Solving Multirate Partial Differential Equations using hat Finite Element basis functions

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Abstract—In this digest a universal approach for solving multirate partial differential equations (MPDEs) is presented. Taking advantage of already known concepts in literature, standard finite element hat functions are used to achieve an efficient solution of a multirate problem in time-domain. The MPDE is solved by a Galerkin-ansatz with hat functions in the fast time-scale and using conventional time discretization for the slow time-scale.

Index Terms—Finite element analysis, Partial differential equations, Numerical simulation, Circuit simulation

I. INTRODUCTION

Multirate partial differential equations (MPDEs) [1]–[3] are an established technique for the solution of high-frequency problems with separable time-scales, as solving them can be computationally much more efficient than conventional time discretization. A more elaborate concept are Warped MPDEs [4], which take advantage of mapping the excitation of a system to a reference signal and thus enabling more complex excitations. This digest focuses on solving MPDEs using a Galerkin-ansatz to discretize the periodic fast time-scale t_2 and a time discretization scheme (e.g. Runge-Kutta) for solving the slow time-scale t_1 . The choice of basis functions used for expanding the solution determines the efficiency of the method. Gyselinck et al. [3] recently proposed for the case of pulsed excitations of electrical circuits the use of PWM basis functions. However for more general problems, more flexible basis functions are needed.

II. GALERKIN-ANSATZ USING HAT FUNCTIONS

We propose the use of standard finite element (FE) hat functions. The j -th state variable thus can be expanded as

$$\hat{x}_j(t_1, t_2) = w_{j,0}(t_1) + \sum_{k=1}^{N_p} p_k(\tau) w_{j,k}(t_1) \text{ with } \tau = \frac{t_2}{T_s} \bmod 1,$$

where $p_k(\tau)$ are the FE hat functions with compact support, $w_{j,k}(t_1)$ are the coefficients and T_s is the period of the highest frequency component in the fast time-scale. The zero-th basis function $p_0(\tau)$ is constant to enable the representation of slowly-varying envelopes in the solution.

III. RESULTS AND OUTLOOK

The principle is validated on an idealized buck converter [3]. Fig. 1 shows the excitation and solution of the MPDE compared to the analytical solution of the buck converter DAE.

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Fig. 1. Solution of the idealized buck converter. The switching cycle of the pulsed signal is $T_s = 2$ ms, the number of basis functions is $N_p = 21$.

Fig. 2. Convergence of the method with respect to the voltage at the capacitor of the idealized buck converter.

The convergence plot in Fig. 2 shows that even with a small number of basis functions the agreement between the analytical solution and MPDE solution is excellent. Choosing practical parameters for the time discretization of the MPDE a total solving time of approximately 7 ms is needed. In contrast, for pure time discretization a total time of approximately 66 ms is necessary to reach the same relative error of the solution compared to the analytical solution in the l_2 -norm.

For the full paper the method will be applied on Warped MPDEs and the efficiency will be discussed.

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