Reduced Order Model for Accounting for High Frequency Effects in Power Electronic Components

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This paper proposes a reduced-order model of power electronic components based on the proper orthogonal decomposition. Starting from a full-wave finite-element model and several snapshots/frequencies, the reduced-order (RO) model is constructed. Local field values (e.g. magnetic flux density, electric current density, magnetic or electric field) and global quantities (e.g. characteristic complex impedance, joule losses) can be determined for the intermediate frequencies with a very low computational cost and high accuracy. Particular attention is paid to the choice of the most suitable snapshots by means of three different greedy algorithms, the performance of which is compared. We adopt an automatic greedy algorithm that only depends on the RO model.

Index Terms—Reduced-order models, proper orthogonal decomposition, full wave, capacitive effects, finite-element methods.

I. INTRODUCTION

THE EVER increasing switching frequencies in modern power electronic converters (from several kHz to several tens of MHz) generate capacitive effects that must be accounted for in early stages of the design for e.g. electromagnetic compatibility issues. In multi-turn windings in DC/DC power supplies, the parasitic capacitance and the leakage inductance present a first resonance around 1 MHz, which is in the operating range of frequencies [1]. Furthermore, at those frequencies the conductors start behaving as transmission lines [1], [2].

In power applications, several approaches have been proposed to approximate the capacitive effects without solving the full-wave Maxwell problem: the coupling of different quasi-static finite-element (FE) formulations [3], or circuit models with parameter extraction based either on the method of moments [2] or the FE method [1] (and references herein).

In this paper, we propose a full-wave FE formulation combined with a reduced-order (RO) model based on the proper orthogonal decomposition (POD). The POD has been widely used in engineering problems [4] and particularly in low-frequency computational electromagnetism (quasi-statics) [5], [6], [7], [8]. The high-frequency RO models for fast frequency sweep are mainly twofold, based on a Taylor expansion or on a singular-value decomposition. See e.g. [9] and literature review therein.

The POD consists in projecting the original basis (constructed using the FE mesh) onto a reduced basis so that the size of the matrix system is highly reduced. The discrete projection operator is most often determined by means of the snapshot technique [5]. Herein, we solve the full-wave problem in the frequency domain for relevant frequencies to determine the snapshots. The obtained projection operator allows us to

dramatically reduce the computational time when solving the problem for intermediate frequencies. The snapshot selection is crucial and can be performed via e.g. a greedy algorithm [10], [11], [12]. We propose an original greedy algorithm, automatic and fully dependent on the RO model. As test case we consider a coil like the one depicted in Fig. 1.

Fig. 1. Coil geometry, mesh and induced current density (real part).

II. FULL WAVE ELECTROMAGNETIC PROBLEM

Let us consider a bounded domain $\Omega = \Omega_c \cup \Omega_c^C$ with boundary Γ , conducting part Ω_c and non-conducting part Ω_c^C . Adopting the complex formalism (frequency f , pulsation $\omega = 2\pi f$), the $\mathbf{a}$ - v strong formulation of Maxwell's equations reads ($\iota = \sqrt{-1}$):

$$\text{curl } \nu \text{ curl } \mathbf{a} - (\omega^2 \epsilon - \iota \omega \sigma) \mathbf{a} + (\iota \omega \epsilon + \sigma) \text{grad } v = \mathbf{j}_s, \quad (1)$$

with $\mathbf{a}$ the magnetic vector potential, v the electric scalar potential, $\mathbf{j}_s$ the source current density (with possibly a charge source ρ_s in the form $-\text{div } \mathbf{j}_s^0 = \rho_s$). For the sake of simplicity, zero total charge is assumed. The reluctivity ν , the permittivity ϵ and the conductivity σ characterize the linear isotropic media. Suitable boundary conditions must be imposed to ensure the uniqueness of the solution [13].

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Integration by parts of (1) yields the weak $\mathbf{a} - v$ formulation: find $\mathbf{a}$ and v such that

$$(\nu \text{curl } \mathbf{a}, \text{curl } \mathbf{a}')_{\Omega} - \omega^2 (\epsilon \mathbf{a}, \mathbf{a}')_{\Omega} + \nu \omega (\sigma \mathbf{a}, \mathbf{a}')_{\Omega_c} + (\nu \epsilon + \sigma) (\text{grad } v, \mathbf{a}')_{\Omega} = (j_s, \mathbf{a}')_{\Omega_s}, \quad (2)$$

$$\nu \omega (\epsilon \text{grad } v, \text{grad } v')_{\Omega} - \omega^2 (\epsilon \mathbf{a}, \text{grad } v')_{\Omega} = 0, \quad (3)$$

holds for all test functions $\mathbf{a}'$ and v' in suitable function spaces. A Silver-Müller absorbing boundary condition is imposed at the outer boundary Γ . After discretizing $\mathbf{a}$ and v with Whitney edge and nodal elements, resp., the linear frequency-dependent matrix system can be written as:

$$\mathcal{A}\mathbf{x} = \mathcal{J} \quad (4)$$

with $\mathbf{x}$ the $N \times 1$ vector of unknowns, $\mathcal{A}$ an $N \times N$ matrix and $\mathcal{J}$ the right-hand-side $N \times 1$ vector with the source.

III. MODEL ORDER REDUCTION

We apply the POD [4] to reduce the matrix system (4). The solution vector $\mathbf{x}$ is then approximated by a vector $\mathbf{x}_r$ in a reduced basis (size $M \ll N$) such that

$$\mathbf{x} \approx \Psi \mathbf{x}_r \quad (5)$$

with Ψ a discrete projector operator. This Ψ operator is typically constructed by applying the snapshot technique [5], i.e. generated from original solutions (full model) either in the time domain or in the frequency domain. Herein the full problem (4) is solved in the frequency domain for a set of M frequencies (snapshots). The snapshot matrix $\mathcal{S}$ is defined by the column vectors $\mathbf{x}_j$, $1 < j \leq M$, the solutions $\mathbf{x}$ at the snapshot/frequency f_j . Applying the singular value decomposition (SVD), this snapshot matrix reads

$$\mathcal{S} = \mathcal{V} \Sigma \mathcal{W}^T, \quad (6)$$

with $\mathcal{V}$ an $N \times N$ matrix, $\mathcal{W}$ an $M \times M$ matrix and Σ an $N \times M$ diagonal matrix containing the singular values. The i^{th} row of $\mathcal{W}$ represents the entries of the i^{th} column of $\mathcal{S}$ projected in the reduced basis formed by the M columns of the matrix $\mathcal{V} \Sigma$. The operator Ψ is obtained by normalizing the matrix $\mathcal{V} \Sigma$ (or $\mathcal{S} \mathcal{W}$). The reduced system to solve is given by

$$\mathcal{A}_r \mathbf{x}_r = \mathcal{J}_r, \quad (7)$$

with $\mathcal{A}_r = \Psi^T \mathcal{A} \Psi$ and $\mathcal{J}_r = \Psi^T \mathcal{J}$. Computing the SVD of $\mathcal{S}$ is expensive (prohibitive). In practice, a thin SVD is performed.

IV. GREEDY ALGORITHMS – SNAPSHOT SELECTION

Being able to approximate the solution vector $\mathbf{x}$ in a small dimensional space $\mathbf{x}_r$ does not mean that a pertinent discrete scheme is easy to find. Such a discrete space comprises solutions of (4) for well chosen parameters, e.g. frequencies f_j , $j = 1, \dots, M$ of the problem in hand. The greedy algorithm provides such an opportunity [10], [11].

Herein, we propose three greedy selection methods to find the minimum number of snapshots M within a prescribed tolerance τ for the error ε on a global quantity of interest, rather than on a local quantity like in [9]. See the pseudo-code hereafter.

Algorithm 1

Stop criteria: convergence of $\mathbf{I}$ (full model dependent)
Snapshot selection: max error on $\mathbf{I}$ (full model dependent)
Input: $\mathcal{A}$, $\mathcal{J}$, $F_k = \{f_k\}$ with $k = 1, \dots, K$, tolerance τ
Output: Ψ
1: choose an initial frequency f_1
2: solve full FE at f_1 and get initial snapshot $\mathbf{x}_1$
 $\mathcal{S} = [\mathbf{x}_1]$, remove f_1 from F_k , $M = 1$
3: generate Ψ (from SVD on $\mathcal{S}$)
4: **While** f_k **do**
i) compute $\mathcal{A}_r$ and $\mathcal{J}_r$
ii) solve $\mathcal{A}_r \mathbf{x}_r = \mathcal{J}_r$
iii) compute $\varepsilon_k = \|\mathcal{A}(f_k)[1, :] \Psi \mathbf{x}_r - \mathcal{J}(f_k)[1]\|$
5: **If** $\varepsilon_{max} = \max(\varepsilon_k) > \tau, \forall k$, pick frequency $f_{\varepsilon_{max}}$ associated to ε_{max} **Else break**
6: solve full FE at $f_{\varepsilon_{max}}$ to get $\mathbf{x}_{\varepsilon_{max}}$
 $\mathcal{S} = [\mathcal{S}, \mathbf{x}_{\varepsilon_{max}}]$, remove $f_{\varepsilon_{max}}$ from F_k , $M++$
7: back to step 3

Algorithm 2

Stop criteria: convergence of $\mathbf{Z}$ (RO model dependent)
Snapshot selection: max error on $\mathbf{I}$ (full model dependent)
Input: $\mathcal{A}$, $\mathcal{J}$, $F_k = \{f_k\}$ with $k = 1, \dots, K$, tolerance τ
Output: Ψ
1: choose an initial frequency f_1
2: solve full FE at f_1 and get initial snapshot $\mathbf{x}_1$
 $\mathcal{S} = [\mathbf{x}_1]$, remove f_1 from F_k , $M = 1$
3: initialize $\mathbf{Z}_0^k \neq 0, \forall k$
4: generate Ψ (from SVD on $\mathcal{S}$)
5: **While** f_k **do**
i) evaluate RO impedance $\mathbf{Z}_M^k = \mathbf{Z}(f_k)$
ii) compute $\varepsilon_k = \left\| \frac{\mathbf{Z}_M^k - \mathbf{Z}_{M-1}^k}{\mathbf{Z}_{M-1}^k} \right\|$
iii) **If** $\varepsilon_{max} < \tau$ **break Else** do steps 4 to 6 in **Algo. 1**
6: back to step 4

Algorithm 3

Stop criteria: convergence of $\mathbf{Z}$ (RO model dependent)
Snapshot selection: max error on $\mathbf{Z}$ (RO model dependent)
Input: $\mathcal{A}$, $\mathcal{J}$, $F_k = \{f_k\}$ with $k = 1, \dots, K$ tolerance τ
Output: Ψ
1: choose an initial frequency f_1
2: solve full FE at f_1 and initial snapshot $\mathbf{x}_1$
 $\mathcal{S} = [\mathbf{x}_1]$, remove f_1 from F_k , $M = 1$
3: initialize $\mathbf{Z}_0^k \neq 0, \forall k$
4: generate Ψ (from SVD on $\mathcal{S}$)
5: **While** f_k **do**
i) evaluate RO impedance $\mathbf{Z}_M^k = \mathbf{Z}(f_k)$
ii) compute $\varepsilon_k = \left\| \frac{\mathbf{Z}_M^k - \mathbf{Z}_{M-1}^k}{\mathbf{Z}_{M-1}^k} \right\|$
6: **If** $\varepsilon_{max} < \tau$ **break Else** pick $f_{\varepsilon_{max}}$
7: solve full FE at $f_{\varepsilon_{max}}$ and get $\mathbf{x}_{\varepsilon_{max}}$
 $\mathcal{S} = [\mathcal{S}, \mathbf{x}_{\varepsilon_{max}}]$, remove $f_{\varepsilon_{max}}$ from F_k , $M++$
6: back to step 4

Algorithm 1 stops the greedy snapshot search when the residual error on the electric current $\mathbf{I}$ is smaller than tolerance τ . The computation of $\mathbf{I}$ involves only one row of matrix $\mathcal{A}$

and one element of vector $\mathcal{J}$, e.g. the first ones $\mathcal{A}[1, :]$, $\mathcal{J}[1]$, resp. [?]. The relative error depends thus on the full model; it reads $\varepsilon = \|\mathcal{A}[1, :]\Psi\mathbf{x}_r - \mathcal{J}[1]\|$. Successive trial frequencies $f_{\varepsilon_{max}}$ are those for which the error on $\mathbf{I}$ is maximum.

Algorithm 2 uses, analogous to Algorithm 1, the residual error $\varepsilon = \|\mathcal{A}[1, :]\Psi\mathbf{x}_r - \mathcal{J}[1]\|$ of the full system for selecting next snapshot. However, the actual convergence is based on the error linked with the complex impedance computation $\mathbf{Z}$. Algorithm 2 evaluates the impedance $\mathbf{Z}$ given by the reduced basis functions at each iteration. Successive trial frequencies $f_{\varepsilon_{max}}$ are those for which the error on $\mathbf{I}$ is maximum, i.e. full model dependent.

Algorithm 3 uses, such as Algorithm 2, the relative error on the complex impedance $\mathbf{Z}$ as stopping criteria. The main difference lays on the fact that it does not require the full matrix $\mathcal{A}$ and the vector $\mathcal{J}$ for the error computation. Here, the next selected snapshot is chosen at the frequency with the biggest variation with regard to previous impedance computation. The method refers to the numerical convergence of the RO $\mathbf{Z}$ and the full model only needs to be evaluated for obtaining new snapshots.

V. APPLICATION EXAMPLE

As test case we consider a coil such as the one in Fig. 1. Made of copper, it has either 2 (model 1) or 4 (model 2) turns with a square section $5 \times 5 \text{ mm}^2$. The gap between successive wires is 5 mm and 15 mm, resp. In both cases, the discretization is fine enough for the highest considered frequency and it is kept invariant for all computations. The full-wave problem has been solved with a classical FE approach (reference) and with a RO approach and different snapshots in a wide range of frequencies $f \in [0.01, 100] \text{ MHz}$.

The mesh of model 1 consists of 13420 prisms and 5360 hexahedra, yielding 46563 complex unknowns with a classical FE approach. The modulus $|\mathbf{Z}|$ and phase $\arg(\mathbf{Z})$ of the coil impedance $\mathbf{Z}$ computed with the reference FE model and a RO approach (snapshots $M = 2, 3, 6, 12$) are compared in Fig. 2. The L_2 -relative error is depicted as well. The resonance frequency is found at 64.49 MHz.

With $M = 2$, $f = [0.01, 100] \text{ MHz}$, we cannot recover $\mathbf{Z}$. Adding the resonance frequency to the set of snapshots $M = 3$, $f = [0.01, 64.49, 100] \text{ MHz}$, clearly improves the approximation. With $M = 6$, $f = [0.01, 0.1, 1, 30, 64.49, 100] \text{ MHz}$, the FE and the RO curves are indistinguishable; the average L_2 -relative error on $\mathbf{Z}$ is $6.7 \cdot 10^{-4}$.

To check the robustness of the proposed greedy algorithms, we consider a 4-turn coil, model 2. Its mesh comprises 15120 prisms and 6800 hexahedra, which yields 54826 complex unknowns. Model 2 exhibits three resonances in the range $[0.01, 100] \text{ MHz}$. The computational time is hardly affected thanks to the small increase in unknowns (18% more in model 2).

First, the performance of the three greedy algorithms is analyzed by means of the L_2 -relative error on $\mathbf{Z}$, see Fig. 3. All algorithms find snapshots that ensure convergence up to a prescribed tolerance τ . Algorithm 3 is clearly the best, if $\tau = 10^{-6}$ convergence is achieved with 25 snapshots. Therefore, this is the algorithm applied hereafter.

Fig. 2. Model 1—Modulus (up) and phase (middle) of the coil impedance $\mathbf{Z}$ versus frequency. Reference FE and RO results (with $M = 2, 3, 6, 12$ snapshots). L_2 —relative error on $\mathbf{Z}$ (down).

Fig. 3. Convergence of the RO approach for the three greedy algorithms vs. the number of snapshots M .

The coil impedance (modulus and phase) computed with the FE model and the RO approach ($M = 5, 10, 15, 25$ snapshots) are depicted in Fig. 4. The L_2 -relative error is represented as well. The resonance frequencies are 17, 50.1 and 83.2 MHz.

With $M = 5$, $\mathbf{Z}$ cannot be recovered in the whole frequency range. Increasing M up to 10, the resonances are clearly detected. With $M = 15$, the RO approach already provides a quite accurate approximation, indeed the average L_2 -relative error in $[0.01, 100] \text{ MHz}$ is $2.94 \cdot 10^{-4}$. With $M = 25$, this average error is further reduced: $1.06 \cdot 10^{-6}$.

The Joule losses are shown in Fig. 5(up) for the FE and RO approaches ($M = 5, 10, 15, 25$). The relative error is represented as well in Fig. 5(down). The accuracy of the RO results improves considerably with M ; the average relative error is 2.59, $6.71 \cdot 10^{-3}$, $8.73 \cdot 10^{-4}$ with $M = 5, 10, 15, 25$, resp.

Analogous convergence behaviour is observed for local quantities, e.g. the magnetic flux density $\mathbf{b}$, the absolute error of which is plotted in Fig. 6. The L_2 -absolute error of $\mathbf{b}$ is $1.04 \cdot 10^{-6}$, $5.95 \cdot 10^{-11}$, $3.77 \cdot 10^{-12}$ and $3.91 \cdot 10^{-14}$ for $M = 5, 10, 15$ and 25, resp.

The average L_2 -relative error for the impedance $\mathbf{Z}$, the Joule losses, the magnetic flux density $\mathbf{b}$ and the current density $\mathbf{j}$ is summarized in Table I according to the RO size M .

With regard to the computational time, let us consider e.g.

Fig. 4. Model 2-Modulus (up) and phase (middle) of the coil impedance Z vs. frequency. Reference FE and RO results (with $M = 5, 10, 15, 25$ snapshots). L_2 -relative error on Z (down).

Fig. 5. Joule losses (up) and relative error vs. frequency.

10000 frequencies, the classical full model approach requires 44h, while the three proposed algorithms allow to solve the problem in 4.5 h, 3.5 h and 19 min, resp. Obviously, the higher the number of frequencies, the higher the gain is. The third algorithm is particularly performing for determining Z at intermediate frequencies, provided well-chosen snapshots.

VI. CONCLUSION

In this paper, we have proposed a POD-based model-order reduction approach to fully characterize power electronic components in a wide frequency range with a very low computational cost. Using a set of well chosen snapshots/frequencies, we are able to compute local and global quantities for the whole spectrum. Three different greedy algorithms are proposed and

TABLE I
AVERAGE L_2 -RELATIVE ERRORS ON GLOBAL QUANTITIES. AVERAGE L_2 -ABSOLUTE ERRORS ON LOCAL QUANTITIES

M	Z	Joule losses	b	j
5	$2.05 \cdot 10^{-1}$	2.59	$1.04 \cdot 10^{-6}$	$4.78 \cdot 10^{-5}$
10	$1.93 \cdot 10^{-2}$	$6.71 \cdot 10^{-3}$	$5.95 \cdot 10^{-11}$	$2.02 \cdot 10^{-9}$
15	$2.94 \cdot 10^{-4}$	$1.99 \cdot 10^{-4}$	$3.77 \cdot 10^{-12}$	$2.05 \cdot 10^{-9}$
25	$1.06 \cdot 10^{-6}$	$8.73 \cdot 10^{-7}$	$3.91 \cdot 10^{-14}$	$1.95 \cdot 10^{-9}$

Fig. 6. Cut (at coil center) of the absolute error of b for $M = 5, 10, 15, 25$ snapshots (from up-left to down-right). Scale range: 10^{-16} (dark blue) to 10^{-5} (dark red).

compared. Algorithm 3, independent of the full system, has proved its robustness in a wide range of frequencies.

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