# Integrating robust timetabling in line plan optimization for railway systems

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**Abstract**

We propose a heuristic algorithm to build a railway line plan from scratch that minimizes passenger travel time and operator cost and for which a feasible and robust timetable exists. A line planning module and a timetabling module work iteratively and interactively. The line planning module creates an initial line plan. The timetabling module evaluates the line plan and identifies a critical line based on minimum buffer times between train pairs. The line planning module proposes a new line plan in which the time length of the critical line is modified in order to provide more flexibility in the schedule. This flexibility is used during timetabling to improve the robustness of the railway system. The algorithm is validated on the DSB S-tog network of Copenhagen, which is a high frequency railway system, where overtakings are not allowed. This network has a rather simple structure, but is constrained by limited shunt capacity. While the operator and passenger cost remain close to those of the initially and (for these costs) optimally built line plan, the timetable corresponding to the finally developed robust line plan significantly improves the minimum buffer time, and thus the robustness, in eight out of ten studied cases.

**Keywords:** railway line planning; timetabling; robustness; mixed integer linear programming.

## 1 Introduction

Railway line planning is the problem of constructing a set of lines in a railway network that meet some particular requirements. A line is often taken to be a route in a high-level infrastructure graph ignoring precise details of platforms, junctions, etc. In our case, a line is a route in the network together with a stopping pattern for the stations along that route, as a line may either stop at or bypass a station on its route (which saves time for bypassing passengers). We define a line plan as a set of such routes, each with a stopping pattern and frequency, which together must meet certain targets such as providing a minimal service at every station.

Timetabling is the problem of assigning precise utilization times for infrastructure resources to every train in the rail system. These times must ensure that trains can follow their routes in the network, stop at appropriate stations where necessary, and avoid any conflicts with other trains. A conflict rises where two trains want to use the same part of the infrastructure at the same time, for example at a switch, platform or turning track. According to Bešinović et al. (2016) a timetable is feasible if all trains are able to adhere to the schedule on their assigned routes, we cite: “if (i) the individual processes are realizable within their scheduled process times, and (ii) the scheduled train paths are conflict free, i.e., all trains can proceed undisturbed by other traffic.” Since in this research the running times, dwell times and turn times of the trains are fixed in advance and thus always realizable, this research focuses on constructing a normative macroscopically feasible timetable. If timetabling is performed separately from line planning, the line plan specifies the lines and the number of hourly trains operating on each line but not the exact times for those trains and not the precise resources that a train on a line will utilize. Those timings and utilizations are decided as part of the timetabling.

Traditionally, a railway line plan is constructed before a timetable is made. However, an optimal line plan does not guarantee an optimal or even a feasible timetable (Kaspi and Raviv, 2013). An integrated approach can overcome this problem. Nevertheless, since line planning and timetabling are both separately already very complex problems for large railway networks (Michaelis and Schöbel, 2009; Goerigk et al., 2013), solving the resulting integrated problem is in most cases not computationally possible (Schöbel, 2015). We propose a heuristic algorithm that constructs a line plan for which a feasible timetable exists. We call a line plan *timetable-feasible* if there exists a normative macroscopically feasible timetable for that line plan. Moreover the algorithm improves the robustness of the line plan by making well chosen changes in the stopping patterns of the lines while the existence of a feasible timetable remains assured.

There are different interpretations of robustness in railway research. According to Dewilde et al. (2011), a railway planning is *passenger robust* if the total travel time in practice of all passengers is minimized in case of frequently occurring small delays. The focus of this definition is twofold, as both short and reliable travel times have to be provided by the planning. Passenger robustness is also what we want to strive for with our approach. However, this objective is not directly included, but implicitly considered by avoiding delay propagation. If delays are less likely to be propagated between trains, fewer passengers will be delayed which positively affects the total passenger travel time in practice.

We have developed an iterative approach to build a line plan and timetable from scratch while taking passenger robustness into consideration. We focus on the integration of both planning problems. A line plan, optimal for a weighted sum of passenger and operator cost, can be created and iteratively updated until a normative macroscopically feasible and passenger robust timetable can be computed while keeping the quality of the line plan high. The main contributions presented in this paper are:

- The integration of line planning, timetabling and passenger robustness.
- An approach that builds coordinated line plans and timetables *from scratch*.
- Two insights and proofs on timetable-infeasibility of line plans.
- The inclusion of limited shunt capacity of terminal stations in line plan and timetable optimization.
- Practical conclusions for the DSB S-tog network in Copenhagen based on experimental results.

The context of this research is a high frequency network. The network can be large but should have a simple structure and trains are forced to turn on their platform in their terminal stations due to a lack of shunting area.

The proposed integrated approach originates from insights on why some line plans do not allow feasible timetables and why some line plans allow more robust timetables. A first insight is that a line can be infeasible on its own, which we call *line infeasibility*. A second insight is that line combinations can be infeasible due to their frequencies. We call this *frequency combination infeasibility*. In Section 3 we explain these insights. Furthermore, we present a technique to develop a line plan that guarantees a feasible timetable. We introduce a timetabling model based on the Periodic Event Scheduling Problem (PESP), introduced by Serafini and Ukovich (1989), to create passenger robust timetables. We illustrate with a case study that a smart and targeted interaction of both techniques develops a line plan from scratch which guarantees a feasible and passenger robust timetable. Moreover, the integrated approach can also be used to improve the robustness of an existing line plan. The line planning and timetabling technique and the integrated approach are explained in Section 4.

Related work and some definitions are discussed initially in Section 2. The case study is described in more detail in Section 5. In Section 6 the results of the case study are presented and examined and the integrated approach is illustrated in an example. The paper is concluded and ideas for future research are suggested in Section 7.

# 2 State of the art

The planning of a railway system consists of several decisions on different planning horizons (Lusby et al., 2011). The construction of railway infrastructure and a line planning are long term decisions. A timetable, a routing plan, a rolling stock schedule and a crew schedule are made several months up to a couple of years in advance. Decisions on handling delays and obstructions in daily operation are made in real time. Each of these decisions affects the performance of the other decisions. Ideally, a model that optimizes all these decisions simultaneously is preferred. Each of the separate decision problems, however, is NP-hard for realistic networks (Schöbel, 2015). In practice these planning decisions are usually made one after the other, although the solution from a previous decision level problem does not even guarantee that a feasible solution exists for the next level problem (Schöbel, 2015). In the case that the output of the previous decision level leads to infeasibility at the next planning step, there are several possible approaches for looking for a feasible solution to both planning levels together. First, the outcome of the previous level can be replaced by a second best outcome in the hope that a feasible solution for the next level exists. Secondly, the outcome of the previous level can be specifically oriented towards making a feasible solution for the next level possible, by using case dependent restrictions specifically for this goal. Thirdly, the constraints on the outcome of the next level can be relaxed. These approaches increase the possibility of finding a feasible solution for the next level, but not necessarily guarantee a good outcome for both levels together. A few integrated approaches for two or three of the typical decision problems in railway research are described in the literature and clearly outperform the hierarchical approach (Goerigk et al., 2013). Most of these solution algorithms are heuristics to overcome the high computation times of an exact approach for a realistic railway network. As in this paper we propose an algorithm towards the integration of line planning and timetabling, we elaborate on existing integrated approaches for these two planning problems in the first part of this literature review. We also introduce some definitions. Thereafter, we explain the place of the individual timetabling and line planning modules that are used in our integrated approach within existing literature on timetabling and line planning.

## 2.1 Integration of line planning and timetabling

This paper is not the first attempt towards an integration of line planning and timetabling in railway scheduling. In Liebchen and Möhring (2007), some line planning decisions are included in the timetabling process. They assume that, for some parts (sequence of tracks) of the network, the number of lines serving each part is known beforehand. On these track sections they put an artificial station in the middle. Every line along this track section is then partitioned into two line segments, before and after the artificial station. They use a

PESP to model the timetabling problem in which they add constraints such that a perfect matching between the arriving and the departing line segments is forced. This is achieved by matching arrival and departure times of the line segments in the artificial station which are assigned by this same model. Here one line corresponds to one train. This approach is deficient if, for some network parts, the number of passing trains is not known beforehand. This is often the case in real world networks.

Kaspi and Raviv (2013) present a genetic algorithm that builds a line plan and timetable from scratch. They start from a given line pool and per line a fixed number of potential trains. A solution consists of three characteristics for each train: the value zero or one, which indicates if the train should be scheduled or not, an earliest start time and a stopping pattern. A member of the initial population is constructed by drawing values for each characteristic from separate Bernoulli distributions. The timetable and line plan are constructed by scheduling trains with value one for the first characteristic according to a fixed priority rule. If a train cannot be scheduled without one or more conflicts with other already scheduled trains, this train is omitted from the solution. For the resulting timetable, the passenger travel time and the operator cost are calculated. These performance results affect the distribution parameters of the Bernoulli distributions from which the next generation will be drawn. This approach uses the performance of the timetable as input for the line planning of the next iteration. This interaction between line planning and timetabling is also the case in our approach. But in contrast to the stochastic approach of Kaspi and Raviv (2013), we use information of the timetable to make some deterministic and tactical changes to the line planning. Also in Goerigk et al. (2013) timetable performance is used to evaluate line plans. However, they do not iterate between the construction phase of the line planning and the timetabling, and they do not use this information to improve the line planning. They only use it to compare different ways to construct a line plan.

Michaelis and Schöbel (2009) offer an integrated approach in which they reorder the classic sequence of line planning, timetabling and vehicle scheduling for bus planning. The different planning steps are, however, performed one after each other such that the approach is still sequential. Vehicle scheduling or rolling stock scheduling are not integrated in our approach, but we take turn restrictions in the terminal stations into account which significantly simplify the rolling stock scheduling. Taking turn restrictions into account is useful if terminal stations are not equipped with enough shunting space for efficient turning during daily operation. In fact, neglecting turn restrictions can lead to infeasible timetables. To the best of our knowledge, no other integrated approach for timetabling and line planning takes turn restrictions during daily operation into account. This is explained in the next section.

Very recently, Schöbel (2015) published a mixed integer linear program (MILP) in which line planning and timetabling are integrated for railway planning. This model is based on the PESP of Serafini and Ukovich (1989). In the model, binary variables are introduced to indicate if a certain line is added to the line plan. There are also big M-constraints added to the PESP model in which these binary variables are used to push event times of lines which are not in the line plan to zero and also to switch off lower bounds of activities for unassigned lines. The objective function minimizes the planned travel time of the passengers. Transfer penalties are not taken into account, but they can easily be introduced as a weight in the objective function. No performance results of this model are presented yet.

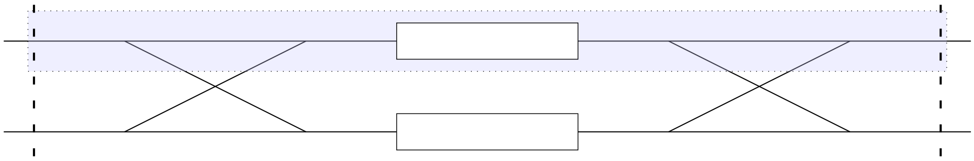
An added value of our approach is that passenger robustness is taken into account when constructing a line plan (and timetable). With our approach we want to shift the focus in current research on integration of line planning and timetabling to the creation of passenger robust line plans (and timetables). The algorithm that we propose constructs a line plan that minimizes planned passenger travel time and operator costs but also prevents unreliable travel times during daily operation in order to provide a short travel time in practice for all passengers. As mentioned in the introduction, a *passenger robust* plan minimizes this total travel time in practice. In order to obtain short travel times in practice, the propagation of delays from one train to another train has to be avoided, among other things. This can be achieved if the line plan allows a timetable with well-placed and large enough buffer times between trains. Also in Kroon et al. (2008); Caimi et al. (2012); Salido et al. (2012); Dewilde et al. (2013); Sels et al. (2016) and Vansteenwegen et al. (2016) the (minimum) buffer times between train pairs are lengthened in order to reduce the propagation of delays.

Another added value of our approach is that trains with the same route are equally spread over the period of the cyclic timetable. Making the reasonable assumption that passengers arrive uniformly in a station of a high frequency network, a constant time interval between two trains with the same route minimizes the average waiting time of the passengers before boarding.

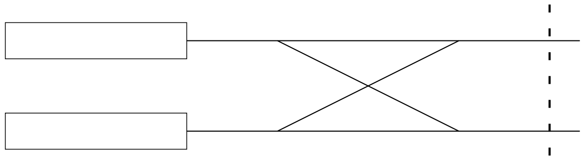
In our heuristic approach, a line planning and timetable module alternate, where each consists of an exact optimization model. We first introduce some definitions and then motivate our choice for the timetable and line planning models that are used and briefly discuss related literature.

## 2.2 Some definitions

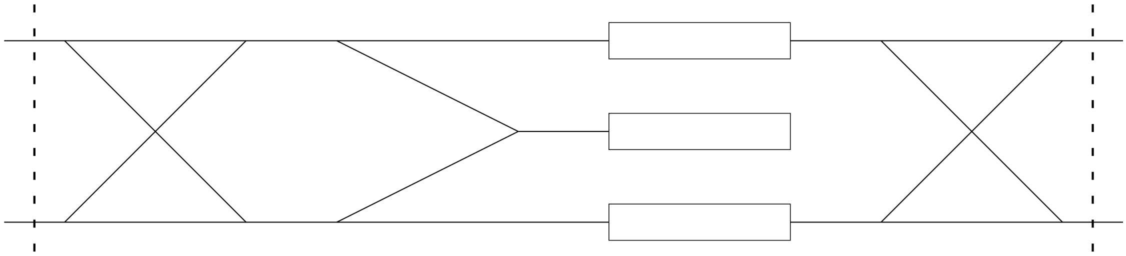
We define a network to be *simple* if (i) in between two succeeding stations, there is one track in each direction, (ii) in each station there is one platform in each direction, (iii) in each intermediate terminal station, there is one extra platform for turning, (iv) the ‘assembling’ of tracks coming from different terminal stations occurs within station areas. Everywhere outside the station areas there are bridges and tunnels to avoid the crossing



(a) Typical intermediate non-terminal station where one of the station areas is indicated by the colored rectangle. A train enters the station area if it enters the colored rectangle.



(b) Terminal station with two platforms, which can both be used for turning.



(c) A station with one extra platform, referred to as an intermediate terminal station. This platform is only connected with the tracks at one side and can be used for turning.

Figure 1: Three station types in a simple network. The vertical dashed lines situate the signals before and after the station. The white rectangles represent the platforms. The crosses at both sides of the platforms represent the switches and tracks that connect both platform areas.

of tracks. Moreover, overtaking is not allowed. For a visual representation of the different station types, see Figure 1.

*A station area* consists of the switches just before and after the station and the platform belonging to one direction to go through the station. So a station in a simple network consists of two station areas, one in each direction. This is illustrated in Figure 1a.

The *occupation interval* of a train in a station area is the time interval that the station area is occupied by that train and no other train can use the station area in this time interval. The occupation interval starts at *the reservation time* and ends at *the release time*. In this paper, the reservation time is a fixed amount of time before the train enters the station area, independent of the station area. A train enters a station area when it passes the vertical dashed line in Figure 1a and enters the colored rectangle. The release time in the model, and thus the occupation interval, is defined in such a way that it allows a next train to reserve the station area immediately after this release time. The release time thus guarantees that the train is already sufficiently far away when a next train wants to reserve the station area. As a result, the occupation interval will be somewhat longer than the time interval that a train will actually be in the station area in practice. The occupation interval of trains not dwelling in a station area, is artificially lengthened such that the occupation time is equal to the occupation time of dwelling trains. This is to avoid undesired overtakings in the planning. *The occupation time* is the length of the occupation interval.

*A conflict* occurs when (at least) two trains want to occupy the same station area at the same moment, so their occupation intervals for this station area overlap.

We define *the necessary turn time* as the time for the train to enter the station area of the terminal station (decreasing speed), stopping at the platform, alighting and boarding of passengers, extra time needed by the driver to move from one side of the train to the other side and the time for the train to leave the station area of the terminal station again (increasing speed). The necessary turn time is in fact the shortest possible occupation time of a train in a terminal station.

*The running time* between two succeeding station areas is the time that a train needs between the release time of the first station area and the reservation time of the next station area.

*The drive time* between two succeeding stations is defined as the occupation time of the first station area and the running time to arrive at the next station area, so it is the time that a train needs between the reservation time of the first station area and the reservation time of the next station area. Since the reservation time of a station area is defined as a fixed amount of time before the entry time of that station, the drive time between two succeeding stations also coincides with the time between the entry times in these two stations. A visual representation is provided in Figure 2.

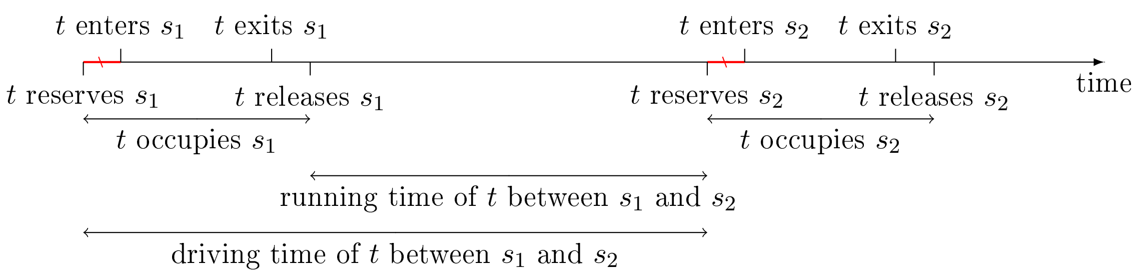



Figure 2: Representation of the reservation time, the entry time, the release time, the exit time, the occupation time, the running time and the drive time of train $t$ for two succeeding stations $s_1$ and $s_2$. The parts indicated in red are equally long, independent of the involved train and stations.

These definitions can be made more general by not only looking at station areas, but at parts of the network bounded by signals.

We define *the buffer time* between two trains on a part of the network as the time between the time instant that the first train releases that part of the network and the time instant that the other train reserves that same part of the network. The buffer interval is the interval between these two time instants. It should be noted that, given the definition of occupation intervals in our paper, buffer times of zero (or more) correspond to a normative macroscopically feasible timetable.

## 2.3 Timetabling

The goal of the timetabling module is to construct a passenger robust timetable. This avoids propagation of delays in case of small delays during daily operation in order to provide reliable travel times to the passengers and is achieved by maximizing the (minimum) buffer times between train pairs. Parbo et al. (2016) give an extensive overview of passenger perspectives in railway timetabling. The PESP model of Serafini and Ukovich (1989) is the foundation of many timetable models (e.g. Schrijver and Steenbeek, 1993; Nachtigall, 1996; Liebchen, 2006; Peeters, 2003; Schmidt and Schöbel, 2015; Großmann, 2011) and is also the framework of our timetabling model. The PESP model schedules events in a period of the cyclic timetable and takes precedence constraints and relations between events into account. Arrivals and departures of trains at stations or reservations and releases of track sections or station areas are events. If two events are related or can affect each other they form an activity. Examples of activities are the arrival and departure of the same train in a station or the reservation times of a shared switch, platform or station area by two different trains. The PESP model constrains each activity time, which is the time between the two events that define the activity. The PESP is originally defined without an objective function, but several objective functions for PESP can be found in the literature. We add an objective function that maximizes the (minimum) buffer times between trains using the same part of the infrastructure, in order to achieve robustness. In our timetabling model, we also have 'turning', 'providing buffer time' and 'station' activities between events besides the usual running and transfer activities. Furthermore, we include extra constraints such that trains of the same line can be equally spread over the period of the cyclic timetable. These constraints coincide with the constraints for the synchronization activities considered in (Siebert and Goerigk, 2013). In that paper the impact of including line frequencies in cyclic timetabling is studied and the authors conclude that it positively and significantly affects the quality of the constructed timetable. In Bešinović et al. (2016) and Goverde et al. (2016) an approach, different from PESP, is developed to obtain a stable robust and conflict-free (and energy-efficient) timetable. This approach iterates between microscopic and macroscopic timetabling. Moreover, this approach includes a delay propagation model to compute delay recovery. However, this approach is more complex and works heuristically in comparison to the exact PESP approach and our approach. A recent and elaborate discussion on timetable literature in general and PESP in specific can be found in Sels et al. (2016).

## 2.4 Line planning

Railway line planning is, generally, the construction of a set of lines to operate in a rail network. There are parallels to line planning problems in bus network design and network design for liner shipping. Line planning for rail takes the physical rail network as a fixed input, and provides a fixed input to subsequent timetabling and rolling stock planning. So when creating the line plan, assumptions can potentially be made about the form or characteristics of timetables, rolling stock and rolling stock planning. Schöbel (2012) gives an overview of different approaches to model and solve the line planning problem, broadly categorizing line planning approaches that are (operator) cost-oriented, and those that are passenger-oriented.

Goossens et al. (2006) focus on minimizing operator cost, for the less-studied case of line planning where lines may not stop at every station. Also in our research the stopping pattern of a line is decided in the line planning problem. The advantage of allowing lines to skip stations is the potential to combine fast lines which only stop at the stations with high demand and slow lines which also stop at stations with low demand (with the classification of stations not specified but decided during line planning). Using fast lines shortens the travel time of a lot of passengers and the slow lines assure a service in every station.

With a passenger focus, a common objective function is to maximize the number of direct travelers, i.e. the number of passengers who have a route from their origin to destination that does not require transfers. The simplest interpretation of this objective is to count the number of passengers for which there exists a line in the solution visiting both their origin and destination. This does not actually find passenger routes and does not guarantee that all counted passengers can actually *use* the line, as there may be insufficient capacity on some lines. Using this objective also has the risk in some networks that the passengers with no direct route may be faced with many transfers. Another disadvantage is that maximizing the number of direct travelers encourages long train lines and, critically in our case, does not favour skipped stations. Bussieck et al. (1997) is one example which uses this direct traveler objective, while ensuring that direct lines also have sufficient capacity to accommodate the passengers.

Another objective function with passenger focus is a travel time objective that takes into account the passenger's time traveling in trains and a penalty for switching trains (transfers). The calculation of this objective requires knowledge on the routing of passengers in the network taking into account travel time and transfers. This routing of the passengers can be modelled as paths in a graph, potentially requiring one path for every pair of stations. Schöbel and Scholl (2006) and Borndörfer et al. (2007) are examples where passengers between a pair of stations are routed by minimizing the sum of the travel time costs of the used paths. This passenger routing objective could be used as part of a weighted sum objective along with some operator cost (Borndörfer et al., 2007), or used alone but with an additional operator cost budget constraint (Schöbel and Scholl, 2006). In some practical problems the inclusion of a budget can be very important when combined with a passenger-oriented objective, as without it, solutions can contain many lines to individually satisfy every type of passenger. Our line plan model uses also the passenger's travel time objective. In our case study, however, there are tight rate limits on the maximum number of trains turning at a terminal station and on the use of certain infrastructure. Thus even without an operator budget consideration we do not risk solutions having particularly many lines.

Operator focused or passenger focused is a first partitioning that can be made. Another partitioning is that a line planning model may be based on a predetermined set of lines (a line pool), or it may find new lines dynamically. An advantage of a predetermined line pool is that all lines in the pool are guaranteed to be feasible in terms of line planning requirements, or advantageously for our case, in terms of timetabling requirements. This latter is explained in the next section. A predetermined pool also has the advantage of limiting the problem size in a useful and dynamic way (because the pool can be limited to be as diverse or as focused as desired). However, it has the disadvantage that the full set of possible lines may be large enough that enumerating them all would be intractable, while taking only a subset of all possible lines risks missing good solutions. Schöbel and Scholl (2006) present a model that takes as input a predetermined pool of lines. In contrast, Borndörfer et al. (2007) present a method where lines are generated dynamically in an infrastructure network as a pricing problem, finding maximum-weighted paths to introduce as lines to a restricted master problem. However, the master problem itself is formulated in terms of a known line pool.

With respect to decision variables, many approaches are similar in using either a binary decision for the presences of each line, or a non-negative or integral decision for the frequency of each line, where a frequency of zero means that the line is not in the solution. In our approach we may only select one of a set of frequencies defined individually for every line, so our model uses a binary decision variable indicating the presence of a (line, frequency)-pair.

Specifically related to the problem we address at DSB S-tog, Rezanova (2015) solves the line planning problem with an operator focus, considering train driving time and a particular competing objective related to new regulations for drivers. The author notes the problem of finding line plan solutions that are not feasible for timetabling, and suggests that an integrated approach would be valuable.

Overall, our modelling approach is similar to the work of Schöbel and Scholl (2006) in the construction of the graph for passenger flows, but differs in the capturing of frequency-dependent costs for passenger travel times. We also model line frequency in a stricter manner which is necessary for our case study, where specific sets of frequencies are valid for each line where in contrast, other work such as Schöbel and Scholl (2006) or Borndörfer et al. (2007) models frequency as a discrete variable over all positive integers for each line.

# 3 Timetable-infeasibility

In this section we explain how limited shunt capacity and certain frequency combinations of lines that share a part of the network can lead to timetable-infeasibility of line plans. Our integrated approach uses these insights to construct line plans that allow normative macroscopically feasible and passenger robust timetables.

## 3.1 Line infeasibility

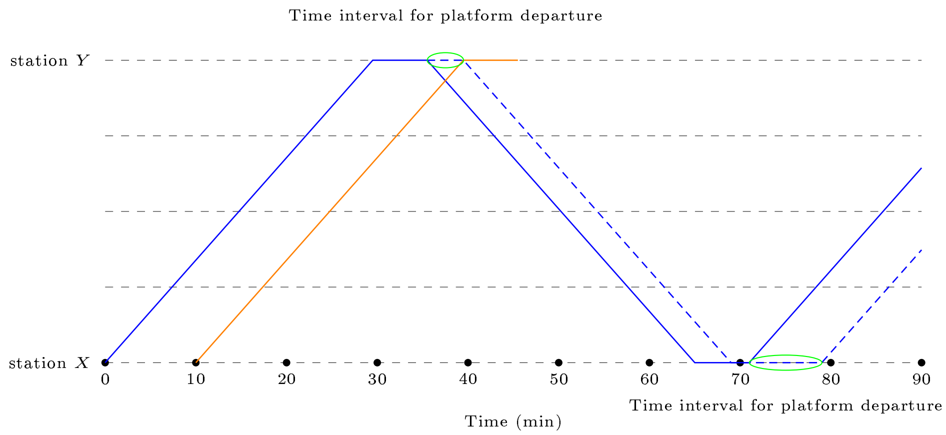



Figure 3: A line can be infeasible on its own

Consider Figure 3, showing a single train line operating at six times per hour between terminal stations $X$ and $Y$. The black dots on the time-axis show the scheduled departures from station $X$ for this line, which is once every ten minutes. We illustrate the first two time-distance graphs; the first departing from station area $X$ at minute zero (solid blue line), and the subsequent train following at minute ten (red line). In this example, the travel time between station area $X$ and $Y$ for the line is 29 minutes. This travel time includes the running times between the stations and the occupation times of the intermediate stations (not in the terminal stations). We assume that the train has to turn on its platform in station area $X$ and $Y$ due to restricted shunt capacity. The subsequent train that departed ten minutes later is therefore entering station area $Y$ ten minutes later as well, so the first train has a well-defined latest departure of that station area which is marked as a dashed blue line. The necessary turn time for this line in station $Y$ is seven minutes. Note that the necessary turn time already includes the occupation time of the terminal station $Y$ for the arriving train and the train driving back to station area $X$, which share the same rolling stock. So, this train is arriving in station area $X$ again between 65 minutes and 68 minutes after its first departure at minute zero. The necessary turn time in station area $X$ is also seven minutes for this line. Thus, the train can leave station area $X$ for the next round trip at 72 minutes after minute zero at the earliest (minute 65 arrival with seven minutes minimum necessary turn time) and 78 minutes at the latest (68 minute arrival with a maximum of ten minutes for dwelling and turning, assuming that the next train enters station area $X$ ten minutes later). However, no train is planned to leave station area $X$ in the interval of 72 to 78 minutes, which can be seen in Figure 3 as no black dot falls in the interval indicated with the green line. Therefore no feasible timetable can be found for this line. We will call this *line infeasibility.*

This insight can be mathematically formulated as: *If there exists no* $k \in \mathbb{Z}^+$ *such that*

$$2\mathcal{T}_l + \mathrm{ntt}_{s_{l,0}} + \mathrm{ntt}_{s_{l,|\mathcal{S}_l|}} \leq \frac{P}{f_l}k \tag{1}$$

*and*

$$\frac{P}{f_l}k \leq 2\mathcal{T}_l + 2\frac{P}{f_l} \tag{2}$$

*are satisfied, then, in case of restricted shunt capacity in its terminal stations, line* $l$ *is infeasible on its own. Here* $\mathcal{S}_l = \{s_{l,0}, \cdots, s_{l,i}, \ldots, s_{l,|\mathcal{S}_l|}\}$ *is the set of all stations on line* $l$ *(independent on an actual stop),* $\mathrm{ntt}_{s_{l,0}}$ *and* $\mathrm{ntt}_{s_{l,|\mathcal{S}_l|}}$ *are respectively the necessary turn time for line* $l$ *in its start station* $s_{l,0}$ *and end station* $s_{l,|\mathcal{S}_l|}$*,* $f_l$ *is the frequency of line* $l$*,* $P$ *is the period length of the cyclic timetable and* $\mathcal{T}_l = \sum_{i=0}^{e-1} \mathrm{run}_{l,s_{l,i},s_{l,i+1}} + \sum_{i=1}^{e-1} \mathrm{occ}_{l,s_{l,i}}$ *is the travel time of line* $l$*, where* $\mathrm{run}_{l,s_{l,i},s_{l,i+1}}$ *consists of the running time between station* $s_{l,i}$ *and* $s_{l,i+1}$*, and* $\mathrm{occ}_{l,s_{l,i}}$ *is the occupation time of station* $s_{l,i}$*. Furthermore, it is assumed that trains of the same line are equally spread over the period and use the same platform in the terminal stations for passenger convenience.*

So in the example above, $\mathcal{T}_l$ is 29 minutes, $\mathrm{ntt}_{s_{l,0}}$ and $\mathrm{ntt}_{s_{l,|\mathcal{S}_l|}}$ are seven minutes, $P$ is 60 minutes and the line frequency $f_l$ is six.

*Proof.* We define a *train cycle* of line $l$ as (i) the trip from its start station to its end station including running and dwelling, (ii) the turn movement in its end station, (iii) the trip from its end station to its begin station including running and dwelling and (iv) the turn movement in its begin station before the train can start a next cycle. The shortest possible duration of a train cycle of line $l$ is the sum of the running and occupation times from the begin station to the end station, $\mathcal{T}_l$, the necessary turn time in its end station, $\mathrm{ntt}_{s_{l,|\mathcal{S}_l|}}$, the travel time from the end station to the begin station, $\mathcal{T}_l$ (the travel time is the same in both directions) and the necessary turn time in its begin station, $\mathrm{ntt}_{s_{l,0}}$. Note that the occupation times of the terminal stations, $\text{occ}_{s_{l,0}}$ and $\text{occ}_{s_{l,|s_l|}}$, are not included in $\mathcal{T}_l$. This shortest possible train cycle length is given in the left hand side (lhs) of formula (1). The longest possible duration differs from the shortest possible duration in the time that the train takes for turning in its terminal stations. Instead of only for the necessary turn time, the train may stay in the station area until the next train of the same line arrives, which is $P/f_l$ minutes after its own arrival. This $P/f_l$ minutes also includes the occupation time of the arriving and departing train (same rolling stock). This maximal train cycle length is represented in the right hand side (rhs) of formula (2). Without loss of generality we can assume that train cycles of line $l$ start at $\{kP/f_l \mid k \in \mathbb{Z}^+\}$. If line $l$ is feasible, then for a train that starts its first cycle at $k_0P/f_l$ for a $k_0 \in \mathbb{Z}^+$, there has to exist a $k \in \mathbb{Z}^+$ for the start of its next cycle such that $kP/f_l \in [k_0P/f_l + (\text{lfs of (1)}), k_0P/f_l + (\text{rhs of (2)})]$. Remark that the latter statement remains true if $k_0P/f_l$ is subtracted from both interval bounds. This proves our mathematical insight by contraposition. As shown in the example, such a $k$ does not always exist. $\square$

### 3.2 Frequency combination infeasibility

Suppose that two lines share a part of the network and that trains of the same line are equally spread in the cyclic timetable. A second insight is that the frequencies of these lines affect the minimum buffer time between these lines. It is straightforward that the higher the frequencies the smaller the buffer time between trains of these lines. But we also make the following claim:

**Claim 1.** *The minimum buffer time between a line at a higher frequency and a lower frequency is no greater than between two lines at the higher frequency.*

**Example** Let $f_l \leq f_{l'}$ be the frequencies of two lines $l$ and $l'$ respectively. If $f_l = f_{l'} = 5$, then on a given infrastructure resource, trains of line $l$ and $l'$ could be planned alternately every six minutes. Without loss of generality, we here assume occupation intervals of length zero, since any larger occupation interval will induce smaller buffer times. However, if we assume $f_l = 4$ and $f_{l'} = 5$. Then, at any infrastructure resource shared by line $l$ and $l'$ and exactly once in the period of the timetable, there will be two succeeding trains of line $l'$ which are planned between two succeeding trains of $l$ (pigeon hole principle). We will refer to the trains of line $l$ and $l'$ that are concerned in this event as $t^1_{l,r}, t^2_{l,r}, t^1_{l',r}$ and $t^2_{l',r}$ respectively, where $l$ and $l'$ are the lines concerned, $r$ represents the shared infrastructure resource and the superscript indicates the order of the trains: $t^1_{l,r}$ ($t^1_{l',r}$) proceeds train $t^2_{l,r}$ ($t^2_{l',r}$). In this example, the time between $t^1_{l,r}$ and $t^2_{l,r}$ to equally spread the trains of line $l$ is 15 minutes and 12 minutes between $t^1_{l',r}$ and $t^2_{l',r}$ for line $l'$. This would lead to the situation in Figure 4, where $a$ is the buffer time between $t^1_{l,r}$ and $t^1_{l',r}$ at $r$. In order to fit $t^1_{l',r}$ and $t^2_{l',r}$ between $t^1_{l,r}$ and $t^2_{l,r}$, $a$ has to be strictly smaller than three. So, the smallest buffer time between a train of line $l$ and line $l'$ at $r$ is smaller than or equal to one-and-a-half minutes, which is much smaller than the six minutes in case $f_l = f_{l'} = 5$. The shared infrastructure resource, that is mostly referred to in this paper, is a station area.

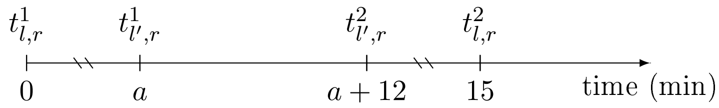



Figure 4: If lines $l$ and $l'$ have frequencies $f_l = 4$ and $f_{l'} = 5$ respectively, then once in 60 minutes two trains ($t^1_{l',r}$ and $t^2_{l',r}$) of line $l'$ will pass in between two trains ($t^1_{l,r}$ and $t^2_{l,r}$) of line $l$ at shared infrastructure resource $r$. Without loss of generality we can assume that this happens in the first quarter. Here $a \in \mathbb{R}$ and $0 < a < 3$.

The minimum buffer time between two lines at a shared infrastructure resource can be bound by the following formula: *The minimum buffer time between line $l$ and line $l'$ with frequencies $f_l \leq f_{l'}$ respectively, on a shared infrastructure resource, is smaller than ($\leq$)*

$$\frac{\frac{P}{f_l} - (\lceil \frac{f_{l'}}{f_l} \rceil - 1)\frac{P}{f_{l'}}}{2} \tag{3}$$

*where $P$ is the period length of the cyclic timetable, $\lceil x \rceil$ equals the smallest integer $y$ with $y \geq x$ and trains that operate on a line are equally spread over the period.*

*Proof.* Let $r$ be a shared infrastructure resource of line $l$ and $l'$. Without loss of generality, we here assume occupation intervals of length zero, since any larger occupation interval will induce smaller buffer times. By the pigeon hole principle, there are two trains of line $l$ in between which $\lceil f_{l'}/f_l \rceil$ trains of line $l'$ are passing at $r$. With the same notation as in the example above, we chronologically have $t^1_{l,r}, t^1_{l',r}, \cdots, t^{\lceil f_{l'}/f_l \rceil}_{l',r}$ and $t^2_{l,r}$. Train $t^1_{l,r}$ and $t^2_{l,r}$ are spread by $P/f_l$ minutes and train $t^1_{l',r}$ and $t^{\lceil f_{l'}/f_l \rceil}_{l',r}$ by $(\lceil f_{l'}/f_l \rceil - 1)P/f_{l'}$ minutes. So, the buffer time between $t^1_{l,r}$ and $t^1_{l',r}$ plus the buffer time between $t^{\lceil f_{l'}/f_l \rceil}_{l,r}$ and $t^2_{l',r}$ equals $P/f_l - (\lceil f_{l'}/f_l \rceil - 1)P/f_{l'}$. Thus at least one of these two buffer times is smaller than half of this value, which is the bound given in (3). $\square$

If the upper bound in (3) is strictly smaller than the minimum necessary buffer time according to safety regulations in the network, then $l$ with frequency $f_l$ and $l'$ with frequency $f_{l'}$ are not feasible together. In the example, if the minimum necessary buffer time according to safety regulations is two minutes, then these lines $l$ and $l'$ cannot be combined at frequencies $f_l = 4$ and $f_{l'} = 5$.

*Proof of Claim 1.* We first show that expression (3) is bounded above by $P/2f_{l'}$. We can write

$$f_{l'} = \alpha f_l + \beta, \tag{4}$$

with $\alpha, \beta \in \mathbb{Z}^+$ and $\beta < f_l$. Then we have:

$$
\begin{aligned}
\frac{\frac{P}{f_l} - (\lceil \frac{f_{l'}}{f_l} \rceil - 1)\frac{P}{f_{l'}}}{2} &= \frac{P}{2f_{l'}}\left(\frac{f_{l'} - (\lceil \frac{f_{l'}}{f_l} \rceil - 1)f_l}{f_l}\right), \\
&= \frac{P}{2f_{l'}}\left(\frac{\alpha f_l + \beta - (\lceil \frac{\alpha f_l + \beta}{f_l} \rceil - 1)f_l}{f_l}\right) \\
&= \frac{P}{2f_{l'}}\left(\frac{\alpha f_l + \beta - (\alpha + \lceil \frac{\beta}{f_l} \rceil - 1)f_l}{f_l}\right) \\
&= \frac{P}{2f_{l'}}\left(\frac{\beta - \lceil \frac{\beta}{f_l} \rceil f_l + f_l}{f_l}\right) \\
&\leq \frac{P}{2f_{l'}}.
\end{aligned} \tag{5}
$$

Formula (3) is maximal in case $f_{l'}$ equals or is a multiple of $f_l$ ($f_{l'} = kf_l$, $k \in \mathbb{Z}^+$):

$$
\frac{\frac{P}{f_l} - (\lceil \frac{kf_l}{f_l} \rceil - 1)\frac{P}{kf_l}}{2} = \frac{\frac{P}{f_l} - (k-1)\frac{P}{kf_l}}{2} = \frac{kP - (k-1)P}{2kf_l} = \frac{P}{2kf_l} = \frac{P}{2f_{l'}}. \tag{6}
$$

□

# 4 Methodology

In this section, we propose an integrated approach that constructs a line plan from scratch that minimizes a weighted sum of operator and passenger cost and allows a feasible and robust timetable. First a timetable-feasible line plan is constructed. Then, iteratively and interactively, a line planning module produces a line plan, and for that line plan, a timetable module produces a timetable that maximizes the (minimum) buffer times between train pairs. In each iteration an analysis of the timetable indicates how the line plan could be adapted in order to allow a more robust timetable. This adaptation increases the flexibility of the line plan which is used, in the timetabling module, to increase the minimum buffer times. The line plan module then calculates a new line plan that includes this adaptation while minimizing the weighted sum of operator and passenger costs. This feedback loop stops when there is no further improvement possible or if there is no improvement during a fixed number of iterations for the minimum buffer times between train pairs.

We first discuss the line planning module and the timetabling module separately and then the integration of both. Both the timetable and the line planning module consist of an exact optimization model, though our combined approach, and the fact that we do not always solve the models to optimality, result in an overall heuristic method.

## 4.1 Line planning module

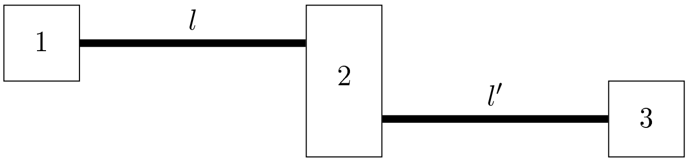
Constructing a line plan consists of selecting a set of lines which meet certain requirements from a pool of predetermined lines. The line pool is not exhaustive; there are many more possible lines than those considered, but the set is reduced to those that meet certain criteria as discussed with the rail operator. This also keeps the problem size small. The model performs three functions: (i) selecting the lines and frequencies and creating a valid plan, (ii) routing passengers between origin and destination stations and (iii) relating passenger routes to line selections.

Let us first define the set of all lines available: $\mathcal{L}$. For every line $l \in \mathcal{L}$ we define a set of valid frequencies for the line: $\mathcal{F}_l$. The operator must meet certain obligations for any valid line plan and must not exceed certain operational limits. These restrictions are referred to as service constraints. We define these all in terms of a set of resources $\mathcal{R}$, and define all limitations as either a minimum ($\mathrm{rmin}_r$) or maximum ($\mathrm{rmax}_r$) number of trains using that resource $r \in \mathcal{R}$ every hour. The subset of lines that make use of resource $r \in \mathcal{R}$ is defined as $\mathcal{L}_r$. Let $c_{l,f}$ be the cost to the operator for operating line $l$ at frequency $f$.

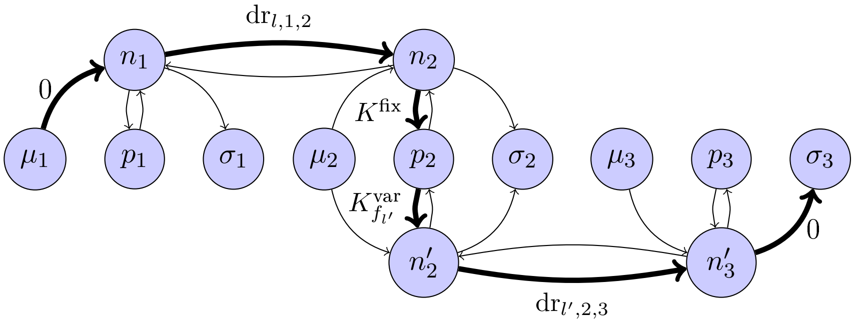
The line planning module starts from a known origin-destination (OD) matrix containing the passenger demand for travel between every origin and destination, where origins and destinations are simply stations in the rail network. Let $\mathcal{S}$ be the set of stations. For two stations $s_1, s_2 \in \mathcal{S}$ we know the demand $d_{s_1,s_2}$. We model passengers as a flow from each origin station to every relevant destination station in a graph. The structure of this graph (nodes and edges) is uniquely determined (i) by the network (stations and station links) and (ii) by the lines considered in the line pool. Furthermore, this graph captures the passenger cost in terms of drive time on lines and estimated transfer time between lines in case a transfer is required (estimated based on the frequency). We refer to this graph as the passenger graph. We now explain the construction of this passenger graph in more detail. An example can be found in Figure 5 for a network with three stations, 1, 2 and 3, and two lines $l$ and $l'$ visiting two of the stations each. A passenger graph contains a *(line, frequency, station)* vertex for every line, frequency, and every station visited by that line. The edges of this graph represent travel possibilities, with the edge cost being the known train driving time or the estimated transfer time. Additionally, for every station $s$ we have a platform vertex $p_s$ with edges from and to every *(line, frequency, s)* vertex, where the costs correspond to an estimate of perceived transfer time which consists of a fixed penalty component and a frequency-dependent component. Finally, this graph contains source $\mu_s$ and sink $\sigma_s$ vertices for every station $s$ where passengers originate from or terminate their travel. These vertices are connected to the appropriate *(line, frequency, station)* vertices with edges representing boarding or alighting from a line. These edges have zero cost. We model source and sink vertices separately to ensure line-to-line transfers are only possible via the platform vertex incurring the frequency-dependent costs.

Let $V$ and $E$ be the set of all vertices and edges of this graph, respectively, and $\tau_e$ be the cost to a single passenger of using edge $e \in E$. In total there are five types of edges.

- Type 1. From $(l, f, s)$ to $(l, f, s')$ for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and $s$ and $s'$ two succeeding




(a) Line structure (two lines, three stations)




(b) Corresponding passenger graph structure

Figure 5: The upper figure shows a simple network with three stations 1, 2 and 3, and two lines $l$ and $l'$. Line $l$ visits stations 1 and 2, and line $l'$ visits stations 2 and 3. Each line operates at just a single frequency ($f_l, f_{l'} \in \mathbb{Z}^+$). The lower figure shows the subsequent passenger graph structure used for this network. We simplified the notation to keep the figure clear: node $n_1$ and $n_2$ represents node $(l, f_l, 1)$ and $(l, f_l, 2)$ respectively and node $n'_2$ and $n'_3$ represents $(l', f_{l'}, 2)$ and $(l', f_{l'}, 3)$ respectively. Costs are labelled on the edges for a passenger travelling from station 1 to station 3, transferring lines at station 2, with used edges in bold. The costs to the passenger are $\mathrm{dr}_{l,1,2}$, travelling (**dri**ving) on line $l$ from station 1 to 2; fixed cost $K^{\mathrm{fix}}$ for a transfer and an additional $K^{\mathrm{var}}_{f_{l'}}$ frequency dependent cost for transferring to line $l'$; and $\mathrm{dr}_{l',2,3}$ travelling on line $l'$ from station 2 to 3.

stations visited by line $l$.

- Type 2. From $(l, f, s)$ to $p_s$ for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and $s$ a station visited by line $l$ and $p_s$ the platform vertex of station $s$.

- Type 3. From $p_s$ to $(l, f, s)$ for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and $s$ a station visited by line $l$ and $p_s$ the platform vertex of station $s$.

- Type 4. From $\mu_s$ to $(l, f, s)$ for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and $s$ a station visited by line $l$ and $\mu_s$ the source vertex of station $s$.

- Type 5. From $(l, f, s)$ to $\sigma_s$ for all lines $l \in \mathcal{L}$, $f \in \mathcal{F}_l$ and $s$ a station visited by line $l$ and $\sigma_s$ the sink vertex of station $s$.

This graph is similar to the *change&go* graph of Schöbel and Scholl (2006), but distinguishes between line transfers that in our case happen to lines with discrete frequencies, with a frequency-dependent cost. A more complex example with multiple frequencies per line can be found in Appendix A.

Let $l_e$ be the line that edge $e$ is related to and $\mathrm{f}_e$ be the frequency of the line that $e$ is related to. This line and frequency of an edge are uniquely defined as the two vertices connected by edge $e$ are either both related to the same line and frequency or only one of them is related to a line and frequency.

Let $a_v^s$ be the flow of passengers originating from station $s$ that enters vertex $v$ minus the flow of passengers originating from station $s$ leaving vertex $v$, where $v$ is a vertex of the passenger graph. For vertices $v$ of type *(line, frequency, station)* or platform vertices, $a_v^s = 0$ for all stations $s \in \mathcal{S}$. All passengers that enter such a vertex, also leave again. For vertices $v$ which are source vertices for a certain station $s$, passengers are only leaving to other stations according to the demand: $a_v^s = -\sum_{s' \in \mathcal{S}} d_{s,s'}$. For vertices $v$ which are sink vertices for a certain station $s$, passengers coming from other stations $s'$ are only entering: $a_v^{s'} = d_{s',s}$ for all stations $s' \in \mathcal{S}$.

For relating passengers to lines, let $C_f$ be the passenger capacity of any line operating at frequency $f$. We are therefore assuming the same rolling stock unit type and sequence for every line, but a higher frequency provides more seats than a lower frequency. We require that no more passengers use a line as the line capacity permits for the frequency the line is operating at.

We use two classes of decision variables: $x_{l,f} \in \{0,1\}$ is a binary decision variable indicating whether or not line $l$ is selected at frequency $f$, and $y_s^e$ decides the number of passengers from origin station $s$ that use edge $e$ in the passenger graph.

The line planning model is:

$$\text{Minimize} \quad \lambda \sum_{l \in \mathcal{L}} \sum_{f \in \mathcal{F}_l} c_{l,f} x_{l,f} + (1-\lambda) \sum_{e \in E} \sum_{s \in \mathcal{S}} \tau_e y_s^e \tag{7}$$

$$\text{s.t.} \quad \sum_{f \in \mathcal{F}_l} x_{l,f} \leq 1 \qquad \forall l \in \mathcal{L} \tag{8}$$

$$\sum_{l \in \mathcal{L}_r} \sum_{f \in \mathcal{F}_l} f x_{l,f} \geq \mathrm{rmin}_r \qquad \forall r \in \mathcal{R} \tag{9}$$

$$\sum_{l \in \mathcal{L}_r} \sum_{f \in \mathcal{F}_l} f x_{l,f} \leq \mathrm{rmax}_r \qquad \forall r \in \mathcal{R} \tag{10}$$

$$\sum_{(u,v) \in E} y_s^{(u,v)} - \sum_{(v,w) \in E} y_s^{(v,w)} = a_v^s \qquad \forall s \in \mathcal{S}, \forall v \in V \tag{11}$$

$$\sum_{s \in \mathcal{S}} y_s^e \leq C_f x_{l_e,\mathrm{f}_e} \qquad \forall e \in E \tag{12}$$

$$x_{l,f} \in \{0,1\} \qquad \forall l \in \mathcal{L}, f \in \mathcal{F}_l \tag{13}$$

$$y_s^e \in \mathbb{R}^+ \qquad \forall s \in \mathcal{S}, e \in E \tag{14}$$

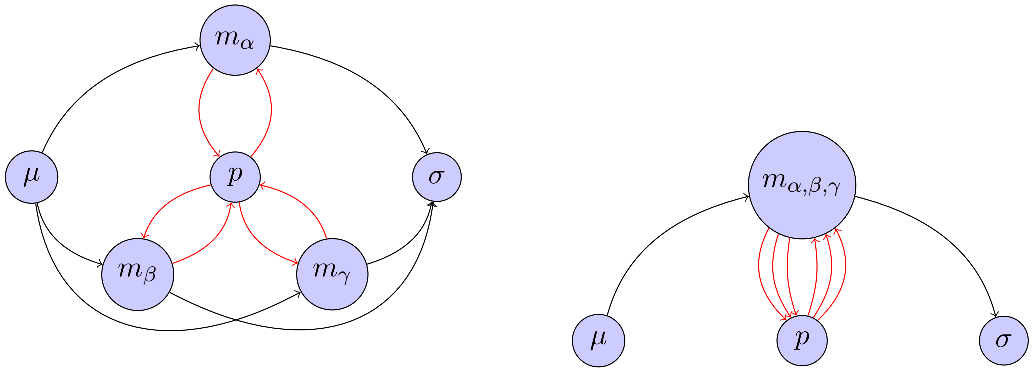
The objective function (7) is a weighted sum of the operator cost and the passenger travel time (drive time and transfer time), using a parameter $\lambda \in [0, 1]$ to determine the importance of one component over the other.

Constraints (8) ensure that a line is chosen with at most one frequency (i.e. combinations of frequencies are not permitted, as if valid a discrete frequency would be present in the frequency set $\mathcal{F}_l$ for the line). Constraints (9) and (10) ensure that the obligatory and operational requirements are met for the line plan. Constraints (11) consist of the flow conservation constraints. The number of passengers leaving from an origin station must flow from that station with the appropriate number arriving at every destination station, such that flow is conserved. Constraints (12) link the flows of passengers to the line decisions. The presence of a positive passenger flow on an edge in the graph is dependent on some line being present in the plan. The maximum flow on that edge depends on the passenger capacity of the corresponding line at the appropriate frequency. Finally, constraints (13) and (14) restrict the line variables and flow variables to be binary variables and positive otherwise unrestricted variables, respectively.

The model requires $|E||\mathcal{S}|$ flow decision variables, which is large due to the many edges in the described passenger graph. However, we observe that many of the vertices and edges in the graph are very similar and differ only in line frequency. For lines with many possible frequencies there is significant duplication. For the edges related to a transfer at a station, the frequency is required to determine the cost to the passenger. However for all other edges the frequency information is redundant. Indeed, the cost of travelling on a line between stations does not depend on the frequency of that line. A first simplification of the model is that for each line and its frequencies, we replace the edges (and vertices) which do not depend on frequency with an edge (and vertex) related only to *line* and *station* instead of *line*, *frequency* and *station*. This is shown in Figure 6. The capacity of the replacement edge (and resulting right hand side of constraints (12)), is given by $\sum_{f \in \mathcal{F}_l} C_f x_{l,f}$.

Figure 6 shows the graph structure for a single station and a single line with three frequencies as originally described (Figure 6a) and with the explained reductions (Figure 6b). Nodes $\mu$ and $\sigma$ are respectively the station source and sink vertices for passengers and $p$ is the platform vertex for that station. The vertices $m_\alpha$, $m_\beta$, $m_\gamma$ are the *(line, frequency, station)* vertices for the three considered frequencies of the line, in that station. The red edges are the transfer edges (though no other lines are shown). Edges connecting these vertices $m_\alpha$, $m_\beta$, and $m_\gamma$ to corresponding vertices at other stations are not shown. Vertex $m_{\alpha,\beta,\gamma}$ is the combination of the vertices $m_\alpha$, $m_\beta$, and $m_\gamma$. The edge between $\mu$ and $m_{\alpha,\beta,\gamma}$, and between $m_{\alpha,\beta,\gamma}$ and $\sigma$, is the combination of the edges between $\mu$ and $m_\alpha$, $m_\beta$, and $m_\gamma$ in (Figure 6a), and $m_\alpha$, $m_\beta$, and $m_\gamma$ and $\sigma$, respectively. In Appendix A a more complex example of a passenger graph reduction can be found.

A second simplification of the model is that we consider transfer edges only at a minimal




(a) Original graph structure (b) Reduced graph structure

Figure 6: The full and reduced graph structure for a single line $l$ with three frequencies, $\alpha$, $\beta$ and $\gamma$ in $\mathbb{Z}^+$, at a single station $s$. We simplified the notation to keep the figure clear: node $m_i$ represents node $(l, i, s)$ for frequency $i \in \{\alpha, \beta, \gamma\}$. Node $m_{\alpha,\beta,\gamma}$ is the replacement node of nodes $m_\alpha, m_\beta$ and $m_\gamma$ in the reduced graph structure.

set of transfer stations. This set of stations is fixed beforehand and suffices to facilitate all optimal passenger flows, when every passenger's origin-destination pair is considered individually. Any solution that is feasible for this restricted problem is feasible if transfers edges are included for any station, but some solutions that are feasible if transfers are permitted anywhere may not be feasible with the restriction (although we have not observed this). At stations where we do not permit transfers we do not include transfer edges, and this reduces the total number of edges in the graph by between 23% and 34% when tested for a range of line pools. Finally, we can determine that only a subset of all edges should be used for the flows from a given origin station; generally it is never true that in an optimal solution passengers will be assigned an edge that travels "towards" the station they originate from. This is a third measure to simplify the model.

By making these three alterations we find that the line planning problem is solvable directly as a MILP, though not to optimality in the time frame we require. For our tests, finding line plans with no other restriction, we use a time limit of one hour, or until a gap between the solution and best lower bound is below 0.5% (in most cases the gap limit is reached, but for some weightings of objectives, one hour is insufficient). However, for a reduced line pool that we use in the integrated approach described later, the problem becomes easier and is solvable to optimality in an acceptable time frame.

The formulation (7)–(14) defines the basic line planning model. However, when searching for line plans that only differ a little from a given line plan we may impose some additional restrictions. The simplest types are the following:

$$\sum_{l\in\mathcal{L}}\sum_{f\in\mathcal{F}_l} f x_{l,f} \geq k_1 \tag{15}$$

$$\sum_{l\in\mathcal{L}}\sum_{f\in\mathcal{F}_l} f x_{l,f} \leq k_2 \tag{16}$$

That is, we require that the total number of (one-directional) trains running in the network per hour is between some upper and lower bound. This may be, for example, to find solutions that do not differ too much from some original solution. We use this because, from the point of view of the timetable module, two solutions that differ only in line frequency but not in line routes can be very different. Without such constraints, when seeking a line plan that is similar but different to a given plan, a change of frequency would not maintain the similarities in timetabling that we seek. Now, suppose we are given a line plan or a partial line plan, in the form $\mathcal{X} = \{(l,f),(l',f'),(l'',f''),...\}$ where every $(l,f)$ in $\mathcal{X}$ is a valid line and frequency combination, and that this (partial) line plan should not be in the solution. Then we may impose the following constraint for every such line plan:

$$\sum_{(l,f)\in\mathcal{X}} x_{l,f} \leq |\mathcal{X}| - 1. \tag{17}$$

Such constraints are used to forbid solutions we have already discovered and do not wish to find again, and also to forbid partial solutions which we already know are problematic for timetabling, i.e. they lead to timetable-infeasibility. Finally, and similarly, we may have some given line plan $\mathcal{X}$ and desire that the solution line plan contains at least $k$ lines from the plan:

$$\sum_{(l,f)\in\mathcal{X}} x_{l,f} \geq k. \tag{18}$$

Such constraints ensure that a discovered line plan is similar to some previous line plan, while differing by some number of (unspecified) lines. If instead the lines that may differ are specified, we can fix the variables of the lines that may not differ and only permit those variables corresponding to the specified lines that may differ to change (along with variables corresponding to lines not in the plan). These extra restrictions are used in the integrated approach when looking for a similar line plan that is more flexible, i.e. allows a more robust timetable.

## 4.2 Timetabling module

The timetable module is based on a PESP model. We indicate our event-activity network as $(\mathcal{E},\mathcal{A})$. The set of trains is indicated as $T$, the set of lines in the line plan (output of the line planning module) as $\mathcal{X}$, the line operated by train $t$ is indicated as $\ell_t$, the set of station areas is $\mathcal{S}$ and the set of station areas on a line $l$ (independent of an actual stop in these stations) is indicated as $\mathcal{S}_l$. As we assume a railway network with limited shunt capacity, our model assumes that all the trains can and must turn on their platform at end stations. The set $T_{\text{turn}}$ contains the train couples $(t, t')$ for which it holds that $t$ becomes train $t'$ after turning on the platform in its end station. Trains $t$ and $t'$ share the same rolling stock. Line $\ell_t$ and $\ell_{t'}$ contain the same stations but in opposite direction. The set $T_{\text{line spread}}$ contains the train couples $(t, t')$ where $t$ and $t'$ are two succeeding trains of the same line, i.e. no other train operating on the same line drives in between them.

The event set $\mathcal{E}$ of the event-activity network consists of the following events.

- The reservation of a station area $s$ by a train $t$ is a reservation event $(t, s, \text{res})$. We define $\mathcal{E}^{\text{res}}$ as $\{(t, s, \text{res}) \,|\, \forall t \in T, s \in S_{\ell_t}\}$.

- The release of a station area $s$ by a train $t$ is a release event $(t, s, \text{rel})$. We define $\mathcal{E}^{\text{rel}}$ as $\{(t, s, \text{rel}) \,|\, \forall t \in T, s \in S_{\ell_t}\}$.

- The reservation of a platform $\rho_{\tilde{s}_t,t}$ by a train $t$ in its terminal station $\tilde{s}_t$ in order to turn is a platform reservation event $(t, \rho_{\tilde{s}_t,t}, \text{res})$. We define $\mathcal{E}^{\text{res},p}$ as $\{(t, \rho_{\tilde{s}_t,t}, \text{res}) \,|\, \forall t \in T\}$.

- The release of a platform $\rho_{\tilde{s}_t,t}$ by a train $t$ in its terminal station $\tilde{s}_t$ in order to turn is a platform release event $(t, \rho_{\tilde{s}_t,t}, \text{rel})$. We define $\mathcal{E}^{\text{rel},p}$ as $\{(t, \rho_{\tilde{s}_t,t}, \text{rel}) \,|\, \forall t \in T\}$.

The following inclusions hold $\mathcal{E}^{\text{res},p} \subset \mathcal{E}^{res} \subset \mathcal{E}$ and $\mathcal{E}^{\text{rel},p} \subset \mathcal{E}^{\text{rel}} \subset \mathcal{E}$ and $\mathcal{E} = \mathcal{E}^{\text{res}} \cup \mathcal{E}^{\text{rel}}$. So platform $\rho_{\tilde{s}_t,t}$ of train $t$ in its terminal station can be interpreted as an extra station where the train arrives after arriving in its terminal station $\tilde{s}_t$. Note that the event set consists here of station reservation and release times instead of the more common arrival and departure times in stations. From a macroscopic viewpoint these reservation and release times of a station area can be used to derive arrival and departure times on the platforms. Since we did not construct the timetable on the signaling level, we cannot fully guarantee that a timetable that is feasible according to our model is conflict-free in practice on the microscopic level. However, all the timetables constructed during our case study that were checked by the railway operator, were found to be suitable to implement in practice.

The activity set $\mathcal{A}$ contains:

- *running activities* between the release of a train in a station and the reservation of this train of the next station on its line. Let $\mathcal{A}^{\text{run}} = \{((t, s, \text{rel}), (t, s', \text{res})) \in \mathcal{E}^{\text{rel}} \times \mathcal{E}^{\text{res}} \,|\, \forall t \in T \text{ and } s \text{ and } s' \text{ succeeding stations of } \ell_t\}$;

- *station activities* between the reservation and the release of a train in a station on its line. Let $\mathcal{A}^{\text{station}} = \{((t, s, \text{res}), (t, s, \text{rel})) \in \mathcal{E}^{\text{res}} \times \mathcal{E}^{\text{rel}} \,|\, \forall t \in T, s \in S_{\ell_t} \setminus \{\rho_{\tilde{s}_t,t}\}\}$;

- *turn activities* between the reservation and the release of a train on its platform in its terminal station. Let $\mathcal{A}^{\text{turn}} = \{((t, \rho_{\tilde{s}_t,t}, \text{res}), (t, \rho_{\tilde{s}_t,t}, \text{rel})) \in \mathcal{E}^{\text{res},p} \times \mathcal{E}^{\text{rel},p} \,|\, \forall t \in T\}$;
- *buffer activities* between the release of one train and the reservation of another train in the same station area. Let $\mathcal{A}^{\text{buffer}} = \{((t, s, \text{rel}), (t', s, \text{res})) \in \mathcal{E}^{\text{rel}} \times \mathcal{E}^{\text{res}} \,|\, \forall t, t' \in T : t \neq t', s \in S_{\ell_t} \cap S_{\ell'_t}\}$;
- *line spread activities* between the reservations of two succeeding trains on the same line in the stations on their line. Let $\mathcal{A}^{\text{line spread}} = \{((t, s, \text{res}), (t', s, \text{res})) \in \mathcal{E}^{\text{res}} \times \mathcal{E}^{\text{res}} \,|\, \forall t, t' \in T : (t, t') \in T_{\text{line spread}}, s \in S_{\ell_t}\}$;
- *turn connection activities* between the release of a train of the platform in its end station and the release of the next train of the opposite line that leaves from that station area. Let $\mathcal{A}^{\text{turn-con}} = \{((t, \rho_{\tilde{s}_t,t}, \text{rel}), (t', \tilde{s}_t, \text{rel})) \in \mathcal{E}^{\text{rel},p} \times \mathcal{E}^{\text{rel}} \,|\, \forall t, t' \in T : (t, t') \in T_{\text{turn}}\}$. This next train is the same physical train.

As mentioned in Section 2, we want to maximize the minimum buffer times between train pairs. In terms of the event-activity graph, we want to maximize the minimum activity time of the buffer activities. Mathematically we have

$$\max \min_{a=(i,j)\in\mathcal{A}^{\text{buffer}}} (\pi_j - \pi_i + k_a P), \tag{19}$$

where $\pi_i$ and $\pi_j$ are the event times of event $i$ and $j$ respectively which define together a buffer activity. However, this objective function is not linear, but as it is a max-min objective function, it can easily be linearized. Therefore, we introduce an auxiliary variable $z \in [0, P]$, where $P$ is the period length of the cyclic timetable. We add the constraints

$$z \leq \pi_j - \pi_i + k_a P \qquad \forall a = (i, j) \in \mathcal{A}^{\text{buffer}} \tag{20}$$

and we change the objective function to the maximization of $z$: $\max z$. The complete model is then the following.

$$\begin{aligned}
\max \quad & z && (21)\\
z \leq \pi_j - \pi_i + k_a P \qquad & \forall a = (i, j) \in \mathcal{A}^{\text{buffer}} && \\
L_a \leq \pi_j - \pi_i + k_a P \leq U_a \qquad & \forall a = (i, j) \in \mathcal{A} && (22)\\
0 \leq \pi_i < P \qquad & \forall i \in \mathcal{E} && (23)\\
k_a \in \{0, 1\} \qquad & \forall a = (i, j) \in \mathcal{A} && (24)
\end{aligned}$$

Constraints (22) bound all activity times from below and above. The term $k_a P$ avoids negative activity times. To ensure a unique value for $k_a$, the value of $U_a$ has to be smaller than the period length $P$. The specific values of $U_a$ and $L_a$ are listed in Table 1 for all activities $a \in \mathcal{A}$. The running activity times are bounded by the time that a train of line $l$

needs between the release of a station $s$ and the reservation of the next station $s'$, indicated as $\text{run}_{l,s,s'}$. The running time between the terminal station of a train and the platform in its terminal station is zero minutes. The station activity times are bounded by the time that is necessary and provided for a line $l$ to occupy a station $s$, indicated as $\text{occ}_{l,s}$. This is the time between the reservation and release time of that station. The turn activity times are bounded by the necessary turn time in the terminal station $s$, which is indicated as $\text{ntt}_s$ and the time from which on a next train can arrive on that platform. Trains that make use of the same turn platform all get the same maximum time to stay on that platform which is equal to the period length of the cyclic timetable divided by the number of trains that turn on platform $p$. The number of trains that turn on platform $p$ is indicated as $\varphi_p$. The buffer activity times have to be positive and smaller than $P - \text{occ}_{\ell_t,s} - \epsilon$ to ensure that occupation intervals do not overlap, independently of the order of both trains that will be assigned. On platforms in terminal stations the upper bound is smaller because trains occupy the platform for a longer time, i.e. the upper bound in our model is $P - \frac{P}{\varphi_{\rho_{\tilde{s}_t,t}}} - \epsilon$. Before initializing the timetable module, a check is necessary to determine if too many trains are scheduled on one platform, i.e. $\frac{P}{\varphi_p} \geq \text{ntt}_s$ must be satisfied. If so, the trains have enough time for turning, otherwise the timetable will be infeasible. The value of $\epsilon$ depends on the time discretization. We use 0.1 minutes. In this model we equally distribute trains of a line over the period, and therefore the line spread activity times have to be equal to the period length divided by the line frequency. The frequency of a line $l$ is indicated as $f_l$. The turn connection activity times have to be equal to zero, ensuring that the 'turning' platform is freed if the next train leaves in the opposite direction.

| Activity | $L_a$ | $U_a$ |
|---|---|---|
| $((t, s, \text{rel}), (t, s', \text{res})) \in \mathcal{A}^{\text{run}}$ | $\text{run}_{\ell_t,s,s'}$ | $\text{run}_{\ell_t,s,s'}$ |
| $((t, s, \text{res}), (t, s, \text{rel})) \in \mathcal{A}^{\text{station}}$ | $\text{occ}_{\ell_t,s}$ | $\text{occ}_{\ell_t,s}$ |
| $((t, \rho_{\tilde{s}_t,t}, \text{res}), (t, \rho_{\tilde{s}_t,t}, \text{rel})) \in \mathcal{A}^{\text{turn}}$ | $\text{ntt}_s$ | $\frac{P}{\varphi_{\rho_{\tilde{s}_t,t}}}$ |
| $((t, s, \text{rel}), (t', s, \text{res})) \in \mathcal{A}^{\text{buffer}}$ | 0 | $P - \text{occ}_{\ell_{t'},s} - \epsilon$ |
| $((t, s, \text{rel}), (t', s, \text{res})) \in \mathcal{A}^{\text{buffer}} : s = \rho_{\tilde{s}_t,t} = \rho_{\tilde{s}_t,t'}$ | 0 | $P - \frac{P}{\varphi_{\rho_{\tilde{s}_t,t}}} - \epsilon$ |
| $((t, s, \text{res}), (t', s, \text{res})) \in \mathcal{A}^{\text{line spread}}$ | $\frac{P}{f_l}$ | $\frac{P}{f_l}$ |
| $((t, \rho_{\tilde{s}_t,t}, \text{rel}), (t', \tilde{s}_t, \text{rel})) \in \mathcal{A}^{\text{turn-con}}$ | 0 | 0 |

Table 1: Lower and upper bounds for the PESP constraints (22)

### 4.3 Integrated approach

Here, we explain how the line planning and timetabling module can be integrated to construct a line plan and timetable that induce a low passenger and operator cost and maximize the buffer times between train pairs in order to provide a passenger robust railway schedule. The line planning and timetabling module work iteratively and interactively. The line planning module creates an initial line plan which is evaluated by the timetabling module. Based on the minimum buffer times between line pairs, a critical line in the line plan is identified. The line planning module then creates a new line plan with at least one different line, i.e. the time length of this critical line is changed. The goal is to create more flexibility in the line plan. This flexibility will be used by the timetabling module to improve its robustness. This heuristic approach which is divided into two parts is now further explained. In Figure 7, a visual overview of the algorithm is presented and in Section 6.2 we apply the approach to an example.

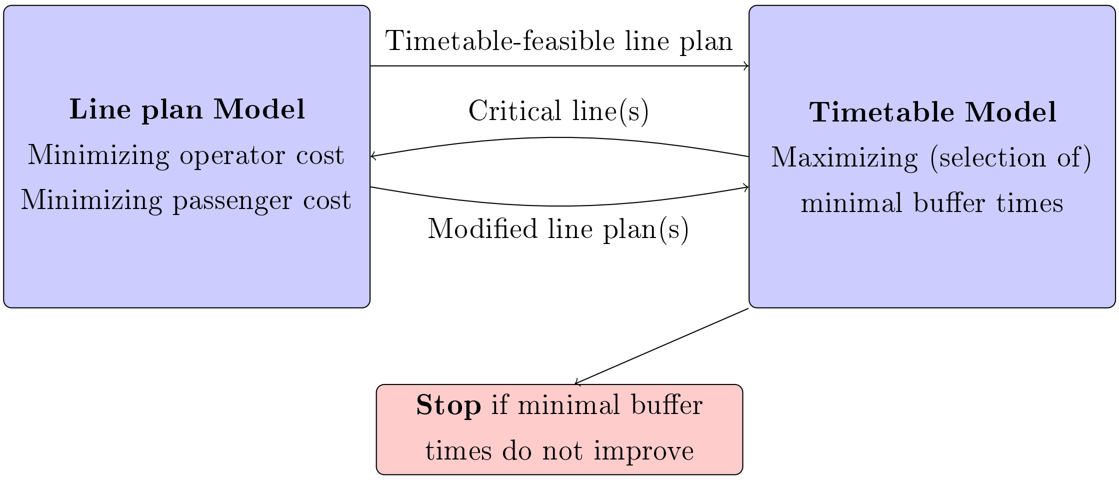



Figure 7: Overview of the integrated approach

**Part 1:** Initialization

**Step 1:** Construct an initial line plan

We construct a line plan that satisfies service constraints and optimizes a weighted sum of the passenger and operator cost with the line planning module. Beforehand, we check for infeasible lines in the line pool as discussed in Section 3. We check with the timetable module if a feasible timetable can be constructed for this line plan. A feasible timetable is a timetable in which no occupation intervals of trains overlap: if a station or platform is occupied by one train, no other train can occupy this station or platform until the first train leaves it. In case the constructed line plan is not timetable-feasible, different strategies can be applied. A straightforward strategy is to take the second best line plan for the weighted sum of the passenger and operator cost and if the second best is not timetable-feasible then the third best and so on. The disadvantage of this strategy is that it is possible that a lot of line plans are to be tested before a timetable-feasible line plan is found, because no insight in the problem is used. We propose another more effective strategy for a network with restricted shunt capacity as is assumed in this research. Due to the restricted shunt capacity in the terminal stations, the occupation of the terminal stations is critical in finding a timetable-feasible line plan. So an effective strategy for looking for a timetable-feasible line plan with a close to optimal objective value is by restricting the number of lines that share a terminal station. If a line using a shared terminal station also passes a different station that may be a terminal station, a close to optimal solution is a line plan in which this line is replaced by one that ends at this alternative terminal station. This decreases the number of lines sharing an end station and in some cases has minimal impact on operator and passenger costs. This new line plan is only feasible in case all service constraints remain fulfilled.

**Part 2:** Iterative steps

**Step 2:** Evaluate the line plan

Construct a timetable with the timetable module that maximizes the minimum buffer times between a selection, or between all the train pairs in the line plan. Calculate the minimum buffer times between all line pairs in the line plan, and the overall minimum buffer time.

Test the following stopping criteria:

- STOP if the minimum buffer time is closer than 5% to the desired minimum buffer time. The *desired minimum buffer time* can be found by identifying the station area or track section which has the highest ratio of occupation time over free time (i.e. buffer times) and dividing the free time by the number of trains that pass by this section or station. This stopping criterion is referred to as 'DES' (from *desired*).
- STOP if the minimum buffer times do not improve the best found value for three successive iterations. This stopping criterion is referred to as 'BFV' (from *best found value*).

Otherwise, select the most critical line from the list. The most critical line is the line that is responsible for the highest number of buffers in the category of smallest buffers in the list. This is illustrated in the example in Section 6.2. In case of a tie, look at the next category of buffer times to identify the most critical line. If there is still a tie, let the decision be made by the line planning module in Step 3, based on the objective values there. The thresholds to categorize the buffer times depend on the operator. Go to Step 3.

**Step 3:** Adapt the line plan by changing the stopping pattern

Make a new line plan that alters the time length of the critical line by adding or removing a stop in a station on that line, such that this line becomes more flexible. This flexibility will be used to improve the buffer times in the timetabling module. This effect can be seen in the results and the example presented in Section 6. There are three important considerations. Firstly, changing the time length can also make a line infeasible as discussed in Section 3 which has to be avoided. Secondly, an extra stop cannot be added to a line in cases where there are no skipped stations on the line. Thirdly, some stations cannot be skipped due to service constraints.

We potentially solve the line plan problem with three different line pools, sequentially, to attempt to find a feasible solution. If a feasible line plan is found, the line plan problem does not need to be solved for the other line pools in the sequence. The three line (and frequency) pools are as follows.

i. All lines of the solution of the previous iteration are fixed, including their frequency, except that of the critical line. We add all lines that differ by one stop from the critical line. For those lines we only allow the frequency of the critical line.

ii. All lines of the solution of the previous iteration are fixed, including their frequency, except that of the critical line. We add lines to the line pool that differ by one stop from the critical line, which we now allow at any frequency.

iii. Solution lines that share no stations with the critical line are fixed. We introduce lines that differ by one stop from the critical line and lines that differ from other non-fixed non-critical lines by one station, at any frequency.

Because the number of lines in the line pool and the number of feasible solutions is much more restricted, the run time for the line planning module is now much shorter. The objective function is the same as in Step 1. For the first line pool, if feasible, the best alternative line will be selected, i.e. the line that provides the lowest passenger and operator costs. For the second line pool, if feasible, one or more of these new lines will be selected, often with a frequency combination that sums to the frequency of the critical line. For the third line pool, one or more lines similar to the critical line will be selected, and other solution lines from the previous iteration may be replaced with one or more similar lines. A simple example of solution from the third line pool is where a stop at a certain station is shifted from the critical line to a line that first skipped this station. The time length of the critical line changes by removing a stop and the station that is now skipped by the critical line is still served, but by another line. Note that in this example, the length of the non-critical line is also changed. A composition resulting from the second line pool is captured in the example in Section 6.2.

In the case that a feasible solution is found, return to Step 2. In the case that no feasible solution is found, and if there is a second most critical line, solve the three line plan problems for the second most critical. Otherwise STOP.

**End**

The selected final solution is the combination of line plan and timetable constructed during the iterative approach that has the best minimal buffer time taken over all iterations. In case of a tie, the best weighted sum of passenger and operator cost is used as criterion. As a result the selected final solution is always the best one found during the search. The intuition behind the integrated approach is the following. Changing the number of stops of a line changes the time length of the line. This time length of a line affects the flexibility of that line. So we alter the stopping pattern of a line to make the line more flexible in order to improve the spreading in the whole network. The station where the stopping pattern is changed is decided by the line planning module, which takes a weighted sum of passenger and operator cost into account. These costs are not taken into account during timetable construction. We note here that in general we do not require that the lines created to modify a line plan are all in the original pool of lines specified for the original problem. This explains why the adapted line plan can have a better weighted sum of passenger and operator cost than the original one. For the stopping criterion 'BFV' we take three non-improving iterations, to both restrict the run time while still allowing improvements that require multiple lines to change before a resultant improvement in minimal buffer time is observed.

# 5 Case study

The railway system on which the approach is tested is the S-tog network in Copenhagen operated by Danish railways operator DSB. This is a cyclic high-frequency network with a one hour period of repetition and which transports 30 000 to 40 000 passengers per hour at peak times between 84 stations. The OD data used comes from the operator, with non-zero demand for 65% of all possible pairings of stations. The network is visualized in Figure 8. It contains a central corridor, indicated in red; five 'fingers', indicated in blue; and a circle track, indicated in yellow. With almost no exception, there are at least two tracks in between every two adjacent stations and mostly exactly one track in each direction. We model the network with exactly one track in each direction between two stations and one platform in each direction in every station. One extra platform for turning is modelled in intermediate stations which can be used as terminal stations, see Figure 1c. The network is built such that crossings in between station areas are avoided by tunnels and bridges and there are only a very few exceptions to this in the real network. Moreover, there are very few locations where trains from opposite directions have to cross each other during normal conditions. In this research we assume that trains in opposite directions only interact with each other in terminal stations. Furthermore, trains are not allowed to overtake each other.

We model that each train occupies all station areas on its route for one minute. In case the train has a stop in the station area this occupation time encloses the dwell time which is observed to be 20 seconds in practice. However, the occupation time is considered the same whether the train stops or not in this station area. This is to rule out that two trains or more can use this station area at the same time, while at most one of them has a stop. This is important as we model only one platform in each station area and trains may not overtake each other. The buffer and occupation intervals of a station area are always disjoint and the buffer and occupation times count up to the length of the period. The driving times (an occupation time in a first station plus running time between this first station and a next station) are given by DSB S-tog and are independent of the line or train passing these stations. Only if a station is skipped, then one minute is subtracted from the driving time between this station and the next station. This rule is also inherited from DSB S-tog. In reality, this is partly the occupation time and partly the running time that is superfluous, but we model this by subtracting one minute from the running time, to stick to the station occupations in order to rule out overtaking conflicts.

During peak hour on weekdays, there is a service requirement of 30 trains per hour through the central corridor in each direction. The minimum *desired buffer time* (as defined in stopping criterion 'BFV') in the DSB S-tog network is therefore one minute, which is (60 min - 30 min)/30, where 60 minutes is the period length of the cyclic timetable and 30 trains occupy a station in the central corridor each for one minute. One requirement specified by the operator is that only lines at frequency three, six, nine or twelve are allowed in the weekday line plan. This restriction decreases the probability of frequency combination infeasibility (though that is not necessarily the intention for the requirement). In order to enable and maintain this high frequency in the central corridor, the spreading of the trains in this part of the network is crucial. Therefore the timetable module will be sequentially used twice with two different objective functions. First, the minimum buffer times in the central corridor are optimized. In a second optimization round the minimum buffer in the rest of the network is optimized while bounding the buffer times in the central corridor by the value found in the first optimization. We also considered one combined weighted objective function, but this proved computationally worse in our experiments, i.e. the run times were significantly higher. The selected final solution is the combination of line plan and timetable constructed during the iterative approach that has the best minimal buffer in the central corridor as a first criterion, has the best minimal buffer overall in the network as a second criterion and has the best weighted sum of passenger and operator cost as a third criterion, taken over all iterations. The second and third criterion are only used in case of a tie (in the first and second criterion, respectively).

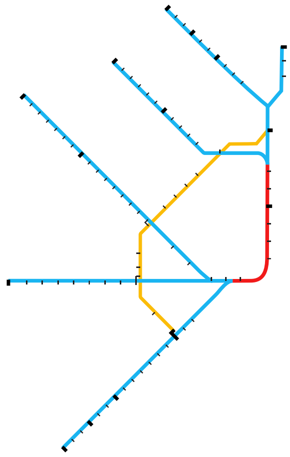


Figure 8: DSB S-tog network of Copenhagen

We test our approach on ten line plans for this network. The approach can be applied to a pre-existing line plan, or applied to first create and then improve a line plan. The full approach is tested for five line plans created as described in Step 1 of the integrated approach, while the other five line plans come from the operator or are created by hand. The first two line plans (1-2) were recently in use for the S-tog network in Copenhagen. We have not considered the current line plan as it is only temporarily active and specifically developed for implementing the new signaling system in the central corridor of the network. The third line plan (3) is a night line plan for weekdays. As the demand during night time is lower, the frequencies of the lines in this line plan are also lower. All other line plans are line plans that are planned with the requirements for use during daytime on weekdays. So, the setting of this third line plan is different from the other ones. This third line plan is also not the current plan in the S-tog network as at the present time a temporary plan is also in use during night time. The fourth up to the eighth line plan (4-8) are created within our algorithm by solving the weighted sum line planning module, using a range of weights that give distinct line plans. For each of these weights, we solve the line planning model with a one hour time limit and to a 0.5% relative gap limit, and terminate when either is reached. We initially solve the line planning module finding distinct solutions with no consideration for the feasibility of timetables except for infeasible lines as explained in Section 3. Then we test whether or not these are timetable-feasible. We find for these considered weights that only a single line plan (4) is feasible for timetabling. This endorses the statement that the output of a previous level in railway planning is not necessarily adequate for the next planning level (Schöbel, 2015). For those that are not feasible, we introduce restrictions on the use of terminal platforms, requiring only one line terminating at each terminal platform a station has. This is described in Step 3 of the integrated approach in Section 4. This is sometimes too conservative, since it can be possible for more than one line to share a single terminal platform. Conversely this alone does not guarantee that a feasible timetable is present for a line plan, but we observe that it is often a sufficient restriction. Applying this restriction we find four other distinct line plans (5-8). We note that, when considering the two line plan objectives of operator cost and passenger cost, none of the final four plans dominates any other. The ninth and the tenth line plan (9-10) are two special 'manually created' line plans, which are each based on one of the weighted-objective line plans (5 and 8 respectively). These paired plans only differ in stopping pattern from the plan they are manually adapted from, as we force every line to stop in every station it passes, while the original line plans contain many skipped stations. We want to investigate if each pair (5 and 9, 8 and 10) converges to a final line plan of similar quality when we modify stopping patterns of lines.

# 6 Results and discussion

In this section we show the results of the integrated approach for all ten line plans described in Section 5. Furthermore, we demonstrate the integrated approach for line plan 2 and include for this line plan the time-distance diagrams for the central corridor for the initial and the finally selected timetable.

## 6.1 Results for ten line plans

A first performance indicator is the estimated operator cost of a line plan. This cost is calculated by the line planning module. The total cost of a line plan is simply the sum of estimated operator costs for each line, which we take as given by the rail operator, here DSB. Each line in the pool has an operating cost associated with each frequency at which it could operate, and in calculating the total cost there are no additional considerations given to the combinations of lines.

A second performance indicator is the estimated passenger cost of a line plan. This cost is calculated by the line planning module. It is the sum of travel time of all passengers in the OD matrix. Because a timetable is not known (by the line planning module), the transfer time is estimated based on the frequency of the line as half of the time between two trains of that commuter line. For each passenger transfer an additional penalty of six minutes is added to the estimated passenger cost as transfers are perceived to be worse than direct connections.

The third performance indicator is the minimum buffer time between train pairs in the central corridor of the DSB S-tog network, optimized by the timetable module. The fourth performance indicator is the minimum buffer between train pairs everywhere in the network, while bounding the minimum buffer time in the central corridor first.

This fourth performance indicator is also optimized by the timetable module. The focus on the minimum buffer time first in the central corridor of the network and thereafter on the minimum buffer time overall is in consultation with DSB S-tog.

A fifth performance indicator is the sum of the inverse of the minimum buffer times between train pairs in each station that they have in common (and pass by in the same direction). We take the inverse minimum buffer times in order to give smaller buffers a higher weight than large buffers. As in Dewilde et al. (2013) a buffer time smaller than the time discretization $\epsilon$ (here 0.1 minute) has a contribution of 15 to the sum of the inverse buffer times. So the lower the sum of the inverse buffer times the better, because this means generally larger buffer times. The results are summarized in Table 2, Table 3 and Table 4.

| | | Min buffer in central corridor (min) | | | Min buffer overall (min) | | | Sum of inverse buffer times (1/min) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Line plan | | initial | final | impro | initial | final | impro | initial | final | impro |
| 1 | real | 0.63 | 1.00 | +58% | 0.00 | 1.00 | $+\infty$% | 2 639 | 2 189 | -17% |
| 2 | real | 0.73 | 1.00 | +36% | 0.00 | 1.00 | $+\infty$% | 2 348 | 2 212 | -6% |
| 3 | real | 3.00 | 3.00 | +0% | 0.70 | 2.55 | +264% | 482 | 382 | -21% |
| 4 | random | 0.33 | 0.64 | +93% | 0.00 | 0.05 | $+\infty$% | 3 293 | 3 323 | +1% |
| 5 | random | 0.17 | 0.83 | +400% | 0.00 | 0.20 | $+\infty$% | 3 840 | 2 365 | -38% |
| 6 | random | 0.37 | 0.99 | +170% | 0.00 | 0.01 | $+\infty$% | 3 211 | 2 929 | -9% |
| 7 | random | 0.23 | 0.23 | +0% | 0.00 | 0.00 | +0% | 4 324 | 4 324 | -0% |
| 8 | random | 0.23 | 0.23 | +0% | 0.00 | 0.00 | +0% | 4 357 | 4 348 | -0% |
| 9 | special | 1.00 | 1.00 | +0% | 0.70 | 1.00 | +43% | 2 318 | 2 203 | -5% |
| 10 | special | 0.92 | 1.00 | +8% | 0.00 | 0.00 | +0% | 3 179 | 3 362 | +6% |

Table 2: The integrated approach significantly improves the buffer times in eight out of ten of the studied line plans.

In Table 2 we observe that there is a significant improvement in the buffer times for eight out of the ten line plans. For three out of the ten line plans, the desired minimum buffer time is reached both in the central corridor and in the rest of the network. For three other line plans the desired minimum buffer time is reached in the central corridor but not in the rest of the network. Furthermore, we see that the sum of the inverse buffer times between train pairs in every station they have in common decreases, which means that the buffer times themselves increase as desired. Moreover, the results on the sum of the inverse buffer times are very similar to the minimum buffer time results in the central corridor and in the overall network. We note that a big absolute improvement of the minimum buffer time in the central corridor (or of the minimum buffer time overall) corresponds to a big improvement in the sum of the inverse buffer times, and vice versa. Unfortunately, for two out of the ten line plans (7 and 8) no improvement in minimum buffer time is achieved. To identify the critical line in Step 2 of the integrated algorithm, we categorize the buffers as zero, smaller than 30 seconds, smaller than one minute and bigger than one minute. We observe that for the timetables corresponding to the initial line plans of 7 and 8 almost half of the minimum buffer times between line pairs are smaller than 30 seconds, while for the other line plans this is at most one third of the minimum buffer times. As a possible explanation, we note that for these line plans almost every line has a pairwise minimum buffer time below half a minute with some other line, and we may therefore expect that multiple lines must be modified to see an improvement. We typically change a single line in every iteration and in such cases, it may take more than three non-improving iterations before seeing an improvement, given that every line plan we consider has between six and ten lines.

The buffer times in Table 2 appear to be small. However, as discussed earlier, the minimum desired buffer time in a daytime week line planning is one minute and this is thus the value of the stopping criterion in Step 2 of the integrated approach. The maximal minimum buffer time everywhere in the network is also restricted by this maximal minimum buffer time in the central corridor. Moreover, even if the buffer time between two trains is zero the timetable is still feasible. A zero buffer time means that the second train enters a station area immediately after the first train releases this station area. Note that the release time of the first train implies that this train is already sufficiently far away. However, a zero buffer time is undesirable and any delay of the first train is immediately propagated to the second. In the case study, a line plan performs best if it allows the desired buffer time of at least one minute between every two trains. As an exception, for line plan 3 the desired buffer time is three minutes (i.e. only 15 trains in each direction have to pass through the central corridor during night time on a weekday). The desired buffer time is achieved for line plan 1, 2 and 9 overall in the network and in line plan 3 and 10 in the central corridor.

One explanation for not reaching the desired value for some line plans could be no further improvement was made, because at each iteration the same line was identified as being critical. Either changing this line was no longer feasible or changing this line was feasible, but did not result in acceptable solutions. If changing the critical line is not feasible, then the second most critical line of the last found line plan is chosen. In the current algorithm, however, if the critical line itself does not give rise to good results, there is no backtracking to a previous iteration to try the second most critical line.

In Table 3 the operator cost and the passenger cost for the initial and final line plan are presented. We observe that for seven out of the ten line plans, the difference in operator cost and passenger cost is smaller than 1.5% when applying the integrated approach. Some

| Line plan | | Operator cost ($\times 10^5$) initial | final | change | Passenger cost ($\times 10^7$) initial | final | change |
|---|---|---|---|---|---|---|---|
| 1 | real | 6.79 | 6.84 | +0.74% | 4.17 | 4.23 | +1.47% |
| 2 | real | 6.84 | 7.21 | +5.40% | 4.22 | 4.21 | -0.12% |
| 3 | real | 3.40 | 3.43 | +0.64% | 1.05 | 1.06 | +1.08% |
| 4 | random | 6.25 | 6.64 | +6.23% | 4.24 | 4.27 | +0.87% |
| 5 | random | 6.48 | 6.80 | +4.94% | 4.27 | 4.29 | +0.36% |
| 6 | random | 6.66 | 6.74 | +1.13% | 4.12 | 4.14 | +0.51% |
| 7 | random | 7.02 | 7.02 | +0.00% | 4.09 | 4.09 | +0.00% |
| 8 | random | 8.27 | 8.32 | +0.71% | 4.05 | 4.04 | -0.22% |
| 9 | special | 7.15 | 7.14 | -0.17% | 4.43 | 4.44 | +0.32% |
| 10 | special | 9.00 | 9.01 | +0.20% | 4.35 | 4.30 | -1.06% |

Table 3: For seven out of the ten line plans, the difference in operator cost and passenger cost is smaller than 1.5% when applying the integrated approach.

plans do improve for one measure but become worse for another and although it is possible for both to improve (which is possible since we allow lines that were not in the original line pool), we do not observe this here. Note that for line plans 4 and 5 we do see a relatively large increase in operator cost (6.23% and 4.94%), combined with an increase in passenger cost which may be a relatively large cost to pay for timetable improvement in terms of robustness. In contrast, for line plan 2 though we see a similarly large increase in operator cost but a reduction in passenger cost. Here the impact must be judged by the perceived relative importance of the two measures together with the expected effect on the robustness of the service. Since for each of the line plans holds that the passenger cost calculated based on the final timetable differs with less than 0.05% from the passenger cost estimated by the line planning module, we did not present these results separately.

In Table 4 some characteristics of the integrated approach are presented. We indicate under which stopping criterion the algorithm was terminated. We see that for three out of the ten line plans the desired minimum buffer time is achieved in the central corridor and in the rest of the network ('DES') and for the remaining seven the algorithm ended with the best found value ('BFV'). The table also reports how many iterations the integrated approach passed through before a stopping criterion was achieved. This value ranges between one and seven. We report the number of out-of-pool lines which are in the final solution, referring to lines that are in the final solution but do not come from the original, restricted, line pool. These out-of-pool lines are similar to a line in the pool but with a

| Line plan | | Stopping criterion | # iterations | # out-of-pool lines | Average run time timetabling (min) |
|---|---|---|---|---|---|
| 1 | real | DES | 4 | 5 | 183.40 |
| 2 | real | DES | 3 | 5 | 4.88 |
| 3 | real | BFV | 2 | 3 | 0.50 |
| 4 | random | BFV | 6 | 5 | 75.71 |
| 5 | random | BFV | 7 | 7 | 385.56 |
| 6 | random | BFV | 7 | 5 | 167.19 |
| 7 | random | BFV | 3 | 0 | 126.75 |
| 8 | random | BFV | 3 | 1 | 9.25 |
| 9 | special | DES | 1 | 2 | 47.50 |
| 10 | special | BFV | 5 | 3 | 346.83 |

Table 4: Characteristics of the integrated approach

modified stopping pattern. We observe that the five line plans with the highest number of out-of-pool lines (line plan 1, 2, 4, 5 and 6) have the greatest relative improvement of the minimum buffer time in the central corridor; have the greatest increase in operator cost; and (with one exception) have the highest increase in passenger cost. Therefore, including new lines in the line pool has the potential to improve the minimum buffer times significantly, but may have a negative effect on passenger and operator costs.

The final characteristic in Table 4 is the run time for timetabling. The total run time for timetabling consists of the creation of the optimal timetable in Step 2 of the integrated approach for each iteration and for determining the initial timetable. As described in Section 5 the timetable is solved sequentially with two objective functions at each iteration. Firstly, the buffer time in the central corridor is maximized, and secondly the buffer time in the rest of the network is increased with a bound on the buffer time in the central corridor fixed by the first step.

For example the algorithm stops after three iterations for line plan 2. This means that eight timetables are calculated: two initially (for the two optimization criteria) and two at each of the three iteration steps. The average run time for timetabling for optimizing line plan 2 is 4.88 minutes, which means that the run time for each calculated timetable in the integrated approach on average is 4.88 minutes. All timetables are calculated with CPLEX 12.6 on an Intel Core i7-5600U CPU @ 2.60 GHz. We observe that there is a high variability in the average run times for the different line plans. Moreover, a high computation time may occur in case where there is both a big improvement (line plans 1 and 5) and also where there is either no improvement (line plan 7) or only a small improvement (line plan 10). Furthermore, the run time for timetabling can differ significantly from one iteration to the next. Even if two line plans are not dissimilar one can be intrinsically more difficult to solve. An explanation could be that due to changes in the stopping pattern, trains of different lines are more or less susceptible to catching up with each other in the fingers of the S-tog network, resulting in it being more complex to spread the trains optimally. The timetable module runs to optimality (relative gap smaller than 0.05%) for about 85% of the timetables. The average run time per timetable optimized within the time limit of 12 hours is 3801 seconds. For the other optimizations a time limit of 12 hours is imposed. The line planning module for the selected line pool determined by the critical line runs to optimality in all instances, taking at most up to ten minutes for cases where many lines are to be changed.

Finally, from Tables 2, 3 and 4, we deduce that line plans 5 and 9 did not converge to the same final line plan and line plans 8 and 10 did not either. We see that the final line plan and timetable for line plan 9 and 10 score better on robustness, i.e. minimum buffer time between line pairs is larger, while line plan 5 and 8 score better on operator and passenger cost. Based on these results, the final decision on which line plan is preferred, rests with the operator. In our opinion, the optimized version of line plan 1 will be the most passenger robust in relation to the corresponding expense for the operator and the passengers.

## 6.2 Illustration

In order to illustrate the integrated approach, we apply it to line plan 2. As this is an existing line plan, we skip Part 1 of the algorithm and only look at the iterative steps in Part 2. The estimated operator cost of this line plan is $6.84 \times 10^5$, the estimated total passenger travel time is $4.22 \times 10^7$. The optimal value for the minimum buffer time for this line plan in the central corridor of the network is 0.73 minutes. The optimal value for the minimum buffer time overall if the minimum buffer time in the central corridor is bounded below by 0.73 minutes is zero minutes. The minimum buffer times between the line pairs are present in minutes in Table 5. The smallest buffer time between line $i$ and $j$ is the same as the smallest buffer time between line $j$ and $i$, so Table 5 is in fact symmetric, but we omitted here the superfluous information. If two lines do not share a part of the network, the minimum buffer time between these lines is indicated as 60 minutes, which is the period length of the cyclic timetable. The smallest buffer time in Table 5 is zero minutes. This buffer time is between line 1 and itself. This means that the turn platform of line 1 in one of its terminal stations is permanently occupied by a train of the first line. Obviously, the critical line here is line 1.

| line | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 1.73 | 0.73 | 0.73 | 2.47 | 2.88 | 3.13 | 2.47 | 60 |
| 1 | - | **0.00** | 2.47 | 0.73 | 1.15 | 2.57 | 0.73 | 60 |
| 2 | - | - | 2.2 | 1.30 | 1.47 | 1.30 | 3.50 | 60 |
| 3 | - | - | - | 7.17 | 0.88 | 1.40 | 6.70 | 60 |
| 4 | - | - | - | - | 4.45 | 7.58 | 1.03 | 60 |
| 5 | - | - | - | - | - | 2.87 | 0.80 | 60 |
| 6 | - | - | - | - | - | - | 6.97 | 60 |
| 7 | - | - | - | - | - | - | - | 0.57 |

Table 5: First iteration: The minimum buffer time overall is zero minutes, if the minimum buffer time in the central corridor is bounded below by 0.73 minutes

The line planning module adds a stop to line 1 by considering only the line pool that contains alternatives for line 1 of the same frequency in the second iteration. The new estimated operator cost increases to $6.99 \times 10^5$ and the new estimated total passenger travel time slightly increases to $4.23 \times 10^7$. The optimal value for the minimum buffer time for this line plan in the central corridor of the network is 1.00 minute. The optimal value for the minimum buffer time overall if the minimum buffer time in the central corridor is bounded below by 1.00 minute is still zero minutes. The minimum buffer times between the line pairs of the first modification of line plan 2 are present in minutes in Table 6. The smallest buffer time between two lines is still zero minutes. This buffer time is now only associated with line 6. Again, this means that the turn platform of line 6 in one of its terminal stations is permanently occupied by a train of line 6. The critical line is line 6.

| line | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 6.17 | 1.01 | 2.99 | 60 | 3.01 | 1.00 | 3.99 | 2.99 |
| 1 | - | 2.14 | 2.99 | 60 | 1.00 | 1.99 | 1.00 | 1.00 |
| 2 | - | - | 0.29 | 60 | 1.00 | 1.00 | 1.01 | 1.00 |
| 3 | - | - | - | 0.67 | 60 | 60 | 60 | 60 |
| 4 | - | - | - | - | 1.82 | 2.99 | 2.99 | 2.99 |
| 5 | - | - | - | - | - | 7.23 | 6.99 | 6.99 |
| 6 | - | - | - | - | - | - | **0.00** | 1.00 |
| 7 | - | - | - | - | - | - | - | 3.02 |

Table 6: Second iteration: The minimum buffer time overall is zero minutes, if the minimum buffer time in the central corridor is bounded below by 1.00 minute

In Step 2 of the third iteration, the line planning module first considers line pools that contain only alternatives for line 6 of the same frequency, but they do not lead to a feasible line plan. We then consider the line pool that contains alternative lines for line 6 for different frequencies. The result is a feasible line plan that does not include original line 6 and 7, each of frequency three, but contains a new line of frequency six. The original line 6 stops at the same stations as the original line 7, but has some additional stops at one end of the line. The new line is the combination of the original lines 6 and 7. The new estimated operator cost is $7.22 \times 10^5$ and the new estimated total passenger travel time is $4.20 \times 10^7$. The optimal value for the minimum buffer time of this line plan in the central corridor of the network remains 1.00 minute. However, the optimal value for the minimum buffer time overall if the minimum buffer time in the central corridor is bounded below by 1.00 minute has now increased to 0.70 minutes. The minimum buffer times between the line pairs in the third iteration are present in minutes in Table 7. The smallest buffer time is now only associated with line 3, so the new critical line is line 3.

| line | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 10.16 | 2.99 | 1.00 | 60 | 2.99 | 8.98 | 1.00 |
| 1 | - | 2.10 | 2.99 | 60 | 1.00 | 2.01 | 1.00 |
| 2 | - | - | 1.15 | 60 | 1.00 | 1.01 | 3.01 |
| 3 | - | - | - | **0.70** | 60 | 60 | 60 |
| 4 | - | - | - | - | 2.06 | 3.01 | 2.99 |
| 5 | - | - | - | - | - | 8.05 | 1.00 |
| 6 | - | - | - | - | - | - | 1.69 |

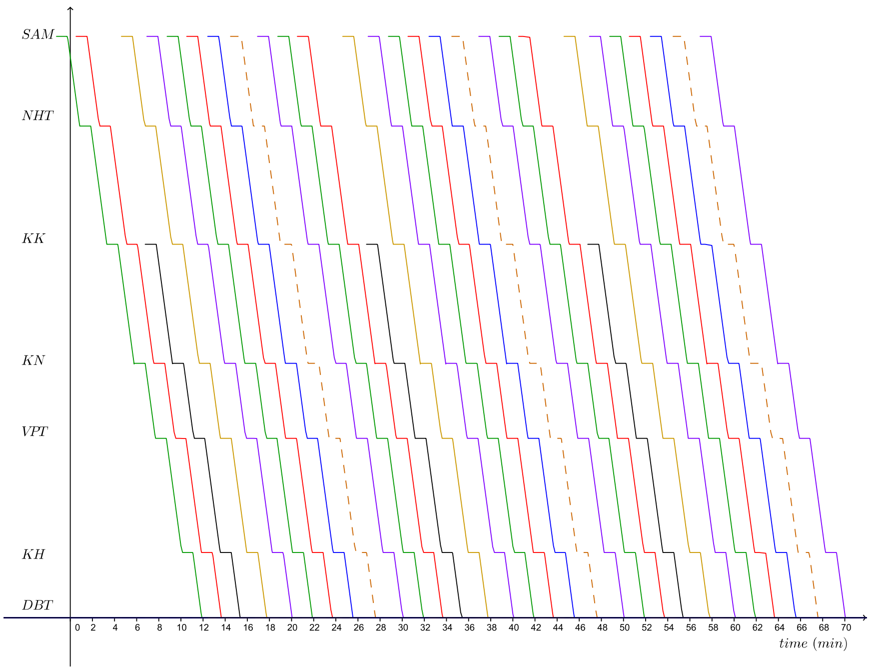
Table 7: Third iteration: The minimum buffer time overall is 0.70 minutes, if the minimum buffer time in the central corridor is bounded below by 1.00 minute

The line planning module skips a stop of line 3 in the fourth iteration. The new estimated operator cost is $7.21 \times 10^5$ and the new estimated total passenger travel time is $4.21 \times 10^7$. The optimal value for the minimum buffer time of this line plan in the central corridor of the network is still 1.00 minute. The optimal value for the minimum buffer time overall if the minimum buffer time in the central corridor is bounded below by 1.00 minute is now 1.00 minute. The minimum buffer times between the line pairs in the fourth iteration are present in minutes in Table 8. This minimum buffer time overall is closer than five percent to the minimum desired buffer time of one minute, so this is the last iteration of the algorithm.

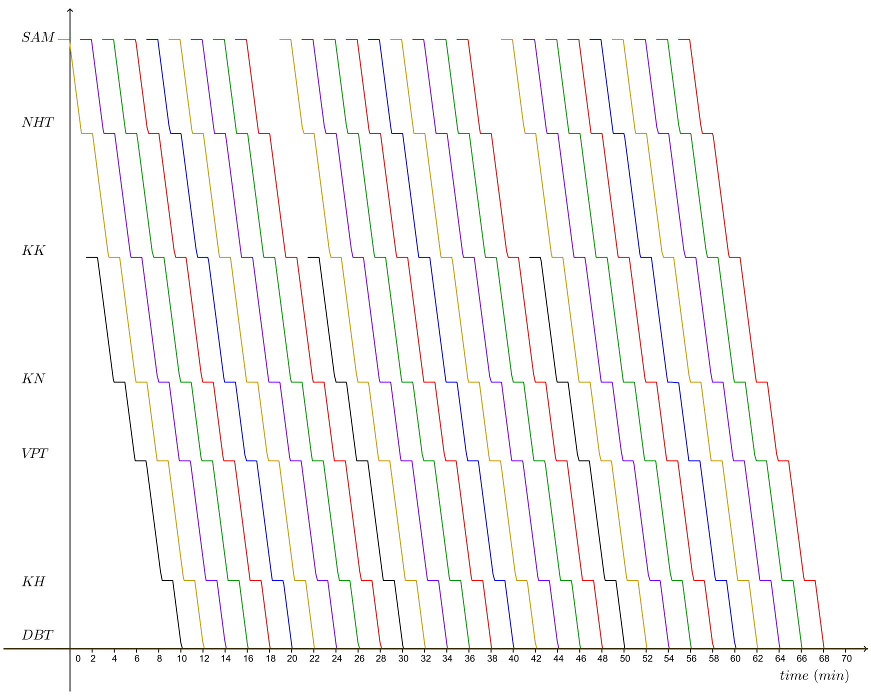
| line | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 10.20 | 3.01 | 1.00 | 60 | 2.99 | 8.98 | 1.00 |
| 1 | - | 2.15 | 2.99 | 60 | 1.00 | 1.99 | 1.00 |
| 2 | - | - | 1.09 | 60 | 1.00 | 1.00 | 2.99 |
| 3 | - | - | - | 1.00 | 60 | 60 | 60 |
| 4 | - | - | - | - | 2.11 | 2.99 | 2.99 |
| 5 | - | - | - | - | - | 8.02 | 1.00 |
| 6 | - | - | - | - | - | - | 1.66 |

Table 8: Fourth iteration: The minimum buffer time overall is 1.00 minute, if the minimum buffer time in the central corridor is bounded below by 1.00 minute

Figure 9 and Figure 10 present the time-distance diagrams of the initial and the finally selected timetable for line plan 2, corresponding to Tables 5 and 8 respectively. Figure 9 zooms in on the timetable for the central corridor, whilst Figure 10 presents the timetable of the central corridor and two fingers of the network. These time-distance diagrams only show the trains driving from north to south. Horizontal parts of a time-distance path represent station activities in station areas where the train stops. The other parts of a time-distance path include running activities between two station areas and station activities in station areas where the train does not stop. Both the initial and the finally selected timetable have 30 trains per hour passing through the central corridor. However, the initial timetable has seven lines passing through the central corridor, while the finally selected timetable has only six different lines passing through the central corridor. The yellow-brown solid line and the brown dashed line of frequency three are combined to one line of frequency six during the line planning phase of the third iteration as discussed above. For both timetables there is one line that does not pass through the central corridor, but uses the circle track. Both timetables also have one line which route starts inside the central corridor. In Figure 9, we can observe that the trains are regularly spread for the finally selected timetable, while this is not the case for the initial timetable. Furthermore, in Figure 10, we see that the final timetable spreads the lines better in the lower finger and equally well in the upper finger. Note that the yellow line has more stops than the blue line in the lower finger since its time-distance paths contain more horizontal parts. By combining the yellow-brown solid and dashed line, it became possible to put the slower yellow-brown line after the faster blue line when leaving the central corridor. This avoids the blue line catching up with the yellow-brown line as is the case in the initial timetable.

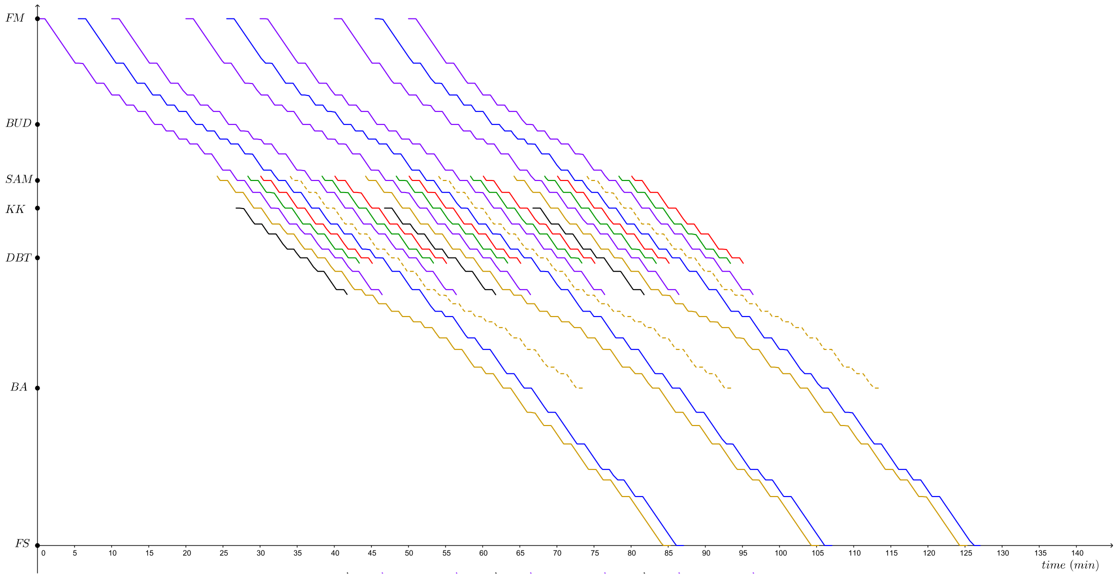



(a) Time distance diagram for the initial timetable of line plan 2.

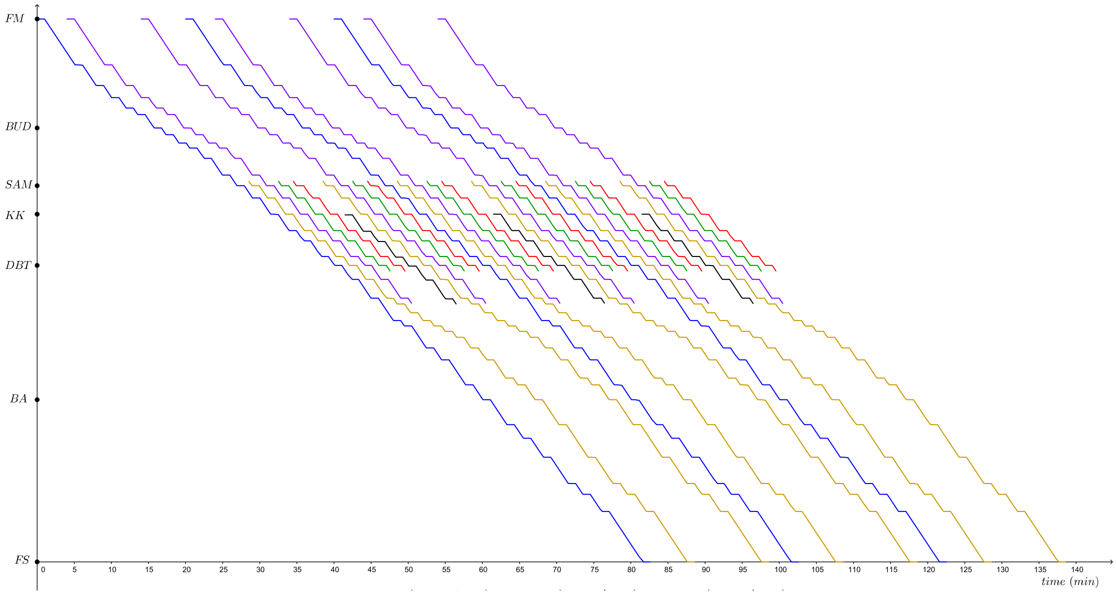



(b) Time distance diagram for the finally selected timetable of line plan 2.

Figure 9: These time-distance diagrams visualize the timetables restricted to the central corridor for the trains from north to south (Svanemøllen (SAM), Nordhavn (NHT), Østerport (KK), Nørreport (KN), Vesterport (VPT), København H (KH), Dybbølsbro (DBT)). The colors indicate the different lines. The same color is used to indicate lines between the same terminal stations in both diagrams.




(a) Time distance diagram for the initial timetable of line plan 2.




(b) Time distance diagram for the finally selected timetable of line plan 2.

Figure 10: These time-distance diagrams visualize the timetables of the finger from Farum (FM) to Svanemøllen (SAM), the central corridor (Svanemøllen (SAM), Nordhavn (NHT), Østerport (KK), Nørreport (KN), Vesterport (VPT), København H (KH), Dybbølsbro (DBT)) and the finger from Dybbølsbro (DBT) to Frederikssund (FS) for the trains from north to south. The colors indicate the different lines. The same color is used to indicate lines between the same terminal stations in both diagrams.

## 7 Conclusion and further research

This paper presents a heuristic algorithm that builds a line plan from scratch resulting in a feasible and robust timetable. Our method iterates interactively, alternating between a line planning module and a timetabling module, improving the robustness of an initially built line plan. Both modules consist of an exact optimization model. The line planning module optimizes a weighted sum of passenger and operator costs, while the timetabling module focuses on improving minimum buffer times between line pairs. Appropriate and sufficiently large buffer times between train pairs are needed to reduce the risk of delays being propagated from one train to the next, thereby obtaining a robust railway schedule. The timetable module identifies a critical line based on the minimum buffer times between line pairs. The line planning module creates a new line plan in which the time length of the critical line is changed. Changing the time length of a line may create more flexibility in the schedule, which may result in improvements in robustness. The approach was tested for ten different line plans on the DSB S-tog network in Copenhagen. This is a high-frequency railway network with 84 stations, currently nine lines and restricted shunt capacity in the terminal stations. For eight out of ten initial line plans the robustness could be significantly improved, while the changes to the line plan resulted in most cases in a change of less than 1.5% to the weighted sum of operator and passenger cost. Ultimately the operator makes the final decision on the preferred criterion, considering the measures we have presented and others we have not captured.

An initial idea for future research is a smart extension of the integrated approach to overcome the situation where a certain line remains critical in each iteration, while keeping the computation time restricted. Another extension would be to allow different shunt characteristics in different terminal stations. In the presented research we had the very strict requirement present in the DSB S-tog system to have a schedule in which no train uses shunt capacity in a terminal station during daily operation. Furthermore, the development of a single integrated exact model that combines line planning and robust timetabling which is solvable in a reasonable amount of time for other real networks (similar to the DSB S-tog network) would be a next noteworthy step. A further idea for future research is to remove the requirement that trains of a line must operate exactly evenly timed (e.g. once every ten minutes for a six-per-hour line). Currently, this requirement is consistent with operation and ensures a regular service for customers. However it is potentially severely restrictive for the timetable given the tight spacing of trains in the central corridor. Relaxing this requirement could increase the complexity of the timetable model, both by expanding the solution space and by requiring new constraints and possibly an objective measure for the evenness of train timings.

# 8 Acknowledgements

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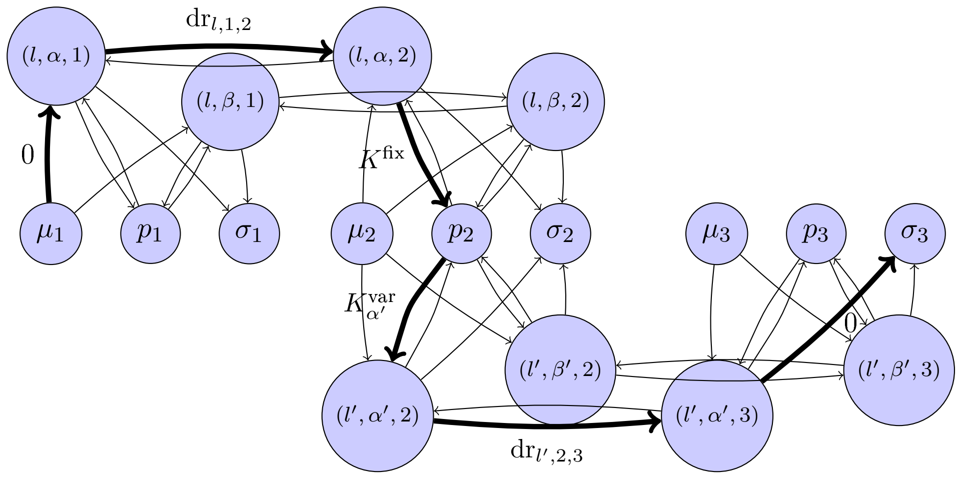
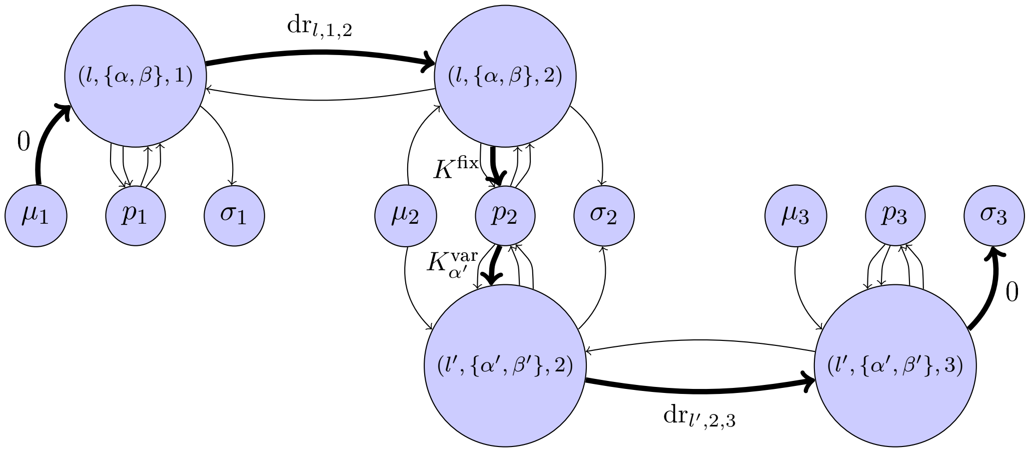
# Appendix A Passenger graph



(a) Full graph structure



(b) Reduced graph structure

Figure 11: For the network in Figure 5a, we now consider line $l$ and $l'$ each operating at two different frequencies, $\alpha, \beta$ and $\alpha', \beta'$ respectively. We show the graph structure before and after an aggregation of the different frequency components for a line. The corresponding (line, frequency, station) vertices are collapsed, leading to multi-edges where the frequency-dependent costs must be considered.

Also here, costs are labelled on the edges for a passenger travelling from station 1 to station 3, transferring lines at station 2, with used edges in bold; considering in Figure 11b that line $l'$ operates at frequency $\alpha'$.

# Appendix B List of symbols

| Sets | |
|---|---|
| $\mathcal{A}$ | Activity set |
| $\mathcal{A}^{\text{run}}$ | Set of running activities |
| $\mathcal{A}^{\text{station}}$ | Set of station activities |
| $\mathcal{A}^{\text{buffer}}$ | Set of buffer activities |
| $\mathcal{A}^{\text{turn}}$ | Set of turn activities |
| $\mathcal{A}^{\text{line spread}}$ | Set of line spread activities |
| $\mathcal{A}^{\text{turn-con}}$ | Set of turn connection activities |
| $\mathcal{E}$ | Event set |
| $\mathcal{E}^{\text{res}}$ | Set of reservation events |
| $\mathcal{E}^{\text{res},p}$ | Set of platform reservation events |
| $\mathcal{E}^{\text{rel}}$ | Set of release events |
| $\mathcal{E}^{\text{rel},p}$ | Set of platform release events |
| $(\mathcal{E}, \mathcal{A})$ | Event-activity network |
| $E$ | Edge set of the passenger graph |
| $\mathcal{F}_l$ | Potential frequencies of line $l$ |
| $\mathcal{L}$ | Line pool |
| $\mathcal{L}_r$ | Subset of lines in $\mathcal{L}$ that make use of resource $r$ |
| $\mathcal{R}$ | Infrastructure resources |
| $\mathcal{S}$ | Station (area) set |
| $\mathcal{S}_l$ | Set of stations on line $l$ |
| $T$ | Set of trains |
| $T_{\text{line spread}}$ | Set of train couples considered for line spreading |
| $T_{\text{turn}}$ | Set of train couples considered for turning |
| $V$ | Vertex set of the passenger graph |
| $\mathcal{X}$ | Line plan solution |
| **Specific elements of a set** | |
| $e$ | Edge |
| $l$ or $l'$ | Line |
| $r$ | (Infrastructure) resource |
| $s$ or $s'$ | Station |
| $t$ or $t'$ | Train |

| | |
|---|---|
| $v$ | Vertex |
| $X, Y$ | Terminal stations |
| Characteristics and properties of elements | |
| $a_v^s$ | Flow of passengers from station $s$ in vertex $v$ |
| $C_f$ | Capacity of line $l$ |
| $c_{l,f}$ | Operator cost of line $l$ at frequency $f$ |
| $d_{s,s'}$ | Demand between stations $s$ and $s'$ |
| $\mathrm{dr}_{l,s,s'}$ | Driving time of line $l$ from station $s$ to station $s'$ |
| $f_l$ | The frequency of line $l$ |
| $\mathrm{f}_e$ | Frequency of edge $e$ |
| $l_e$ | Line of edge $e$ |
| $\ell_t$ | Line operated by train $t$ |
| $L_a$ | Lower bound for activity $a$ |
| $\mu(_s)$ | Source vertex (of station $s$) in the passenger graph |
| $\mathrm{ntt}_s$ | Necessary turn time in station $s$ |
| $\mathrm{occ}_{l,s}$ | Occupation time of line $l$ in station $s$ |
| $p(_s)$ | Platform vertex (of station $s$) in the passenger graph |
| $\rho_{\tilde{s}_t,t}$ | Platform of train $t$ in terminal station $\tilde{s}_t$ |
| $\mathrm{rmin}_r$ | Min resource usage of resource $r$ |
| $\mathrm{rmax}_r$ | Max resource usage of resource $r$ |
| $\mathrm{run}_{l,s,s'}$ | Running time of line $l$ between stations $s$ and $s'$ |
| $\sigma(_s)$ | Sink vertex (of station $s$) in the passenger graph |
| $s_{l,i}$ | The i-th station of line $l$ |
| $\tilde{s}_t$ | Terminal station of train $t$, i.e. $s_{\ell_t,\lvert\mathcal{S}_{\ell_t}\rvert}$ |
| $\tau_e$ | Passenger cost for edge $e$ of the passenger graph |
| $\mathcal{T}_l$ | Travel time on line $l$ from begin to end station |
| $t_{l,r}^i$ | The i-th train of line $l$ considered on resource $r$ |
| $U_a$ | Upper bound for activity $a$ |
| $\varphi_p$ | Usage frequency of platform $p$ |
| Parameters | |
| $\epsilon$ | Time discretization |
| $\lambda$ | Weight for the operator cost in the line planning objective |
| $P$ | Period length of the cyclic timetable |
| Decision variables | |
| $k_a$ | Variable to induce a positive activity time for activity $a$ |
| $\pi_i$ | Event time of event $i$ |

| | |
|---|---|
| $x_{l,f}$ | Line decision variable of line $l$ at frequency $f$ |
| $y_s^e$ | Flow decision variable of station $s$ along edge $e$ |
| $z$ | Minimum overall buffer time |

## References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