The natural number bias and its role in rational number understanding in children with dyscalculia. Delay or deficit?

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ABSTRACT

Background: Previous research indicated that in several cases learners' errors on rational number tasks can be attributed to learners' tendency to (wrongly) apply natural number properties. There exists a large body of literature both on learners' struggle with understanding the rational number system and on the role of the natural number bias in this struggle. However, little is known about this phenomenon in learners with dyscalculia.

Aims: We investigated the rational number understanding of learners with dyscalculia and compared it with the rational number understanding of learners without dyscalculia.

Method: Three groups of learners were included: sixth graders with dyscalculia, a chronological age match group, and an ability match group.

Results: The results showed that the rational number understanding of learners with dyscalculia is significantly lower than that of typically developing peers, but not significantly different from younger learners, even after statistically controlling for mathematics achievement.

Conclusion: Next to a delay in their mathematics achievement, learners with dyscalculia seem to have an extra delay in their rational number understanding, compared with peers. This is especially the case in those rational number tasks where one has to inhibit natural number knowledge to come to the right answer.

What this paper adds

While there exists a large body of literature both on learners' struggle with understanding the rational number domain and on the role of the natural number bias in this struggle, little is known about the rational number understanding of learners with dyscalculia (LWD). Nonetheless, a better insight of the understanding of rational numbers of LWD is important to provide adaptive (remedial) instruction with the aim to increase LWD's understanding of the rational number system. Moreover, the studies that did already investigate LWD's rational number understanding only focused on their understanding of the magnitude of rational numbers. The present study includes a wider range of problem types, including both magnitude, operations, and density tasks, enabling us to provide a more complete picture of the rational number understanding. The results of the present study add insight in the rational number understanding of LWD by suggesting that LWD have an additional delay on their rational number understanding. While this is the case for all types of rational number tasks, the difference in accuracy level between LWD and their peers is much higher in those rational number tasks where natural number reasoning leads to an incorrect answer. So, this study suggests that LWD are more affected by the natural number bias compared to their peers, leading to even more difficulties understanding the rational number

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system.

1. Introduction

Rational numbers are known to form a stumbling block for many people. Not only elementary school children, but also secondary school students, adults, and even (prospective) teachers have trouble understanding the rational number domain (e.g., Cramer, Post, & delMas, 2002; Mazzocco & Devlin, 2008; Vamvakoussi, Van Dooren, & Verschaffel, 2012; Vamvakoussi & Vosniadou, 2004). For instance, in a recent study of Van Hoof, Verschaffel, and Van Dooren (2015), only 57% of a large group of eighth graders solved the question “What is half of $1/8$?” correctly. The most frequent erroneous answer was $1/4$. It is quite worrying that so many learners have troubles understanding rational numbers, some of which even last until the end of secondary school. This is especially worrying since the understanding of rational numbers has been shown to relate to later mathematics achievement in general and for performance in important domains of the mathematics curriculum in particular such as algebra (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; Siegler, Thompson, & Schneider, 2011). For example, Booth and Newton (2012) found that learners’ understanding of fractions is a better predictor of learners’ algebra readiness than their whole number magnitude knowledge.

1.1. The natural number bias

There are several distinct reasons why learners are found to struggle so much with the understanding of rational numbers (e.g., Van Hoof, Vamvakoussi, Van Dooren, & Verschaffel, 2017). For example, learners need to understand the several distinct conceptual meanings of fractions such as fractions as parts of a whole, fractions as ratios, fractions as numbers (Lamon, 2001) and the fact that different representations can represent the same magnitude (e.g., $\frac{1}{2}$, $\frac{3}{6}$, 0.5, 0.50, 50%) (Moss, 2005). Siegler, Fazio, Bailey, and Zhou (2013) discuss the possible confusion when conducting arithmetic operations with rational numbers. They give the example that adding and subtracting fractions with the same denominator will result in a fraction with the same denominator, whereas this is not the case for multiplication and division. This leads to mistakes such as “ $2/5 \times 3/5 = 6/5$ ”.

One frequently recurring explanation for learners’ difficulty to understand the rational number system is the natural number bias, a phenomenon that has generated substantial research interest both in the fields of mathematics education and cognitive psychology (Jordan et al., 2013; Merenluoto & Lehtinen, 2004; Ni & Zhou, 2005; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Vamvakoussi, Christou, Mertens, & Van Dooren, 2011; Vamvakoussi, Van Dooren, & Verschaffel, 2012; 2013; Van Hoof, Janssen, Verschaffel, & Van Dooren, 2015; Van Hoof, Lijnen, Verschaffel, & Van Dooren, 2013; Van Hoof, Vandewalle, Verschaffel, & Van Dooren, 2015). Ni and Zhou (2005) define the natural number bias as the tendency to apply natural number properties in tasks with rational numbers, even when this is inappropriate. The phenomenon is described as follows: the rational number system is the first new number system learners encounter after the natural number system. Therefore, to comprehend the rational number system, learners have to construct an entirely new number concept and new procedures to handle numbers, which are not always in accordance with their prior knowledge of (natural) numbers. In line with this description, numerous studies indeed indicated that learners make systematic and predictable errors in rational numbers tasks where the use of prior natural number knowledge leads to the incorrect answer (further referred to as inconsistent items) while at the same time they are much more accurate in rational number tasks where reliance on prior natural number knowledge leads to the correct answer (further referred to as consistent items) (Vamvakoussi et al., 2012; Van Hoof, Vandewalle et al., 2015).

Previous research on the natural number bias mainly focused on three aspects in which rational numbers differ from natural numbers and where errors are known to occur: their dense structure, the way their numerical magnitude can be determined, and the effect of the four basic operations.

Regarding the first aspect, rational numbers are dense. Contrary to natural numbers, rational numbers do not obey the successor principle. While with natural numbers one can always point out which number follows a given number, it is impossible to say which number follows a given rational number, because between any two given rational numbers are always infinitely many other numbers. Several studies report the struggle learners have with understanding the dense structure of rational numbers (Merenluoto & Lehtinen, 2004; Vamvakoussi & Vosniadou, 2004, 2010).

Concerning the second aspect, the numerical magnitude of rational numbers, you cannot always rely on the numerical value of the natural numbers involved to determine the magnitude of rational numbers. With fractions, it is not always true that a fraction’s numerical value increases when its denominator, numerator or both increase. In the same line, with decimal numbers, longer decimals are not always larger and shorter decimals are not always smaller. These differences between rational and natural numbers in the way their numerical magnitude can be determined are known to lead to many mistakes by learners (Christou & Vosniadou, 2012; Resnick et al., 1989).

Regarding the last aspect, the effect of arithmetical operations, with rational numbers, multiplication and addition do not always lead to a larger outcome and division and subtraction do not always lead to a smaller outcome, while this is the case with natural numbers. Several studies indicated that learners still rely on the natural number rules while solving rational number tasks, leading to systematic errors (Vamvakoussi et al., 2012; Van Hoof, Vandewalle et al., 2015; Christou & Vosniadou, 2012).

In sum, these three aspects on which natural numbers differ from rational numbers are reported to lead to many misconceptions about rational numbers, which lead in their turn to those systematic errors in inconsistent rational number tasks.

1.2. Rational number understanding in learners with dyscalculia

While learners' difficulties with rational numbers are widely documented, much less is known about the rational number understanding of learners with dyscalculia (further abbreviated with LWD, also termed having a Mathematical Learning Disability (MLD)) (Hecht & Vagi, 2010; Mazzocco & Devlin, 2008). Nonetheless, Mazzocco and Devlin (2008) stated that it is "of concern to both educators and researchers [...] to determine [...] how this difficulty varies in children with and without MLD" (p. 681). After all, a better understanding of LWD's rational number understanding is important to provide adaptive (remedial) instruction with the aim to increase their understanding of the rational number system (Hecht & Vagi, 2010). A study that did investigate rational number understanding in LWD is the longitudinal study by Mazzocco and Devlin (2008). 147 learners were given the Ranking Proportions Test in 6th, 7th, and 8th grade. In this test learners were asked to order rational numbers with respect to their magnitude. They were presented in decimal form, fractional form, a combination of both fractions and decimals, and in pictures. Based on standardized mathematics achievement scores, three different groups of learners were formed: learners with mathematical learning disabilities, learners with a low mathematics achievement but without mathematical learning disabilities, and learners with a typical mathematics achievement. The main conclusion was that in learners with mathematical learning disabilities the accuracy on the Ranking Proportions Test was significantly lower on all subtests compared to the other two groups of learners. In a subsequent study, Mazzocco, Myers, Lewis, Hanich, and Murphy (2013) longitudinally monitored 122 learners' rational number knowledge from Grade 4 to Grade 8 using the Symbolic Magnitude Comparison of Fractions test. In this test, learners had to compare 60 fractions pairs and were instructed to circle the larger one or put an equal sign if both fractions are equally large. All items were either Arabic numbers or shaded boxes and no time limit was used to avoid automatized responses. Their main conclusion was that although many learners still struggle with understanding fraction magnitudes by grade 8, the nature and trajectories of those difficulties differ between learners with and without a mathematics learning disability. For example, one-half items were consistently found to be easier compared to non-half items for learners with a typical or low mathematics achievement. However, this was not found in the group of learners with a mathematics learning disability.

1.3. The present study

The goal of the present study is to extend the study of Mazzocco and Devlin (2008) in several ways.

First, previous studies investigating LWD's rational number understanding only look at learners' understanding of the magnitude of rational numbers. However, as stated above, there are three aspects on which natural numbers differ from rational numbers that are reported to lead to many misconceptions about rational numbers: magnitude, operations, and density. In the present study, we will provide a diversity of problem types, including both magnitude, operations, and density tasks, enabling us to measure a broader and complete rational number understanding.

Second, in the study of Mazzocco and Devlin (2008) learners were solely matched on their 'chronological age'. In this study we will also match learners on their mathematical ability level. This allows to investigate whether there is a 'deficit' or a 'delay' in LWD's rational number understanding (Torbeys, Verschaffel, & Ghesquière, 2004). If LWD's rational number understanding is significantly lower than that of learners of the same age (= chronological age match), but not significantly different from learners with the same mathematics achievement level (= ability match), who are typically younger, this implies that LWD's rational number understanding matches their mathematics achievement level in a more general sense, and thus that the development of LWD's rational number understanding is only characterized by a 'delay' rather than 'deficit'. However, if LWD's rational number understanding is not only significantly lower than that of learners of the same age but also significantly lower than (younger) learners with the same mathematics achievement level, the development of LWD's rational number understanding does not match their mathematics achievement level, thus their rational number understanding is characterized by a 'deficit' (Torbeys et al., 2004). This leads to the first research question (Research Question 1), namely whether LWD's rational number understanding is characterized by a 'delay' or a 'deficit' compared to learners without dyscalculia. No specific prediction could be made based on the available research literature.

Third, while Mazzocco and Devlin (2008) measured learners' understanding of the magnitude of rational numbers in a rather general sense, we will pay particular attention to the differences between consistent and inconsistent rational number tasks, in order to get an idea of a possible natural number bias in the three groups of learners. As stated above, numerous studies indicated that learners make systematic and predictable errors especially in those rational numbers tasks where the use of prior natural number knowledge leads to the incorrect answer (inconsistent items). At the same time, they are much more accurate in rational number tasks where reliance on prior natural number knowledge leads to the correct answer (consistent items). By focusing on learners' accuracy on inconsistent as compared to consistent items, we tried to answer our second research question (Research Question 2): Is the strength of the natural number bias in LWD comparable with that of typically developing children? Again, based on the available research literature, no specific prediction could be made concerning the comparison between both groups' accuracy on inconsistent items, because two outcomes seem possible: One could argue that because LWD have less strong intuitive concepts regarding natural numbers (e.g., Berch, 2005), they may be less biased by the natural numbers in rational number tasks. However, the natural numbers that are present in the rational number representation (fraction or decimal) unavoidable need to be processed to give a correct answer to a task, while the inclination to give an answer that is merely based on the reasoning about natural numbers needs to be inhibited in inconsistent rational number tasks. Given that LWD are known to have low inhibition capabilities and are especially known to struggle with number inhibition tasks (Szucs, Devina, Soltesz, Nobes, & Gabriel, 2013; Wang, Tasi, & Yang, 2012; Zhang & Wu, 2011) one could also argue that they will have a stronger natural number bias because they are less able to inhibit their natural number knowledge in rational number tasks. As a measure of the strength of the natural number bias, we used learners' accuracy level on the

inconsistent rational number tasks. However, this needs to be compared to the accuracy on consistent rational number tasks, in which natural number knowledge leads to the correct answer.

2. Method

2.1. Participants

Three different groups of learners were included in this study. A first group consisted of sixth graders with dyscalculia ($n = 16$). To be part of this group, learners had to meet four conditions. First, all the learners of this group had to have an official clinical diagnosis of dyscalculia, given by independent experts. This way, they all met the criterion of lag (the level of general math achievement is significantly lower compared to peers), persistence (the lag is persistent even after remediation and instruction), and exclusivity (while there can be other problems such as reading problems, these factors do not explain learners' low general math achievement) (Bally et al., 2014). Second, all the learners with dyscalculia needed to have a normal intelligence level. Third, the learners of this group were not allowed to be exempted from rational number instruction. Fourth, the home language of the learners needed to be Dutch.

Because it is very difficult to find learners with only dyscalculia as a learning deficit and a clinical diagnosis, comorbidities were allowed. However, the diagnostic process was such that comorbidity would not be a sufficient explanation for learners' difficulty with mathematics; a specific mathematical disability was required (see exclusivity criterion above). Next to the LWD, we included two control groups, a chronological age match and an ability match group. The first control group were sixth graders without dyscalculia ($n = 56$), further referred to as the sixth grade control group. The mean age (in months) of the LWD was 143.97 ($SD = 10.39$), while the mean age of the sixth grade control group was 142.13 ($SD = 3.53$). An independent samples *t*-test showed that this difference in age was not significant, $t(70) = 1.13$, $p = 0.26$. The ability-match control group were fourth graders ($n = 51$), further referred to as the fourth grade control group. We chose to include this age group because we needed younger learners with a mathematics achievement level that we could expect to be comparable to that of sixth graders with dyscalculia, but who would also be able to solve the rational number test. The mean mathematics achievement level (for a description of this measure, see below) of the LWD was 92.81 ($SD = 40.13$), while the mean mathematics achievement level of the fourth grade control group turned out to be much higher, i.e., 134.75 ($SD = 29.56$). An independent samples *t*-test showed that this difference was significant; $t(65) = -4.53$, $p < 0.001$. This result shows that we did not succeed in creating an ability-matched group. A possibility to address this problem would have been to include even younger learners, who would then have a lower mathematics achievement level and be able to serve as control group. However, this was not possible because learners from third grade or younger would not have been able to solve the rational number test, which was based on the Flemish mathematics curriculum. Therefore, we chose to take into account learners' mathematical ability as a control variable instead. By statistically controlling for learners' mathematics achievement, we were able to investigate whether there was still a difference between both groups' (LWD and fourth grade control group) rational number understanding that could not be explained by a difference in mathematics achievement level, but by having dyscalculia.

2.2. Instruments

2.2.1. Rational number understanding

Learners completed a shortened version of the "Rational Number Sense Test" (RNST; Van Hoof, Verschaffel et al., 2015) as a measure of their rational number understanding. This test instrument was constructed based on a broad literature with the aim to measure learners' natural number bias in rational number tasks. The RNST was first used in a larger research project aimed at mapping the development of rational number understanding throughout elementary and secondary school education and has been found a valid and reliable test instrument to measure learners' rational number knowledge and the natural number bias (e.g., Van Hoof, Verschaffel et al., 2015). The shortened test consisted of 49 items. Both consistent ($n = 16$) and inconsistent items ($n = 33$) from the three aspects that were discussed in the introduction (density, size, and operations) were included in the study. Further, rational numbers were presented both in their decimal as in their fractional form. Examples of items can be found in Fig. 1. This test was administered collectively. Moreover, the RNST is characterized by a high reliability (Cronbach's alpha = 0.87, see Van Hoof, Verschaffel et al., 2015). The subtest is also characterized by a high reliability (Cronbach's alpha = 0.83).

	Consistent	Inconsistent
Density	Write a number between $1/4$ and $3/4$	Write a number between 3.49 and 3.50
Size	Choose the largest number: 4.4 or 4.50	Choose the largest number: $3/2$ or $9/8$
Operations	Is the outcome of $50 * 3/2$ smaller or larger than 50?	Is the outcome of $40 * 0.99$ smaller or larger than 40?

Fig. 1. Examples of consistent and inconsistent items.

2.2.2. Mathematics achievement

It is known that one of the core difficulties LWD have is automatizing basic arithmetic facts (e.g., Bull & Johnston, 1997; Munro, 2003). Therefore, we chose to include the Tempo Test Automatiseren (De Vos, 1992) as a measure of learners' mathematics achievement. This test measures the automated knowledge of the four basic operations (addition, subtraction, multiplication, and division). For each basic operation, learners were asked to complete as accurately and as fast as possible arithmetic problems during two minutes. Afterwards, for each learner, the sum score was calculated by adding the scores on each of four subtests. This test was administered collectively.

2.2.3. Intelligence

2.2.3.1. Raven's progressive matrices. To be sure that the learners indeed met the exclusivity criterion and their low math achievement level could not be explained by their IQ, we decided to include two IQ measures. We used Raven's Progressive Matrices test (Raven, Court, & Raven, 1995) as a measure of learners' non-verbal intelligence. In total, 24 items were used: all items of Set D and all items of Set E. All 24 items were scored as correct or incorrect. This test was administered collectively.

2.2.3.2. SiBO. We used the SiBO test as a second measure of learners' intelligence. While the Raven's Progressive Matrices Test was a measure of learners' non-verbal intelligence, this test measured their verbal intelligence. This test was developed as part of a large-scale longitudinal study investigating learners' educational careers from the first year of primary education to the first year of secondary education (Hendrikx, Maes, Magez, Ghesquière, & Van Damme, 2007). It consists of three subtest: contradictions, logical connections, and separation. From every subtest, only the first 10 items were used in this study. All items were scored as correct or incorrect. This test was also administered collectively.

2.2.4. Reading achievement

Because we allowed comorbidity in our sample, we also included two measures of reading achievement as control variables. This way, we could be sure that the results found in this study were not due to lower reading achievement.

2.2.4.1. One minute test. The één-minuut test (one minute test, further abbreviated with EMT) was used as a measure of learners' word recognition ability. The test consists of a list of 116 words ordered in four columns. The goal of the test is that learners correctly read out loud as many words as possible within one minute. Standardized scores were used based on existing norm tables (Brus & Voeten, 1972). This test was administered individually.

2.2.4.2. Klepel. We used the Klepel as a second measure of learners' reading ability. Like the EMT, the test consists of a list of 116 words ordered in four columns. However, contrary to the EMT, the words in this test are non-existing words. Further, instead of one minute (as in the EMT), learners get two minutes to correctly read out loud as many non-words as possible. Standardized scores were used based on the existing norm tables (van den Bos, Spelberg, Scheepstra, & de Vries, 1994). This test was also administered individually.

3. Results

3.1. Descriptive statistics

Table 1 presents the descriptive statistics for the variables used in this study. Concerning the two measures of IQ (Raven and

Table 1
Descriptive statistics.

		LWD	4th graders	6th graders
Age (months)	mean	143.97	119.34	142.13
	SD	10.39	3.54	3.532
Raven	mean	10.06	11.71	13.50
	SD	3.11	5.05	4.62
SiBo	mean	27.19	27.92	28.73
	SD	2.17	1.64	1.26
IQ (Raven + SiBo, z-scores)	mean	−0.99	0	0
	SD	1.12	0.79	0.83
EMT (standardized scores)	mean	5.69	8.82	9.55
	SD	3.98	2.85	2.62
KLEPEL (standardized scores)	mean	8.56	11.16	12.09
	SD	3.83	2.33	2.51
Reading achievement (EMT + KLEPEL, standardized scores)	mean	7.13	9.99	10.82
	SD	3.78	2.49	2.48
Mathematics achievement	mean	92.81	134.75	171.27
	SD	40.13	29.56	22.30

Fig. 2. Learners' accuracy on consistent and inconsistent rational number tasks per group. Note. * = $p < 0.01$, ** = $p < 0.001$.

SiBo), the lowest scores were found in the LWD, while the highest scores were found in the sixth grade control group. The same trend was found for learners' performance on the measures of reading achievement (EMT and KLEPEL).

Because high correlations were found between both IQ measures (Raven and SiBo) ($r = 0.41$, $p < 0.001$), we created one general intelligence score by first calculating z-scores for every measure separately and then taking the mean of these two scores for each learner. We first calculated the z-scores of the fourth grade control group and the sixth grade control group separately. Subsequently, the z-scores of the LWD were calculated based on the mean and standard deviation of the sixth grade control group. Also for the two reading achievement measures, high correlations were found ($r = 0.92$, $p < 0.001$). Therefore, we also calculated each learner's mean standardized score on the two measures as a general score of reading achievement. Both learners' general IQ score and their general reading achievement score were used as covariates in the further analyses.

3.2. Comparison between LWD and sixth grade control group

3.2.1. Consistent items

As can be seen in Fig. 2, LWD's accuracy on consistent items ($M = 12.06$, $SD = 2.41$ with a possible maximum of 16) was lower than the accuracy of the sixth grade control group ($M = 13.82$, $SD = 1.77$). This difference was significant, $F(1.70) = 10.40$, $p < 0.01$, $\eta^2 = 0.13$ and was characterized by a medium effect size.

Because our aim was to have a chronological age match design, in a next step we added learners' age (in months) as control variable in the comparison between both groups' accuracy on consistent rational number tasks. Moreover, as stated above, the LWD and the sixth grade control group differed in their general IQ and reading achievement. Therefore, we also included these two as control variables. An ANCOVA indicated that, even after controlling for learners' age, IQ, and reading achievement, the sixth grade control group still significantly outperformed LWD on consistent rational number tasks, $F(1.67) = 4.35$, $p = 0.04$, $\eta^2 = 0.06$, but the effect size was only small.

3.2.2. Inconsistent items

LWD's accuracy on inconsistent items ($M = 7.00$, $SD = 5.48$ with a possible maximum of 33) was also far lower than the accuracy of the sixth grade control group ($M = 20.52$, $SD = 4.10$ with a possible maximum of 33), see Fig. 2. This difference was significant $F(1.70) = 115.69$, $p < 0.001$, $\eta^2 = 0.62$ and was characterized by a large effect size.

In a next step we again additionally controlled for learners' age, IQ, and reading achievement. An ANCOVA indicated that, even after controlling for these variables, the sixth grade control group still significantly outperformed LWD on inconsistent rational number tasks, $F(1.67) = 102.16$, $p < 0.001$, $\eta^2 = 0.60$, and that the effect size remained large.

3.2.3. Comparison between consistent and inconsistent items

If we look at the partial eta squared values, we see that the difference in accuracy between LWD and the sixth grade control group is much higher in inconsistent than in consistent rational number items. Even when taking into account the various control variables, 60% of the variance in learners' accuracy on inconsistent items can be explained by whether a learner is in the group of learners with

dyscalculia or not, while this was only 6% of the variance for the consistent rational number items. This suggests that the natural number bias (as shown in learners' accuracy level on inconsistent rational number tasks) is much stronger in LWD compared to their peers.

3.3. Comparison between LWD and fourth grade control group

3.3.1. Consistent items

As can be seen in Fig. 2, LWD's accuracy on consistent items ($M = 12.06$, $SD = 2.41$ with a possible maximum of 16) was a bit higher than the accuracy of the fourth grade control group ($M = 11.16$, $SD = 3.10$ with a possible maximum of 16). This difference between the two groups was however not significant $F(1.65) = 1.15$, $p = 0.29$, $\eta^2 = 0.02$. As stated above, we did not succeed in realizing an ability match. Therefore, in a next step we added learners' mathematics achievement as a control variable in the comparison between both groups' accuracy on consistent rational number tasks. Moreover, because the LWD and the fourth grade control group also differed in their general IQ and reading achievement, we also included these two measures as control variables. An ANCOVA indicated that, after controlling for learners' mathematics achievement, IQ, and reading achievement, no significant difference was found between both groups' accuracy on consistent rational number tasks, $F(1.62) = 3.69$, $p = 0.08$, $\eta^2 = 0.03$.

3.3.2. Inconsistent items

LWD's accuracy on inconsistent items ($M = 7.00$, $SD = 5.48$ with a possible maximum of 33) is lower than the accuracy of the fourth grade control group ($M = 8.98$, $SD = 4.61$ with a possible maximum of 33), see Fig. 2. This difference between the two groups was however not significant $F(1.65) = 2.06$, $p = 0.16$, $\eta^2 = 0.03$. For the same reasons as above, in a next step we additionally controlled for learners' mathematics achievement, IQ, and reading achievement. An ANCOVA indicated that, after controlling for these three measures, no significant difference was found between both groups' accuracy on inconsistent rational number tasks, $F(1.62) = 1.75$, $p = 0.19$, $\eta^2 < 0.01$.

3.3.3. Comparison between consistent and inconsistent items

Even though the LWD and fourth grade control group differ significantly in their mathematics achievement level, they do not significantly differ in their rational number understanding. Based on the accuracy levels on the inconsistent rational number tasks, we can conclude that the strength of the natural number bias is similar in the LWD compared with the fourth grade control group. Moreover, even though LWD have a significantly lower mathematics achievement level compared to the fourth grade control group, they do not significantly differ in their rational number understanding. This result suggests that while LWD have a serious delay in their rational number understanding compared to their peers, they might have a lead compared to younger learners with the same mathematics achievement level.

4. Discussion

In the present study, we further investigated LWD's rational number understanding as we saw several valuable extensions to the study of Mazzocco and Devlin (2008). First, previous studies investigating LWD's rational number understanding only looked at learners' understanding of the magnitude of rational numbers. In the present study, we included a diversity of problem types, including besides magnitude tasks also items about operations and density, enabling us to measure a more broad and complete rational number understanding.

Second, next to a 'chronological age' match, the aim in this study was also to match on learners' mathematical ability to investigate whether there is a 'deficit' or a 'delay' in LWD's rational number understanding (Research question 1). However, we did not succeed to match learners on their mathematics achievement. Therefore, we chose to take into account learners' mathematics achievement as a control variable instead. By statistically controlling for learners' mathematics achievement, we could investigate whether there was still a difference between both groups' rational number understanding that could not be explained by a difference in general mathematics achievement, but by having dyscalculia. Third, while Mazzocco and Devlin (2008) measured learners' understanding of the magnitude of rational numbers understanding in a general sense, we made a distinction between consistent and inconsistent rational number tasks. By focusing on learners' accuracy on inconsistent rational number tasks, we investigated whether the strength of the natural number bias in LWD is comparable with that of typically developing learners (Research question 2). To answer both research questions, we first compared LWD's accuracy on rational number tasks with that of learners without dyscalculia of the same age and with that of younger learners without dyscalculia to investigate whether there is just a delay in LWD's rational number understanding or a deficit. As a second step we focused on learners' accuracy level on inconsistent items to investigate whether the strength of the natural number bias in LWD is comparable with that of typically developing learners.

Concerning the first research question results showed that LWD's rational number understanding is significantly lower than that of typically developing learners of the same age, but not significantly different from younger learners, even after statistically controlling for mathematics achievement, both in consistent and inconsistent rational number tasks. These results suggest that the development of LWD's rational number understanding is characterized by a delay instead of a deficit. Next to the fact that LWD are known to have a delay in their general mathematics achievement level, this study suggests that they have an additional delay in their rational number understanding compared with their peers.

Concerning the second research question, there was a big difference in accuracy level between LWD and their peers on inconsistent rational number tasks, while no significant difference could be found with fourth graders' accuracy on the same inconsistent

rational number tasks. The difference in accuracy on inconsistent rational number tasks revealed that the strength of the natural number bias is higher in LWD compared to typically developing learners of the same age, but is not different compared to younger learners. This finding points out that LWD's rational number understanding is characterized by a delay instead of a deficit.

In sum, the results of the present study suggest that LWD have a delay on their rational number understanding. While this is the case both for consistent and for inconsistent rational number tasks, the difference in accuracy level between LWD and their peers is rather small for consistent items, but much higher in inconsistent items where one has to inhibit one's natural number knowledge to come to the correct solution. One possible interpretation of this finding is that previous studies have indicated that LWD have low inhibition capabilities and are especially known to have low accuracy levels on number inhibition tasks (Szucs et al., 2013; Wang et al., 2012; Zhang & Wu, 2011). This finding suggests that LWD are more affected by the natural number bias compared to their peers, which leads to even more difficulties understanding the rational number system. We further found that LWD's rational number understanding is not significantly different from younger learners with a higher mathematics achievement level. This suggests that the role of dyscalculia is less strong for learners' accuracy levels on (both consistent and inconsistent) rational number tasks compared to learners' accuracy level on a mathematics achievement test measuring learners' automated arithmetic knowledge. Moreover, the finding that LWD's rational number understanding is not significantly different from younger learners with a higher mathematics achievement level, gives us reason to hypothesize that LWD have a lead in their rational number understanding compared to even younger learners with the same mathematics achievement.

Our findings may have some implications for mathematics education. As stated above, a better understanding of LWD's rational number understanding is important to provide adaptive (remedial) instruction with the aim to increase LWD's understanding of the rational number system (Hecht & Vagi, 2010). We found that LWD struggle even more with rational numbers than their peers, confirming the finding of Mazzocco and Devlin (2008), and we found that this is particularly the case for inconsistent items (where even the peers without dyscalculia have difficulties). It is quite worrying that LWD struggle so much with understanding the rational number domain, especially because previous research indicated the importance of a good rational number understanding for later mathematics achievement (Bailey et al., 2012; Booth & Newton, 2012; Siegler et al., 2011). Although our results pointed out that LWD struggle even more with inconsistent rational number tasks than their peers, they also suggested that this struggle may not be characterized by a deficit but rather by a more general delay. This implies that it would be possible for LWD to develop more insight in the rational number system and, therefore, more (remedial) instructional attention should go to the enhancement of LWD's understanding of the rational number system.

There are several principles for instruction coming from the conceptual change perspective on learning (see, for example, Greer, 2004; Vosniadou, Ioannides, Dimitrakopoulou, & Papademetriou, 2001) that could be usefully implemented. For example, learners could be encouraged to become aware of their intuitions of numbers; and of the differences between natural number and rational numbers, and differences between both types of numbers could be made explicit to learners through the use of refutational texts (Tippett, 2010). Refutational texts have two main parts: The first part is a statement of a commonly held misconception and the second part consists of an explicit refutation of that misconception while emphasizing the correct scientific explanation (Tippett, 2010). The usefulness of refutational texts to change learners' wrong intuitions about mathematical concepts have amply been shown in previous research (e.g., Lem, Onghena, Verschaffel, & Van Dooren, 2017; Tippett, 2010). However, it should be noted that previous studies investigating the usefulness of refutational texts follow from research with typically developing learners and that future research is necessary to investigate its' usefulness also in LWD.

Moreover, important to note is that next to emphasizing the dissimilarities, instructional attention should also go to the similarities between natural and rational numbers. For example, both natural and rational numbers can be placed on number lines, since they both have magnitudes (Schneider & Siegler, 2010; Siegler et al., 2011; Vamvakoussi & Vosniadou, 2010). Previous studies moreover showed that a good understanding of the magnitude of rational numbers forms a prerequisite to gain insight in the aspects of operations and possibly of density (e.g., Van Hoof, Verschaffel et al., 2015). This is in line with the integrated theory of numerical development (Siegler et al., 2011), which states that understanding the numerical sizes of fractions forms a crucial step in the understanding of fractions.

In sum, when learners have more insight in both the similarities and the differences between both type of numbers, this might increase their rational number understanding. However, we recognize that these principles for instruction follow from research with typically developing learners and that further research is needed to confirm that they also account for LWD.

Some caution is also required with interpreting our results. First, we had a relatively small sample of LWD. Future studies, including more participants, are needed to further strengthen our results, conclusions, and recommendations. Second, our results are based on a cross-sectional design. Future studies would benefit from including a longitudinal design to capture how LWD's rational number understanding evolves over time. For example, results of the previously mentioned longitudinal study of Mazzocco et al. (2013) were in line with those of the present study by showing that in the sixth grade, LWD struggle more with fraction comparison tasks with common numerators (= inconsistent tasks in the present study, e.g. $1/2$ vs $1/3$) compared to fraction comparison tasks with common denominators (= consistent tasks in the present study, e.g., $1/3$ vs $2/3$). However, from grade 7 on, and especially in grade 8 LWD's accuracy on fraction comparison tasks with common numerators (= inconsistent tasks) was much higher than their accuracy levels on fraction comparison tasks with common denominators (consistent tasks). Moreover, LWD's accuracy level on fraction comparison tasks with common denominators (= consistent tasks) significantly decreased from sixth to eighth grade. The results of Mazzocco et al. (2013) suggest that LWD's rational number understanding is not only characterized by a delay, but that at certain points in time and for specific tasks there might also be qualitative differences in LWD's rational number understanding when compared with their peers. This suggestion should be further investigated in future longitudinal studies.

Third, we hypothesized that LWD have a lead in their rational number understanding compared to even younger learners with the

same mathematics achievement, based on the finding that LWD's rational number understanding is not significantly different from younger learners with a higher mathematics achievement level. Further research could reveal whether this is the case. An alternative explanation could be that because the group of LWD already had two more years of rational number instruction, this difference in instruction time explains why no differences were found between LWD and the fourth graders.

At last, future studies should investigate whether this delay in LWD's rational number understanding can also be found in older age groups, or whether, at a certain point in time, LWD do not have a specific delay anymore compared to their peers. An additional benefit of including older students would be that it allows to have an ability match design too.

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