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Non-Linear Acoustic Transfer Impedance of Micro-Perforated Plates with Circular Orifices

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Abstract

A practical description of the transitional behaviour of micro-perforated plates (MPPs) is provided between the linear and strongly non-linear regimes. Micro-perforated plates are efficient sound absorbers whose application areas vary from room acoustics to duct acoustics. Although there are accurate models for the linear and strongly non-linear acoustic behaviour of MPPs, the transition from one to another has not been a focus of interest so far. A series of measurements are performed with MPP samples for various excitation amplitudes. The deviation from the linear impedance is found to be a function of excitation amplitude and oscillating viscous boundary layer thickness, expressed in terms of the Strouhal number and the Shear number. Typical for MPPs is a Shear number of order unity, implying that the viscous boundary layer thickness is in the order of the perforation radius. Using the measurement data, expressions are proposed for calculating the non-linear acoustic resistance and reactance for circular perforations with sharp square edges. Some additional data is provided for the higher Shear number range. The behaviour at low amplitudes for high Shear numbers deviates strongly

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from the typical MPP behaviour. This is due to local vortex shedding at the sharp edges of the perforation.

Keywords:

Acoustic Transfer Impedance, Micro-Perforated Plates, Non-Linear Acoustics, Impedance Tube Measurements, Transition Regime

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1. Introduction

Maa [1] introduced the micro-perforated plates (MPPs) as promising sound absorbers in the presence of a supporting air cavity. MPPs are plates with small perforations whose diameter is in the order of a millimeter and low porosity, *i.e.* $\sim 1\%$. Due to these features MPPs have high acoustic resistance and low reactance. Moreover, they can be produced from any material, so that their durability and weight can be adjusted according to the application. There are a large number of possible application fields such as room acoustics, duct acoustics and thermo-acoustics.

In many practical applications, even in case of moderate intensity of the incident sound, the acoustic particle velocity in the perforations can reach high values [2]. This results in flow separation and vortices at the sharp edges of the perforations. Vortex formation takes energy from the acoustic wave and as a result increases the acoustic resistance of the perforation. Sivian [3] was the first scientist to observe this phenomenon experimentally. Inspired by his findings, Ingard and Labate [4] have performed experiments to conclude that the mechanisms causing extra resistance are flow circulations and these vortices are visualized. Later on, Ingard and Ising [5] have measured quantitatively the non-linear acoustic resistance through an orifice. The orifice used in their experiments had sharp edges. Guess [6] proposes a design method for perforated liners with a backing cavity under high acoustic excitation and subjected to grazing flow. Nevertheless, non-linear effects are included only for the resistance in his method. Later, Disselhorst and van Wijngaarden [7] have measured and described theoretically the amount of dissipated energy due to vortex formation at an open pipe termination and have investigated the influence of rounding off the edges. Cummings and Eversman [8] have improved the quasi-steady model describing the behaviour of perforations at high Shear numbers and very high amplitudes of the acoustic particle velocity. In their model, the acoustic flow separates at the sharp

edges and forms a free jet with a cross-section smaller than the perforation area, this is called a *vena-contracta*. Testud *et al.* [9] report that the separated flow reattaches for thick orifices, whose thickness is larger than twice the diameter. Aurégan and Pachebat [10] have studied rigid porous materials both in the moderate and high intensity acoustic amplitudes. One of the most significant observations of this particular study is that in the moderate excitation case the relation between the resistance and the Reynold's number is quadratic. Another contribution to the quasi-steady approach for high amplitude acoustic excitation has been provided by Hofmans *et al.* [11]. Instead of a circular perforation, they have investigated a slit geometry and a method is proposed for obtaining the *vena-contracta* factor as a function of geometry and Mach number of the acoustic jet. Shortly after the study of Hofmans *et al.*, Jing and Sun [12] have proposed an empirical model for the non-linear acoustical behaviour of the in-duct orifices. Their focus has been on very high excitation amplitudes. Leung *et al.* [13] have carried out some numerical experiments with an in-duct orifice with and without flow. They have observed that vortices are shed both upstream and downstream in absence of main flow, but shedding can take place only in the downstream when there is a bias flow present. Buick *et al.* [14] have performed a quite comprehensive study including numerical, analytical and experimental results altogether for explaining the acoustic losses in the open termination of a tube. Although the models do not converge perfectly, all of them result in that there are two separate cases in non-linear acoustic absorption: one case in which vortices are formed locally and remain close to the edges; and the other case in which the vortices are shed far from the orifice. Their observations agree with those of Aurégan and Pachebat [10]. A more practical study has been carried out by Park [15]. A design method for MPPs is proposed including the non-linear effects for the impedance. The non-linear resistance term is updated in this study, however for the reactance, an expression from early works of Maa [2] is used. Ji and Zhao [16] have applied the Lattice Boltzmann Method for modelling the non-linear acoustic losses of an in-duct orifice. They have successfully reproduced the experimental results of Jing and Sun [12] and their method promises a less expensive computation compared to classical Navier-Stokes solvers. Nevertheless, none of these studies address MPPs directly. Also, they mainly focus on the regime where strong non-linear effects are observed.

With this study, we focus on non-linear acoustic behaviour of MPPs with circular orifices with sharp square edges (90° angle). Besides, we extend

our study to plates with perforations whose diameters are larger than those of MPPs. In this way, we link our results with previous studies on orifices whose diameter is larger than the MPP range. Furthermore, we limit the scope of this study to the transition between linear and non-linear regimes, to bridge the gap between these two regimes. Moreover, we investigate both the non-linear acoustic resistance and reactance.

Our results are obtained from open-end transfer impedance measurements. The dimensions of the samples used in these measurements cover both the typical MPP range and slightly beyond. We propose practical formulas forming a bridge between linear and non-linear regimes in MPPs. These formulas are expressed in terms of dimensionless parameters, introduced in the next section.

2. Theoretical Background

In the linear regime the absorption of the acoustic energy takes place in the Stokes layers shown in Figure 1a. These layers form due to the presence of the solid-walled plate [17]. The thickness of the oscillating Stokes layer is $\delta_v = \sqrt{\omega\rho_0/\mu}$ where $\omega = 2\pi f$ is the angular frequency, ρ_0 is the density of air (1.18 kg/m³ @25°C, 1 atm, dry air) and μ is the dynamic viscosity of air (1.85 × 10⁻⁵ kg/m s @25°C).

The Stokes layer thickness relative to the perforation diameter d_p is the main parameter that defines a micro-perforated plate (MPP) and is referred to as the *Shear number*,

$$Sh = \frac{d_p}{2\delta_v} = d_p \sqrt{\frac{\omega\rho_0}{4\mu}}. \quad (1)$$

In an MPP the Stokes layers cover almost completely the perforation, *i.e.* $Sh = \mathcal{O}(1)$. For low enough excitation amplitudes, the linear transfer impedance of an MPP is calculated as a function of perforation geometry and frequency only [18].

When the excitation amplitude is larger than a critical value, vortices start forming at the sharp corners of the perforation [5]. This is schematically shown in Figure 1b. The formation of vortices is a non-linear mechanism. This takes up energy from the acoustic waves resulting in additional absorption. The criterion for these vortices to start forming is based on the *Strouhal number*, which is calculated as

Figure 1: Orifice geometry (a) and the quasi-steady acoustic flow (b) with orifice diameter d , plate thickness t , channel diameter D , Stokes layer thickness δ_v , (complex) acoustic pressure $\hat{p}$ and (complex) acoustic particle velocity $\hat{u}$ where the subscripts p and j imply perforation and jet flow respectively.

$$Sr = \frac{\omega d_p}{|\hat{u}_p|}, \quad (2)$$

where $|\hat{u}_p|$ is the cross-sectional surface averaged acoustic velocity amplitude at the orifice.

For $Sr \gg 1$, the particle displacement is smaller than the orifice diameter and vortices are unlikely to be observed. In other words, the acoustic system is linear. In the opposite limit, $Sr \ll 1$, the particle displacement in the vicinity of the orifice is so large that vortices are blown away by the acoustic flow, which can be described as a free jet. In between these two limiting cases, *i.e.* $Sr = \mathcal{O}(1)$, vortices form at the edges of the perforations but they remain local. In our study, to focus on the transition regime, we limit ourselves to the range $0.05 < Sr < 10$. The limit cases are now discussed in more detail.

2.1. Strongly Non-linear Regime: $Sr \ll 1$

If the particle displacement is much larger than the orifice diameter, *i.e.* $Sr \ll 1$, the non-linear resistance due to vortex shedding dominates the absorption mechanism as shown by Ingard and Ising [5]. Assuming that

the acoustic pressure is negligible on the downstream side of the orifice, the pressure amplitude on the upstream is expressed by

$$|\hat{p}_{up}| = \rho_0 |\hat{u}_p|^2, \quad (3)$$

according to Ingard and Ising's experiments [5], where $\hat{p}_{up}$ is the complex pressure amplitude at the upstream of the orifice. Note that the derivation of this formula by Ingard and Ising [5] is not correct.

Cummings and Eversman [8], propose a simple theoretical approach to represent the acoustic flow through the orifice in a duct in the high Sh number limit. They assume a quasi-steady behaviour in which the relation between the pressure change and the particle velocity can be derived from the Bernoulli equation. They find:

$$|\hat{p}_{up}| \simeq \rho_0 |\hat{u}_p|^2 \frac{1 - \sigma^2 C_{vc}^2}{2C_{vc}^2}, \quad (4)$$

where σ is the porosity and C_{vc} is the vena-contracta factor. Considering the typical properties of MPPs, the term $\sigma^2 C_{vc}^2$ can be neglected due to low values of σ .

Bearing in mind that Ingard and Ising [5] used an orifice with a sharp-edge profile, both of the expressions given earlier in this section (Eqs. (3) and (4)) are in practice equivalent because the vena-contracta factor for sharp edges (see Figure 2) is $C_{vc} \simeq 0.7$. Yet, Cummings and Eversman's model takes other types of edge profile into account as well, because another value of *vena-contracta* can be used.

Figure 2: Edge profiles described in (a) Ingard and Ising's [5], (b) Disselhorst and van Wijngaarden's [7] and (c) current study.

2.2. Almost Linear Regime: $Sr > 1$

Disselhorst and van Wijngaarden [7] give the power absorbed by vortices, $\bar{P}$, in the high Sr limit as

$$\bar{P} = \alpha \left[\rho_0 \pi \left(\frac{D}{2} \right)^2 |\hat{u}_D|^3 / 2 \right] \frac{Sr^{1/3}}{\sqrt[3]{2}}, \quad (5)$$

for an open tube termination. In Eq. (5), α is a constant changing between 0.6 and 1.0 depending on the number of point vortices used to describe the flow [20] and $\hat{u}_D$ is the acoustic particle velocity amplitude at the tube termination. Written in terms of acoustic pressure amplitude, Eq. (5) takes the following form:

$$|\hat{p}_{up}| = 2\alpha\rho_0|\hat{u}_D|^2 \frac{Sr^{1/3}}{\sqrt[3]{2}}. \quad (6)$$

According to Disselhorst and van Wijngaarden's study the higher Sr gets, the larger normalized non-linear resistance becomes. Yet it should be noted that this model is based on a high Sh limit and does not apply to MPPs for which $Sh = \mathcal{O}(1)$.

2.3. Transition Regime: $Sr = \mathcal{O}(1)$

The study of Aurégan and Pachebat [10] focuses on acoustic resistance in porous materials. They quantify the non-linearity in the results with Reynolds number instead of Strouhal number, which is defined as $Re = D|\hat{u}_p|\rho_0/\mu$. Their main result is that for $Re \ll 1$, the non-linear correction to Darcy's law takes the following form:

$$R_{NL} = R_L(1 + C_2 Re^2), \quad (7)$$

where $R = \Re\{Z_t\}$ is the acoustic resistance and C_2 is a constant defined empirically. By Eq. (7), one concludes that the non-linear resistance in the transition regime must be quadratically related to the particle velocity in the low Sh number limit valid for MPPs.

3. Experiments

3.1. Properties of the setup

The experimental setup we use in this study is an impedance tube with 6 pre-polarized 1/4 in microphones (type BWSA, sensitivity 50 mV/Pa). The

schematic description of the setup, including the dimensions of the tube and positions of the microphones, is shown in Figure 3.

Figure 3: Impedance tube used in the measurements: (a) schematic drawing of the components which are: (1) the tube, (2) the sample, (3) (hollow) sample holder, (4) microphones, (5) microphone amplifier, (6) analyser and computer, (7) loudspeaker amplifier and (8) loudspeaker; (b) dimensions of the tube: $a_1 = 50$ mm, $a_2 = 175$ mm, $a_3 = 10$ mm, $D = 50$ mm and $L_t = 1$ m.

The measurements have been conducted in a semi-anechoic room to avoid problems related to room resonances. The air in the tube is excited by a 3.5 in, 25 W loudspeaker. The microphone closest to the tube termination is calibrated by a B & K piston-phone at 1000 Hz for 1 Pa. The relative calibration of the remaining microphones with respect to the first one is performed using an apparatus, which is displayed in Figure 4 specially designed for this purpose. Using the apparatus, we measure simultaneously the response of the 6 microphones to an identical pressure fluctuation. For each frequency step, the pressure amplitude and the phase of each microphone are adjusted to the reference microphone's by means of a calibration factor.

Figure 4: Placement of the microphones for the relative calibration process. All the microphones are located axially at the same distance from the loudspeaker and the closed tube termination.

3.2. Properties of the DAQ system

For data acquisition, an *NI PCIe-6361 X-Series* DAQ card with 16 analog input and 2 analog output channels is used. A script built in *NI Lab-View* software controls the signal processing and data acquisition during the measurements. The post-processing of the microphone readings is done separately in a user-generated Matlab script with a *Lock-in* method [19]. The sampling rate is 20 kHz for the excitation signal and 10 kHz for recording the input signal. Each frequency step is measured for 10 seconds. To avoid transient effects in the measurements, the first and last 2 seconds are omitted in the post-processing. The amplitude of the excitation signal is adjusted automatically until it satisfies the pre-determined pressure value for the reference microphone, *i.e.* the microphone closest to the sample.

3.3. Measurement Method

The idea behind the open-end transfer impedance measurement is displayed in Figure 5. Since the samples have relatively low porosity, *i.e.* $\sigma = \mathcal{O}(1\%)$, the open-end radiation impedance (see Figure 5b) is expected to be negligible compared to the impedance measured for the sample loaded termination (see Figure 5a). Nevertheless, to take open end radiation effects into account, the acoustic transfer impedance of a sample is calculated as $Z_t = Z_S - Z_R$, where Z_S is the impedance for the loaded case and Z_R is the measured open-end radiation impedance.

Figure 5: Open-end impedance tube measurements are performed in 2 steps. First, the impedance values for the sample loaded (a), then the open-end (b) configurations are measured. The transfer impedance of the sample is calculated from these values by $Z_t = Z_S - Z_R$.

To measure the impedance, the relation between the reflection coefficient, ζ , and the impedance, Z is used [21]. Based on this relation, the impedance in each measurement is calculated as $Z = \rho_0 c_0 (1 + \zeta) / (1 - \zeta)$. Then the

measured radiation impedance Z_R is subtracted from the sample loaded end termination impedance Z_S to find the transfer impedance, $Z_t = Z_S - Z_R$.

The calculation of R is performed by the wave decomposition described by Jang and Ih [22]. In our study, this method is improved by employing the complex wave number approximation of Peters *et al.* [23].

In this study, the measurements and the calibration are performed for $60 \text{ Hz} < f < 240 \text{ Hz}$. The frequency step is 20 Hz and multiples of the grid frequency, *i.e.* 100 Hz and 200 Hz, are excluded from the calculations for the sake of reliability. The deviation from the theoretical limit is less than 0.5% for the closed-end reflection coefficient measurements in the frequency span of interest.

In our experiments, we perform 6 sets of measurements for each sample: 1 set in the linear and 5 sets in the non-linear regime. To make sure the measurement set with the lowest excitation amplitude is in the linear regime, we follow an empirical method. Measurements start with an arbitrary excitation amplitude and the resulting reflection coefficient is recorded for the lowest frequency of interest. As non-linear effects decrease with increasing frequency [13], the amplitude is gradually reduced until two successive measurements give the same (or very close) reflection coefficient value. For this amplitude value, the linear transfer impedance Z_{t-L} is calculated from the reflection coefficient throughout the frequency span of interest. Next, the amplitude is doubled and the non-linear transfer impedance values Z_{t-NL} in the frequency span are recorded. Finally, this last step is repeated until the 6 sets are completed.

3.4. Measuring $|\hat{u}_p|$

The required acoustic particle velocity amplitude to calculate Sr at a specific frequency and excitation amplitude is deduced from the pressure amplitude in the tube.

The wave decomposition provides the amplitudes of right and left travelling pressure waves, $\hat{p}^+$ and $\hat{p}^-$ respectively (see Figure 1b). By Euler's law, the relation between the amplitudes of the pressure waves and the particle velocity is given by

$$\hat{u}_D = \frac{\hat{p}^+ \exp(jk^+x) - \hat{p}^- \exp(jk^-x)}{\rho_0 c_0}, \quad (8)$$

where $\hat{u}_D$ is the particle velocity in the tube, $j = \sqrt{-1}$ is the imaginary number, x is the distance from the reference plane, k is the complex wave number

and superscripts ‘+’ and ‘−’ indicate the right and left travelling waves respectively. In case of no mean flow $k = k^+ = k^-$, and is approximated by Peters *et al.* [23] as

$$k = \frac{\omega}{c_0} \left[1 + \frac{1-j}{\sqrt{2}} \frac{1}{Sh_D} \left(1 + \frac{\gamma-1}{Pr^{0.5}} \right) \right] - \frac{\omega}{c_0} \frac{j}{Sh_D^2} \left(1 + \frac{\gamma-1}{Pr^{0.5}} - \frac{1}{2} \gamma \frac{\gamma-1}{Pr} \right), \quad (9)$$

where γ and Pr are respectively the ratio of the specific heats and Prandtl number for air. Since the calculation is performed just before the sample in the upstream region, $x = 0^-$ is valid. Considering continuity, one calculates the acoustic particle velocity in the perforation as

$$|\hat{u}_p| = \frac{|\hat{p}^+ - \hat{p}^-|}{\rho_0 c_0 \sigma}. \quad (10)$$

3.5. Specifications of the Samples

Throughout this study we have used 4 samples with different diameters and comparable porosity values. Schematic drawings of these samples are given in Figure 6 and their specifications are provided in Table 1.

Figure 6: Schematic drawings of samples used. Samples A, B and C have multiple orifices where Sample D has a single orifice. All of the orifices are assumed to have square edge profiles.

The *vena-contracta* coefficient C_{vc} given in Table 1 depends on both orifice edge profile and Reynolds number [24]. In our case, the Reynolds number of the acoustic flow in the perforation is $Re_p = |\hat{u}_p| d_p \rho_0 / \mu$ and for our measurement cases $Re_p = \mathcal{O}(10)$.

Table 1: Specifications of the samples.

Sample	d_p [mm]	t_p [mm]	σ	C_{vc}
Sample A	0.3	1.0	0.77%	0.70
Sample B	0.8	1.6	0.74%	0.70
Sample C	1.6	1.6	0.72%	0.70
Sample D	4.2	4.0	0.71%	0.70

All of the samples are made of brass by drilling. The outer diameter of the samples is 70 mm. To ensure that there is no air leakage between the samples and the impedance tube o-rings are used for mounting the samples.

4. Results and Discussions

We are interested in the non-linear effects, hence we subtract the linear contribution from the non-linear resistance and reactance that are measured in the tube just before the sample. To calculate a single perforation, this quantity is multiplied with the porosity, σ . Furthermore, the change in resistance is normalized with the non-linear limit proposed by Ingard and Ising [5] where for reactance it is normalized by the fluctuating mass term proposed by Ingard [25]. By these normalizations, we obtain non-dimensional resistance and reactance changes due to non-linearity, ΔR_{NL} and $\Delta \delta_{NL}$ respectively. These parameters are given in Eqs. (11a) and (11b).

$$\Delta R_{NL} = \frac{(\Re\{Z_{t-NL}\} - \Re\{Z_{t-L}\}) \sigma}{|\hat{u}_p| \rho_0}, \quad (11a)$$

$$\Delta \delta_{NL} = \frac{(\Im\{Z_{t-NL}\} - \Im\{Z_{t-L}\}) \sigma}{\omega \rho_0 d_p / 2}. \quad (11b)$$

Recall that, Ingard and Ising's [5] resistance expression is only valid for orifices with sharp edge profiles. For a universal non-dimensional parameter, the effect of the orifice edge should be corrected via Eq. (4) as,

$$\Delta R_{NL-C} = 2C_{vc}^2 \Delta R_{NL}. \quad (12)$$

We provide the plots of ΔR_{NL-C} and $\Delta \delta_{NL}$ as functions of Sr in Figures 7 to 9. In these graphs, for each Sh value, there is a separate curve (symbol).

In Figure 7, it is seen that for $Sh = \mathcal{O}(1)$, the effect of Shear number cannot be ignored both for ΔR_{NL-C} and $\Delta \delta_{NL}$. Nevertheless, as the Shear

Figure 7: The effect of Sh number for Sample A: $\bullet$ $Sh = 0.75$, $\circ$ $Sh = 0.86$, $\blacksquare$ $Sh = 1.05$, $\square$ $Sh = 1.14$, $\blacktriangle$ $Sh = 1.22$, $\triangle$ $Sh = 1.29$, $\blacklozenge$ $Sh = 1.43$, $\diamond$ $Sh = 1.49$.

number increases this dependence vanishes rapidly both for the resistance and reactance change due to non-linearity in MPPs. Moreover, in the resistance case, ΔR_{NL-C} approaches an asymptote which is determined by C_{vc} . We see that for high Strouhal and low Shear numbers ($Sr \gg 1$, $Sh = \mathcal{O}(1)$), the non-linear normalized resistance is proportional to the inverse of the Strouhal number (Figure 7). Hence the non-linear contribution to the resistance is proportional to the square of the acoustic amplitude (and Reynolds number) as predicted by Aurégan and Pachebat [10].

On the other hand, deriving conclusion on the reactance is not as straightforward as the resistance case. We observe that δ_{NL} approaches 0 for high the Sr limit regardless of the Sh value, as expected from the definition given in Eq. (11b). Yet, for the low Sr limit, we observe no clear trend for the interval of Sr where we perform the measurements. For example, in Figure 7b, the case $Sh = 0.75$, $\Delta \delta_{NL}$ has a tendency of having a plateau; yet this behaviour cannot be easily generalized by looking at other Sh number cases.

The distributions of ΔR_{NL-C} and $\Delta \delta_{NL}$ in Figures 8 and 9 agree with the previous observations for Sample A in Figure 7. As the Shear number increases, the deviation between the measurement points becomes smaller for both the normalized resistance and reactance.

Based on the measurement data presented in Figures 7–9, we propose fits which are functions of Sr and Sh . The fits representing the change in normalized resistance and reactance are given as F_C and G_C respectively.

(a) Change in Normalized Resistance.

(b) Change in Normalized Reactance.

Figure 8: The effect of Sh number for Sample B: $\bullet$ $Sh = 1.99$, $\circ$ $Sh = 2.29$, $\blacksquare$ $Sh = 2.81$, $\square$ $Sh = 3.04$, $\blacktriangle$ $Sh = 3.25$, $\triangle$ $Sh = 3.44$, $\blacklozenge$ $Sh = 3.81$, $\diamond$ $Sh = 3.98$.

$$F_C(Sr, Sh) = \frac{1}{1 + 2Sr[1 + 0.06 \exp(3.74/Sh)]}. \quad (13)$$

$$G_C(Sr, Sh) = \begin{cases} 0.20 - \frac{0.50}{Sr} \left(1 - \frac{0.42}{Sh^2}\right) + \frac{0.05}{Sr^2} \left(1 - \frac{0.68}{Sh^2}\right), & \text{if } Sr \leq 1. \\ -0.05 \left[\frac{1}{Sr} \left(1 - \frac{1}{Sh}\right)\right] - 0.60 \left[\frac{1}{Sr} \left(1 - \frac{1}{Sh}\right)\right]^2, & \text{otherwise.} \end{cases} \quad (14)$$

The expressions in Eqs. (13) and (14) are obtained by curve fitting of the experimental data. For these fits, 118 data points were used. The correlation coefficients r_F and r_G for F_c and G_c are given as $r_F^2 = 0.92$ and $r_G^2 = 0.85$ respectively.

The comparison between the proposed expressions and the experimental data is given in Figure 10. In this figure, to maintain simplicity, 3 Shear number cases are selected. These cases are chosen in such a way that experimental data with low ($Sh = 1.05$), medium ($Sh = 3.81$) and high ($Sh = 7.61$) values for the Shear number are compared with the proposed expressions. As it is seen in Figure 10, the proposed expressions represent the Sr and Sh number dependence in the transition regime.

As shown in Figure 10, the proposed expressions represent the general behaviour of the measurements well in the transition regime. With these expressions, one can calculate the non-linear resistance and reactance as

(a) Change in Normalized Resistance.

(b) Change in Normalized Reactance.

Figure 9: The effect of Sh number for Sample C: $\bullet$ $Sh = 3.98$, $\circ$ $Sh = 4.59$, $\blacksquare$ $Sh = 5.62$, $\square$ $Sh = 6.07$, $\blacktriangle$ $Sh = 6.49$, $\triangle$ $Sh = 6.88$, $\blacklozenge$ $Sh = 7.61$, $\diamond$ $Sh = 3.98$.

$$\Re\{Z_{t-NL}\} = \Re\{Z_{t-L}\} + \frac{F_c(Sr, Sh)|\hat{u}_p|\rho_0}{2C_{vc}^2\sigma}, \quad (15a)$$

$$\Im\{Z_{t-NL}\} = \Im\{Z_{t-L}\} + \frac{G_c(Sr, Sh)\omega\rho_0 d_p}{2\sigma}. \quad (15b)$$

For the resistance, as Sr increases, the dependence between the dimensionless ΔR_{NL-C} and acoustic particle velocity amplitude becomes almost linear according to the proposed expression in Eq. 13. This behaviour corresponds to the prediction for porous media by Aurégan and Pachebat [10].

Although it is not in the focus of this study, we have carried out some experiments with orifices which have considerably larger diameters than typical MPPs ($Sh \gg 1$). For example with Sample D, we have observed some peculiar behaviour both for the resistance and the reactance. The measurement results for this sample with three different Sh levels are given in Figure 11.

For Sample D, when the Stokes layer is thin compared to the diameter of the orifice, the resistance increases as Sr becomes very large. Considering the results from Disselthorst and van Wijgaarden [7] and Peters and Hirschberg [20], similar behaviour has been observed with tube terminations with sharp edges. An increase following $Sr^{1/3}$ is expected for the normalized non-linear contribution to the resistance, following Eq. (6). Yet, further research and measurements are required to support this explanation.

Figure 10: Comparison of the proposed expressions F_c and G_c with experiment results. In sake of simplicity, one Sh value is chosen from Samples A, B and C as examples: $\bullet$ $Sh = 1.05$ (Exp), $-\circ-$ $Sh = 1.05$ (Fit); $\blacksquare$ $Sh = 3.81$ (Exp), $-\square-$ $Sh = 3.81$ (Fit); $\blacktriangle$ $Sh = 7.61$ (Exp), $-\triangle-$ $Sh = 7.61$ (Fit).

We also observe an overshoot to positive values in Figure 11b for the non-linear contribution to the inertia. As the Shear number increases, the overshoot becomes stronger. This corresponds to the weak non-linear behaviour reported by Ingard and Labate [4]. At such amplitudes, vortices are formed at the edges of the orifice, but they remain close to the edge. The formation of a jet flow is not reached there. The flow separates from the orifice edges, but reattachment of this flow occurs soon downstream. The reattachment of the flow also occurs for relatively thin orifices [4]. The schematic description of reattachment is illustrated in Figure 12.

The local contraction of the acoustic flow due to vortex formation occurs at the upstream edge. This corresponds to a local increase of flow velocity due to the contraction of the flow. Further increase of the inertia is due to the increased effective length of the perforation caused by the vortex shedding at the downstream edge of the perforation.

In case of Sample A, we observe some slightly positive valued data points for the high Sr limit. Yet, for this sample, the Stokes layer thickness is comparable with the orifice radius which could be a reason for the suppression of the overshoot.

(a) Change in Normalized Resistance.

(b) Change in Normalized Reactance.

Figure 11: The effect of Sh for Sample D. As the shear layer gets thin, different non-linear mechanisms are activated: $\bullet$ $Sh = 10.4$, $\square$ $Sh = 14.8$ and $\blacktriangle$ $Sh = 18.1$.

Figure 12: Sketch of the acoustic flow with flow separation at sharp edges and reattachment.

5. Conclusions

This study focuses on the acoustic response of micro-perforated plates (MPPs) with circular orifices and square sharp edges in the linear to non-linear transition regime. In this regime, we are interested in the deviation from the linear resistance and reactance due to the increase in the excitation amplitude. To measure this, we use an impedance tube setup. We propose dimensionless functions of the Strouhal (Sr) and Shear (Sh) numbers both for non-linear resistance and reactance, F_c and G_c respectively.

The non-linear effects are initiated around $Sr = \mathcal{O}(1)$. In our investigation, to see the transition from linear to non-linear regimes, we vary the excitation amplitude for Strouhal number values $Sr \in [0.05, 10]$. Further-

more, to meet the typical micro-perforated plate range and to observe the perforations slightly larger than this range Shear number values are limited to within the range $0.75 < Sh < 8$.

The most prominent result of the study is that in the transition regime, the non-linear corrections both for resistance and reactance depend mainly on Sr and Sh for MPPs. The dependence on Sh is significant for $Sh = \mathcal{O}(1)$, and becomes negligible for higher values of $Sh > 3$.

The non-linear correction for the resistance, F_c , is selected in such a manner that it vanishes for $Sr \gg 1$ and approaches to an asymptote defined by C_{vc} for $Sr \ll 1$. In other words, F_c satisfies the limits for both the linear and quasi-steady approach. Moreover, in the transition regime, the relation between the resistance and the particle velocity is quadratic according to F_c . This is in agreement with previous analytical models [10, 26, 27]. For higher Shear numbers and $Sr \gg 1$, a more complex behaviour is observed which has been described by Disselhorst and van Wijngaarden [7].

The proposed non-linear correction function for the inertia G_c also vanishes for $Sr \gg 1$. Yet, for the $Sr \ll 1$, it does not have a limit. The inertance is negligible in the limit $Sr \ll 1$. Moreover, especially for high Sh and high Sr cases, we have observed an overshoot to positive values for the non-linear correction for the inertia. We suspect the mechanism responsible for this is local vortex shedding as described by Ingard and Labate [4]. This results in a local increase in flow velocity and correspondingly of inertia.

With the correction functions we propose, a link between the linear and non-linear acoustic behaviour for MPPs is provided for practical applications.

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