. Mathematical activities through the use of mathematical software not only extends students' mathematical technique but shapes their sense of the mathematical entities involved (Ruthven, 2002). The use of mathematical software together with the possibility to quickly switch from an application to another, i.e. from the graphic view to a spreadsheet, from the Internet to the algebraic elaboration, provides the teacher with a flexible learning environment that, if well exploited, improves students' participation and understanding and favours students' instrumental genesis. The mathematical software Geogebra, with its extended potentialities (graph sheet, calculus sheet, spreadsheet) was particularly useful in the *Problem solving* pattern, where it provided students with a powerful instrument to carry out their elaborations.
However, preparation of the lessons by IWB intrinsically requires more time than normal no-IWB frontal lessons do. This may slow the lesson pace and extend the teaching and learning time. There is consequently a need for accurate calibration of these activities, also considering curriculum constraints. It could be useful to collaborate with other colleagues, at least in the preparation of the lessons, both in the same school and in teams involving different schools. Sharing ideas and projects would be a fruitful way to extend and increase the exploitation of digital resources in the daily instructional practice.

5.3 *Conclusions*

From a methodological perspective, the instrumental orchestration framework (Drijvers et al., 2013) turned out very useful in analysing the dynamics of learning and teaching in an IWB environment. Drijvers framework was integrated in this study by the analysis of the orchestration sequences, that allowed a deeper understanding of how orchestrations are connected and how their succession is exploited in raising a powerful reasoning line.
The framework contributed to a detailed picture of the relationships occurring in the classroom between teacher and students and how these relationships involve the IWB. Though Drivers et al. (2013) do not consider the list of orchestrations as exhaustive, in this study the orchestrations allowed to classify all the actions carried out by the students and by the teacher. Nonetheless, it is possible that zooming in on the details of the work by the teacher other kinds of orchestration could be included in the framework.
Nevertheless, the instrumental orchestration has been used in this study only in one aspect of its meaning. Instrumental orchestration can be also seen as a process of *instrumental genesis* (Artigue, 2002; Drijvers & Gravemeijer, 2005). The *instrumental genesis* is the process through which the use of a material resource, object or artefact is elaborated as it becomes a functional instrument for the user (in this case the learner). In this study, the instrumental orchestration framework was used to analyse and document the classroom activities, not addressing the way in which the learnings of individual learners are put at the core, how instrumental orchestration fosters and guides individual students' instrumental genesis, i.e., the development of schemes in which ICT techniques and mathematical knowledge and insights co-emerge. During *instrumental genesis*, bilateral relationships between the artefact and the user are established: while the student's knowledge guides the way the tool is used and in a sense shapes the tool; the affordances and constraints of the tool influence the way the student carries out a task and the emergence of the corresponding conceptions. This study

did not deal with this issue. On one side this can be considered as a limit, on the other one it provides the opportunity to develop further studies. It would be useful to investigate in depth the quality of interactions, using the instrumental orchestration framework as a microscopical lens, that zooms in on the details of the work by the teacher, and investigating the students' instrumental genesis with the applications in use on the IWB and the way in which these are used in the teaching (the teacher's *didactical performance*).

Combining these complementary approaches would provide a complete representation of classroom educational practice, helping to better understand the role of technology in mathematics education and contributing to a better strategy in teaching and learning.

A further methodological reflection concerns the decision to act both as teacher and as researcher. This decision allows to make an accurate planning and to directly control the enactment of the lessons. The direct engagement of the teacher-researcher permits systematic, continuous and deliberate attention to the methods, to the students' perceptions and understandings, and to the whole didactical process. The teacher-researcher has the opportunity to straightly govern the course of the lessons and in the same time to monitor the educational process, introducing corrections when needed and improving the didactical level when there is the opportunity. This way, he can carry out a critical and reflective self-checking about his own theoretical and practical approach.

From the attained results, it appears possible to design and enact lessons in which the IWB acts as a useful instrument for students' discussion and contribution to the collective construction of mathematical knowledge. In both patterns, observed through the lens of the instrumental orchestration framework, the IWB revealed to be effective in involving students and making them participate in the didactical activities. The IWB sustained students' learning and helped them to develop a clear articulation of thinking and a productive conceptualization of mathematical activities. More, it becomes an active tool managed directly by the students, who take advantage of its affordances under the teacher's scaffolding.

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