Double wall barriers for the reduction of ground vibration transmission

C. Van hoorickx^{a,*}, M. Schevenels^b, G. Lombaert^a

^a*KU Leuven, Department of Civil Engineering, Structural Mechanics Section,
Kasteelpark Arenberg 40, 3001 Leuven, Belgium*

^b*KU Leuven, Department of Architecture, Architectural Engineering,
Kasteelpark Arenberg 1, 3001 Leuven, Belgium*

Abstract

Stiff wall barriers can be effective in reducing the transmission of environmental ground vibration. Up to now, single wall barriers have mostly been studied. In building acoustics, however, double walls are used in order to realize a high level of sound insulation. In this paper, the potential of using double walls in reducing ground vibration transmission is investigated by means of numerical simulations. Two cases are studied: jet-grout walls and concrete walls in a homogeneous soil with elastic properties representative of a sandy soil. For both cases, the three-dimensional free field response due to a point load is computed using a 2.5D finite element methodology. Subsequently, the free field response is computed for a simplified train load. Double jet-grout wall barriers are found to be slightly more effective than single wall barriers, in particular when the thickness of the walls and the intermediate soil matches a quarter Rayleigh wavelength. The largest increase in vibration reduction is found for the area closest to the vibration source, where the vibration levels have the highest values. The performance of concrete wall barriers, however, is mainly determined by the stiffness of the walls, and almost no difference in performance is found for single and double walls.

Keywords: Elastodynamic wave propagation, Vibration reduction measures, Wave barriers, Double wall, 2.5D FE methodology

1. Introduction

Railway induced vibrations in the built environment are a major environmental concern. In the frequency range between 1 and 80 Hz, building vibration is perceived as mechanical vibration of the human body, whereas between 16 and 250 Hz, ground-borne vibrations can cause re-radiated or structure-borne noise by vibrating walls and floors.

In order to reduce environmental ground vibration due to railway traffic, mitigation measures at the source, the transmission path and the receiver can be applied. Mitigation measures at the source include rail grinding [1], soft railpads [2], under-sleeper pads [3], ballast mats [4] and floating slab tracks [5]. Mitigation measures at the receiver, such as foundation design and base isolation [6], are only effective for the structures they are applied to.

Mitigation measures on the transmission path aim at impeding propagation of ground vibrations from source to receiver. They are particularly appealing in situations with existing track and buildings, since these can be left unmodified. The measures include open trenches [7], soft and stiff buried wall barriers [8, 9, 10, 11], subgrade stiffening [12], wave impeding blocks [13, 14], and wave reflectors [15, 16].

This paper focuses on the application of stiff wall barriers to hinder ground vibration propagation from source to receiver. A stiff wave barrier can be realized by introducing a concrete slab [17] or a sheet pile

*Corresponding author. Tel.: +32 16 32 22 40

Email addresses: cedric.vanhoorickx@kuleuven.be (C. Van hoorickx), mattias.schevenels@kuleuven.be (M. Schevenels), geert.lombaert@kuleuven.be (G. Lombaert)

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wall [18] into the soil, or by stiffening the soil for example by jet-grouting [11]. The performance of the wall barrier depends on the soil properties and the size of the wall. Two phenomena play a role. First, for deep walls, incident waves are reflected, reducing wave transmission [17]. Second, the bending stiffness of the wall prevents the propagation of waves when the trace wavelength (the distance between two wave fronts in the direction along the barrier) is smaller than the bending wavelength of the barrier, similar to the coincidence phenomenon in building acoustics [19].

Up to now, single wall barriers with a rectangular cross section have mostly been studied. In building acoustics, however, it is well known that double walls can reduce sound levels much better than single walls. For single walls, the sound reduction index increases 6 dB per octave over the entire frequency range, while for double walls, the sound reduction index increases to 18 dB per octave between the double wall resonance frequency and the first cavity resonance frequency [20, 21]. In this contribution, the potential of double wall barriers for environmental ground vibration is studied.

To assess the effectiveness of double wall barriers for three-dimensional wave propagation, a two-and-a-half-dimensional (2.5D) finite element methodology is adopted. By using the Fourier transform from the longitudinal coordinate to the wavenumber domain, the three-dimensional radiated wave field can be represented on a two-dimensional mesh [22]. At the boundaries of the finite element mesh, perfectly matched layers are added to prevent spurious wave reflections [23].

The paper is organized as follows. In section 2, the model is described and the 2.5D methodology is explained. Using this model, the three-dimensional free field response is computed both for jet-grout wall barriers and for concrete slurry walls. The performance of single and double jet-grout wall barriers is compared in section 3. Next, these results are interpreted using wave dispersion relations (section 4) and the similarity with the double wall behavior in acoustics is investigated using a one-dimensional model (section 5). These insights facilitate the interpretation of the influence of the geometry, which is investigated with a parametric study in section 6. The influence of the wall material properties is examined by studying the performance of concrete double wall barriers. Finally, the effectiveness of the wall barriers is analyzed for a simple train in section 7.

2. 2.5D model of a double wall barrier

2.1. Geometry and material properties

The geometry of the longitudinally invariant problem is shown in figure 1. This problem consists of a longitudinally invariant (double) wall barrier in a homogeneous halfspace. In order to define the geometry, the right-handed Cartesian frame of reference in figure 1 is chosen, where the origin coincides with the position of the source point.

The halfspace is assumed to be homogeneous to facilitate physical interpretation. The chosen material properties are typical for sandy soils, and are listed in table 1. It is excited at the surface by a point source. The aim is to reduce the vibration levels for three receivers located at $(x = 15 \text{ m}, y = 0 \text{ m}, z = 0 \text{ m})$, $(x = 15 \text{ m}, y = 5 \text{ m}, z = 0 \text{ m})$, and $(x = 15 \text{ m}, y = 15 \text{ m}, z = 0 \text{ m})$. These positions will be referred to as receiver 1, receiver 2, and receiver 3, respectively.

Table 1: Material properties of the original homogeneous halfspace and the jet-grout wall barrier.

	Homogeneous halfspace	Jet-grout wall barrier
Mass density ρ [kg/m ³]	1800	2000
Longitudinal wave velocity C_p [m/s]	300	950
Shear wave velocity C_s [m/s]	150	550
Longitudinal damping ratio β_p [%]	2.5	2.5
Shear damping ratio β_s [%]	2.5	2.5

A double wall barrier is introduced between the source and the receivers to reduce the transmission of ground vibration. The barrier is made out of material which is stiffer than the surrounding soil. A jet-grout wall barrier is considered with material properties given in table 1. The double wall barrier is located

Figure 1: The three-dimensional wave propagation problem with a double wall barrier. The mesh is added as illustration only and is not the actual finite element mesh.

symmetrically with respect to the center of the source and the first receiver. The double wall barrier is characterized by its depth, in this case a constant value of 7.5 m, the thicknesses of the two walls, denoted by t , and the spacing between the walls, denoted by d . The performance of the double wall barrier is compared with the performance of a single wall barrier with a thickness of $2t$, having the same volume as the double wall barrier.

2.2. 2.5D finite element formulation

For longitudinally invariant geometries, a computationally efficient two-and-a-half-dimensional (2.5D) method can be used to calculate the three-dimensional wave field in the soil. In this approach, the longitudinal coordinate y is transformed to the wavenumber k_y by means of a forward Fourier transform, and the response is calculated in the frequency-wavenumber domain. The response in the spatial domain is then obtained by performing an inverse Fourier transform from the wavenumber k_y to the longitudinal coordinate y .

The stresses $\hat{\sigma}_i$ and $\hat{\tau}_{ij}$ and the displacements $\hat{u}_i$ satisfy the following equilibrium equations:

$$\begin{cases} \frac{\partial \hat{\sigma}_x}{\partial x} + \frac{\partial \hat{\tau}_{xy}}{\partial y} + \frac{\partial \hat{\tau}_{zx}}{\partial z} = -\omega^2 \rho \hat{u}_x \\ \frac{\partial \hat{\tau}_{xy}}{\partial x} + \frac{\partial \hat{\sigma}_y}{\partial y} + \frac{\partial \hat{\tau}_{yz}}{\partial z} = -\omega^2 \rho \hat{u}_y \\ \frac{\partial \hat{\tau}_{zx}}{\partial x} + \frac{\partial \hat{\tau}_{yz}}{\partial y} + \frac{\partial \hat{\sigma}_z}{\partial z} = -\omega^2 \rho \hat{u}_z \end{cases} \quad (1)$$

where a hat above a variable denotes its representation in the frequency domain. By separating the in-plane

coordinates x and z and the out-of-plane coordinate y , the equilibrium equations become:

$$\left(\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & 0 & \frac{\partial}{\partial z} \\ 0 & 0 & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & 0 & \frac{\partial}{\partial x} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \frac{\partial}{\partial y} \right) \begin{Bmatrix} \hat{\sigma}_x \\ \hat{\sigma}_y \\ \hat{\sigma}_z \\ \hat{\tau}_{xy} \\ \hat{\tau}_{yz} \\ \hat{\tau}_{zx} \end{Bmatrix} = -\omega^2 \rho \begin{Bmatrix} \hat{u}_x \\ \hat{u}_y \\ \hat{u}_z \end{Bmatrix} \quad (2)$$

which can be written as:

$$\left(\mathbf{L}_1^T + \mathbf{L}_2^T \frac{\partial}{\partial y} \right) \hat{\boldsymbol{\sigma}} = -\omega^2 \rho \hat{\mathbf{u}} \quad (3)$$

A linear elastic constitutive behavior is assumed, leading to the following relation between the stresses and strains:

$$\hat{\boldsymbol{\sigma}} = \mathbf{C} \hat{\boldsymbol{\epsilon}} \quad (4)$$

where $\mathbf{C}$ is the constitutive tensor and the strains $\hat{\boldsymbol{\epsilon}}$ are calculated from the displacement vector $\hat{\mathbf{u}}$:

$$\hat{\boldsymbol{\epsilon}} = \left(\mathbf{L}_1 + \mathbf{L}_2 \frac{\partial}{\partial y} \right) \hat{\mathbf{u}} \quad (5)$$

The domain is discretized using finite elements (figure 1). In this paper, two-and-a-half-dimensional eight-node elements are used with an element size of 0.25 m, corresponding to ten elements per shear wavelength at a frequency of 80 Hz. As the domain is longitudinally invariant, the displacement vector $\hat{\mathbf{u}}$ is discretized as:

$$\hat{\mathbf{u}}(x, y, z, \omega) \approx \mathbf{N}(x, z) \hat{\mathbf{u}}(y, \omega) \quad (6)$$

where $\mathbf{N}(x, z)$ are shape functions defined over the cross section and $\hat{\mathbf{u}}(y, \omega)$ is the discretized displacement vector which is a function of the longitudinal coordinate y and the frequency ω .

Neumann boundary conditions $\hat{\boldsymbol{\sigma}} \cdot \mathbf{n} = \hat{\mathbf{t}}^n = \bar{\mathbf{t}}^n$ are applied with $\mathbf{n}$ the unit outward normal vector of the boundary Γ_t . The equilibrium equations in Eq. (3) and the Neumann boundary conditions are formulated in a weak form and a standard Galerkin procedure is followed, resulting in the following finite element formulation of the equilibrium equations [23]:

$$-\omega^2 \mathbf{M} \hat{\mathbf{u}}(y, \omega) + \mathbf{K}_0 \hat{\mathbf{u}}(y, \omega) + \mathbf{K}_1 \frac{\partial \hat{\mathbf{u}}(y, \omega)}{\partial y} + \mathbf{K}_2 \frac{\partial^2 \hat{\mathbf{u}}(y, \omega)}{\partial y^2} = \hat{\mathbf{f}}(y, \omega) \quad (7)$$

The differential equation (7) is solved by a Fourier transform of the longitudinal coordinate y to the longitudinal wavenumber k_y :

$$\mathcal{F}[f(y), k_y] = \int_{-\infty}^{\infty} e^{+ik_y y} f(y) dy \quad (8)$$

This results in the following equation:

$$(-\omega^2 \mathbf{M} + \mathbf{K}_0 - ik_y \mathbf{K}_1 - k_y^2 \mathbf{K}_2) \tilde{\mathbf{u}}(k_y, \omega) = \tilde{\mathbf{f}}(k_y, \omega) \quad (9)$$

where a tilde above a variable denotes its representation in the frequency-wavenumber domain. An inverse Fourier transform from the wavenumber k_y to the coordinate y yields the solution in the spatial domain.

At the boundaries of the finite element mesh, perfectly matched layers (PMLs) are added to prevent spurious wave reflections (figure 1). The modified elastodynamic equations include complex-valued coordinate stretching along the direction(s) normal to the interface [23]. The resulting element mass and stiffness matrices of the PML layers are included in Eq. (9).

3. Analysis of a double wall barrier

In this subsection, the performance of the double wall barrier is studied for a point load. In a homogeneous halfspace excited by a point load, cylindrical wave fronts propagate through the soil. The amplitude decreases with the distance to the point source as a result of geometric attenuation and material damping in the soil. Figures 2a and 3a show the real part of the vertical receptance respectively at 30 Hz and 60 Hz for the homogeneous halfspace. The wave propagation is altered when the single or double wall is introduced in the halfspace. Figures 2 and 3 show the real part of the vertical receptance and the corresponding insertion loss for both cases at 30 Hz and 60 Hz respectively, with the insertion loss $\hat{\text{IL}}$ defined as:

$$\hat{\text{IL}}(x, y, z, \omega) = 20 \log_{10} \frac{|\hat{u}_z^{\text{ref}}(x, y, z, \omega)|}{|\hat{u}_z(x, y, z, \omega)|} \quad (10)$$

where $\hat{u}_z^{\text{ref}}(x, y, z, \omega)$ is the vertical displacement before and $\hat{u}_z(x, y, z, \omega)$ the vertical displacement after introducing the considered mitigation measure in the soil.

Figure 2: Real part of the vertical receptance $\hat{u}_z$ at 30 Hz for (a) the original homogeneous halfspace, (b) the single wall barrier, and (c) the double wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m). Corresponding insertion loss $\hat{\text{IL}}$ for (d) the single wall barrier and (e) the double wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m).

The single wall barrier has a thickness of 2 m and the center axis is located at a distance of 7.5 m from the point source. The barrier depth influences two effects: reflection and stiffness. Waves are reflected by the barrier if the depth is larger than 0.6 times the Rayleigh wavelength. Furthermore, wave propagation is prevented by the bending stiffness of the wall at frequencies larger than [19]:

$$\omega_c \approx \frac{C_R^2}{h} \sqrt{\frac{12\rho}{E}} \quad (11)$$

where h is the height, ρ the mass density, and E the elastic modulus of the barrier and C_R is the Rayleigh wave velocity of the soil. A barrier depth of 7.5 m is chosen. For frequencies above 1.7 Hz (Eq. (11)), the

Figure 3: Real part of the vertical receptance $\hat{u}_z$ at 60 Hz for (a) the original homogeneous halfspace, (b) the single wall barrier, and (c) the double wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m). Corresponding insertion loss $\hat{I}L$ for (d) the single wall barrier and (e) the double wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m).

barrier depth is large enough to mobilize the bending stiffness of the barrier around the x -axis, while for frequencies above 11.2 Hz, it is also large enough to reflect incoming waves.

Figures 2 and 3 show that the waves are partly reflected, resulting in a reduction of the response behind the barrier. The reflected waves interfere constructively and destructively with the incoming waves, causing the alternating regions of low and high insertion loss at the surface. The insertion loss behind the wall barrier is positive at the surface, meaning that the transmission of Rayleigh waves is reduced. A comparison of figures 2d and 3d shows that the positive insertion loss at the surface is restricted to a shallower depth for the highest frequency, as the depth of the Rayleigh waves is smaller. At the surface, the insertion loss is higher at larger distances in the y -direction.

The two walls of the double wall barrier have a depth of 7.5 m and a thickness t of 1 m, resulting in the same volume of material as for the single wall barrier. The center axis of the both walls is located at a distance of respectively 5 m and 10 m from the point source ($d = 4$ m). As shown in figure 2, interference patterns again occur in front of the first wall. In between the two walls, there is a similar interference pattern caused by interference of waves traveling in both directions. Behind the barrier, there is a high insertion loss at the surface.

Figure 4 shows the amplitude of the vertical receptance and the insertion loss as a function of the frequency for the three receiver points (figure 1). At low frequencies, the depth of the wall barrier is small compared to the penetration depth of the incoming surface waves, resulting in low insertion losses. From about 20 Hz, corresponding to a shear wavelength equal to 7.5 m, an insertion loss between 6 dB and 20 dB is obtained.

For receiver 1, the insertion loss for the double wall barrier is higher than for the single wall barrier for most of the frequency range. However, the improvement becomes less significant at larger distances in the y -direction, for example for receiver 2. At even larger distances, for example for receiver 3, the single and

Figure 4: Amplitude of the vertical receptance $\hat{u}_z$ for the original halfspace (dotted line), the single wall barrier (dashed line) and the double wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m; solid line) in the different receiver points: (a) receiver 1, (c) receiver 2, and (e) receiver 3 (figure 1). Insertion loss $\hat{I}L$ for the single wall barrier (dashed line) and the double wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m; solid line) in the different receiver points: (b) receiver 1, (d) receiver 2, and (f) receiver 3.

double wall barrier have a similar performance over the frequency range considered.

To conclude, the double wall barrier is mainly advantageous in the area close to the source in the y -direction, which is also the area with the highest vibration levels. This can be mainly attributed to the weaker performance of single wall barriers for points close to the line going through the source point and perpendicular to the barrier, as was reported before [19]. The double wall barrier improves the performance in the important area close to the source while maintaining the effectiveness of the single wall barrier at larger distances.

4. Physical interpretation by results in the frequency-wavenumber domain

The results in the previous paragraph can be physically interpreted by studying the dispersion relations for free vibration of a single wall and for the standing waves in between the two walls. In subsection 4.3, these dispersion curves are used to analyze the simulated results from section 3 in the frequency-wavenumber domain.

4.1. Free vibration of a single wall

Without any excitation, Eq. (9) becomes:

$$(-\omega^2 \mathbf{M} + \mathbf{K}_0 - ik_y \mathbf{K}_1 - k_y^2 \mathbf{K}_2) \tilde{\mathbf{u}}(k_y, \omega) = 0 \quad (12)$$

The dispersion curves of the beam are calculated by computing the longitudinal wavenumber k_y for multiple values of the frequency ω . Since Eq. (12) is a quadratic equation in k_y , this equation is reformulated to a linear equation as follows [24]:

$$\begin{bmatrix} -\omega^2 \mathbf{M} + \mathbf{K}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \tilde{\mathbf{u}}_1 \\ \tilde{\mathbf{u}}_2 \end{Bmatrix} = k_y \begin{bmatrix} i\mathbf{K}_1 & \mathbf{K}_2 \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \tilde{\mathbf{u}}_1 \\ \tilde{\mathbf{u}}_2 \end{Bmatrix} \quad (13)$$

where $\tilde{\mathbf{u}}_1 = \tilde{\mathbf{u}}$ and $\tilde{\mathbf{u}}_2 = k_y \tilde{\mathbf{u}}$. This corresponds to a generalized eigenvalue problem of the form $\mathbf{A}\mathbf{x} = \lambda \mathbf{B}\mathbf{x}$ where the eigenvalues λ represent the wavenumbers k_y and the first half of the eigenvectors $\mathbf{x}$ represent the corresponding mode shapes.

In this way, the dispersion relations and the corresponding mode shapes are calculated for the single wall barrier. The dispersion curves are computed for frequencies f from 0 Hz to 100 Hz in steps of 2 Hz. For $f = 0$ Hz, the system in Eq. (13) becomes ill-conditioned. Therefore, this frequency is replaced with a small value. The results are shown in figure 5, where the slowness p_y is defined as k_y/ω . At low frequencies, four eigenvalues are found. For the corresponding modes, two shear-bending modes, a torsion mode and a longitudinal mode are distinguished.

The dispersion relations for these modes have also been computed by means of the following analytical expressions found in the literature. For the longitudinal waves, the following expression holds [25]:

$$-\rho A \omega^2 + E A k_y^2 = 0 \quad (14)$$

where A is the area of the cross section, ρ is the mass density, and E is Young's modulus of the beam. For the bending waves, Timoshenko's theory leads to the following dispersion relation [26]:

$$-\rho A \omega^2 + E I k_y^4 - \rho I \left(1 + \frac{E}{\kappa G} \right) \omega^2 k_y^2 + \frac{\rho^2 I}{\kappa G} \omega^4 = 0 \quad (15)$$

with I the moment of inertia, G the shear modulus of the beam, and κ the shear coefficient. For rectangular solid beams, $\kappa = 10(1 + \nu)/(12 + 11\nu)$ [27]. The dispersion relation for the torsional waves can be described by Barr's theory [28]:

$$-\rho I_p \omega^2 + G I_t k_y^2 - \rho I_\phi \frac{I_p}{I_p - I_t} \left(\kappa' + \frac{E}{G} \right) \omega^2 k_y^2 + E I_\phi \kappa' \frac{I_p}{I_p - I_t} k_y^4 + \rho^2 \frac{I_\phi}{G} \frac{I_p}{I_p - I_t} \omega^4 = 0 \quad (16)$$

Figure 5: (a) Dispersion curves for the single wall. Dispersion curves computed from Eq. (13) are indicated with a dot. The analytically calculated dispersion curves are superimposed: the longitudinal mode (solid line), Timoshenko's bending mode around the x -axis (dashed line), Timoshenko's bending mode around the z -axis (dotted line), and Barr's torsion mode (dash-dotted line). The associated modes at $f \approx 0$ Hz (from high to low value of k_y) are displayed for (b) the shear-bending mode around the z -axis, (c) the shear-bending mode around the x -axis, (d) the torsion mode, and (e) the longitudinal mode.

In this equation, I_p is the polar second moment of inertia, I_t is the torsion constant and I_ϕ is the warping constant. The non-dimensional parameter κ' adjusts the wave velocity to the Rayleigh wave velocity at zero wavelength [29] and is equal to 0.84 for $\nu = 0.25$. These dispersion curves are plotted in figure 5 as well, and clearly match the dispersion curves found by solving Eq. (13).

Since the loading is vertical, mainly the bending mode around the x -axis will determine the displacement and therefore the mitigation mechanism of the wall barrier. The single and the double wall barrier both have the same bending stiffness around the x -axis and therefore the free bending dispersion curves of both barriers coincide. When the wavenumber k_y exceeds the free bending dispersion curve k_b of the barrier, which corresponds to a trace wavelength λ_y smaller than the bending wavelength λ_b , the bending stiffness of the barrier will hinder the wave transmission [19].

4.2. Standing waves

At specific frequencies, standing waves occur in the space between the two walls as a result of constructive and destructive interference of the waves traveling in this space. This leads to an increase of the transmitted wave field, which is referred to as cavity resonance in building acoustics [21].

Standing waves appear at the surface when the distance between the two walls is equal to an integer number (m) times half the wavelength in the x -direction:

$$d = \frac{m\lambda_x}{2} = \frac{m\pi}{k_x} = \frac{mC_R}{2f \cos \theta} \quad (17)$$

Here, use is made of the relation $k_x = k_R \cos \theta$ (figure 6). After reformulation, where use is made of the relation $\sin \theta = k_y/k_R$, the (positive) dispersion relation for the standing waves becomes:

$$k_y = k_R \sqrt{1 - \left(\frac{mC_R}{2df} \right)^2} \quad (18)$$

These dispersion relations are shown in figure 7. The cut-off frequencies f_{co} , i.e. the frequencies for which $k_y = 0$, are given by:

$$f_{co} = \frac{mC_R}{2d} \quad (19)$$

Figure 6: Definition of the wavenumber components for Rayleigh waves.

For small distances d , the first cut-off frequency is high. For distances d smaller than 0.7 m, the first cut-off frequency f_{co} is larger than 100 Hz, meaning that there are no standing waves in the frequency range considered. For increasing frequency, or decreasing wavelength, the wavenumber k_y approaches the Rayleigh wavenumber $k_R = \omega/C_R$.

Figure 7: Dispersion curves for the standing waves in the space (distance 4 m) between the double wall (gray line) and the Rayleigh wave slowness (black line).

4.3. The response computed with the 2.5D model in the frequency-slowness domain

Since the calculations are performed in the frequency-wavenumber domain, the insertion loss in a point can easily be plotted as a function of the frequency f and the longitudinal slowness $p_y = k_y/\omega$. This is done in figure 8 for both the single and double wall barrier.

When the longitudinal wavenumber k_y exceeds the Rayleigh wavenumber k_R , corresponding to a slowness $p_R = 1/C_R = 7.16 \times 10^{-3}$ s/m, the lateral wavenumber $k_x = \sqrt{k_R^2 - k_y^2}$ is imaginary and the wave in the x -direction becomes evanescent. Consequently, the free field response is very small for these wavenumbers. Therefore, the plots in figure 8 are limited to a slowness of 8×10^{-3} s/m.

For slownesses up to the P-wave slowness, $p_p = 1/C_p = 3.33 \times 10^{-3}$ s/m, there is a phase shift between wave fronts in the reference case and in the case with the wall barrier, since the P-wave travels faster in the stiffer wall barrier than in the surrounding soil. This phase shift can be expressed for $n = 0 \dots \infty$ by [19]:

$$\left(\sqrt{k_{R1}^2 - k_y^2} - \sqrt{k_{p1}^2 - k_y^2} \right) \Delta x - \omega \frac{w}{C_{p2} - C_{p1}} = 2\pi n \quad (20)$$

Figure 8: Insertion loss $\hat{I}\tilde{L}(x = 15 \text{ m}, k_y, z = 0 \text{ m}, \omega)$ for (a) the single wall barrier and (b) the double wall barrier (wall thickness $t = 1 \text{ m}$ and spacing $d = 4 \text{ m}$). Superimposed is the analytical dispersion curve for Timoshenko's bending mode around the x -axis (black dashed line). For the single wall barrier, also the frequency-slowness curves are added which show the phase shift between wave fronts in the reference case and in the case of stiffening (black dotted lines). For the double wall barrier, the dispersion curves caused by the standing waves between the walls are added (gray solid lines). The Rayleigh wave slowness is shown as well (black solid line).

where w is the thickness of the wall barrier (equal to $2t$) and Δx the distance between source and receiver.

For the single wall barrier (figure 8a), the highest insertion losses are found for slownesses above a specific value in the frequency-slowness plots. The interpretation is explained in detail by Coulier et al. [19], and is here briefly recapitulated. The insertion losses become very high when the wavenumber k_y exceeds the value k_b corresponding to free bending waves around the x -axis of the barrier, or, reciprocally, for wavelengths λ_y smaller than the bending wavelength λ_b . The wave transmission is then dominated by the stiffness of the wall, resulting in a reduction of the displacements. The waves with a large wavenumber k_y are more strongly reduced than waves with a small wavenumber k_y . As a result, the single wall barrier has the best performance at receiver points located at larger distances y along the barrier, as was indicated before.

A similar analysis can be made for the double wall barrier (figure 8b). The insertion loss again becomes large for wavenumbers k_y greater than the wavenumber k_b corresponding to free bending around the x -axis. In addition, the insertion loss is affected by resonance caused by the standing waves between the two walls. At the corresponding frequency-slowness pairs given by Eq. (18), the transmitted wave field is increased, resulting in a lower insertion loss. One can therefore expect a smaller insertion loss and therefore a weaker performance for the double wall barrier as compared with the single wall barrier. In section 3, it was shown, however, that the considered double wall barrier performs better than the single wall barrier. This was mainly the case for small distances along the barrier (y close to 0). When comparing figures 8a and 8b, a difference in the insertion loss is visible for small slownesses p_y , where higher insertion losses are observed for the double wall barrier. Analysis of the behavior of the barrier at these wavenumbers, more particularly at $k_y = 0$, which corresponds to the behavior due to a line load (the two-dimensional case), allows understanding the difference in performance of the single and double wall barriers.

5. Equivalent one-dimensional model for physical interpretation

In figure 9, the insertion loss computed for a line load, obtained with a two-dimensional (2D) model, is compared with the insertion loss computed for a point load, obtained with the two-and-a-half-dimensional (2.5D) model. The displacements for the line load are obtained using Eq. (9) where the longitudinal wavenumber k_y is set to 0. Since the insertion losses obtained for a line load and for a point load are similar, the nature of the difference in performance between the single and double wall barrier is mainly two-dimensional. At low frequencies, the difference between the performance of both barriers is rather small,

while at higher frequencies, there is a clear difference between the single and double barrier. At higher frequencies, the depth of the wall is large compared to the penetration depth of the incoming Rayleigh wave. It can therefore be expected that an equivalent one-dimensional model of the wave transmission can give insights in the interpretation of the response due to a line and point load. A one-dimensional equivalent model furthermore allows to verify the similarity with double wall behavior in acoustics.

Figure 9: Insertion loss $\hat{\text{IL}}$ at receiver 1 (figure 1) for the single wall barrier (dashed line) and the double wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m; solid line), for both a point load (2.5D model, black line) and line load (2D model, gray line).

The equivalent one-dimensional model is shown in figure 10. The model consists of a homogeneous full space which is intersected by a (double) wall barrier. An incoming wave with amplitude I arrives at the wall barrier and is partly reflected (R) and partly transmitted (T). The insertion loss $\hat{\text{IL}}_{1\text{D}}$ is calculated as:

$$\hat{\text{IL}}_{1\text{D}} = 20 \log_{10} \left| \frac{I}{T} \right| \quad (21)$$

The direct stiffness method [30] is used to obtain an (exact) solution of the wavefield.

Figure 10: The one-dimensional wave propagation problem with a double wall barrier.

Two cases are considered. First, the acoustical performance of the double wall is investigated by inserting it into air. Second, the response of the double wall barrier due to ground vibrations is investigated by inserting it into the soil considered in this paper. By following this two-step approach, the physical mechanism of double walls in elastodynamics can be compared with the mechanism in acoustics.

In the first case, the double wall barrier is put into air, with a wave velocity C_1 of 340 m/s and a mass density ρ_1 of 1.2 kg/m³. A damping ratio of 2.5% is assumed. The material properties of the double wall barrier are given in table 1. In the one-dimensional model, only the wave velocity in the direction of the

wave transmission plays a role. In order to enable a comparison with the results of the second case, the Rayleigh wave velocity of the barrier is used in the model. The results are shown in figure 11 for both the single and double wall barrier.

Figure 11: Insertion loss for the one-dimensional problem $\hat{\text{IL}}_{1\text{D}}$ as a function of the frequency for the single wall barrier (dashed line) and the double wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m; solid line) in air. Superimposed are the mass law (thin line with circles), the value of the resonance frequency (dotted vertical line), and the theoretical values for frequencies above this frequency (thin line with triangles).

The performance of the single wall with a thickness $2t$ follows the well-known mass-law [31], which can be written as

$$\hat{\text{IL}}_{1\text{D}} = 10 \log_{10} \left[1 + \left(\frac{\omega \rho_2 t}{\rho_1 C_1} \right)^2 \right] \approx 20 \log_{10} \left(\frac{\omega \rho_2 t}{\rho_1 C_1} \right) \quad (22)$$

where a subscript 1 stands for the air and the subscript 2 stands for the wall. This relation corresponds to an increase of 6 dB per octave band.

At low frequencies, the performance of the double wall (with spacing $d = 4$ m) is similar. However, from a certain frequency onwards, the two performances diverge. This frequency is the double wall resonance frequency f_0 , which can be calculated by considering two masses connected by a spring:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{\rho_1 C_1^2}{d} \frac{2}{\rho_2 t}} \quad (23)$$

For frequencies above the double wall resonance frequency, the performance is theoretically approximated by:

$$\hat{\text{IL}}_{1\text{D}} \approx 20 \log_{10} \left[\frac{\omega^3 d \rho_2^2 t^2}{2 \rho_1^2 C_1^3} \right] \quad (24)$$

which corresponds to an increase of 18 dB per octave band. This phenomenon, leading to better performance of a double wall as compared to a single wall with the same total thickness ($2t$), is called the double wall phenomenon. At higher frequencies, cavity resonance appears, adversely affecting the performance, which is still better, however, than for the single wall.

In soil, the physics are different however. This is illustrated by considering again the problem in figure 10, with the properties in table 1 for both the wall barrier and the surrounding soil. The wave velocity corresponding to the Rayleigh waves in the three-dimensional case is used in this equivalent one-dimensional model. The resulting performance is shown in figure 12 for both the single and double wall barrier.

For the single wall barrier, the insertion loss is very low at low frequencies. For higher frequencies, the insertion loss increases from 5 dB at a frequency of 10 Hz to 38 dB at 100 Hz. Therefore, the insertion loss is much lower than in the case of the wall barrier located in air. Furthermore, the performance of the single wall doesn't follow the mass law. The performance of the single wall in soil depends on the impedance ratio for the original soil and the wall barrier and on the wave speed in and the thickness of the wall barrier.

Figure 12: Insertion loss for the one-dimensional problem $\hat{I}_{L1D}$ as a function of the frequency for the single wall barrier (dashed line) and the double wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m; solid line) in the soil. Superimposed are the stopband frequencies in Eq. (25) (dotted vertical lines).

Similarly, also the double wall barrier behaves differently in soil. The insertion loss goes up to 58 dB at 100 Hz and therefore exceeds the insertion loss for single wall barriers at higher frequencies. Moreover, larger values are observed at particular frequencies, where less energy is transmitted, in a way which is similar to stopbands in periodic media. For multiple periodic cells consisting of a layer with thickness d and wave velocity C_{R1} and a layer with thickness t and wave velocity C_{R2} , it can be shown that following frequencies are located in a stopband ($n = 1 \dots \infty$) [32]:

$$f_{\text{stop}} = \frac{n}{2 \left(\frac{d}{C_{R1}} + \frac{t}{C_{R2}} \right)} \quad (25)$$

These frequencies correspond to the local maxima in the insertion loss for the 1D model, as shown in figure 12. The first stopband frequency and the spacing between the stopband frequencies becomes larger for higher wave velocities in the soil and in the barrier and for smaller wall thicknesses and spacings between the walls. The insertion loss at these frequencies depends on the number of periodic cells (in this case 2), the material properties (the insertion loss increases for a higher impedance ratio $(\rho_2 C_{R2})/(\rho_1 C_{R1})$) and the thicknesses of the layers.

Table 2 compares these stopband frequencies with the local maxima observed in the insertion loss for the different models (1D, 2D, and 2.5D). A good agreement is found between the different models for the local maxima in the insertion loss. Although no local maximum is found beneath 30 Hz for the 2.5D model, the insertion loss in figure 9 shows a change in curvature around 16 Hz, indicating an increase resulting from this stopband effect.

Table 2: Theoretical frequencies calculated with Eq. (25) compared with local maxima in the insertion loss for respectively the 1D, 2D, and 2.5D model.

Stopband frequencies f_{stop}	Local maxima 1D	Local maxima 2D	Local maxima 2.5D
16 Hz	18 Hz	15 Hz	
33 Hz	33 Hz	31 Hz	30 Hz
49 Hz	49 Hz	49 Hz	46 Hz
65 Hz	65 Hz	63 Hz	62 Hz
82 Hz	82 Hz	80 Hz	81 Hz
98 Hz	99 Hz	100 Hz	100 Hz

6. Parametric study

The insertion loss depends on the geometry and the material properties of the halfspace and the wall barriers. Relevant geometrical parameters include the thickness of the walls, the distance between the walls and the number of walls. In this section, only the former two are investigated as the focus is on double wall barriers.

6.1. A jet-grout wall barrier: influence of the geometry

In figure 13, the amplitude of the vertical receptance and the insertion loss (Eq. (10)) at receiver 1 (figure 1), in the case of a point load at the source point, is shown for multiple values of the wall thickness t and the spacing d between the walls. For a small wall thickness ($t = 0.5$ m), the insertion loss for a double wall is similar to the insertion loss for the single wall: the results are relatively insensitive to the spacing d between the walls. Since the wall thickness t of each wall is small compared to the transmitted wavelength, the performance of the wall barrier is mainly determined by its bending stiffness around the x -axis, which is the same for the single and double wall. For a wall thickness of $t = 1$ m, the performance of the double walls differs from the one of the single wall barrier, as was observed before (section 3). The spacing d between the walls also influences the performance, where the largest spacing considered ($d = 4$ m) leads to the best results for most of the frequency range considered. For a wall thickness of $t = 2$ m, an even larger difference between the performance of the single wall barrier and double wall barrier is observed. At particular frequencies, such as at 65 Hz for the wall with a spacing of $d = 4$ m, the insertion loss reaches a very high value (40 dB).

The one-dimensional model is adopted to explain the influence of the geometry. Figure 14 shows the insertion loss for the one-dimensional problem for wall thicknesses $t \in [0, 3]$ and spacings $d \in [0, 4]$ at three different frequencies: 35 Hz, 50 Hz, and 65 Hz. For each frequency, there are local optima at certain thickness-distance pairs. These optima fulfill the so-called quarter wave-stack condition [32], where the thickness of the walls (t) and the intermediate soil (d) are equal to the quarter of the Rayleigh wavelength ($m = 1 \dots \infty$, $n = 1 \dots \infty$):

$$\begin{cases} t^{\text{opt}} = \frac{(2p-1)\lambda_{R2}}{4} = \frac{(2p-1)C_{R2}}{4f} \\ d^{\text{opt}} = \frac{(2q-1)\lambda_{R1}}{4} = \frac{(2q-1)C_{R1}}{4f} \end{cases} \quad (26)$$

At 35 Hz, the optimal geometry consists of $t^{\text{opt}} = (2p-1) \times 4.20$ m and $d^{\text{opt}} = (2q-1) \times 1.16$ m. Since the optimal thickness t^{opt} is much larger than the maximum value of 2 m considered in the 2.5D computation, the quarter wave-stack condition cannot be satisfied, and the geometry has only a small effect on the insertion loss. At 50 Hz, the optimal geometry consists of $t^{\text{opt}} = (2p-1) \times 2.52$ m and $d^{\text{opt}} = (2q-1) \times 0.70$ m. For $p = 1$ and $q = 2$, we have an optimal wall thickness of $t^{\text{opt}} = 2.52$ m and an optimal spacing of $d^{\text{opt}} = 2.10$ m. This geometry is close to the 2.5D layout with $t = 2$ m, $d = 2$ m in figure 13, where a high value was found at 50 Hz. At 65 Hz, the optimal parameters become $t^{\text{opt}} = (2p-1) \times 1.94$ m and $d^{\text{opt}} = (2q-1) \times 0.54$ m. For $p = 1$ and $q = 4$, we have an optimal wall thickness of $t^{\text{opt}} = 1.94$ m and an optimal spacing of $d^{\text{opt}} = 3.76$ m. This geometry is close to the 2.5D layout with $t = 2$ m, $d = 4$ m in figure 13, where a high value was found at 65 Hz. At this frequency, also the layout with $t = 2$ m, $d = 0.5$ m, close to the optimal layout with $p = q = 1$, leads to a higher value for the insertion loss.

To conclude, the one-dimensional model helps to understand the difference in performance between the double wall barriers and the single wall barriers. The geometry determines the frequencies with higher insertion loss, approximated by Eq. (25). This insertion loss has a particularly high value when the quarter wave-stack condition in Eq. (26) is fulfilled.

6.2. A concrete double wall barrier

It is now investigated how the performance changes when the jet-grout walls are replaced by concrete slurry walls. The wall has a mass density $\rho_{\text{conc}} = 2500 \text{ kg/m}^3$, a longitudinal wave velocity $C_{p,\text{conc}} =$

$\cdots$ original halfspace $---$ single wall barrier
 double wall barriers: --- $d = 0.5$ m --- $d = 1$ m --- $d = 2$ m --- $d = 4$ m

Figure 13: Amplitude of the vertical receptance $\hat{u}_z$ and insertion loss $\hat{\mathcal{I}}L$ at receiver 1 (figure 1) for a wall thickness of (a)-(b) 0.5 m, (c)-(d) 1 m, and (e)-(f) 2 m. For the original halfspace, the amplitude is shown with a dotted line, for the single wall barrier with a dashed line. The results are plotted for different distances between the walls: 0.5 m, 1 m, 2 m, and 4 m (from light gray to black solid lines).

Figure 14: Insertion loss for the one-dimensional problem $\hat{I}_{1D}$ for a one-dimensional double wall barrier as a function of the wall thicknesses and the distance between the walls at (a) 35 Hz, (b) 50 Hz, and (c) 65 Hz.

3795 m/s, a shear wave velocity $C_{s,\text{conc}} = 2191$ m/s, and damping ratios $\beta_{p,\text{conc}} = \beta_{s,\text{conc}} = 2.5\%$. A concrete wall thickness t of 0.5 m is considered here. The distance between the walls is again $d = 4$ m.

Figure 15 shows the amplitude of the vertical receptance and the corresponding insertion loss at receiver 1. Again, Eq. (25) allows predicting at which frequencies higher values of the insertion loss are found, resulting from the periodicity. Although the insertion loss has rather large values in both the case of the single and double wall barrier, the double wall barrier does not perform better than the single wall barrier over the frequency range considered. Because of the high stiffness of the wall, a large wall thickness would be needed to satisfy the quarter wave-stack condition in the frequency range.

Figure 15: (a) Amplitude of the vertical receptance $\hat{u}_z$ at receiver 1 (figure 1) for the original halfspace (dotted line), the single concrete wall barrier (dashed line) and the double concrete wall barrier (wall thickness $t = 0.5$ m and spacing $d = 4$ m; solid line). (b) Insertion loss $\hat{I}_L$ at receiver 1 for the single concrete wall barrier (dashed line) and the double concrete wall barrier (wall thickness $t = 0.5$ m and spacing $d = 4$ m; solid line).

In figure 16, the insertion loss is shown as a function of the frequency f and the slowness p_y . The insertion loss again has high values above the bending wave dispersion curve, and is for the double wall barrier reduced by the standing waves in between the two walls. Because of the large stiffness of the wall, the bending wave dispersion curve is situated in the lower part of the slowness range, resulting in a high insertion loss for a large slowness range and therefore in a high insertion loss as observed in figure 15.

It should be noted that the effectiveness of the barrier depends, for given barrier properties, on the material properties of the soil. For a softer soil, the Rayleigh wavenumber k_R shifts to values of k_y which are higher relative to the values in figures 8 and 16. As a consequence, the response at the receiver positions

Figure 16: Insertion loss $\tilde{I}_L(x = 15 \text{ m}, k_y, z = 0 \text{ m}, \omega)$ for (a) the single concrete wall barrier and (b) the double concrete wall barrier (wall thickness $t = 0.5 \text{ m}$ and spacing $d = 4 \text{ m}$). Superimposed are the dispersion curve of Timoshenko's bending mode around the x -axis (dashed black line) and, in case of the double wall, the dispersion curves for standing waves in between the walls (gray solid lines). Also the Rayleigh wave slowness is shown (black solid line).

is increasingly dominated by the effect of the bending stiffness. This results in increased performance of the barrier.

7. Effectiveness of the double wall barrier for a train loading

Up to now, the performance of the double wall barrier has been studied for a point source at the surface. However, the goal is to mitigate ground vibration caused by passing trains. The displacement field resulting from a passing train is calculated in a simplified way, applying the dynamic axle loads at fixed positions as a series of incoherent point loads of the same magnitude, as was done in [16]. Previous studies have indicated that the stationary part of the response can be well approximated in this way [33].

The axle positions are taken from a typical four-car EMU train, with a car length of 26.4 m, and a total length of 105.6 m. The bogie centre distance is 19 m and the distance between two axles of a bogie is 2.7 m. The center of the train is located at the position of the point source indicated in figure 1, and the train is oriented parallel to the y -axis. The y -coordinates of the axle loads are shown in figure 17.

Figure 17: The axle positions of a typical four-car EMU train.

The resulting amplitude of the vertical receptance and insertion loss at receiver 1 are shown in figure 18 for the considered single and double jet-grout wall barriers (with a wall thickness of $t = 1 \text{ m}$ and a spacing $d = 4 \text{ m}$ between the two walls of the double wall barrier). As a result of geometric attenuation and material damping in the soil, the excitation points closest to the receiver point contribute most to the response at the receiver point. Therefore, the insertion loss is similar to the insertion loss found for a point source at the closest axle position, and about the same local maxima are found. For frequencies above 24 Hz, the double wall barrier performs slightly better than the single wall barrier over the entire frequency range considered.

Further analysis shows that, as can be deduced from section 6, the performance of the double wall barrier can be further improved by increasing the thickness t of the walls to, for example, 2 m. For a smaller

Figure 18: (a) Amplitude of the vertical receptance $\hat{u}_z$ at receiver 1 (figure 1) resulting from a train load for the original halfspace (black line), the single jet-grout wall barrier (dashed line) and the double jet-grout wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m; solid line). (b) Insertion loss $\hat{\text{IL}}$ at receiver 1 resulting from the train load for the single jet-grout wall barrier (dashed line) and the double jet-grout wall barrier (wall thickness $t = 1$ m and spacing $d = 4$ m; solid line).

thickness t of, say, 0.5 m, however, the difference in performance between the double and single wall barrier decreases as the thickness t becomes small compared to the wavelength of the passing wave.

Figure 19 shows the amplitude of the vertical receptance and the corresponding insertion loss at receiver 1 for the considered concrete single and double wall barrier (with a wall thickness of $t = 0.5$ m and a spacing $d = 4$ m between the two walls of the double wall barrier). Similar to the case of a point load, the single and double wall barrier have more or less the same performance over the frequency range considered. The high insertion loss of both barriers results from the high bending stiffness.

Figure 19: (a) Amplitude of the vertical receptance $\hat{u}_z$ at receiver 1 (figure 1) resulting from a train load for the original halfspace (black line), the single concrete wall barrier (dashed line) and the double concrete wall barrier (wall thickness $t = 0.5$ m and spacing $d = 4$ m; solid line). (b) Insertion loss $\hat{\text{IL}}$ at receiver 1 resulting from the train load for the single concrete wall barrier (dashed line) and the double concrete wall barrier (wall thickness $t = 0.5$ m and spacing $d = 4$ m; solid line).

8. Conclusion

In building acoustics, double walls are used when a high sound reduction is needed from lightweight structures. In this paper, the effectiveness of a double wall barrier in a homogeneous soil is investigated in reducing the transmission of environmental ground vibration. This is done by computing the free field response due to a point load and a train load based on numerical simulations using a 2.5D finite element methodology. The performance of double jet-grout wall barriers and double concrete wall barriers are examined and compared with the performance of single wall barriers with the same total thickness.

For jet-grout wall barriers, the double wall barrier is found to perform slightly better than the single wall barrier at smaller distances along the barrier (y close to zero). This also leads to a lower response for the double wall barrier in case of a train load. The slightly better performance is, however, much less significant than in acoustics, where an increase of 18 dB is expected for the insertion loss over a part of the frequency range. As was shown with a one-dimensional equivalent model, the elastodynamic and acoustic performance of the double wall barrier are quite different. The better elastodynamic performance of the double wall barrier as compared with the single wall barrier is caused by the periodicity (twofoldness) of the geometry. For large thicknesses t and spacings d between the walls, a higher insertion loss is found, especially when the thickness of the walls (t) and the intermediate soil (d) are close to a quarter of the Rayleigh wavelength. The performance is however affected by standing waves between the walls which increase the ground vibration transmission.

Because of the high stiffness of concrete walls, a large thickness t of the walls would in this case be needed to satisfy the quarter-wave stack condition in the frequency range of interest, and the performance is mainly determined by the bending stiffness. The decoupling of the single wall into two walls with the same overall thickness $2t$ therefore does not alter the performance, as the single and double wall barrier have the same total bending stiffness around the x -axis.

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