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Distributed Observer and Controller Design for Spatially Interconnected Systems

Xueji Zhang^{ID}, *Student Member, IEEE*, Kristian Hengster-Movrić, Michael Šebek, *Senior Member, IEEE*, Wim Desmet, and Cassio Faria

Abstract—This paper tackles networked distributed observer and controller design problem over directed graph topology for spatially interconnected systems. Traditional centralized design methods suffer from a lack of adaptability to graph variations incurred by network reconfiguration, communication failures, and redundant sensors integration. In this paper, to handle the foregoing limitations imposed by centralized design, state observers are designed in a distributed manner facilitated by pinning control precepts. On the one hand, this novel approach adds fault tolerance with respect to communication link failures. On the other hand, the proposed approach brings flexibility of integrating additional sensors into the network. In addition, this approach affords a reduction of computational cost. A sufficient condition to guarantee stability of the closed-loop system is derived. The controllers, though in the end implemented in a distributed way, are designed in a centralized framework, where linear-quadratic-regulator theory is adopted to handle the fact that separation principle fails to hold in the networked observer and controller design. Numerical simulation results of a piezoelectric actuated smart flexible system are presented, and the effectiveness of the proposed design is thereby verified.

Index Terms—Consensus, distributed control, flexible structures, large-scale systems, networked control, pinning control, vibration damping.

I. INTRODUCTION

TODAY is witnessing a more and more automated and networked world, and the concepts of smart factory,

smart cities, automated highway, and Internet of Things have paved the way for a more intelligent future world. With rapid advances and integration of computing, communication, and smart sensing technologies, small-size and low-cost sensing devices empowered with embedded processing and communication capabilities have been deployed in a wide range of environments [1]–[4]. These technical achievements have attracted researchers from a variety of disciplines to the emerging field of networked control systems [5]–[7], and their synonymous cyber-physical systems in which cyber networks interact with humans and physical plants intensively [8]–[10].

The nascent networked/distributed control systems exhibit many advantages. First, communication *via* a network provides the potential to improve the overall system performance compared to purely decentralized control architectures, since fusion of global information enables the distributed control stations (decision makers) to gain deeper insights into the considered plant and thereafter make more intelligent decisions. Second, it reduces the implementation costs and complexity: for large-scale plants—a large number of system states and inputs/outputs or a geographically distributed plant—it might be costly to have a complete network with all-to-all links [11] or send all sensory information to a centralized controller, which would require a high communication bandwidth. Peer-to-peer networks, instead, can mitigate the communication overhead since each *node* or *agent* in the network communicates (transmits and/or receives information) only with its neighborhood. Physical wires are further eliminated in wireless actuator and sensor networks [12], [13]. Furthermore, with redundant communication links, networked/distributed control architecture can add the fault-tolerance property or retain graceful degradation in the case of single components (like sensors, actuators, and control stations) failures [5]. In contrast, a centralized controller might suffer a potential catastrophic failure. Additionally, the networked system benefits from *scalability* or *flexibility* if extra components such as actuators, sensors, or even control stations are integrated into the network in a plug-and-play fashion.

Versatile as it is, networked control paradigm induces many challenges as well [5], [6], of which distributed estimation/observation of system states is a fundamental problem, especially when observer-based control strategies are exploited to improve the system performance. In the past decade, a variety of related results have been reported [14]–[17]. In particular, with the theoretical framework of consensus protocols introduced in [18] and [19], consensus-based estimation has been extensively reported for linear systems (see [20]–[29]).

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Fig. 1. Networked architecture equipped with distributed actuators and sensors over a large-scale plant.

Effort has also been devoted to tackling multiagent systems with nonlinear dynamics for consensus reaching and tracking [30]–[34].

In this paper, we design networked observers that fuse state estimates and controllers for linear-time-invariant spatially interconnected plants. Examples of such plants are furnished by large-scale flexible structures such as aircraft fuselage, truss bridges, and lattice towers. Mechanical and structural vibrations could be detrimental to the functioning and safety of these plants, hence effective vibration suppression is desired in many occasions. A schematic of the architecture in the networked control context is depicted in Fig. 1. Each agent possesses all or some of the following functions: sensing, actuation, communication, and information processing. Design of state observers and controllers for all the agents is a crucial problem for the considered networked architecture. As is widely known, observability is a measure for how well internal states of a system can be inferred from the knowledge of external system outputs [35]. In the networked system shown in Fig. 1, each measurement can have a different contribution from different system internal states. For example, a sensor placed at a given location on a flexible structure can detect some vibration modes more easily than others [36]. Hence, in this paper, each observer builds part of state estimates based on its local measurements, and the remaining part based on the consensus algorithm. Specifically, each observer has a communication matrix with a predefined structure depending on its local observability. Facilitated by the concept of synchronizing region in pinning control theory [37], [38], a distributed design procedure for the observers is proposed. The controllers, though in the end implemented in a distributed way, are designed in a centralized framework, where linear-quadratic-regulator theory is adopted to handle the fact that the *separation principle* fails to hold in the networked observer and controller design.

This paper differs from the spirit of many prior results. Our scheme allows the sensors in each agent to observe only some portions of the total plant states and requires only *global observability*, i.e., observability from all outputs taken together. This should be contrasted with [15], where each agent necessarily observes all the unstable states of the system. However, unlike [15], we do not consider the process and measurement noises, as this paper mainly focuses on the

convergence and stability of designed distributed observers and controllers. Since we want all the observers' states to reach consensus asymptotically and to track the plant states, in consequence, each observer needs to estimate the total plant states, which is different from [25] where heterogeneous subspaces of the plant state are estimated among the distributed observers. On the one hand, we do not need to consider the projection matrices to match the heterogeneous subspaces of states of neighboring observers. On the other hand, however, we do require *global observability* of the network of distributed observers. Compared to [29], we do not guarantee optimality in any sense, but our proposed approach is distributed since the observer design equations are single-agent based. This leads to *scalability* or *flexibility* in terms of integrating additional sensors to the existent network and *robustness* to communication link failures as long as the connectivity requirements are satisfied. An additional merit of our distributed approach, in contrast with [29], is reduction of computational effort, as the total computational complexity grows linearly with the number of agents, while for each single agent, the effort depends only on the order of the plant. An early-stage work along these lines has been reported in [39], which relies on an assumption [39, Assumption 1] adopted for mathematical simplicity. This assumption presumes that the internal states of the considered plant can be decomposed into disjoint groups according to the measured outputs, and it is found to be at odds with some important realistic cases, in particular flexible structures. This paper therefore attempts to relax this assumption, so that the design procedure can be more successfully applied to vibration damping for flexible structures. The main contributions of this paper are as follows.

- 1) The communication graph topology is separated from the observer/controller design. And the requirements on graph topology are relaxed from (connected) undirected graphs to more general directed graphs, compared with [15], [28], and [34].
- 2) Since the observer design is single-agent based, it leads to *flexibility* in terms of integrating additional sensors to the existent network and graceful degradation in case of communication link failures as long as connectivity requirements remain satisfied. These properties are not expected in [29].

This paper is organized as follows. Necessary mathematical preliminaries and problem formulation are given in Section II. Distributed observer design without considering controls is elaborated in Section III. Distributed controller design is analyzed in Section IV. Numerical simulations of vibration reduction for a representative smart flexible structure are presented in Section V. Finally, Section VI concludes this paper.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Preliminaries on Graph Theory and Notation

The communication topology for state estimates is represented by a directed graph [40]. A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ consists of a set of *nodes* $\mathcal{V} \triangleq \{1, 2, \dots, p\}$,

a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and an associated adjacency matrix $\mathcal{A} = [a_{ij}]$. An edge (i, j) is graphically depicted by an arrow with the head *node* i and the tail *node* j , indicating that the information flows from *node* j to *node* i . Each entry a_{ij} of $\mathcal{A}$ is the weight associated with the pair (i, j) , and in this paper, it is taken that $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$; $a_{ij} = 0$, otherwise. We consider simple graphs with no self-edges, namely, $a_{ii} = 0$ and $\forall i \in \mathcal{V}$. The set of neighbors of *node* i is denoted as $\mathcal{N}_i \triangleq \{j | (i, j) \in \mathcal{E}\}$. Define the in-degree of *node* i as $d_i \triangleq \sum_{j=1}^p a_{ij}$ and in-degree matrix as $\mathcal{D}_{\text{in}} \triangleq \text{diag}\{d_i\}$. The graph Laplacian matrix is then defined as $\mathcal{L} \triangleq \mathcal{D}_{\text{in}} - \mathcal{A}$.

A sequence of successive edges in the form of $\{(i, k), (k, l), \dots, (m, j)\}$ is a directed path from *node* j to *node* i . A directed tree is an acyclic directed graph with a root node such that there is a unique directed path from that root node to every other *node* in the directed tree. A spanning tree of $\mathcal{G}$ is a directed tree that contains all the *nodes* of $\mathcal{G}$ [40]. A directed forest is a disjoint union of directed trees. A spanning forest of $\mathcal{G}$ is a directed forest that contains all the *nodes* of $\mathcal{G}$. A graph $\mathcal{H}$ is a subgraph of $\mathcal{G}$ if $\mathcal{V}(\mathcal{H}) \subseteq \mathcal{V}(\mathcal{G})$, $\mathcal{E}(\mathcal{H}) \subseteq \mathcal{E}(\mathcal{G})$ and $\mathcal{A}(\mathcal{H})$ is a restriction of $\mathcal{A}(\mathcal{G})$ [40]. A square matrix is an *M-matrix* if all its off-diagonal elements are nonpositive and all its eigenvalues have positive real parts [41], [42].

Throughout this paper, $\otimes$ is used to denote the Kronecker product [43]. $\mathbf{1}_p$ denotes a vector of dimension of p with all entries equal 1. I_p denotes an identity matrix of dimension $p \times p$. $\mathbf{0}$ stands for a zero matrix with a compatible dimension in context. $\mathbf{0}_{m \times n}$ denotes a zero matrix with dimension $m \times n$. $\lambda(A)$ denotes the eigenvalue of the matrix A . $\text{Re}(\lambda)$ represents the real part of λ . $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are adopted to denote the minimum and maximum eigenvalue of a real symmetric matrix A , respectively. $|\mathcal{V}|$ denotes the cardinality of the set $\mathcal{V}$.

B. Problem Formulation

The considered dynamics of the plant is in the form of

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{i=1}^p B_i u_i(t) = Ax(t) + Bu(t) \\ y_i(t) = C_i x(t), i = 1, 2, \dots, p \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state of the plant, $u_i \in \mathbb{R}^{m_i}$ and $i \in \mathcal{V}$, are the control inputs of agent i , $y_i \in \mathbb{R}^{p_i}$ and $i \in \mathcal{V}$ are the system outputs which are measurements by different agents, and

$$B = [B_1 \ B_2 \ \dots \ B_p] \in \mathbb{R}^{n \times m}$$

$$u = [u_1^T \ u_2^T \ \dots \ u_p^T]^T \in \mathbb{R}^m.$$

To allow for a distributed design in Section III, we need to transform $A \in \mathbb{R}^{n \times n}$ to a block diagonal matrix. For example, one widely used technique is Jordan decomposition [44]. In fact there always exists a nonsingular real matrix $\Pi = [\Pi_1 \ \Pi_2 \ \dots \ \Pi_l] \in \mathbb{R}^{n \times n}$ such that $A\Pi_i = \Pi_i A_i$ and

$A\Pi = \Pi\tilde{A}$, where

$$\tilde{A} = \begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \ddots & \\ & & & A_l \end{bmatrix} \in \mathbb{R}^{n \times n}, \quad \Pi_i \in \mathbb{R}^{n \times n_i}. \quad (2)$$

$A_i \in \mathbb{R}^{n_i \times n_i}$ is the Jordan block which has one of two forms [45]. However, it should be pointed out that the Jordan decomposition here serves only as an example for the block-diagonalization of the real matrix A ; it is by no means the unique approach. The main motivation for focusing on block-diagonal system matrices here is our consideration of the eigenmodes in flexible structures. For the problem considered in this paper, we suppose that Π is given or can be found without difficulty such that $\tilde{A} = \Pi^{-1}A\Pi$ is block diagonal. With the linear transformation $x = \Pi z = \sum_{i=1}^l \Pi_i z_i$, where $z = [z_1^T \ z_2^T \ \dots \ z_l^T]^T \in \mathbb{R}^n$, $z_i \in \mathbb{R}^{n_i}$, and $\sum_{i=1}^l n_i = n$, the dynamics of the plant becomes

$$\begin{cases} \dot{z}(t) = \tilde{A}z(t) + \sum_{i=1}^p \tilde{B}_i u_i(t) \\ y_i(t) = C_i \Pi z(t) \end{cases} \quad i \in \mathcal{V}. \quad (2)$$

Herein, $\tilde{B}_i = \Pi^{-1}B_i$. Furthermore, for conciseness and clarity of notation, throughout the rest of the paper, we define the set $\mathcal{S} \triangleq \{1, 2, \dots, l\}$. In addition, we make the following assumption.

Assumption 1: $C_{ij} \triangleq C_i \Pi_j \neq \mathbf{0}$, where $i \in \mathcal{V}$ and $j \in \mathcal{S}$, if and only if the state group z_j is observable from the measurement y_i .

Remark 1: In general, *Assumption 1* is a strong assumption: if z_j is observable from y_i , certainly $C_{ij} \neq \mathbf{0}$, but not necessarily *vice versa*. *Assumption 1* is equivalent to saying that a state-group permutation alone can bring the system to a Kalman decomposed form [46]. This is trivially satisfied if all A_j blocks are distinct and have the dimension of 1: $n_j = 1$ and $\forall j \in \mathcal{S}$. For A_j blocks of dimension larger than 1, this assumption is reasonable in this paper due to two facts; the first one is that *Assumption 1* is approximately satisfied by flexible structure dynamics, as we consider the eigenmodes which are dynamically independent, and this induces a partition of each output C_i into C_{ij} matrices; the second fact is that even if *Assumption 1* is only approximately satisfied, in the sense that if $C_{ij} \neq \mathbf{0}$ but small in magnitude, such C_{ij} can be neglected completely for design, and hence the pertaining z_j is classified as unobservable from output y_i . Our proposed design in Section III is robust enough to withstand putting such z_j simply in the set of unobservable state groups (see the discussion in Section III-B).

Definition 1: The *observable set* of agent i , $i \in \mathcal{V}$, is defined as $\mathcal{O}_i \triangleq \{j \in \mathcal{S} | C_{ij} \neq \mathbf{0}\}$; the *unobservable set* of agent $i \in \mathcal{V}$ is defined as $\bar{\mathcal{O}}_i \triangleq \{j \in \mathcal{S} | j \notin \mathcal{O}_i\}$.

Definition 2: The *converse observable set* of z_j , $j \in \mathcal{S}$, is defined as $\mathcal{D}_j \triangleq \{i \in \mathcal{V} | C_{ij} \neq \mathbf{0}\}$; the *converse unobservable set* of z_j is defined as $\bar{\mathcal{D}}_j \triangleq \{i \in \mathcal{V} | i \notin \mathcal{D}_j\}$.

Note that *Definition 1* and *Definition 2* are conjugate: $j \in \mathcal{O}_i \iff i \in \mathcal{D}_j$; $j \in \bar{\mathcal{O}}_i \iff i \in \bar{\mathcal{D}}_j$.

Assumption 2: The system is globally observable

$$\mathcal{O}_1 \cup \mathcal{O}_2 \cup \dots \cup \mathcal{O}_p = \mathcal{S}.$$

This simply means that the entire system is observable from all outputs taken together.

As introduced in Section I, p agents are assigned to the plant. The structure of agent i , $i \in \mathcal{V}$, is in the form of

$$\begin{cases} \dot{\hat{z}}_i(t) = \tilde{A}\hat{z}_i(t) + L_i(y_i(t) - \hat{y}_i(t)) \\ \quad + \mathcal{F}_i \cdot \left(\sum_{k \in \mathcal{N}_i} a_{ik}(\hat{z}_k(t) - \hat{z}_i(t)) \right) + \tilde{B}\hat{u}_i(t) \\ \hat{y}_i(t) = C_i \Pi \hat{z}_i(t) \\ \hat{U}_i(t) = K \hat{z}_i(t) \\ u_i(t) = K_i \hat{z}_i(t) \end{cases} \quad (3)$$

where $\hat{z}_i \in \mathbb{R}^n$ is the state estimate vector of observer i , $L_i \in \mathbb{R}^{n \times p_i}$ is the Luenberger-like observer gain [47], $\mathcal{F}_i \in \mathbb{R}^{n \times n}$ is the communication matrix, and $\tilde{B} = \Pi^{-1}B$. Note that $\hat{z}_i$ has a different dimension from $z_i \in \mathbb{R}^{n_i}$, since $\hat{z}_i$ is the state estimate of the total system; while z_i is a subset of the system states. Note also that all the observers share the same drift dynamics $\tilde{A}$ which equals that of the plant. This way, each observer replicates the dynamics of the entire plant and if synchronized to the plant state, they would all continue to track it with no need for further communication. The agent i estimates the system states via two information sources: one is the local measurement y_i , accounted by the term $L_i(y_i - \hat{y}_i)$, and the other is the state estimate from neighboring agents, appearing in the term $\mathcal{F}_i \cdot [\sum_{k \in \mathcal{N}_i} a_{ik}(\hat{z}_k(t) - \hat{z}_i(t))]$. It is considered that the controller in each agent generates an estimate-based local control action to the plant, $u_i(t)$, as shown in (3). Herein, $K_i \in \mathbb{R}^{m_i \times n}$ is the local feedback matrix to be designed for agent i . $\hat{U}_i \in \mathbb{R}^m$ is the estimation, by agent i , of the overall control action applied to the plant, and it is constructed based on the local state estimate: $\hat{U}_i = K \hat{z}_i$, where $K \in \mathbb{R}^{m \times n}$ is the global feedback matrix

$$K = [K_1^T \ K_2^T \ \dots \ K_p^T]^T. \quad (4)$$

Remark 2: Different possibilities for $\hat{U}_i$ exist, for example, neighboring estimates could be used. However, in all such distributed cases, unless all the local state estimates $\hat{z}_i$, $\forall i \in \mathcal{V}$, reach consensus, each control estimation $\hat{U}_i$ is different from the actual control action u applied to the system. This mismatch leads to a feature not appearing in the conventional centralized observer and feedback design. Specifically speaking, the controller design is not completely separated from the observer design, making the problem complex as elaborated later in Section IV.

Note that even though in (3) the local control action u_i appears for each agent, in practice depending on matrix B or $\tilde{B}$, it is allowed that some agents apply no control actions, as long as the following global controllability holds.

Assumption 3: The pair (A, B) (or equivalently the pair $(\tilde{A}, \tilde{B})$) is controllable.

Assumption 4: The communication graph $\mathcal{G}$, given a priori, satisfies the following condition: for any $j \in \mathcal{S}$, the

subgraph $\mathcal{G}_j$, formed by the nodes belonging to $\mathcal{D}_j$, has outgoing edges pinning into all the roots of a spanning forest of the subgraph $\bar{\mathcal{G}}_j$, formed by the nodes belonging to $\bar{\mathcal{D}}_j$.

Remark 3: Note that we consider general directed graph, whereas many existing results [15], [28], [34] focus only on undirected graphs.

In summary, the problem addressed in this paper is to design distributed observers (parameters: $\{L_i\}, \{\mathcal{F}_i\}$ in (3)) and controllers (parameters: $\{K_i\}$ in (3)) to achieve asymptotic state omniscience [48] and stabilization, namely

$$\begin{cases} \lim_{t \rightarrow \infty} (\hat{z}_i(t) - z(t)) = 0, \forall i \in \mathcal{V} \\ \lim_{t \rightarrow \infty} z(t) = 0. \end{cases} \quad (5)$$

III. DISTRIBUTED OBSERVER DESIGN

This section proposes a distributed design for the distributed observers. Observers are designed here without applying any control actions u_i s and also without control signal estimates $\hat{U}_i$ s, namely, in (3) let $K_i = \mathbf{0}$ and $\forall i \in \mathcal{V}$.

A. Main Results

The dynamics of the autonomous plant becomes

$$\begin{cases} \dot{z}(t) = \tilde{A}z(t) \\ y_i(t) = C_i \Pi z(t) = \sum_{j \in \mathcal{O}_i} C_{ij} z_j(t), i \in \mathcal{V}. \end{cases} \quad (6)$$

Accordingly, the dynamics of the observers then reads

$$\begin{cases} \dot{\hat{z}}_i(t) = \tilde{A}\hat{z}_i(t) + L_i(y_i(t) - \hat{y}_i(t)) \\ \quad + \mathcal{F}_i \cdot \left(\sum_{k \in \mathcal{N}_i} a_{ik}(\hat{z}_k(t) - \hat{z}_i(t)) \right) \\ \hat{y}_i(t) = C_i \Pi \hat{z}_i(t) \end{cases} \quad i \in \mathcal{V}. \quad (7)$$

Similar to the internal states of the plant, states of each observer are divided accordingly into l groups

$$\hat{z}_i \triangleq \begin{bmatrix} \hat{z}_{i1} \\ \hat{z}_{i2} \\ \vdots \\ \hat{z}_{il} \end{bmatrix} \in \mathbb{R}^n, \quad \forall i \in \mathcal{V}, \quad \hat{z}_{ij} \in \mathbb{R}^{n_j}, \quad \forall j \in \mathcal{S}. \quad (8)$$

We rearrange the states of each observer i , $i \in \mathcal{V}$, such that the new state vector $\hat{z}_i^{\text{new}}$ is a tandem of observable state vector $\hat{z}_{i\mathcal{O}} \in \mathbb{R}^{n_{i\mathcal{O}}}$, containing z_{ij} s, for $j \in \mathcal{O}_i$ and unobservable state vector $\hat{z}_{i\bar{\mathcal{O}}} \in \mathbb{R}^{n_{i\bar{\mathcal{O}}}}$, containing z_{ij} s, for $j \in \bar{\mathcal{O}}_i$. Specifically, for each $\hat{z}_i$, there is a permutation matrix T_i such that $\hat{z}_i^{\text{new}} = [\hat{z}_{i\mathcal{O}} \ \hat{z}_{i\bar{\mathcal{O}}}] = T_i \hat{z}_i$. Correspondingly, in the new coordinates, the dynamical and output matrices are

$$\begin{cases} T_i \tilde{A} T_i^{-1} = T_i \tilde{A} T_i^T = \begin{bmatrix} A_{i\mathcal{O}} & \\ & A_{i\bar{\mathcal{O}}} \end{bmatrix} \\ C_i \Pi T_i^T = [C_{i\mathcal{O}} \ \mathbf{0}_{p_i \times n_{i\bar{\mathcal{O}}}}]. \end{cases} \quad (9)$$

Remark 4: After this permutation of state groups, the plant is in Kalman decomposed form, as made possible by Assumption 1. It can be concluded that for $\hat{z}_{i\mathcal{O}}$, a Luenberger-like

observer can be designed relying only on the local measurements y_i to estimate the corresponding part of the plant states.

However, to make the state $\hat{z}_{i\bar{o}}$ converge to the corresponding part of system state which is unobservable for agent i , the local measurements are to no avail, so the observer i needs information from the communication network. The dynamics of each component of $\hat{z}_{i\bar{o}}$, $\hat{z}_{ij}$, is considered separately. Hence, we propose to decompose the dynamics of each observer i , $i \in \mathcal{V}$, in

$$\begin{cases} \dot{\hat{z}}_{io}(t) = A_{io}\hat{z}_{io}(t) + L_{io}(y_i(t) - \hat{y}_i(t)) \\ \dot{\hat{z}}_{ij}(t) = A_j\hat{z}_{ij}(t) \\ \quad + c_j F_j \left(\sum_{k \in \mathcal{N}_i} a_{ik}(\hat{z}_{kj}(t) - \hat{z}_{ij}(t)) \right), j \in \bar{\mathcal{O}}_i \\ \hat{y}_i(t) = \sum_{j \in \mathcal{O}_i} C_{ij}\hat{z}_{ij}(t) = C_{io}\hat{z}_{io}(t). \end{cases} \quad (10)$$

Herein L_{io} is the observer gain matrix, $c_j > 0$ is the scalar coupling gain, and $F_j \in \mathbb{R}^{n_j \times n_j}$ is the communication matrix. For the observer i , L_{io} , $\{c_j\}$ and $\{F_j\}$, $j \in \bar{\mathcal{O}}_i$ are the parameters to be designed. The rationale of decomposing the dynamics of each observer as shown in (10) is to allow for a distributed design later.

Remark 5: In (10), c_j and F_j can take a more general form, reflected by notation c_{ij} and F_{ij} . However, with the first subscript omitted here, different observers share some of the design parameters. For example, if $\bar{\mathcal{O}}_i \cap \bar{\mathcal{O}}_k \neq \emptyset$, then for $j \in \bar{\mathcal{O}}_i \cap \bar{\mathcal{O}}_k$, observer i and k will share the same parameters c_j , F_j . The motivation for omitting the first subscript is to adopt the synchronizing region theory for distributed design as illustrated later in this section.

For system integration, it is worth to explicitly relate (7) with (10) via the following equations:

$$L_i = T_i^T \begin{bmatrix} L_{io} \\ \mathbf{0}_{n_{i\bar{o}} \times p_i} \end{bmatrix} \quad (11)$$

$$\mathcal{F}_i = T_i^T \begin{bmatrix} \mathbf{0}_{n_{io} \times n_{io}} & \mathbf{0}_{n_{io} \times n_{i\bar{o}}} \\ \mathbf{0}_{n_{i\bar{o}} \times n_{io}} & \text{diag}\{c_j F_j\} \end{bmatrix} T_i. \quad (12)$$

For convergence analysis, define the observation error of observer i for the state z_j as

$$\delta_{ij} \triangleq \hat{z}_{ij} - z_j. \quad (13)$$

For observer i , stack all the δ_{ij} values with $j \in \mathcal{O}_i$ together, and denote them as δ_{io} . These are the observation errors of observer i for state groups z_j s which observer i estimates locally.

Proposition 1: The dynamics of δ_{io} , $i \in \mathcal{V}$ is

$$\dot{\delta}_{io} = (A_{io} - L_{io}C_{io})\delta_{io}. \quad (14)$$

The proof is immediate from (6) and (10).

Proposition 2: For $j \in \bar{\mathcal{O}}_i$, the dynamics of δ_{ij} is

$$\begin{aligned} \dot{\delta}_{ij} &= A_j\delta_{ij} + c_j F_j \left[\sum_{k \in \mathcal{N}_i} a_{ik}(\delta_{kj} - \delta_{ij}) \right] \\ &\quad + c_j F_j \left[\sum_{\substack{k \in \mathcal{N}_i, \\ k \in \bar{\mathcal{D}}_j}} a_{ik}(\delta_{kj} - \delta_{ij}) + \sum_{\substack{k \in \mathcal{N}_i, \\ k \in \mathcal{D}_j}} a_{ik}(\delta_{kj} - \delta_{ij}) \right] \\ &= A_j\delta_{ij} + c_j F_j \left[\sum_{k \in \bar{\mathcal{D}}_j} a_{ik}\delta_{kj} - \sum_{k \in \mathcal{N}_i} a_{ik}\delta_{ij} \right] \\ &\quad + c_j F_j \sum_{k \in \mathcal{D}_j} a_{ik}\delta_{kj}. \end{aligned} \quad (15)$$

The proof is immediate from (6) and (10). We denote the last term in (15) as $v_{ij} = c_j F_j \sum_{k \in \mathcal{D}_j} a_{ik}\delta_{kj}$.

Fixing j , stack all the δ_{ijs} with $i \in \bar{\mathcal{D}}_j$, denoted as $\delta_{\bar{o}j} \in \mathbb{R}^{n_j |\bar{\mathcal{D}}_j|}$. These are the observation errors with respect to state z_j for all those observers that do not estimate z_j directly, relying not on the local measurements but rather on the information from the network.

Proposition 3: The dynamics of $\delta_{\bar{o}j}$, $j \in \mathcal{S}$, is

$$\dot{\delta}_{\bar{o}j} = (I_{|\bar{\mathcal{D}}_j|} \otimes A_j - c_j \mathcal{L}_j \otimes F_j) \delta_{\bar{o}j} + v_j \quad (16)$$

where v_j is a stack of v_{ijs} with all $i \in \bar{\mathcal{D}}_j$. $\mathcal{L}_j$ is obtained via deleting the k th row and column from the original Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$, for all $k \in \mathcal{D}_j$.

Proof: From (15), we know for $i \in \bar{\mathcal{D}}_j$ (or equivalently $j \in \bar{\mathcal{O}}_i$)

$$\begin{aligned} \dot{\delta}_{ij} &= A_j\delta_{ij} + c_j F_j \left[\sum_{k \in \bar{\mathcal{D}}_j} a_{ik}\delta_{kj} - \sum_{k \in \mathcal{N}_i} a_{ik}\delta_{ij} \right] \\ &\quad + c_j F_j \sum_{k \in \mathcal{D}_j} a_{ik}\delta_{kj} \\ &= A_j\delta_{ij} - c_j F_j \left[d_i\delta_{ij} + \sum_{k \in \bar{\mathcal{D}}_j} (-a_{ik})\delta_{kj} \right] + v_{ij}. \end{aligned} \quad (17)$$

The coefficients appearing in the term $[d_i\delta_{ij} + \sum_{k \in \bar{\mathcal{D}}_j} (-a_{ik})\delta_{kj}]$, d_i , and $-a_{iks}$, form the i th row of $\mathcal{L}$ with the i_x th columns removed for $i_x \in \mathcal{D}_j$. Fixing j , stacking δ_{ijs} for all $i \in \bar{\mathcal{D}}_j$ to construct $\delta_{\bar{o}j}$, one immediately gets (16) from (17). ■

Remark 6: Note that for construction of $\delta_{\bar{o}j}$, we only select δ_{ijs} with $i \in \bar{\mathcal{D}}_j$. However, the dynamics of $\delta_{\bar{o}j}$ is also influenced by δ_{ijs} with $i \in \mathcal{D}_j$, and all such δ_{ij} values are included in v_j in (16), hence the constructed $\mathcal{L}_j$ is a square matrix.

Lemma: Under Assumption 4, $\mathcal{L}_j$ constructed in (16) is a nonsingular M -matrix. □

Proof: From the original graph, a group of nodes ($i \in \mathcal{V}$ and $i \in \mathcal{D}_j$) is excluded. The effect of these excluded nodes on the remaining ones, as reflected by $\mathcal{L}_j$, is the same as if the

remaining *nodes* were pinned by a single leader, with appropriately summed pinning gains.¹ In the latter instance, the pertaining pinned Laplacian matrix is a nonsingular M -matrix if the single leader pins into all the *roots* of a *spanning forest* [49]. In the former, original, instance, this condition is equivalent to the excluded group of *nodes* pinning with their outgoing edges into all *roots* of the *spanning forest* of the subgraph $\bar{\mathcal{G}}_j$, formed by the *nodes* belonging to $\bar{\mathcal{D}}_j$. According to *Assumption 4*, $\mathcal{L}_j$ is a nonsingular M -matrix. ■

Then, define the total observation error for observer i as $\delta_i = \hat{z}_i - z$. Put these observation errors for all observers together in a compact form $\delta = [\delta_1^T \ \delta_2^T \ \dots \ \delta_p^T]^T$.

Proposition 4: The dynamics of δ is given by

$$\dot{\delta} = A_\epsilon \delta \quad (18)$$

where $A_\epsilon = \text{diag}\{\tilde{A} - L_i C_i \Pi\} - \mathcal{F} \cdot (\mathcal{L} \otimes I_n)$, and $\mathcal{F} = \text{diag}\{\mathcal{F}_i\}$.

Proof: From (6) and (7), the dynamics of δ_i can be derived as

$$\dot{\delta}_i = (\tilde{A} - L_i C_i \Pi) \delta_i + \mathcal{F}_i \cdot \sum_{k \in \mathcal{N}_i} a_{ik} (\delta_k - \delta_i). \quad (19)$$

The proof of *Proposition 4* is immediately completed by stacking the dynamics of all the δ_i values together. ■

Proposition 5: A_ϵ is Hurwitz iff the dynamics of $\delta_{io}, \forall i \in \mathcal{V}$, and $\delta_{\bar{o}j}, \forall j \in \mathcal{S}$ are asymptotically stable.

This result can be easily proved by noticing that the components of δ include all the components of $\delta_{io}, \forall i \in \mathcal{V}$, and $\delta_{\bar{o}j}, \forall j \in \mathcal{S}$, and *vice versa*.

The following theorem proposes a distributed design of the parameters for all the observers to estimate the plant states when no controls or their estimates are present.

Theorem 1: Under *Assumptions 1, 2, and 4* the following hold.

1) Let each $L_{io}, i \in \mathcal{V}$, be selected such that the matrix $A_{io} - L_{io} C_{io}$ is Hurwitz.

2) Let $F_j = \hat{R}_j^{-1} \hat{P}_j, j \in \mathcal{S}$, where $\hat{P}_j = \hat{P}_j^T > 0 \in \mathbb{R}^{n_j \times n_j}$ is the unique positive definite solution of the following control algebraic Riccati equation:

$$A_j^T \hat{P}_j + \hat{P}_j A_j - \hat{P}_j \hat{R}_j^{-1} \hat{P}_j + \hat{Q}_j = 0 \quad (20)$$

where $\hat{Q}_j \geq 0 \in \mathbb{R}^{n_j \times n_j}$ and $\hat{R}_j > 0 \in \mathbb{R}^{n_j \times n_j}$ are given.

3) Let $c_j, j \in \mathcal{S}$, satisfy the condition

$$c_j \geq \frac{1}{2\underline{\lambda}_{R_j}}, \quad j \in \mathcal{S} \quad (21)$$

with $\underline{\lambda}_{R_j} = \min_k \text{Re}(\lambda_{jk})$, and $\lambda_{j\bullet}$ are the eigenvalues of $\mathcal{L}_j$. Then with the dynamics of (6) and (7), $\lim_{t \rightarrow \infty} (\hat{z}_i(t) - z(t)) = 0, \forall i \in \mathcal{V}$. □

Proof: Since each matrix $A_{io} - L_{io} C_{io}$ is Hurwitz, the dynamics (14) of all the $\delta_{io}, \forall i \in \mathcal{V}$ is asymptotically stable.

Under *Assumption 2*, all $\delta_{io}, \forall i \in \mathcal{V}$ consist of all δ_{kj} s, where $\forall j \in \mathcal{S}$ and $\forall k \in \mathcal{D}_j$. Therefore, $v_{ij} = c_j F_j \sum_{k \in \mathcal{D}_j} a_{ik} \delta_{kj}$ in (15) vanishes asymptotically in time.

¹All incoming edges $a_{ik}, k \in \mathcal{D}_j$ of a pinned node i , are summed as if originating from a single pinning leader, namely, $\sum_{k \in \mathcal{D}_j} a_{ik}$.

Hence the stability of $\delta_{\bar{o}j}$ is determined by the matrix $I_{|\bar{\mathcal{D}}_j|} \otimes A_j - c_j \mathcal{L}_j \otimes F_j$ in (16).

Based on the *Lemma*, $\mathcal{L}_j$ is a nonsingular M -matrix, hence satisfying preconditions on pinned Laplacians for synchronization [38, Assumption 1]. According to [38, Th. 1], the matrix $I_{|\bar{\mathcal{D}}_j|} \otimes A_j - c_j \mathcal{L}_j \otimes F_j$ for all $j \in \mathcal{S}$ is Hurwitz, if all F_j s are designed based on (20) and all c_j values satisfy (21). Therefore the dynamics of $\delta_{\bar{o}j}$ for all $j \in \mathcal{V}$ is asymptotically stable.

Given that the dynamics of both $\delta_{io}, \forall i \in \mathcal{V}$ and $\delta_{\bar{o}j}, \forall j \in \mathcal{S}$ is asymptotically stable, according to *Proposition 5*, A_ϵ is Hurwitz. This completes the proof. ■

Remark 7: *Theorem 1* is one of the main results of this paper. It provides a distributed observer design for A_ϵ to be Hurwitz. One specific way of selecting L_{io} to obtain a Hurwitz matrix $A_{io} - L_{io} C_{io}$ is by using LQR theory: let $L_{io} = P_i C_{io}^T R_i^{-1}, i \in \mathcal{V}$, where $P_i = P_i^T > 0 \in \mathbb{R}^{n_{io} \times n_{io}}$ is the unique solution of the observer algebraic Riccati equation

$$A_{io} P_i + P_i A_{io}^T + Q_i - P_i C_{io}^T R_i^{-1} C_{io} P_i = 0, \quad (22)$$

where $Q_i \geq 0 \in \mathbb{R}^{n_{io} \times n_{io}}$ and $R_i > 0 \in \mathbb{R}^{p_i \times p_i}$ are properly chosen. Any other design of a stabilizing L_{io} , pole-placement for example, is equally applicable.

Remark 8: In *Theorem 1*, (20) and (21) are chosen with an eye toward the synchronizing region results familiar from cooperative control theory. The aim in cooperative control is to render matrices like A_ϵ Hurwitz. However, that cannot be generally achieved by straightforward classical pole-placement if the graph is allowed to vary or is imperfectly known. Synchronizing region approach achieves asymptotic stability while providing a certain level of robustness to varying graphs. Namely, the subsystems, in our case A_j values, are considered separately from the detailed graph topology. In particular, the distributed gain designed as in (20) and (21) yields an unbounded synchronizing region, allowing for a wide class of communication graphs. Hence this design can handle less-than-perfectly reliable graphs, with possible agent or link failures. Conventional pole placement would generally not result in these specific properties.

Remark 9: Furthermore, the additional merit of this distributed approach is to reduce the computational effort needed for design. The total computational complexity grows linearly in the number of agents, while for each single agent, the effort depends only on the order of the plant. This should be contrasted with centralized designs existing in the literature (see [29] and *Remark 16* in Section V-A).

Remark 10: Nevertheless, other approaches to yield a Hurwitz A_ϵ do exist; for example, as proposed in [28], let $M = [M_1^T, M_2^T, \dots, M_p^T]^T \in \mathbb{R}^{n \times n}$ such that the matrix $\tilde{A} - M\tilde{C}$ is Hurwitz, where $\tilde{C} = [C_1^T, C_2^T, \dots, C_p^T]^T \Pi$, then in (7) with $L_i = p M_i$ and $\mathcal{F}_i = \gamma I_n$ with a sufficiently large positive scalar γ , the matrix A_ϵ defined in *Proposition 4* is Hurwitz. However, note that the approach in [28] is restricted to undirected graphs.

B. Discussion

In the practical design, one may encounter several issues with the developed distributed method, and some of them are thereby discussed more clearly in the following.

- 1) *Assumption 1* may be only approximately satisfied in the sense that for *agent i*, the magnitude of some C_{ij} is nonzero but quite small, hence the corresponding state group z_j is costly to estimate. One realistic approach to tackle this is to neglect small C_{ij} values, considering those states corresponding to small C_{ij} values as being in the *unobservable set* $\bar{\mathcal{O}}_i$. Such terms would only introduce a small vanishing disturbance which a properly designed asymptotically converging observer is robust to (see [50]). In such a case, after the design of distributed observers, it needs to be checked if the resulting matrix $\tilde{A} - L_i C_i \Pi$ is still Hurwitz. If it is not Hurwitz, then Q_i and R_i in (22) should be tuned to make L_{io} smaller in magnitude.
- 2) The instance of an observer i with all $C_{ij}s$, $j \in \mathcal{S}$, relatively large does not cause problems for our approach. This only means that the observer i can reconstruct all the plant states based on its local measurements y_i . It sends its state estimates to the network, but does not receive any state estimates from the network.
- 3) *Theorem 1* also allows for some *agent i* to have $C_{ij} = \mathbf{0}$, $\forall j \in \mathcal{S}$. In this case that *agent* does not have a local sensing device and reconstructs all the states based only on the information from the network. It would serve as a linking node or a router in the network, and possibly as an actuator as well.

Remark 11: The systems we focus on in this paper are flexible structures which can be described to a high degree of accuracy by linear models. However, in principle, the design philosophy proposed in *Theorem 1* is applicable to a class of nonlinear systems along the lines of [32]–[34], if it is assumed, similar to *Assumption 1*, that by pure permutation of state groups, for every $i \in \mathcal{V}$, the plant dynamics can be cast into the corresponding decomposed form. Particularly, one would then have, similar to (10), for observer i

$$\begin{cases} \dot{\hat{z}}_{io} = f_{io}(\hat{z}_{io}, t) + O_i^{-1}(z_{io}) M_i [y_i(t) - h_{io}(\hat{z}_{io}(t))] \\ \dot{\hat{z}}_{ij} = f_j(\hat{z}_{ij}, t) + c_j F_j \left[\sum_{k \in \mathcal{N}_i} a_{ik} (\hat{z}_{kj}(t) - \hat{z}_{ij}(t)) \right], j \in \bar{\mathcal{O}}_i \end{cases} \quad (23)$$

where $O_i(z_{io})$ has full rank and is Lipschitz continuous [51], $M_i \in \mathbb{R}^{n_{io}}$ is some finite gain vector. If conditions of [51, Th. 1] are fulfilled, $\hat{z}_{io}$ would converge to z_{io} . And if each f_j satisfies special Quadratic condition [33] for $\hat{z}_{ij}, z_j \in \ker(\hat{P}_j \hat{R}_j^{-1} \hat{P}_j)$

$$(\hat{z}_{ij} - z_j)^T \hat{P}_j (f_j(\hat{z}_{ij}, t) - f_j(z_j, t)) \leq -(\hat{z}_{ij} - z_j)^T \hat{Q}_j (\hat{z}_{ij} - z_j) \quad (24)$$

where $\hat{Q}_j = \hat{Q}_j^T > 0$, then with sufficiently large c_j and $F_j = \hat{R}_j^{-1} \hat{P}_j$, each $\hat{z}_{ij}$ would converge to the corresponding plant state z_j .

IV. DISTRIBUTED CONTROLLER DESIGN

Section III proposes a distributed design for observers without any controls acting, resulting in a Hurwitz matrix A_ϵ . In this section, we return to the original problem of (2)

and (3) building on results of Section III and considering controls and their estimates. Here we propose a design of feedback matrix K which stabilizes the original system while the distributed observers maintain estimates' convergence.

Proposition 6: When controls and their estimates are applied, the dynamics of the global observation error δ for all *agents* is given by

$$\dot{\delta} = A_\epsilon \delta + B_\epsilon(K) \delta \quad (25)$$

where $B_\epsilon(K) = I_p \otimes \tilde{B}K + \mathbf{1}_p \otimes \Upsilon(K)$ and

$$\Upsilon(K) = [-\tilde{B}_1 K_1 \quad -\tilde{B}_2 K_2 \quad \dots \quad -\tilde{B}_p K_p].$$

Proof: From (2) and (3), the dynamics of $\delta_i = \hat{z}_i - z$ can be derived as

$$\begin{aligned} \dot{\delta}_i = (\tilde{A} - L_i C_i \Pi) \delta_i + \mathcal{F}_i \sum_{k \in \mathcal{N}_i} a_{ik} (\delta_k - \delta_i) \\ + \tilde{B}K \delta_i - \sum_{j=1}^p \tilde{B}_j K_j \delta_j. \end{aligned} \quad (26)$$

The proof of *Proposition 6* is immediately completed by stacking the dynamics of all the δ_i values together. ■

Remark 12: When there are no control actions, the error dynamics is given by (18), which is here considered as the *nominal case*. Note that if $K = \mathbf{0}$, $B_\epsilon(K) = I_p \otimes \tilde{B}K + \mathbf{1}_p \otimes \Upsilon(K) = \mathbf{0}$. As shown in (25), the mismatch between the real control actions u and the estimated control actions $\hat{U}_i$, $i \in \mathcal{V}$ generates a vanishing perturbation [50], for the dynamics of observation error δ .

The following theorem gives a sufficient condition to solve the problem (5) posed in Section II.

Theorem 2: Under *Assumption 1–4*, with the plant dynamics described by (2) and the *agent* dynamics described by (3), let the conditions in *Theorem 1* be satisfied. If the matrix K is designed such that $\tilde{A} + \tilde{B}K$ is Hurwitz and $\|K\|_\infty < (\lambda_{\min}(Q)/2\sqrt{np}(1+p)\|\tilde{B}\|_\infty\lambda_{\max}(P))$, where $P = P^T > 0$ is the unique solution for the Lyapunov equation $A_\epsilon^T P + P A_\epsilon = -Q$ with a given $Q = Q^T > 0$, then the dynamics of plant (2) is asymptotically stable, and all the observers reach consensus converging to the true plant states. □

Proof: From (2) and (3), expressing $\hat{z}_i$ as $(z + \delta_i)$, the dynamics of z is

$$\dot{z} = \tilde{A}z + \sum_{i=1}^p \tilde{B}_i K_i (z + \delta_i) = (\tilde{A} + \tilde{B}K)z - \Upsilon(K)\delta. \quad (27)$$

Define the augmented state vector $\xi = [z^T, \delta^T]^T$. Based on (27) and *Proposition 6*, the closed-loop dynamics of ξ is

$$\dot{\xi} = \begin{bmatrix} \tilde{A} + \tilde{B}K & -\Upsilon(K) \\ \mathbf{0} & A_\epsilon + B_\epsilon(K) \end{bmatrix} \xi. \quad (28)$$

This is a hierarchical system; if $\tilde{A} + \tilde{B}K$ is Hurwitz, the stability of the closed-loop dynamics is determined by the matrix $A_\epsilon + B_\epsilon(K)$, i.e., the dynamics of (25).

A_ϵ is guaranteed to be Hurwitz by *Theorem 1*. Based on [50, Lemma 9.1, p. 341], the dynamics of (25) is asymptoti-

cally stable if

$$\begin{cases} \|B_\epsilon(K)\delta\|_2 \leq \gamma \|\delta\|_2 \\ \gamma = \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}. \end{cases} \quad (29)$$

Since

$$\begin{aligned} \|B_\epsilon(K)\|_\infty &= \|I_p \otimes \tilde{B}K + \mathbf{1}_p \otimes \Upsilon(K)\|_\infty \\ &\leq \|I_p \otimes \tilde{B}K\|_\infty + \|\mathbf{1}_p \otimes \Upsilon(K)\|_\infty \\ &= \|\tilde{B}K\|_\infty + \|\Upsilon(K)\|_\infty, \end{aligned}$$

and $\|\Upsilon(K)\|_\infty \leq \sum_{i=1}^p \|\tilde{B}_i K_i\|_\infty \leq p\|\tilde{B}\|_\infty\|K\|_\infty$, one has $\|B_\epsilon(K)\|_\infty \leq (1+p)\|\tilde{B}\|_\infty\|K\|_\infty$. Based on matrix analysis, $\|B_\epsilon(K)\|_2 \leq \sqrt{np}\|B_\epsilon(K)\|_\infty \leq \sqrt{np}(1+p)\|\tilde{B}\|_\infty\|K\|_\infty$. Hence if $\sqrt{np}(1+p)\|\tilde{B}\|_\infty\|K\|_\infty < (\lambda_{\min}(Q)/2/\lambda_{\max}(P))$, that is

$$\|K\|_\infty < \frac{\lambda_{\min}(Q)}{2\sqrt{np}(1+p)\|\tilde{B}\|_\infty\lambda_{\max}(P)} \quad (30)$$

is satisfied, then (29) holds and the dynamics of (28) is asymptotically stable, which means $z \rightarrow 0$ and $\delta \rightarrow 0$. ■

Remark 13: Condition (30) reveals the fact that if the magnitude of K is bounded under a certain level, the observation error dynamics (25) remains asymptotically stable. However, it should be noted that this condition is conservative.

To ensure the matrix $\tilde{A} + \tilde{B}K$ is Hurwitz, while not yielding an unstable matrix $A_\epsilon + B_\epsilon(K)$, K can be designed using, e.g., the LQR framework for the system

$$\begin{cases} \dot{z}(t) = \tilde{A}z(t) + \tilde{B}u(t) \\ u(t) = Kz(t) \end{cases} \quad (31)$$

where $K = -\mathcal{R}^{-1}\tilde{B}^T\mathcal{P}$, and $\mathcal{P} = \mathcal{P}^T > 0 \in \mathbb{R}^{n \times n}$ is the unique solution of the following algebraic Riccati equation:

$$\tilde{A}^T\mathcal{P} + \mathcal{P}\tilde{A} + \mathcal{Q} - \mathcal{P}\tilde{B}\mathcal{R}^{-1}\tilde{B}^T\mathcal{P} = 0 \quad (32)$$

and $\mathcal{Q} \geq 0 \in \mathbb{R}^{n \times n}$ and $\mathcal{R} > 0 \in \mathbb{R}^{m \times m}$ are given matrices.

Remark 14: Any design of stabilizing K for the dynamics (28) is, in principle, applicable. However, the LQR approach is adopted here mainly because of its neat parameterization with $\mathcal{Q}$ and $\mathcal{R}$ matrices. Through imposing a large $\mathcal{R}$, the control action would be limited to a safe range which could not deteriorate the convergence of observation errors. Note that eigenvalues of $A_\epsilon + B_\epsilon(K)$ are continuous functions of K , i.e., $\lambda(A_\epsilon + B_\epsilon(K))$. When $K = \mathbf{0}$, which is the *nominal case*, the real part of the eigenvalues $\text{Re}(\lambda(A_\epsilon + B_\epsilon(K))) = \text{Re}(\lambda(A_\epsilon)) < 0$, if A_ϵ is designed based on *Theorem 1*. Hence by imposing large $\mathcal{R}$ in (32), $A_\epsilon + B_\epsilon(K)$ is guaranteed to be Hurwitz, due to the resulting small $\|K\|_\infty$.

Remark 15: Note that in (28), K relates both observer $[A_\epsilon + B_\epsilon(K)]$ and controller $(\tilde{A} + \tilde{B}K)$ design, imposing their interdependence. The main results of this section reveal a partial separation principle of sorts. LQR approach is proposed to partially separate the observer and controller design. Observers are designed without considering controls in Section III, while controls are designed here as with perfect state information (31). This differs from the spirit of [29] where observer and controller design are inseparable.

Fig. 2. Piezoelectric actuated beam clamped at both ends; four piezo actuators are distributed along the beam.

TABLE I
PARAMETER TABLE

Parameter	Value
beam length	0.5 m
beam width	0.03 m
Poisson ratio	0.33
beam thickness	0.003 m
beam density	2700 kg/m ³
Young's Modulus	69 × 10 ⁹ Pa
Piezo type	PZT 5H
actuator size	0.03 × 0.03 m ²
actuator thickness	0.001 m
1st resonance frequency	18.5 Hz
2nd resonance frequency	52.0 Hz
3rd resonance frequency	104.2 Hz
4th resonance frequency	176.2 Hz
5th resonance frequency	270.5 Hz
6th resonance frequency	380.9 Hz
η	0.112
ζ	2.769 × 10 ⁻⁷

V. APPLICATION EXAMPLE

Active control of smart flexible structures has received extensive attention in recent years. Exemplary applications lie in the aeronautic, aerospace (satellites, telescopes, and so on), and civil engineering (earthquake-resistant buildings, lattice towers, and so on). This section gives numerical simulations to demonstrate the efficacy of the proposed distributed design of observers and centralized design of controllers applied to vibration reduction of a flexible structure. Section V-A describes a representative flexible structure and simulation results are given in Section V-B.

A. System Modeling

We consider an example of a clamped aluminum beam with four surface-bonded piezoelectric actuators as the representative smart flexible structure. The distribution of the piezoelectric actuators and sensors on the beam is depicted in Fig. 2. The parameters of the smart structure are shown in Table I. A detailed finite element (FE) model of the smart flexible structure is built using a procedure similar to the one in [53]. The beam is divided into 100 elements equidistantly. The dynamics of the sensors is neglected. The piezoelectric actuators are polarized in the thickness direction, and the electric field is assumed to be constant along the thickness of the actuator. The equation of motion of the smart flexible structure is derived using the Hamilton principle, yielding

$$[M]\ddot{q}(t) + [D]\dot{q}(t) + ([K_s] - \Theta_{q\phi}\Theta_{\phi\phi}^{-1}\Theta_{q\phi}^T)q(t) = -\Theta_{q\phi}u(t) \quad (33)$$

where q denotes the nodal displacement vector, consisting of transverse displacement and rotation of cross section of all the

nodes [54], $[D]$, $[M]$, and $[K_s]$ are the damping matrix, mass matrix and stiffness matrix, respectively. $\Theta_{q\phi}$ and $\Theta_{\phi\phi}$ are the piezoelectric coupling matrix and the piezoelectric capacity matrix respectively, and u is the input vector of voltages driving the piezoelectric actuators. The *Rayleigh damping* is adopted in the model [55]

$$[D] = \eta[M] + \zeta[K_s] \quad (34)$$

where the values of η and ζ are listed in Table I.

The FE model constructed has been cross-validated by LMS Samcef Field, a commercial finite element modeling solver suite from Siemens PLM Software. Velocities are considered as the measured outputs

$$y(t) = H_o \dot{q}(t) \quad (35)$$

where H_o is the location matrix for measurement channels. In practice, velocity information can be obtained either by noncontact laser Doppler vibrometers or by numerical integration of bandpass filtered outputs of accelerometers.

The first six vibration modes are extracted via mode displacement method [56]. Specifically, consider the following coordinates transformation:

$$q = \Phi_r q_m \quad (36)$$

herein Φ_r stands for the first six columns of Φ , where Φ is the matrix of the ordered natural mode shapes. Let $x = [q_m \ \dot{q}_m]$. The dynamics of the piezoelectric actuated beam expressed by (33) and (35) can be written into state-space representation

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (37)$$

where

$$\begin{aligned} A &= \begin{bmatrix} \mathbf{0}_{6 \times 6} & I_6 \\ -(\Phi_r^T [M] \Phi_r)^{-1} \Phi_r^T [K_{aug}] \Phi_r & -(\Phi_r^T [M] \Phi_r)^{-1} [D] \end{bmatrix} \\ B &= \begin{bmatrix} \mathbf{0}_{6 \times 4} \\ -(\Phi_r^T [M] \Phi_r)^{-1} \Theta_{q\phi} \end{bmatrix} \\ C &= L_o \Phi_r [\mathbf{0}_{6 \times 6} \ I_6] \end{aligned}$$

and

$$[K_{aug}] = [K_s] - \Theta_{q\phi} \Theta_{\phi\phi}^{-1} \Theta_{q\phi}^T.$$

Obviously, A is not block diagonal, we block-diagonalize it mode-wisely, by putting $z = \Pi^{-1}x$ in the form [36]

$$z = [(q_{m1}, \dot{q}_{m1}), \dots, (q_{m6}, \dot{q}_{m6})]^T = [z_1, \dots, z_6]^T. \quad (38)$$

Then $\tilde{A} = \text{diag}\{A_1, A_2, \dots, A_6\}$ with $A_i = [0 \ 1 \ -\omega_i^2 \ -2\xi_i\omega_i]$, ω_i and ξ_i are the i th modal frequency in radian per second and modal damping ratio, respectively.

The 2-norms of C_{ij} for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4, 5, 6$ are depicted in Fig. 3. The observability of modes compared to the 2-norms of C_{ij} s is found to satisfy *Assumption 1*. In particular, the second sensor is placed in the middle of the clamped beam, hence it can observe only the first, third, and fifth mode groups, which is consistent with the $\|C_{2j}\|_2$ profile in Fig. 3. Also, as discussed in Section III-B, when the

Fig. 3. $\|C_{ij}\|_2$, $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4, 5, 6$.

Fig. 4. Communication topology.

magnitude of some C_{ij} is relatively small, for the distributed design, one can treat that corresponding state group as being in the unobservable set $\bar{\mathcal{O}}_i$ as long as *Assumption 2* still holds, because this will only introduce small vanishing disturbances to Luenberger-like observers. Hence the *observable set* for each *agent* is allocated as follows: $\mathcal{O}_1 = \{1, 2\}$, $\mathcal{O}_2 = \{1, 3\}$, $\mathcal{O}_3 = \{1, 2\}$, and $\mathcal{O}_4 = \{4, 5, 6\}$.

The communication topology of the four *agents* is shown in Fig. 4. It can be verified that the given graph satisfies *Assumption 4*. The corresponding Laplacian matrix is

$$\mathcal{L} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

Obtaining $\mathcal{L}_i$, $i \in \mathcal{S}$ is straightforward; for example, for $i = 3$, $\mathcal{D}_3 = \{2\}$, by eliminating the second row and column from $\mathcal{L}$, we get

$$\mathcal{L}_3 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

L_{1o}, L_{2o}, L_{3o} , and L_{4o} are here designed based on (22): for $i = 1, 2, 3$, $Q_i = I_{n_{io}}$, $R_i = 1 \times 10^{-3}$; for $i = 4$, $Q_4 = I_6$, $R_4 = 1 \times 10^{-2}$. F_1, F_2, F_3, F_4, F_5 , and F_6 are designed based on (20): for $i = 1, 2, 3, 4, 5, 6$, $\hat{Q}_i = \hat{R}_i = I_2$. In (21), $\underline{L}_{R_1} = \underline{L}_{R_2} = \underline{L}_{R_4} = \underline{L}_{R_5} = \underline{L}_{R_6} = 1$; $\underline{L}_{R_3} = 0.3820$. For $j = 1, 2, 3, 4, 5, 6$, $\{c_j\}$ values are chosen as $c_j = 10$ to reach the estimates' consensus fast enough. L_i values and $\mathcal{F}_i$ values are constructed based on (11) and (12), respectively. A_e is checked if it is Hurwitz following the observer design. The controllers are designed based on (32), and $\mathcal{Q}$ and $\mathcal{R}$ are assigned as $\mathcal{Q} = I_{12}$ and $\mathcal{R} = 3 \times 10^{-5} I_4$.

Remark 16: Note that as mentioned in *Remark 9*, the complexity of our design procedure depends linearly on the

Fig. 5. State observation error of agents in time-domain.

Fig. 6. Piezoelectric transducer actuation voltages.

number of agents, while for each single agent, the complexity depends only on the order of the plant. To carry out a fair comparison with [29], we have resorted to linear matrix inequality method to solve algebraic Riccati equations involved in the above design (see [57]). The computational complexity for agent i , $i \in \mathcal{V}$, and the total system is

$$J_i = O(n_{io}^2) + \frac{n_{i\bar{o}}}{2} O(2^2) \leq O(n^2) \quad (39)$$

$$J_{\text{total}} = \sum_{i=1}^p J_i + O(n^2) \leq (1+p)O(n^2). \quad (40)$$

Note, in practice, the design of c_j values in (21) is to choose a sufficiently large number, and hence its computational cost is negligible. Whereas the computational complexity for the centralized design in [29] is

$$J' = O\left(\frac{n^*(n^*+1)}{2}\right) + O\left(n \sum_{i=1}^p p_i\right) + O\left(n \sum_{i=1}^p m_i\right) + O\left(n^2 \sum_{i=1}^p d_i\right) \quad (41)$$

where $n^* = n(p+1)$. Apparently, in our distributed design, the computational complexity scales better with a growing number of agents. In fact, the computational complexity for each agent does not depend on the total agent number p .

B. Simulations

Numerical simulations are run in MATLAB/Simulink environment. The initial condition for the plant states is arbitrarily chosen as $z(0) = [0.005, 1, 0.005, 1, 0, 1, 0, 1]^T$. All initial states for the observers are set to zero. As depicted in Fig. 5, the local observation error $\delta_i = \hat{z}_i - z$, $i = 1, 2, 3, 4$ converges to zero within 2 s. The voltage transient to drive the piezoelectric actuators is shown in Fig. 6. To demonstrate the damping performance, an open-loop measurement and closed-loop measurement are compared in Fig. 7. The slow decay of the measurement signals for the open-loop case is due to the inherent low damping of the flexible beam.

Fig. 7. Comparison of measurement signals in time-domain between open and closed-loop.

1) *Failure of Communication Link ④ → ③*: To demonstrate that our distributed approach is *robust* against varying graphs to a certain degree, we consider a communication failure from agent 4 to agent 3. From Fig. 4, we know that the remaining graph still satisfies *Assumption 4*. The local observation error $\delta_i = \hat{z}_i - z$, $i = 1, 2, 3, 4$ is given in Fig. 8. Compared with Fig. 5, no critical degradation in terms of convergence time is observed, indicating the *robustness* property of the design approach. It is natural to deduce that the more redundant communication links are, the more *robust* the NCS will be.

2) *Integrating an Extra Agent to the Network*: This simulation scenario is to present how to integrate extra sensors to the network with only a small number of additional parameters to design. We consider adding an additional *agent 5* with sensing capability into the well-established network. The sensor is added to measure the location 0.20 m from the left clamped end in Fig. 2. The augmented graph is depicted in Fig. 9.

We only need to design parameters for *agent 5*. The 2-norms of C_{ij} for $j = 1, 2, 3, 4, 5, 6$ are depicted in Fig. 10. The *observable set* for *agent 5* is allocated as $\mathcal{O}_5 = \{1\}$. L_{5o} is designed based on (22) with $Q_5 = I_2$, $R_5 = 1 \times 10^{-3}$. Note that even though the graph topology has changed, c_j values for $j = 1, 2, 3, 4, 5, 6$ still satisfy the condition (21),

Fig. 8. State observation error of agents in time-domain, after a communication link failure.

Fig. 9. Communication topology, after adding agent 5.

 Fig. 10. Profile of $\|C_{5j}\|_2$, $j = 1, 2, 3, 4, 5, 6$.

since their values are initially chosen large enough. L_5 and $\mathcal{F}_5$ are constructed based on (11) and (12), respectively. The new A_ϵ is checked if it is Hurwitz after this additional observer design. Note that the *flexibility* demonstrated in this example stems from inherent robustness of the synchronizing region design.

3) *Distributed Estimation on a More Complex Graph*: This simulation scenario is to verify the proposed distributed estimation scheme on a more complex directed graph, specifically one comprising 11 agents. The sensors are distributed on the beam with the nodal index 5, 15, 25, 35, 45, 50, 60, 70, 80, 90, 95, respectively. The *observable sets* are allocated as: $\mathcal{O}_1 = \{5, 6\}$, $\mathcal{O}_2 = \{1, 2, 3, 4\}$, $\mathcal{O}_3 = \{1, 2\}$, $\mathcal{O}_4 \sim \mathcal{O}_7 = \{1\}$, $\mathcal{O}_8 = \{1, 2\}$, $\mathcal{O}_9 = \{1, 2, 3\}$, $\mathcal{O}_{10} = \{1, 2, 3, 4, 5, 6\}$, $\mathcal{O}_{11} = \{5, 6\}$. The graph topology is shown in Fig. 11, and it can be easily verified that it satisfies *Assumption 4*. As shown in Fig. 12,

Fig. 11. Generalized communication topology with 11 agents.

 Fig. 12. Convergence of $\|\delta_i\|_2$, where $\delta_i = \hat{z}_i - z$, $i = 1, 2, \dots, 11$.

convergence of estimates of all the 11 agents to the true states verifies the efficacy of our distributed observers on this more complex graph topology. Note that this graph is more general, or less restrictive, than strongly connected graphs, therefore adding redundant edges on this graph would strengthen its connectivity and facilitate convergence of states of distributed observers.

VI. CONCLUSION

In this paper, distributed observers and controllers are designed for a class of spatially interconnected systems. Each observer estimates the system states *via* two information sources: one is the local measurement and the other is the shared state estimate from the neighboring nodes. By virtue of synchronizing region methods in pinning control theory, the observers are designed in a distributed fashion. This leads to *flexibility* in integrating additional sensors to the existent network and graceful degradation in case of communication link failures as long as the connectivity requirements remain satisfied. The observer-based feedback laws are designed in a centralized fashion, though in the end they are implemented locally. LQR approach is adopted due to its neat parameterization to partially separate the interdependent design of distributed observers and controllers. Numerical simulation of a piezoelectric actuated smart flexible system verifies the efficacy of the proposed approach. A reduction of the computational cost is given as an additional merit of our distributed approach, compared with existing centralized design. Moreover, the *robustness* of our proposed approach is demonstrated

by a simulated communication link failure. By adding an extra sensor to the simulated network, we also present the flexibility of integrating additional sensors into the existing architecture, which only requires to design a small number of additional parameters. To demonstrate the effectiveness of our proposed distributed observer design on more complex graphs, distributed estimation performance with 11 agents on a generalized directed graph is examined.

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