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Landau damping of Langmuir twisted waves with kappa distributed electrons

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The kinetic theory of Landau damping of Langmuir twisted modes is investigated in the presence of orbital angular momentum of the helical (twisted) electric field in plasmas with kappa distributed electrons. The perturbed distribution function and helical electric field are considered to be decomposed by Laguerre-Gaussian mode function defined in cylindrical geometry. The Vlasov-Poisson equation is obtained and solved analytically to obtain the weak damping rates of the Langmuir twisted waves in a nonthermal plasma. The strong damping effects of the Langmuir twisted waves at wavelengths approaching Debye length are also obtained by using an exact numerical method and are illustrated graphically. The damping rates of the planar Langmuir waves are found to be larger than the twisted Langmuir waves in plasmas which shows opposite behavior as depicted in Fig. 3 by J. T. Mendonça [Phys. Plasmas **19**, 112113 (2012)]. © 2015 AIP Publishing LLC.
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I. INTRODUCTION

It is well known that the momentum of an electromagnetic field has both linear and angular contributions. The angular momentum of electromagnetic radiation consists of two distinct parts, i.e., the intrinsic part associated with the wave polarization or spin, and the extrinsic part associated with the orbital angular momentum (OAM) and it depends on the spatial distribution of the electromagnetic field.¹ The OAM of electromagnetic radiation has been studied extensively at optical frequencies.² The OAM has also been identified in the beams of laser light with a Laguerre-Gaussian (LG) amplitude distribution in paraxial approximation theory.³ The transfer of OAM carried by ring shaped laser beams to trapped particles has been seen in experiment.⁴ The effects of photon OAM on interaction with atmosphere turbulence have also been studied theoretically in optical communications.⁵ A few years ago, the effect of orbital angular momentum was transferred from quantum optics to various regimes, such as radio waves,^{6,7} neutrino physics,⁸ inverse Faraday effect,⁹ and plasmas.¹⁰

Shukla *et al.*¹¹ studied the stimulated scattering instabilities of coherent circularly polarized electromagnetic (CPEM) waves having OAM in dense quantum plasmas with degenerate electrons and non-degenerate ions. One year later, Shukla¹² showed that due to the magnetic flux ropes carrying orbital angular momentum (OAM) or plasma density whirls, a tornado-like geometrical structure is formed by the three dimensional modified kinetic Alfvén waves (3D m-KAWs) in a magneto-plasma known as Alfvénic tornadoes. The two fluid model and Ampere's law are used to derive the equation of the three dimensional modified kinetic Alfvén wave and these 3D m-KAWs have Laguerre Gaussian Alfvénic vortex beam-like solutions. The dynamical behavior of shear Alfvénic magnetic flux ropes depends on the plasma density whirls.

Mendonça *et al.*¹³ predicted that the stimulated Raman and Brillouin backscattering cause the exchange of orbital angular momentum between electromagnetic and electrostatic waves in a plasma. The linear dust acoustic waves (DAWs) and twisted dust acoustic vortex beam carrying OAM are taken into account by Shukla.¹⁴ The application of a twisted dust acoustic vortex in transport and trapping of dust particles is investigated. Mendonça¹⁵ studied the paradigmatic characteristics of the twisted electrostatic and electromagnetic plasma modes carrying OAM.

Shukla¹⁶ found that a dispersive shear Alfvén wave (DSAW) in a magnetoplasma behaves like a twisted shear Alfvén wave carrying OAM. The wave equation is obtained by employing generalized ion vorticity and the magnetic field aligned electron momentum equation to couple the scalar and vector potentials of the DSAW. The obtained twisted shear Alfvén vortex beam is capable of trapping and transporting plasma particles and energy in magnetized plasma, such as Earth's auroral zone, in the solar atmosphere, and in the Large Plasma Device (LAPD) at the University of California, Los Angeles.

The polarization of radio waves is a well known phenomenon, however the OAM of the electromagnetic field at radio frequency was unknown. Leyser *et al.*¹⁷ demonstrate experimentally for the first time the formation of a radio beam with a helical wave front carrying OAM and pumped it into an ionospheric plasma under the High frequency Active Auroral Research Program (HAARP). The optical emissions from pumped plasma turbulence show the characteristics of ring shaped morphology due to pumped radio beams containing OAM.

Shukla¹⁸ showed the presence of twisted electrostatic ion-cyclotron (ESIC) modes carrying OAM in a magneto dusty plasma. These braided ESIC wave structures can trap and transport particles in magnetized plasma, such as Saturn's F-ring.

Yadav and Sharma¹⁹ investigated the modulational instability due to nonlinear interaction between a 3D KAW

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and an ion acoustic wave (IAW) in solar wind plasmas. The presence of OAM of wave eigen-modes is indicated by the propagation of a 3D KAW in the form of a vortex beam in magnetized plasma. Ayub *et al.*²⁰ studied the collisionless phonon modes with OAM in unmagnetized plasma using fluid theory, and LG beam solutions are employed to obtain paraxial equations. Leyser *et al.*²¹ predicted a new method of remote sensing. They provide the possibility for the generation of twisted electromagnetic beams that inherit OAM by azimuthal phase variation. For incoherent scatter radars, twisted beams having relevance with the study of the ionosphere and auroral arcs are investigated. Shahzad *et al.*²² have used fluid equations to study the effect of OAM states and LG potential on plasma consisting of cold and hot electrons.

Recently, the kinetic theory for the electron plasma and ion acoustic twisted modes has been developed by considering the distribution of particles to be thermal (Maxwellian).^{23,24} These waves could be excited by nonlinear decay of laser beams with OAM or rotating electron beams. These complex helical wave structures develop new features of the Landau resonance and instabilities. Also, in most of the space plasmas and in some laboratory experiments, the charged particles have shown non-Maxwellian or nonthermal distribution.^{25–27} These non-thermal charged particles in most of the cases are electrons and can be described well by using generalized Lorentzian or kappa (non-Maxwellian) distribution function.²⁸

In this paper, kinetic theory is applied to study the electron plasma waves in the presence of a perturbed electric field carrying orbital angular momentum for a non-Maxwellian plasma described by the generalized Lorentzian or kappa distribution function. The paper is composed of various sections. Section II contains the set of dynamic equations (i.e., the Vlasov equation, Poisson equation, and paraxial equation) for the theoretical modeling. In Section III, the modified plasma dispersion function with the electric field having orbital angular momentum is derived for kappa distributed electrons by decomposing the potential and distribution function in LG modes. The dispersion relation of twisted Langmuir waves with nonthermal electrons and weak Landau damping rates are calculated analytically. In the last section, Section IV, the numerical plots for the weak Landau damping rate and the exact numerical solution for strong Landau damping effects by using the Newton Raphson method are obtained; the conclusion is presented in the same section.

II. THEORETICAL MODEL

In this section, the Vlasov equation is employed for the description of the electron's distribution function $f(\mathbf{r}, \mathbf{v}, t)$. Consider the perturbation $f = \hat{f} + f_0$ of the distribution function for an electron plasma wave. Linearization of the Vlasov equation with respect to perturbation is given as follows:

$$(\partial_t + \mathbf{v} \cdot \partial_{\mathbf{r}})\hat{f} - e\mathbf{E} \cdot \partial_{\mathbf{p}}f_0 = 0, \quad (1)$$

where $\mathbf{p}$ is the momentum such that $\mathbf{p} = m\mathbf{v}$ and $\mathbf{E}$ represents the electric field $\mathbf{E} = -\nabla\phi$. The electric field will be calculated from the Poisson equation given below:

$$\nabla^2\phi = 4\pi en_0 \int \hat{f}(\mathbf{v})d\mathbf{v}, \quad (2)$$

here, the distribution function f is assumed to be normalized with unity. The electron plasma wave is assumed to propagate along the z -axis, i.e., $\exp(ikz)$ with a slowly varying amplitude. In this situation for $\nabla^2\phi$, we have

$$\nabla^2\phi = (\partial_{\perp}^2 - k^2 + 2ik\partial_z)\phi. \quad (3)$$

Here, the transverse Laplacian operator is written in cylindrical coordinates as $\partial_{\perp}^2 = (1/r)\partial/\partial r(r\partial/\partial r) + (1/r^2)\partial^2/\partial\theta^2$. We can assume that the field varies only gradually along the z -axis such that $\partial_z^2\phi \ll 2k\partial_z\phi$. Under these conditions, the electrostatic wave has a finite orbital angular momentum corresponding to the potential ϕ and it satisfies the paraxial equation given by

$$(\partial_{\perp}^2 + 2ik\partial_z)\phi = 0. \quad (4)$$

The requirement for the existence of an electron plasma wave is that it must satisfy the Poisson equation (2) which gives

$$-k^2\phi = 4\pi en_0 \int \hat{f}(\mathbf{v})d\mathbf{v}. \quad (5)$$

Equations (1)–(5) are the basis of the kinetic theory for a twisted electron plasma wave with orbital angular momentum of the perturbed electric field.

III. PLASMA DISPERSION FUNCTION WITH ORBITAL ANGULAR MOMENTUM OF ELECTRIC FIELD

In order to obtain the modified plasma dispersion function with orbital angular momentum, we will illustrate the solutions of paraxial equation (4). Such solutions can be obtained as the superposition of LG functions described in cylindrical coordinates $\mathbf{r} = (r, \theta, z)$. The electric potential $\phi(\mathbf{r}, t)$ can be written in terms of LG function $F_{pl}(r, z)$ (which is described in the Appendix with its properties as well, please see the Appendix for detail) as follows:

$$\phi(\mathbf{r}, t) = \sum_{pl} \tilde{\phi}_{pl} F_{pl}(r, z) e^{i\theta} e^{ikz - i\omega t}. \quad (6)$$

Here, $\tilde{\phi}_{pl}$ is the mode amplitude and integers p and l are the radial and angular mode numbers, while θ denotes the azimuthal angle. Using the spatial structure of a given potential described in Eq. (6), we can write the electric field as $\mathbf{E} = -i\mathbf{k}_{eff}\phi$, where k_{eff} is the effective wave number and it is defined by the following expression:

$$\mathbf{k}_{eff} = -\frac{i}{F_{pl}} \partial_r F_{pl} \mathbf{e}_r + \frac{l}{r} \mathbf{e}_{\theta} + \left(k - \frac{i}{F_{pl}} \partial_z F_{pl} \right) \mathbf{e}_z. \quad (7)$$

Therefore, the electric field can also be written in the components form as follows:

$$E_r = -\frac{\partial \phi}{\partial r} = -\frac{i}{F_{pl}} \partial_r F_{pl}, \quad (8)$$

$$E_\theta = -\frac{l}{r} \frac{\partial \phi}{\partial \theta} = -\frac{il}{r} \phi, \quad (9)$$

and

$$E_z = -\frac{\partial \phi}{\partial z} = -\left(ik + \frac{1}{F_{pl}} \partial_z F_{pl}\right) \phi. \quad (10)$$

It can be seen from the above set of equations (8)–(10) that the electric field shows helical structures instead of straight lines (as in case of ordinary plane wave solutions) along the z -direction. Here, the axial component of the electric field is still dominant in accordance with the paraxial approximation.

For the solution of the coupled Vlasov and Poisson equations (1) and (5), we can decompose the perturbed distribution function in LG modes such that

$$\tilde{f}(\mathbf{v}) = \sum_{pl} \tilde{f}_{pl}(\mathbf{v}) F_{pl}(r, z) e^{i\theta} e^{ikz - i\omega t}. \quad (11)$$

By using the above distribution function (11) in the Vlasov equation (1), an integration is performed in rdr and multiplication is done by F_{pl} , and employing the orthogonality condition described in the Appendix, we obtain the following relation:

$$\tilde{f}_{pl} = \frac{e}{m} \frac{\tilde{\phi}_{pl}}{(a + ib)} (\mathbf{q}_{eff} \cdot \partial_{\mathbf{v}} f_0), \quad (12)$$

where

$$a = (\omega - kv_z) - lq_\theta v_\theta, \quad b = (q_r v_r + q_z v_z), \quad (13)$$

such that $a + ib = (\omega - \mathbf{q}_{eff} \cdot \mathbf{v})$ and

$$\mathbf{q}_{eff} = -iq_r \hat{\mathbf{e}}_r + lq_\theta \hat{\mathbf{e}}_\theta + (k - iq_z) \hat{\mathbf{e}}_z. \quad (14)$$

Here, the parameters q_j and q_θ are defined as

$$q_j = \int_0^\infty F_{pl} \partial_j F_{pl} r dr \quad \text{where } (j = r, z), \quad q_\theta = \int_0^\infty F_{pl}^2 dr. \quad (15)$$

Now putting the value of the decomposed perturbed distribution function from (12) into the Poisson equation (5), we get the plasma dielectric function for the Langmuir twisted wave with nonthermal distributed electrons as follows:

$$\epsilon(\omega, k, lq_\theta) = 1 + \frac{\omega_{pe}^2}{k^2} \int \frac{\mathbf{q}_{eff} \cdot \partial_{\mathbf{v}} f_0}{(\omega - \mathbf{q}_{eff} \cdot \mathbf{v})} d\mathbf{v}, \quad (16)$$

or it can be written as

$$\epsilon(\omega, k, lq_\theta) = 1 + \chi(\omega, lq_\theta, k), \quad (17)$$

where $\chi(\omega, k, lq_\theta)$ is the electron susceptibility defined as

$$\chi(\omega, k, lq_\theta) = \frac{\omega_{pe}^2}{k^2} \int \frac{\mathbf{q}_{eff} \cdot \partial_{\mathbf{v}} f_0}{(\omega - \mathbf{q}_{eff} \cdot \mathbf{v})} d\mathbf{v},$$

and $\omega_{pe} = \sqrt{4\pi n_0 e^2 / m_e}$ is the electron plasma frequency.

The above equation (17) is analogous to those for the usual planar electric field perturbations and can also be written in the form $1 + \chi(\omega, lq_\theta, k) = 0$ to find the plasma mode. It can be seen from Eq. (16) that the condition for the Landau resonance is modified for electron Langmuir waves in the presence of a helical (twisted) electric field in comparison with a planar electric field case as follows:

$$\omega = kv_z + lq_\theta v_\theta \pm i(q_r v_r + q_z v_z). \quad (18)$$

The physical meaning of the new resonance condition can be well understood by neglecting the imaginary part $b = (q_r v_r + q_z v_z)$. This is applicable only if the condition $(|q_r|, |q_z| \ll |q_\theta|)$ is satisfied. So the resonance condition becomes $\omega = kv_z + lq_\theta v_\theta$, which resembles the Landau cyclotron resonance for the magnetized plasma case.²⁹

The electron susceptibility can be written in more simplified form as

$$\chi(\omega, k, lq_\theta) = \frac{\omega_{pe}^2}{k^2} Z_{\kappa, lq_\theta}(\xi_z, \xi_\theta), \quad (19)$$

where the modified plasma dispersion function $Z_{\kappa, lq_\theta}(\xi_z, \xi_\theta)$ for the kappa distributed non-Maxwellian plasmas with orbital angular momentum of the electric field is defined as

$$Z_{\kappa, lq_\theta}(\xi_z, \xi_\theta) = \int \left[\frac{\partial_{\mathbf{v}} f_0}{u_z - v_z} + \frac{\partial_{\mathbf{v}} f_0}{u_\theta - v_\theta} \right] d\mathbf{v}, \quad (20)$$

here, $\xi_z = u_z / \theta_\parallel$ and $\xi_\theta = u_\theta / \theta_\parallel$ and $u_z = \left(\frac{\omega}{k} - \frac{lq_\theta v_\theta}{k} \right)$ and $u_\theta = \left(\frac{\omega}{lq_\theta} - \frac{kv_z}{lq_\theta} \right)$.

Equation (17) can be written in terms of the plasma dielectric function as follows:

$$\epsilon(\omega, k, lq_\theta) = 1 + \frac{\omega_{pe}^2}{k^2} Z_{\kappa, lq_\theta}(\xi_z, \xi_\theta). \quad (21)$$

The definition of the three dimensional non-Maxwellian (kappa) distributed electrons is given by the expression²⁸

$$f_0 = \frac{n_0}{\pi^{3/2} \theta_\perp^2 \theta_z} \frac{\Gamma(\kappa + 1)}{\kappa^{3/2} \Gamma(\kappa - 1/2)} \left[1 + \frac{v_z^2}{\kappa \theta_\parallel^2} + \frac{v_\perp^2}{\kappa \theta_\perp^2} \right]^{-\kappa-1}, \quad (22)$$

where κ is the spectral index and $\theta_{(\parallel, \perp)} = \sqrt{(2\kappa - 3)/\kappa} v_{T(\parallel, \perp)}$; $\kappa > 3/2$ is the effective thermal velocity for nonthermal electrons such that $v_{T(\parallel, \perp)} = \sqrt{T_{(\parallel, \perp)} / m}$ is the thermal velocity of the electrons along the parallel and perpendicular directions, respectively.

The modified plasma dispersion function $Z_{\kappa, lq_\theta}(\xi_z, \xi_\theta)$ in Eq. (21) can be expanded under the condition $(\xi_z, \xi_\theta \gg 1)$ for twisted electron plasma waves with kappa distributed electrons, which gives

$$\epsilon(\omega, k, lq_\theta) = 1 - \frac{\omega_{pe}^2}{\omega^2} \left[1 + \frac{k^2 \theta_{||e}^2}{\omega^2} \frac{3\kappa}{(2\kappa - 3)} \right] - \frac{\omega_{pe}^2}{\mu^2 \omega^2} \left[1 + \frac{k^2 \theta_{||e}^2}{\mu^2 \omega^2} \frac{3\kappa}{(2\kappa - 3)} \right] + 2i\sqrt{\pi} \frac{\omega \Gamma(\kappa + 1)}{\kappa^{3/2} \Gamma(\kappa - 1/2)} \frac{\omega_{pe}^2}{k^3 \theta_{||e}^3} \left[\left(1 + \frac{\omega^2}{\kappa k^2 \theta_{||e}^2} \right)^{-\kappa-1} + \frac{\mu}{k} \left(1 + \mu^2 \frac{\omega^2}{\kappa k^2 \theta_{||e}^2} \right)^{-\kappa-1} \right], \quad (23)$$

where $\mu = k/lq_\theta$.

The plasma dielectric function $\epsilon(\omega, k, lq_\theta)$ contains real and imaginary parts such that $\epsilon(\omega, k, lq_\theta) = \text{Re}[\epsilon(\omega, k, lq_\theta)] + i\text{Im}[\epsilon(\omega, k, lq_\theta)]$. Here, $\omega = \omega_r + i\gamma$ is the temporal angular frequency of the wave. The expression for ω_r can be obtained by equating the real part of the dielectric function $\epsilon(\omega, k, lq_\theta)$ to zero, i.e., $\text{Re}[\epsilon(\omega, k, lq_\theta)] = 0$ which yields

$$\omega_r^2 = \omega_{pe}^2 \frac{(1 + \mu^2)}{\mu^2} + \frac{3\kappa k^2 \theta_{||e}^2}{(2\kappa - 3)} \frac{(1 + \mu^4)}{\mu^2 (1 + \mu^2)}. \quad (24)$$

The above equation is the dispersion relation for electron plasma waves in a helical (twisted) electric field with kappa distributed nonthermal electrons. In the case of the Maxwellian plasma limit $\kappa \rightarrow \infty$, the above dispersion relation of Eq. (24) becomes the same as described in Ref. 23.

The weak Landau damping rate γ for electron plasma waves with kappa distributed electrons in the presence of a twisted electric field can be obtained from the relation $\gamma = -[\text{Im}(\epsilon)/(\partial \text{Re}(\epsilon)/\partial \omega)]_{\omega=\omega_r}$. Using the respective values of $\text{Im}(\epsilon)$ and $\text{Re}(\epsilon)$ in the expression of γ , we have

$$\gamma = -\frac{\sqrt{\pi} \omega_r^4 \mu^2}{k^3 \theta_{||e}^3 (1 + \mu^2)} (\gamma_z + \gamma_\theta), \quad (25)$$

where γ_z is the conventional damping factor as it appeared in the planar electric field case for nonthermal kappa distributed electrons and γ_θ presents the contributions of the damping factor due to a helical (twisted) electric field with kappa distributed electrons, which are described as follows:

$$\gamma_z = \frac{\Gamma(\kappa + 1)}{\kappa^{3/2} \Gamma(\kappa - 1/2)} \left(1 + \frac{\omega_r^2}{\kappa k^2 \theta_{||e}^2} \right)^{-\kappa-1}, \quad (26)$$

$$\gamma_\theta = \frac{\Gamma(\kappa + 1) \mu}{\kappa^{3/2} \Gamma(\kappa - 1/2)} \left(1 + \mu^2 \frac{\omega_r^2}{\kappa k^2 \theta_{||e}^2} \right)^{-\kappa-1}. \quad (27)$$

In the Maxwellian electron plasma limit, $\kappa \rightarrow \infty$, the damping rate becomes the same as described in Ref. 23.

In the limit, when the azimuthal wave number approaches zero, i.e., $lq_\theta \rightarrow 0$ or $\mu \rightarrow \infty$, we will obtain the same results as those of the plane wave electric field perturbations. To understand this, we have to analyze the terms containing μ in Eqs. (25) and (27). For $\mu \rightarrow \infty$, the power term in Eq. (27) decays rapidly to zero, which results in the vanishing of γ_θ (i.e., damping due to the azimuthal wave number) from Eq. (25). Also in the limit $\mu \rightarrow \infty$, the term $\mu^2/(1 + \mu^2) = 1$ in Eq. (25) and gives the damping rate of the Langmuir wave as follows:

$$\gamma = -\frac{\sqrt{\pi} \omega_r^4}{k^3 \theta_{||e}^3} \frac{\Gamma(\kappa + 1)}{\kappa^{3/2} \Gamma(\kappa - 1/2)} \left(1 + \frac{\omega_r^2}{\kappa k^2 \theta_{||e}^2} \right)^{-\kappa-1}, \quad (28)$$

which is the expression obtained in the absence of orbital angular momentum of the electric field perturbations in nonthermal kappa distributed electrons plasma. Here, in Eqs. (25)–(28), $\omega_r = \omega_{pe} \sqrt{(1 + \mu^2)/\mu^2}$ has been used for further study under assumption $\omega_{pe}^2 (1 + \mu^2)/\mu^2 \gg 3k^2 v_{Te}^2 (1 + \mu^4)/\mu^2 (1 + \mu^2)$.

From Eq. (25), we obtain the following normalized weak damping rate of the Langmuir twisted wave with kappa distributed electrons, which is given by

$$\frac{\gamma}{\omega_r} = -\frac{\sqrt{\pi}}{k^3 \lambda_{De}^3} \frac{(1 + \mu^2)^{1/2}}{\mu} \frac{\kappa^{3/2}}{(2\kappa - 3)^{3/2}} (\gamma_z + \gamma_\theta), \quad (29)$$

where the simplified form of conventional damping factor γ_z and twisted damping factor γ_θ can be written as follows:

$$\gamma_z = \frac{\Gamma(\kappa + 1)}{\kappa^{3/2} \Gamma(\kappa - 1/2)} \times \left[1 + \frac{(1 + \mu^2)}{\mu^2 (2\kappa - 3)} \frac{1}{k^2 \lambda_{De}^2} + \frac{3}{(2\kappa - 3)} \frac{(1 + \mu^4)}{\mu^2 (1 + \mu^2)} \right]^{-\kappa-1}, \quad (30)$$

and

$$\gamma_\theta = \frac{\Gamma(\kappa + 1) \mu}{\kappa^{3/2} \Gamma(\kappa - 1/2)} \times \left[1 + \frac{(1 + \mu^2)}{(2\kappa - 3)} \frac{1}{k^2 \lambda_{De}^2} + \frac{3}{(2\kappa - 3)} \frac{(1 + \mu^4)}{(1 + \mu^2)} \right]^{-\kappa-1}. \quad (31)$$

IV. NUMERICAL PLOTS AND CONCLUSION

In this section, the weak damping rates of twisted Langmuir waves for nonthermal kappa distributed electron plasmas are plotted as described in Eq. (29), along with (30) and (31), as well. It can be seen that the damping rate of the twisted Langmuir wave is weak for wavelengths larger than the Debye length for a fixed kappa value. As the wavelength of the wave decreases, the damping rate increases rapidly and it becomes strong as it approaches to the Debye length. The expression obtained for weak damping effects, i.e., Eq. (25), is valid for longer wavelengths in comparison with the Debye length and it fails when the wavelength of the wave approaches the Debye length of the system. Therefore, in order to find the plots for strong damping rates of the

FIG. 1. (a) The plot of normalized Landau damping rate γ/ω_r obtained analytically and described in Eqs. (29)–(31) is presented against the normalized wave number $k\lambda_{De}$, for the twisted Langmuir mode using different values of $\mu = k/lq_0$, i.e., ($\mu = 0.5$: blue dotted curve), ($\mu = 1$: green dashed curve), and ($\mu = 1.5$: red dotted-dashed curve) as well as for the planar Langmuir mode, i.e., $\mu \rightarrow \infty$ (black solid curve) are shown for the Maxwellian distributed electrons case, i.e., at $\kappa = 100$. (b) The exact numerical results are shown for the normalized Landau damping rate γ/ω_r obtained from Eq. (21) by putting $\epsilon(\omega, k, lq_0) = 0$ against the normalized wave number $k\lambda_{De}$, for the twisted Langmuir mode using different values of $\mu = k/lq_0$, i.e., ($\mu = 0.5$: blue dotted curve), ($\mu = 1$: green dashed curve), and ($\mu = 1.5$: red solid curve) as well as for planar Langmuir mode, i.e., $\mu \rightarrow \infty$ (Black solid curve) at $\kappa = 100$.

Langmuir twisted wave, the exact numerical solutions are obtained from Eq. (21) by putting $\epsilon(\omega, k, lq_0) = 0$, respectively, which are also shown in this section. In order to find the imaginary roots (or damping roots) of the wave frequency from Eq. (21), the well known Newton Raphson method^{30,31} has been used. The obtained results are discussed below.

Figures 1(a) and 1(b) present the plots of the analytical results and exact numerical values of normalized Landau damping rates γ/ω_r vs normalized wave number of the twisted Langmuir wave at different values of the azimuthal wave number lq_0 for the Maxwellian electron plasma case ($\kappa \rightarrow \infty$). In particular, the normalized Landau damping rates γ/ω_r are plotted using three distinct values of $\mu = k/lq_0$, i.e., (i) for the azimuthal wave number lq_0 greater than the parallel wave number k such that $\mu = 0.5$, (ii) when the azimuthal wave number lq_0 is equal to the parallel wave number k such that $\mu = 1$, and (iii) when the azimuthal wave number lq_0 is less than parallel wave number k such that $\mu = 1.5$. It can be seen from Figs. 1(a) and 1(b) that the damping rate of the Langmuir twisted wave is greater for

the case $\mu < 1$ and then less for $\mu > 1$ and it has least value for the $\mu = 1$ case in comparison with each other, respectively. The physical reason for high damping rates for the $\mu < 1$ case corresponds to the large number of particles taking energy from the wave. It means that the Landau damping rate of the twisted Langmuir wave is greater for larger values of the azimuthal wave number lq_0 and less for smaller values of the azimuthal wave number lq_0 in comparison with the parallel wave number k . But when azimuthal wave number lq_0 and parallel wave number k are equal such that $\mu = 1$, the wave-particle interaction is least in comparison with $\mu > 1$ and $\mu < 1$ cases. The damping rate of the planar Langmuir wave is also plotted in the same figure in which azimuthal wave number $lq_0 \rightarrow 0$ so that $\mu \rightarrow \infty$. It is found that the damping rate of the Langmuir wave is much higher in the case of planar wave electric fields in comparison with twisted electric fields containing orbital angular momentum. This result is quite different from Ref. 23 as shown in Figure 3. It can also be seen from Eq. (24) that in the presence of the azimuthal wave number, i.e., with the finite value of μ , the value of ω_r increases in comparison with ω_r for plane wave electric field perturbations at large values of μ , which decrease the damping rate of the twisted wave as indicated from Eqs. (29)–(31). It must be noted here that our model does not remain valid for $\mu \rightarrow 0$. Now, we will compare the results of analytical plots in Fig. 1(a) with the exact numerical solution as shown in Fig. 1(b). The analytical results for weak damping rates are shown in Fig. 1(a) and when the value of the normalized wave number $k\lambda_{De}$ reaches about 0.65, the damping profiles start to show a rising trend instead of falling down, which deviates from the physical interpretation of the increasing damping rate of the wave. This is due to the fact that we have employed weak damping wave approximations for obtaining the analytical results of the Landau damping of the twisted Langmuir wave. After $k\lambda_{De} = 0.65$, the analytical results obtained are not able to explain well the Landau damping rate of the Langmuir twisted mode. Therefore, the exact numerical results are also calculated and shown in Fig. 1(b) for the same cases as given in Fig. 1(a). It can be seen that the damping rate of the twisted Langmuir wave for the above cases of azimuthal wave numbers lq_0 is obtained in an exact numerical study when there is an increase in the wave number and that the curves do not cross each other now. This fact appears in the plot shown in Fig. 1(a), which was obtained through analytical study of the weak damping wave approximation.

In Figures 2(a) and 2(b), the effect of azimuthal wave number lq_0 on the analytical and exact numerical results of normalized Landau damping rates γ/ω_r of the twisted Langmuir wave is shown for the nonthermal kappa distributed electrons plasma, i.e., at $\kappa = 5$. We have again taken into account the effect of azimuthal wave number lq_0 defined in terms of $\mu = k/lq_0 = 0.5, 1$, and 1.5 , respectively, and also planar wave case with $\mu \rightarrow \infty$ on the twisted Langmuir waves, but now spectral index $\kappa = 5$ has been taken for nonthermal kappa distributed electrons. It can be observed that the magnitude of the damping rate of twisted Langmuir waves in nonthermal electrons is larger for $\mu < 1$ in

FIG. 2. (a) The plot of normalized Landau damping rate γ/ω_r obtained analytically described in Eqs. (29)–(31) is presented against the normalized wave number $k\lambda_{De}$, for the twisted Langmuir mode using different values of $\mu = k/lq_0$, i.e. ($\mu = 0.5$: blue dotted line), ($\mu = 1$: green dashed curve), and ($\mu = 1.5$: red solid curve) as well as for planar Langmuir mode i.e., $\mu \rightarrow \infty$ (black solid curve) with nonthermal electrons, i.e., at $\kappa = 5$. (b) The exact numerical results are shown of the normalized Landau damping rate γ/ω_r obtained from Eq. (21) by putting $\epsilon(\omega, k, lq_0) = 0$ against the normalized wave number $k\lambda_{De}$, for the twisted Langmuir modes using different values of $\mu = k/lq_0$, i.e. ($\mu = 0.5$: blue dotted curve), ($\mu = 1$: green dashed curve), and ($\mu = 1.5$: red solid curve) as well as for planar Langmuir mode, i.e., $\mu \rightarrow \infty$ (black solid curve) with nonthermal electrons at $\kappa = 5$.

FIG. 3. The exact numerical plot of normalized Landau damping rate γ/ω_r is presented against the normalized wave number $k\lambda_{De}$, for the twisted Langmuir modes, i.e., $\mu = k/lq_0 = 1$ using different values of spectral index kappa κ , i.e. ($\kappa = 3$: sky blue dotted curve) for kappa distributed electrons and ($\kappa = 100$: yellow solid curve) for Maxwellian distributed electrons.

comparison with $\mu > 1$ and $\mu = 1$ cases, which indicates that a greater number of particles is taking energy in the $\mu < 1$ case instead of the $\mu > 1$ and $\mu = 1$ cases as happened in the Maxwellian electrons plasma case. Also, the damping rate of the planar Langmuir wave is also plotted in the same figure in which azimuthal wave number $lq_0 \rightarrow 0$ so that $\mu \rightarrow \infty$ at $\kappa = 5$. It is found that the damping rate of the Langmuir wave is much larger in the case of planar wave electric fields in comparison with twisted electric fields containing orbital angular momentum for nonthermal kappa distributed electrons plasma. The damping rate profiles shown in Fig. 2(a) deviate from the original path of having a trend of increasing damping rates at large values of $k\lambda_{De}$. The rates cross each other at about $k\lambda_{De} = 0.73$ and they show a decreasing trend instead of an increasing one. In order to resolve this issue, the exact numerical results are included in Fig. 2(b), which shows the strong damping rate behavior of the Langmuir twisted and planar waves in non-Maxwellian plasmas as the wavelength of the Langmuir wave approaches the Debye length in plasma.

Figure 3 shows the exact numerical results for the comparison of the normalized Landau damping rate γ/ω_r of the twisted Langmuir wave in kappa distributed Lorentzian plasma (i.e., $\kappa = 3$) and Maxwellian distributed plasma (i.e., $\kappa \rightarrow \infty$) at $\mu = 1$. It clearly shows that the solid curve for $\kappa \rightarrow \infty$ (Maxwellian electrons plasma case) is above the dotted curve when $\kappa = 3$. Therefore, it is evident from the figure that the Landau damping rate of the non-Maxwellian electrons plasma is greater in comparison with the Maxwellian electrons plasma case. In non-Maxwellian plasma, the number of superthermal particles increases in the tail of the energy distribution function at smaller values of the spectral index kappa. Hence, more energetic particles for the wave-particle interaction are available in the tail of the energy spectrum to take energy of the Langmuir wave, which results in increasing the damping rate of the twisted Langmuir wave with kappa distributed electrons.

To conclude, we have studied the damping rate of the Langmuir twisted wave in nonthermal kappa distributed electrons plasma. The twisted Langmuir wave in nonthermal electrons plasma is investigated using the Vlasov-Poisson equation and the plasma dielectric function is derived in the presence of the OAM of the helical electric field perturbations with kappa distributed electrons. The perturbed distribution function and electric field are decomposed into LG modes described in cylindrical coordinates. The analytical results for weak damping rates of the twisted and planar Langmuir waves are obtained and plotted numerically. The exact numerical results for strong wave damping effects for the Langmuir wave at short wavelengths, i.e., wavelengths approaching the Debye length, are also shown for planar as well as helical (twisted) electric field perturbations.

The physical explanation of Landau damping effect considers the phenomenon from an energetic point of view as a result of the interaction of a plasma wave with particles of nearby velocity. The dominant exchange occurs with the particles which are trapped by the wave because their velocity is close to the wave velocity. If the distribution function is a decreasing function of $|v|$, then among trapped particles more are accelerated than are decelerated, so the wave loses the energy to the plasma and the wave damping phenomenon

occurs. This damping is extremely weak for large wave numbers in comparison with the Debye length of the system. Then a question arises: does this phenomenon hold for an infinite or long time. The Landau damping effects hold for a long time only when the distribution of the particles remains close to the original equilibrium value for all time, which cannot happen because the particle trapping phenomenon occurs at resonance and the system does not remain linear for long.^{32,33} Backus³⁴ noticed that linear and nonlinear evolution break apart after some time (which is not very long) and then questions of the validity of the linearization remain there.³⁵ O'Neil³⁶ argues that relaxation holds in the “quasi-linear regime” on large time scales, when the “trapping time” (which is proportional to the inverse square root of the size of the perturbation) is much smaller than the damping time. Kaganovich³⁷ studied that nonlinear effects may affect Landau damping phenomena by several orders of magnitude. The “quasi-linear relaxation theory” uses second order approximation of the Vlasov equation to predict the convergence of the spatial average of the distribution function. This procedure involves averaging over statistical ensembles and diffusion equations with discontinuity coefficients, acting only near the resonance velocity for particle-wave exchanges. Because of these discontinuities, the predicted asymptotic state is discontinuous and collisions are taken to restore the smoothness. Our results are general but applicable to laser plasma interactions and space plasmas where the helical electric field perturbations with nonthermal electrons can exist.

APPENDIX: LAGUERRE GAUSSIAN MODE FUNCTION

The LG mode function $F_{pl}(r, z)$ is defined as²⁴

$$F_{pl}(r, z) = C_{pl} X^{|l|} L_p^{|l|}(X) \exp(-X/2),$$

where $X = r^2/w^2(z)$ such that $w(z)$ is the beam waist. Here, integers p and l are the radial and angular mode numbers, while θ denotes the azimuthal angle.

We can define the normalization constants C_{pl} and the associated Laguerre polynomials $L_p^{|l|}(X)$ by the following expressions:

$$C_{pl} = \sqrt{(l+p)!/4\pi p!},$$

and

$$L_p^{|l|}(X) = \exp(X) d^p/dX^p [X^{l+p} \exp(-X)]/p!X.$$

The orthogonality condition for the LG modes is given by the equation

$$\langle F_{p'l'} \cdot |F_{pl}\rangle = \int_0^{2\pi} d\theta \int_0^\infty r dr F_{p'l'} F_{pl} \exp[i(l-l')\theta] = \delta_{pp'} \delta_{ll'}.$$

- ¹V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics* (Butterworth-Heinemann, Oxford, 1982).
- ²G. Molina-Terriza, J. P. Torres, and L. Torner, *Nat. Phys.* **3**, 305 (2007).
- ³L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, *Phys. Rev. A* **45**, 8185 (1992).
- ⁴H. He, M. E. J. Friese, N. R. Heckenberg, and H. Rubinsztein-Dunlop, *Phys. Rev. Lett.* **75**, 826 (1995).
- ⁵C. Paterson, *Phys. Rev. Lett.* **94**, 153901 (2005).
- ⁶B. Thide, H. Then, J. Sjöholm, K. Palmer, J. Bergman, T. D. Carozzi, Ya. N. Istomin, N. H. Ibragimov, and R. Khamitova, *Phys. Rev. Lett.* **99**, 087701 (2007).
- ⁷M. Harwit, *Astrophys. J.* **597**, 1266 (2003).
- ⁸J. T. Mendonca and B. Thide, *Europhys. Lett.* **84**, 41001 (2008).
- ⁹S. Ali, J. R. Davies, and J. T. Mendonca, *Phys. Rev. Lett.* **105**, 035001 (2010).
- ¹⁰J. T. Mendonca, S. Ali, and B. Thide, *Phys. Plasmas* **16**, 112103 (2009).
- ¹¹P. K. Shukla, B. Eliasson, and L. Stenflo, *Phys. Rev. E* **86**, 016403 (2012).
- ¹²P. K. Shukla, *J. Geophys. Res.: Space Phys.* **118**, 1–4 (2013).
- ¹³J. T. Mendonca, B. Thide, and H. Then, *Phys. Rev. Lett.* **102**, 185005 (2009).
- ¹⁴P. K. Shukla, *Phys. Plasmas* **19**, 083704 (2012).
- ¹⁵J. T. Mendonca, *Plasma Phys. Controlled Fusion* **54**, 124031 (2012).
- ¹⁶P. K. Shukla, *Phys. Lett. A* **376**, 2792–2794 (2012).
- ¹⁷T. B. Leyser, L. Norin, M. McCarrick, T. R. Pedersen, and B. Gustavsson, *Phys. Rev. Lett.* **102**, 065004 (2009).
- ¹⁸P. K. Shukla, *Phys. Rev. E* **87**, 015101 (2013).
- ¹⁹N. Yadav and R. P. Sharma, *Sol. Phys.* **289**, 1803–1814 (2014).
- ²⁰M. K. Ayub, S. Ali, and J. T. Mendonca, *Phys. Plasmas* **18**, 102117 (2011).
- ²¹T. B. Leyser, F. Waldemarsson, and S. C. Buchert, *Radio Sci.* **47**, RS5004 (2012).
- ²²K. Shahzad and S. Ali, *Astrophys. Space Sci.* **353**, 3–8 (2014).
- ²³J. T. Mendonca, *Phys. Plasmas* **19**, 112113 (2012).
- ²⁴S. A. Khan, A. Rehman, and J. T. Mendonca, *Phys. Plasmas* **21**, 092109 (2014).
- ²⁵K. Arshad and S. Mahmood, *Phys. Plasmas* **17**, 124501 (2010).
- ²⁶K. Arshad, Z. Ehsan, S. A. Khan, and S. Mahmood, *Phys. Plasmas* **21**, 023704 (2014).
- ²⁷K. Arshad, S. Mahmood, and A. M. Mirza, *Phys. Plasmas* **18**, 092115 (2011).
- ²⁸D.-H. Ki and Y. Jung, *Phys. Plasmas* **18**, 014506 (2011).
- ²⁹A. F. Alexandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics* (Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1984).
- ³⁰R. L. Burden and J. D. Faires, *Numerical Analysis*, 7th ed. (Brooks/Cole, 2001).
- ³¹A. Iserles, *A First Course in the Numerical Analysis of Differential Equations*, 2nd ed. (Cambridge University Press, 2009).
- ³²H. Schamel, *Phys. Plasmas* **22**, 042301 (2015).
- ³³C. Mouhot and C. Villani, *Acta Math.* **207**, 29 (2011).
- ³⁴G. Backus, *J. Math. Phys.* **1**, 178 (1960).
- ³⁵G. Belmont, F. Mottez, T. Chust, and S. Hess, *Phys. Plasmas* **15**, 052310 (2008).
- ³⁶T. M. O'Neil, *Phys. Fluids* **8**, 2255 (1965).
- ³⁷D. I. Kaganovich, *Phys. Rev. Lett.* **82**, 327 (1999).