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Reverse engineering of deep drawn components with an isogeometric framework

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Abstract

In this work, the framework of isogeometric shell analysis is applied to the simulation of the dynamic behaviour of thin components measured by a 3D laser scanner. To this end, a B-spline surface is reconstructed to describe the mid-surface of the components and is used for the analysis. The exact thickness distribution is also described through the B-spline basis functions and is considered in the numerical integration. Two components manufactured through deep drawing are studied and the surface fitting procedure is adapted to their particular geometry. The results are finally validated through experimental testing of these components.

Keywords: point clouds; B-spline fitting; isogeometric analysis; thin shells; experimental validation

1. Introduction

Isogeometric analysis (IGA) is a numerical modelling methodology that has been recently introduced to bridge the gap between computer aided design (CAD) and numerical analysis [1, 2]. As the word isogeometric suggests, the same basis functions that describe the geometry of a component are used directly for the analysis, avoiding the meshing procedure that is needed in the classical finite element approach.

In the original version of IGA these functions are non-uniform rational B-splines (NURBS) [3], although more recently other methodologies have

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also been considered in order to improve the geometric flexibility of IGA, including T-splines [4, 5, 6], locally refined B-splines [7], hierarchical [8] and truncated hierarchical [9] B-splines, subdivision schemes [10, 11] and other spline methodologies based on unstructured meshes [12, 13, 14], to name some of the most relevant contributions.

Besides avoiding the meshing procedure, IGA allows one to easily increase the order of the basis functions and, therefore, it is particularly well suited for problems governed by high-order partial differential equations [15, 16, 17, 18, 19]. In particular, in the field of structural mechanics, isogeometric shell formulations have been recently developed following both the Reissner-Mindlin [20] and the Kirchhoff-Love theory [21]. Since then, other formulations and applications emerged, see e.g. [22, 23, 24, 25, 26, 27].

The Reissner-Mindlin formulation is normally employed for the description of thick shells where, as a rule of thumb, the ratio between the curvature radius of the surface R and the thickness t is smaller than 20 and transverse shear deformations need to be taken into account. On the other hand, for thin shells ($R/t \geq 20$), the Kirchhoff-Love formulation can be used, which assumes that transverse shear deformations are negligible.

In practice, although many shell structures could be regarded as thin shells, Reissner-Mindlin models are a lot more common in the FE community. In fact, these models assume also rotations as degrees of freedom (DOFs) and can be implemented using the classical FE C^0 inter-element continuity. This is not the case for Kirchhoff-Love models because they do not consider rotational DOFs and, therefore, need C^1 inter-element continuity. This continuity is not easily obtained when employing conventional Lagrangian shape functions, but needs more complex and expensive implementations, see for instance [28].

Kirchhoff-Love shell formulations became more popular with the introduction of IGA, because NURBS functions are in general much smoother and allow by construction a higher inter-element continuity. On removing rotational DOFs, the size of the resulting system matrix is significantly reduced and its units are consistent, which improves the condition of the mass and stiffness matrices. In addition, the shear locking issues typically encountered in (low-order) Reissner-Mindlin elements [29] are also avoided. For these reasons, the Kirchhoff-Love formulation is considered in this work while the interested reader is referred to [30] for a more detailed overview on both formulations. Another important advantage coming from IGA with respect to the classical FE approach, is that the geometrically exact character of IGA can be of crucial advantage for shell models, which can be very sensitive to small imperfections [31]. However, for NURBS based IGA,

the aforementioned considerations on the C^1 continuity are valid for single patch geometries, while for complex models constituted by non-conforming multi-patch configurations, the C^1 continuity needs to be enforced at their interface [32, 18, 30, 33].

As an application of IGA shell analysis, this work focuses on the reverse engineering of thin components manufactured through a deep drawing process and the simulation of their dynamic behaviour. In particular, a B-spline model is reconstructed starting from laser scan measurements on the surface of the components and, following the isogeometric framework, it is used directly for the numerical analysis.

Coordinate measuring machines (CMMs) are nowadays useful tools for reverse engineering and quality inspection applications [34, 35, 36] and, within this family, laser CMMs can produce large amounts of point data in a short time period without entering in contact with the scanned surface and are widely adopted for many applications in industry. However, the point cloud itself is most of the time not sufficient for an accurate description of the component or for the purpose of numerical analysis and, therefore, a surface has to be reconstructed through a reverse engineering process [37, 35, 38]. In this context, point cloud fitting with parametric curves and surfaces, such as B-splines and NURBS, has been employed starting much earlier than the introduction of IGA [3, 39] because of the wide use of these representations in the CAD industry and also, from a mathematical point of view, because of the high-order and tailorable smoothness of splines functions.

Combining the B-spline reconstruction with IGA shell analysis can therefore be a useful tool for the numerical modelling and simulation of a component and this work considers the specific case of deep drawn components. The outline of the paper is the following: B-spline basis functions and surfaces are introduced in Section 2; then the surface reconstruction procedure is discussed in Section 3.1 and the method used in this work is presented; a brief discussion of IGA shell analysis is given in Section 4; two practical examples are studied in Section 5, where the IGA results are compared with experiments; concluding remarks follow in Section 6.

2. Fundamentals of B-spline curves and surfaces

In one dimension, a B-spline function of polynomial order p is defined by a knot vector $\Xi = \{\xi_1 \leq \dots \leq \xi_{n+p+1}\}$, with n the number of basis functions forming the B-spline. The i^{th} ($i = 1, \dots, n$) B-spline basis function $N_i^p(\xi)$ is

derived from the knot-vector using the Cox-De Boor recursive formula [3]:

$$N_i^0(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1}, \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

$$N_i^p(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_i^{p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1}^{p-1}(\xi), \quad \text{for } p \geq 1.$$

Given two univariate B-spline basis functions $N_i^p(\xi)$ ($i = 1, \dots, n$) and $M_j^q(\eta)$ ($j = 1, \dots, m$), of order p and q respectively, and associated to the two knot vectors $\Xi = \{\xi_1 \leq \dots \leq \xi_{n+p+1}\}$ and $\mathcal{H} = \{\eta_1 \leq \dots \leq \eta_{m+q+1}\}$, a bivariate B-spline basis function is defined as

$$R_{i,j}^{p,q}(\xi, \eta) = N_i^p(\xi) M_j^q(\eta) \quad (2)$$

and B-spline surfaces $\mathcal{S}(\xi, \eta)$ are defined by

$$\mathcal{S}(\xi, \eta) = \sum_{i=1}^n \sum_{j=1}^m R_{i,j}^{p,q}(\xi, \eta) \mathbf{B}_{i,j}, \quad (3)$$

where $\mathbf{B}_{i,j}$ are the control points. Or, with $k = i + (j-1)m$ a single counter replacing i and j ,

$$\mathcal{S}(\xi, \eta) = \sum_{k=1}^N R_k(\xi, \eta) \mathbf{B}_k. \quad (4)$$

3. B-spline surface fitting

3.1. Point cloud fitting

In the context of B-spline and NURBS surface fitting, least squares methods have been the most used approach, starting from the first works dated back to the seventies [40, 41]. The main idea of these methods is to consider a reference surface and to update the position of its control points such that the error between the surface and the point cloud is minimized.

Given a cloud to be fitted composed of M points denoted by $\mathbf{P}_l$, this consists in imposing M equations

$$\mathbf{P}_l = \sum_{k=1}^N R_k(\xi_l, \eta_l) \mathbf{B}_k, \quad l = 1, \dots, M \quad (5)$$

or in matrix form

$$\mathbf{R}\mathbf{B} = \mathbf{P}, \quad (6)$$

where $\mathbf{P}_l = (x_l, y_l, z_l)$ and ξ_l and η_l are the parametric coordinates associated to $\mathbf{P}_l$, whose computation is discussed in the following.

Since in general $M \gg N$, Eq. (6) corresponds to an overdetermined system of equations, which is solved in a least square sense [42]. Such a choice is also motivated by statistical reasoning, as it effectively reduces the influence of random errors in measurements. In practice, this corresponds to obtaining a square system by pre-multiplying both sides by $\mathbf{R}^T$:

$$\mathbf{R}^T \mathbf{R} \mathbf{B} = \mathbf{R}^T \mathbf{P}. \quad (7)$$

The solution of this system minimizes the functional

$$f_s = \|\mathbf{R} \mathbf{B} - \mathbf{P}\|_2^2, \quad (8)$$

corresponding to the sum of the squares of the distance from the points to the surface. Other approaches, based on different norms such as the L^1 and the L^∞ are also possible [43, 44].

In the numerical practice, the direct solution of system (7) results to be accurate only in some limited cases, such as when a regular distributed set of points is considered. However, the point clouds coming from 3D scanners are composed by randomly oriented points, and regions with a lower density of points or holes can also be present, which results in a poor conditioning and sometimes even in a rank deficiency of $\mathbf{R}^T \mathbf{R}$. As a consequence, the fitted surface can present an irregular and oscillatory behaviour.

Therefore, in order to improve the quality of the fitting, regularizing terms are often added to functional (8). In particular, a common practice in curve and surface fitting consists in fairing by means of energy terms [45, 46, 47, 48, 49]. In fact, a spline surface can be seen as a mathematical representation of a membrane or an elastic plate and when it oscillates, the corresponding membrane and bending energy becomes higher. These energy terms are in general a non-linear function of the surface derivatives and, by following a common practice of surface fairing, the corresponding linearised terms are considered in this work:

$$E_m(\mathbf{S}) = \int \int \|\mathbf{S}_\xi\|^2 + \|\mathbf{S}_\eta\|^2 d\xi d\eta, \quad (9)$$

$$E_b(\mathbf{S}) = \int \int \|\mathbf{S}_{\xi\xi}\|^2 + 2\|\mathbf{S}_{\xi\eta}\|^2 + \|\mathbf{S}_{\eta\eta}\|^2 d\xi d\eta, \quad (10)$$

where E_m and E_b are the membrane and bending energy respectively, while $\mathbf{S}_\xi$ denotes the first derivative of the surface with respect to ξ and so on.

Terms involving higher order derivatives as well as more advanced functionals [50] have also been a topic of research. However, in this work the fairing is completed with a functional depending on first order terms that has been proposed in a recent study [51]. The main idea is to keep the control points as close as possible to the surface in order to improve its regularity. Therefore, the square distance from the control points $\mathbf{B}_k$ to a corresponding point $\mathcal{S}(\xi_k, \eta_k)$ on the surface is minimized by considering the functional:

$$f_c = \|\mathbf{B} - \mathbf{S}_P\|_2^2. \quad (11)$$

As suggested in [51], the surface points $\mathcal{S}(\xi_k, \eta_k)$ are computed by assigning the Greville abscissae [52] to ξ_k and η_k .

The fitting surface is therefore computed by minimizing the functional f , a Pareto combination of the aforementioned terms:

$$f = f_s + \alpha_c f_c + \alpha_m E_m + \alpha_b E_b. \quad (12)$$

Expressing f in matrix form,

$$f = \|\mathbf{R}\mathbf{B} - \mathbf{P}\|_2^2 + \alpha_c \|\mathbf{B} - \mathbf{S}_P\|_2^2 + \alpha_m \frac{1}{2} \mathbf{B}^T \mathbf{K}_m \mathbf{B} + \alpha_b \frac{1}{2} \mathbf{B}^T \mathbf{K}_b \mathbf{B}, \quad (13)$$

and considering its minimization, the following linear system of equations is obtained:

$$\mathbf{A}\mathbf{B} = \mathbf{b}, \quad (14)$$

where

$$\mathbf{A} = \mathbf{R}^T \mathbf{R} + \alpha_c (\mathbf{C} - \mathbf{I})^T (\mathbf{C} - \mathbf{I}) + \alpha_m \mathbf{K}_m + \alpha_b \mathbf{K}_b, \quad (15)$$

$$\mathbf{b} = \mathbf{R}^T \mathbf{P}. \quad (16)$$

The interested reader is referred to [46, 48, 51] for the details about the computation of the matrices $\mathbf{C}$, $\mathbf{K}_m$ and $\mathbf{K}_b$.

An important aspect that also needs to be discussed is how to properly choose the parametrization of the fitted points, i.e. how to assign them the parametric coordinates ξ_l and η_l . As suggested in [53] and further elaborated in [54], these coordinates are computed by defining a base surface that is used as a first approximation of the final surface and by associating to the points in the cloud the parametric coordinates of their projection on the base surface.

The initial base surface can either be defined by fitting a plane to the entire point cloud or constructed starting from a curve fitting on the boundary of the cloud. If the former approach is employed, the resulting fitting

surface needs to be trimmed at the boundaries [48]. Although dealing with trimmed geometries is possible for IGA [55, 56, 57, 58], an additional implementation effort is needed in this case. Therefore, in this work the second approach is followed such that a non-trimmed surface is computed as a bi-linear Coons patch starting from the four curves defining the boundary [54]. For the same reason, a single patch surface is used. Multi-patch models are also possible but particular care is required for their use in IGA, especially for Kirchhoff-Love shell problems, where the C^1 continuity between patches needs to be enforced [18, 32, 30, 33]. Given the simple topology of the components considered in this work, which is a typical feature of many parts manufactured through the deep drawing process, a fitting with a single patch B-spline surface appears as the most convenient choice. On the other hand, for components with a more complex topology, multi-patch B-spline models or fitting with spline methodologies defined on unstructured meshes are needed. The latter family of approximants [6, 12, 13, 14] would also allow to avoid the singularities in the geometric map at the boundary, coming from the initial representation of the base surface with a Coons patch, which may negatively affect the accuracy of the final solution. Another possible alternative that could be extremely useful for the parametrization of most of the geometries of deep drawn components is the recent technology of smooth polar splines [59].

Once the base surface is computed, problem (14) is solved in an iterative way by progressively decreasing the contribution of the fairing terms. In fact, although a direct solution is possible, a better choice of the parametric values associated to the points is obtained if the base surface moves progressively and smoothly towards the points in the cloud. Therefore, Eq. (15) is modified in the following way

$$\mathbf{A} = \mathbf{R}^T \mathbf{R} + \lambda[\alpha_c(\mathbf{C} - \mathbf{I})^T(\mathbf{C} - \mathbf{I}) + \alpha_m \mathbf{K}_m + \alpha_b \mathbf{K}_b] \quad (17)$$

and the value of λ is progressively decreased. The particular values to assign to α_m , α_c and α_b depend on the application and will be discussed in Section 5. Note that all the fairing terms remain constant during the iterative process and hence can be precomputed. On the other hand, the computation of the matrix $\mathbf{R}$, which does change while the fitting surface moves towards the point cloud, takes most of the total computation time. In order to improve the convergence of the process, it is also possible to consider the tangent and the curvature of the surface during the projection procedure [60, 61]. Surface fitting with a NURBS surface where the weights associated to the control points are updated as well can be also implemented — see

for instance [53, 62, 63]. However, it is not considered here for the sake of simplicity.

3.2. Mid-surface definition and thickness approximation

Within the shell framework, the constitutive equations are normally expressed with respect to the mid-surface (see Section 4). In order to obtain this surface, one could in principle implement the fitting procedure for both the inner and the outer surface and use them to derive the mid-surface. However, this procedure is not straightforward and the influence of errors in the fitting process can be eventually amplified.

Therefore, another approach is employed in this work that consists in fitting the B-spline surface directly to a point cloud corresponding to the mid-surface of the component. To this end, the inner and the outer surfaces are pre-processed using the open source tool *meshlab* [64]. In particular, for each point $\mathbf{P}_l$ on the inner surface, the Hausdorff distance [65] from the outer surface is computed and is then used to approximate the thickness of the component. The normal to the surface in $\mathbf{P}_l$ is also computed and, together with the thickness information, is used to define a new cloud, corresponding to the mid-surface. In fact, if $\mathbf{n}_l$ is the unit normal in $\mathbf{P}_l$ and t_l the thickness, the point $\mathbf{P}'_l$ on the mid-surface can be computed as

$$\mathbf{P}'_l = \mathbf{P}_l + \frac{t_l}{2} \mathbf{n}_l.$$

For more details about the procedure, the interested reader is referred to [66], where this approach is used for shell finite element analysis.

Once the mid-surface is defined, a B-spline surface is computed with the approach described in the previous section and the model is completed by fitting also the thickness. In particular, by following the IGA paradigm, the thickness is parametrized with the same B-spline basis functions

$$t(\xi, \eta) = \sum_{k=1}^N R_k(\xi, \eta) \mathbf{T}_k, \quad (18)$$

and is fitted to the values t_l on the mid-surface by solving

$$\mathbf{A} \mathbf{T} = \mathbf{R}^T \mathbf{t}, \quad (19)$$

where $\mathbf{T}$ contains the unknown thickness values associated to the control points and $\mathbf{t}$ the thickness values of the points on the mid-surface. In this procedure, the same matrix $\mathbf{A}$ coming from the surface fitting is used such that a smooth thickness distribution is obtained. However, we note that if the matrix $\mathbf{R}^T \mathbf{R}$ has full rank and is well-conditioned, the fairing terms can be omitted in $\mathbf{A}$.

4. Isogeometric shell analysis

According to the introduction, this work considers dynamic shell analysis using isogeometric Kirchhoff-Love elements, as introduced by Kiendl et al. [21]. Consider a thin shell structure of thickness t , and of a homogeneous, isotropic and linearly elastic material of density ρ , Young's modulus E and Poisson's coefficient ν . The undeformed configuration of the structure is denoted in Cartesian coordinates by $\mathbf{X} = \{X_1, X_2, X_3\}^T$ and its deformation is described by the displacement $\mathbf{u} = \{u_1, u_2, u_3\}^T$ of the mid-surface.

For the sake of simplicity, the equation of motion are presented only for flat shells here (i.e. plates). In this case, restricting the analysis to small deformations, the in-plane and out-of-plane motions of the flat geometry are governed by uncoupled dynamic equations. If X_3 is the out-of-plane coordinate (such that u_3 represents the transverse displacement), the steady-state dynamic bending motion (using a harmonic time-dependency $u(\mathbf{X}, t) = u(\mathbf{X}) \cdot e^{j\omega t}$ for a given angular frequency ω) is governed by the following equation:

$$\nabla^4 u_3(X_1, X_2) - k_b^4 \cdot u_3(X_1, X_2) = \frac{p(X_1, X_2)}{D}, \quad (20)$$

with ∇^4 being the biharmonic operator and $p(X_1, X_2)$ the external (normal) load. The bending stiffness D and the bending wavenumber k_b are defined as:

$$D = \frac{Et^3}{12(1 - \nu^2)}, \quad (21)$$

$$k_b = \sqrt[4]{\frac{\rho t \omega^2}{D}}. \quad (22)$$

The in-plane or membrane motion is described by $u_1(X_1, X_2)$ and $u_2(X_1, X_2)$ and is governed by the following dynamic equations of motion :

$$\begin{cases} \frac{\partial^2 u_1}{\partial X_1^2} + \frac{1 - \nu}{2} \frac{\partial^2 u_1}{\partial X_2^2} + \frac{1 + \nu}{2} \frac{\partial^2 u_2}{\partial X_1 \partial X_2} + \frac{1 - \nu^2}{E} b_{X_1} + \frac{\rho(1 - \nu^2)\omega^2}{E} u_1 = 0 \\ \frac{\partial^2 u_2}{\partial X_2^2} + \frac{1 - \nu}{2} \frac{\partial^2 u_2}{\partial X_1^2} + \frac{1 + \nu}{2} \frac{\partial^2 u_1}{\partial X_1 \partial X_2} + \frac{1 - \nu^2}{E} b_{X_2} + \frac{\rho(1 - \nu^2)\omega^2}{E} u_2 = 0, \end{cases} \quad (23)$$

with b_{X_i} the body force acting on the shell in direction X_i .

For shells, the bending and membrane phenomena are no longer decoupled. Although a shell formulation is not simply obtained by combining a

plate and a membrane problem, describing the deformation as a function of convective curvilinear coordinates allows the stresses and strains in the curved shell to be separated into bending and membrane contributions. This theory is not discussed in detail here, but more information concerning the employed Kirchhoff-Love shell formulation can be found in [21].

On adopting the IGA framework, the basis functions making up a B-spline geometry are also used for approximating the field variables for that geometry. Therefore, if $u(\xi, \eta)$ is the primary variable, its approximation $\hat{u}(\xi, \eta)$ can then be written as:

$$u(\xi, \eta) \approx \hat{u}(\xi, \eta) = \sum_{i=1}^n \sum_{j=1}^m R_{i,j}^{p,q}(\xi, \eta) \cdot d_{i,j} = \sum_{k=1}^N R_k(\xi, \eta) \cdot d_k, \quad (24)$$

where d_k are the unknown control variables to be solved for. Following a Galerkin approach, these same shape functions are then also used as test functions in a weighted residual formulation [21]. In addition, the approximated thickness distribution is also taken into account during the numerical integration, by computing the proper thickness value at each Gauss point.

5. Numerical examples

As a validation of the simulation framework described in the previous sections, the method is applied to the shell analysis of two components manufactured through a deep drawing process. The analysis results are then compared with experiments done on the dynamical testing of the components.

5.1. Component 1

The first component considered is an oil-pan look-alike structure that has been employed in previous studies in order to understand the vibration and noise radiation characteristics of this type of components [67, 68]. In fact, the radiated noise coming from an oil-pan can have a significant contribution towards the total noise emitted by an automobile engine and it was shown in the literature that oil-pan noise reduction up to 1.5 dB(A) can be achieved by structural optimization [69, 70] or through resorting to passive damping treatments.

The component is shown in Figure 1. It is made of IS 513 steel and its length, width and depth are 850 mm, 460 mm and 140 mm respectively. The component is manufactured through deep drawing starting from a blank with a nominal thickness of 1.5 mm and has wall angles of 35° and 30° on

Figure 1: The component 1: an oil-pan look-alike structure.

either sides for ease of manufacturability. It can be seen from the figure that the component is made of several flat surfaces. These flat surfaces are designed as effective sources of vibration and noise and studying their abatement in the component can give insights regarding the design of quieter vehicles. However, in order to obtain these solutions in a reasonable time, fast and accurate simulation methodologies are needed and IGA can be a very useful tool for this purpose.

5.1.1. Surface reconstruction

The surface reconstruction procedure is outlined in Figure 2. A B-spline surface is generated starting from a mid-surface point cloud composed of roughly 100 000 nodes. The initial base surface (Figure 2b) is computed as a bi-linear Coons patch starting from the four curves that define the boundary, as described in [54]. A number of 36 and 90 control points is used in each direction. The resulting final surface is shown in Figure 2c.

According to Eq. (17), the base surface is progressively shaped to the point cloud by decreasing the value of λ from 10^3 to 1. On the other hand, the following values are used for the fairing coefficients: $\alpha_c = 1$, $\alpha_m = 0.1$ and $\alpha_b = 1 \cdot 10^{-4}$. These values are chosen such that the approximation error of the fitting surface, defined as the average distance of the points from the surface, is in the order of magnitude of the error due to the noise in the laser measurements. In fact, as discussed e.g. in [48] and [51], it is difficult to define a general rule but their choice depends rather on the sensibility of the analyst in a given application and on some eventual error criteria on the fitting surface.

The situation becomes even more complicated when different fairing terms are combined and their ratio needs to be defined. For this particular case, it was found that using too high values of the energy terms associated to α_m and α_b can lead to an irregularly distributed control net which also affects the smoothness of the final surface. Therefore, the aforementioned values of the fairing coefficients were determined with a trial and error approach until obtaining a smooth surface with a well distributed control net (outlined in Figure 2d). Determining the optimal ratio between α_c , α_m and α_b with a more automatic framework appears a challenging problem in the general case and can be the topic of future research.

As discussed in Section 3.2, the thickness distribution is also reconstructed within the surface fitting procedure and the results are outlined in Figure 3. The changes with respect to the nominal thickness of 1.5 mm due to the thinning phenomenon [71] are clearly visible.

Figure 2: Surface reconstruction on component 1; (a) the point cloud; (b) the base surface; (c) the reconstructed surface and (d) the control net;.

Figure 3: The reconstructed thickness field on component 1.

5.1.2. Experimental testing

The component is tested in free-free boundary condition by suspending it using a set of soft bungee cords as shown in Figure 4. An impact testing using a roving hammer (Onosokki GK 3100) technique is chosen for carrying out modal testing in order to avoid mass-loading effects on the sheet metal component.

The test geometry is represented by 102 points selected from the surfaces of the component. It is ensured that these 102 points are already sufficient to avoid spatial aliasing of maximum frequency of interest. The component is impacted at the 102 points in the normal direction of each surface, as out-of-plane behaviour is expected to dominate the dynamic response. The modal testing includes seven accelerometer locations. The accelerations at response locations are measured using seven tri-axial (*PCB 356A02* type) accelerometers. These accelerometers have a low transverse sensitivity and a frequency range valid up to 5000 Hz with $\pm 5\%$ error. The weight of each accelerometer is 10.5 g.

A force window (1%) on the input signal along with a Hanning window on the response signal is used to minimize the noise contamination of the results. In addition, in order to minimize noise in response measurements, ten inputs are chosen at each impact location to obtain an averaged FRF. These averaged FRFs at each node are used in curve-fitting of the modal test data to obtain eigenfrequencies and mode shapes. The sampling frequency used for acquiring data is 4098 Hz and a frequency resolution of 0.5 Hz is used. The hardware used for data acquisition is *LMS SCADAS* [72] and the

Figure 4: Modal testing arrangement of component 1 in free-free boundary condition.

LMS Test.Lab software [73] is used for carrying out modal test data analysis.

5.1.3. Results and discussion

The accuracy of the IGA model is assessed by means of a correlation between the numerical and experimental results. As a preliminary comparison, the first 10 natural frequencies are considered. A comparison between the frequencies computed with the numerical model and the ones extrapolated from the experiment is reported in Table 1. The error is in the order of a few percent digits.

The material parameters used in the simulation are a Young's modulus E of 205 GPa, a Poisson coefficient $\nu = 0.29$ and a density $\rho = 8.0 \cdot 10^3$ kg/m³. These parameters play a key role in the computation of the natural frequencies, since they are proportional to $\sqrt{\frac{E}{\rho}}$. Thus, as a first approximation, an error in the order of 2% on E or ρ corresponds to an error of 1% on the predicted natural frequency. The values indicated above are those given by the material supplier, however it is quite common to have an uncertainty in the order of $\pm 2\%$ only for the Young's modulus [74]. Based on these considerations, the results reported in Table 1 can be assumed accurate given the uncertainty limits and the other effects not accounted for in the numerical model, such as the residual stresses coming from the manufacturing process.

Together with the natural frequencies, the error on the corresponding shapes is considered by means of the Modal Assurance Criterion (MAC)

Table 1: First ten frequencies computed with the IGA model and their error with respect to the measurements for component 1.

Experiment	IGA simulation	Err%
91	89	-2.25
114	112	-1.93
247	253	2.60
271	268	-1.04
328	323	-1.55
355	364	2.61
385	383	-0.55
421	399	-5.23
438	443	1.11

Figure 5: MAC results of the IGA simulation compared to the measurements.

[75, 76]. MAC has been widely used in the literature for correlating finite element models to experimental results and recently also in the context of IGA [77]. In particular, the MAC value for a pair of modal vectors $\{\phi_r\}_i$ and $\{\phi_a\}_j$ is a scalar number indicating the degree of consistency between these two vectors and is defined by:

$$MAC = \frac{|\{\phi_r\}_i^H \{\phi_a\}_j|^2}{(\{\phi_r\}_i^H \{\phi_r\}_i)(\{\phi_a\}_j^H \{\phi_a\}_j)}, \quad (25)$$

where $\{\}^H$ denotes the Hermitian matrix. A value close to unity indicates a good correlation between the experimental mode shape vector $\{\phi_r\}_i$ and the numerical eigenvector $\{\phi_a\}_j$.

The experimental mode shapes are obtained from the set of 102 points discussed in the previous section. The resulting MAC matrix is plotted in Figure 5 where a perfect correlation is observed for the first 5 modes, with the accuracy decreasing toward higher frequencies. These results are in agreement with those reported using the traditional FEM approach in [67], where a similar accuracy is obtained.

The validation of the model is completed comparing the experimental and numerical frequency response functions (FRFs), when the component is excited in the center of its upper part, denoted as Point 2 in Figure 4. In order to reduce the effect of the measurement noise, the sum of the FRFs of the 102 points is considered first in Figure 6. If damping is not included in the numerical model, a good agreement is observed in the first part of the spectrum but the peaks of the IGA solution become too high in the higher frequency region.

On implementing a reverse engineering procedure, the exact damping behaviour of a structure is not known *a priori* and hypotheses need to be made to properly describe it. An approach often adopted is to make the assumption of modal damping [78] and to use the same coefficient for each mode, normally in the range of 1-2% for steel. However, in this case, the exact damping coefficients are also extrapolated from the measurements and used for the purpose of the validation. In particular, the first 20 modal damping coefficients are calculated and a value of 1.5% is used for all the remaining modes. The resulting simulation gives a better agreement also in the higher frequency region.

The accuracy of the model is confirmed also by considering the FRFs on three single points, which are reported in Figure 7. Although the noise in the measurements is more evident, a good agreement is observed in the first part of the spectrum and the numerical FRFs are still able to globally match the experiments toward higher frequencies.

5.2. Component 2

The second component considered for the validation of the method, shown in Figure 8, is a steel cup in the material DP600. The cup has been produced by deep drawing at OCAS¹ on an Erichsen 145-60 press starting from a circular blank with a diameter of 100 mm and a nominal thickness of 0.97 mm. The drawing depth is 24 mm. These components find applications in fuel cells.

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(a) Without damping

(b) Using modal damping

Figure 6: FRF comparison between measurements and IGA simulation results for component 1. A point force excitation is applied in Point 2 (see Figure 4) and the sum of the FRFs for the acceleration in 102 points distributed over its surface is considered.

(a) Point 1

(b) Point 2

(c) Point 3

Figure 7: FRF comparison between measurements and IGA simulation for component 1. A point force excitation is applied in Point 2 and the FRF is computed in Point 1, 2 and 3 (see Figure 4).

Figure 8: The component 2: a cup for fuel cell applications.

5.2.1. Surface reconstruction

The surface reconstruction procedure is outlined in Figure 9. The B-spline model is generated starting from a cloud of 50 000 nodes and a circular shaped base surface is generated, starting from the boundary of the cup. A number of 32 control points is used along each edge.

For this component, it is found that better results in terms of smoothness of the final surface are obtained if a bigger weight is associated to the membrane and bending energy contributions. In particular, the following values are used: $\alpha_c = 0.1$, $\alpha_m = 0.5$ and $\alpha_b = 1 \cdot 10^{-3}$.

The thickness approximation is also plotted in Figure 10, where the thinning phenomenon is again visible, with the cup become thinner on the flanks and thicker on the upper part, with respect to the initial blank thickness of 0.97 mm.

5.2.2. Experimental testing

The low mass of the component, with a weight of approximately 60 g, and its particular shape result in having the first natural frequencies at approximately 4 kHz, which poses some challenges for its dynamical testing. Therefore, instead of the measurement set-up with accelerometers on the component, it is opted to measure the structural displacement with a laser micrometer, in order to avoid the addition of mass. In particular, a *Polytec PSV-500 Scanning Vibrometer* is used.

Also in this case, it is chosen to mimic free-free boundary conditions. This is obtained by fixing the cups with two thin nylon strings as shown in Figure 11. The test bench is described in detail in [79]. Some preliminary mode shape calculations on cup-like structures indicate that the first

Figure 9: Surface reconstruction on component 2; (a) the point cloud; (b) the base surface; (c) the reconstructed surface and (d) the control net;.

Figure 10: Thickness reconstruction on component 2.

Figure 11: Dynamical testing of component 2 with the laser vibrometer. The cup is hammered in correspondence of the blue circle (referred to as point 1) and the FRF is considered in correspondence of the red circles (referred to as points 2, 3 and 4). The reflecting labels used for the laser vibrometer are also visible.

modeshapes have nodal lines at the location of the strings, motivating this choice.

With reference to Figure 11, where the reflecting labels used for the measurements are visible, the cup is excited using a hammer with impedance head type *PCB 086C03* in point 1 (outlined with a blue circle) and the FRF is measured in points 2, 3 and 4 (outlined in red). Since the vibrometer can measure only one point at a time, the FRF results of these three points are gathered and used to validate the numerical model.

5.2.3. Results and discussion

The natural frequencies of the component computed with the IGA model are compared with those coming from the measurements in Table 2. Also in this case, the error is in the order of a few percent digits for the first nine frequencies.

The material parameters are a Young's modulus E of 200 GPa, a Poisson coefficient $\nu = 0.3$ and a density $\rho = 7.8 \cdot 10^3 \text{ kg/m}^3$. Since 20 exemplars of this component are available, the density was calculated by measuring their weight and considering the nominal volume of the blank, with an estimated standard deviation of 0.5%. On the other hand, the value given by the material supplier is used for the Young's modulus. For this type of steel, we note that in literature different values in the range of 200-210 GPa have been

Table 2: First nine frequencies computed with the IGA model and their error with respect to the measurements for component 2.

Experiment	IGA simulation	Err%
3913	3898	-0.39
4024	3939	-2.12
4113	4116	0.08
4130	4164	0.82
4262	4208	-1.26
4914	4824	-1.83
4925	4911	-0.30
5022	5060	0.77
5065	5081	0.32

used for the Young’s modulus [80, 81, 82], which confirms the uncertainty on his value.

As far as FRFs are concerned, taking into account the different behaviour of this structure and the lack of experimental data on damping, no damping is included in the numerical simulation.

A comparison between the experimental and numerical FRFs in the three measured points is reported in Figure 12. Since the laser vibrometer provides measurements on the velocity, the displacements FRFs are obtained from them and used for the comparison. Contrary to the previous case, the natural frequencies of this component start at around 4 kHz and, therefore, it can be observed how the numerical model is able to correctly represent its behaviour in the low frequency region. On the other hand, some frequency shifts start to occur towards higher frequencies, which may hint again at a slight difference in stiffness due to the uncertainty in the material parameters or due to effects not accounted for in the numerical model, as discussed above.

Also for this application, the accuracy of the results is in line with that found using the standard FEM approach in [66]. However, the advantage of employing the IGA framework is evident when comparing the number of DOFs used by the two methodologies. In particular, in the case of FEM, a mesh containing 50’000 nodes is used to properly capture and approximate the geometry. This is not the case of IGA, where the $32 \times 32 = 1024$ control points model is already able to properly represent the shape of the component. The advantages are then evident for the FRF computation, where a linear system of equations has to be solved at each frequency.

(a) Point 2

(b) Point 3

(c) Point 4

Figure 12: FRF comparison between measurements and IGA simulation results for component 2. A point force excitation is applied in Point 1 (see Figure 11) and the FRFs in points 2, 3 and 4 are considered.

6. Concluding remarks

This work considered the simulation of the dynamic behaviour of deep drawn components with a reverse engineering framework, based on isogeometric analysis. In particular, a B-spline surface was generated from the measured point clouds of the components and used directly for the analysis, with the isogeometric Kirchhoff-Love shell formulation.

Two manufactured components were considered and the results of the numerical simulation were validated by comparing them with experimental tests. The results were accurate both in the prediction of the natural frequencies and in the FRF computation and were in line with those found in previous studies using the classical FEM approach.

However, as discussed in the paper, the isogeometric framework introduces some advantages with respect to the traditional FEM fitting and simulation:

- the Kirchhoff-Love rotation-less formulation can be directly applied;
- an accurate smooth geometric representation of the fitted component can be obtained with a reduced number of degrees of freedom;
- the fitted surface can be directly refined for the purpose of the analysis, for instance in the case of high frequency FRF calculations.

These considerations make the methodology appealing for different applications. Geometries with a more complex topology can be the topic of future research, where both multi-patch B-spline models or splines defined on unstructured grids can be applied. The recent technology of smooth polar splines appears also particularly suited for most of the geometries of deep drawn components and its application within the framework developed in this study can also be the topic of future studies.

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