# Identification of post-necking strain hardening behavior of pure titanium sheet


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ABSTRACT

This paper deals with the identification of the post-necking strain hardening behavior of pure titanium sheet. Biaxial tensile tests using a servo-controlled multi-axial tube expansion testing machine revealed that commercial pure titanium sheet exhibits significant differential work hardening (DWH). The latter phenomenon implies that the shapes of the work contours significantly change during plastic deformation which is accurately measured in the first quadrant of the stress space up to an equivalent plastic strain of approximately 0.3. In this paper we focus on the plastic material behavior beyond the point of maximum uniform strain in a quasi-static tensile test. To this purpose, the material is subjected to a post-necking tensile experiment during which the strain field in the diffuse necking zone is measured using a dedicated Digital Image Correlation (DIC) system. The key point in the identification of the post-necking strain hardening is the minimization of the discrepancy between the external work and internal work in the necking zone. In this study, we scrutinize the influence of DWH in the pre-necking regime on the identification of the post-necking strain hardening behavior of pure titanium sheet. Finally, a strain hardening model which enables disentangling pre –and post-necking hardening behavior is presented.



**Key words** Post-necking strain hardening, tensile test, differential work hardening, pure titanium, DIC


INTRODUCTION

Pioneering work of Bridgman [1] resulted in a solution for the problem of diffuse necking in a tensile specimen. Bridgman considered his method as a second level of approximation because the method is only concerned with the distribution of stress and strain across the diffuse neck. He also envisioned a third level of approximation, also referred to as the “complete solution” of the general problem, which takes the material state and the shape of the whole deforming specimen into account. Several researchers arrived at such complete solutions using finite-element based inverse methods. From a practical point of view, however, the coupling between the experimentally measured quantities and the numerically computed response can be a burden. Moreover, the iterative FE simulations to predict the plastic instability are usually very time-consuming. In order to avoid the shortcomings of the FE-based inverse method, a new method based on the complete solution to retrieve the strain hardening behavior hidden in the diffuse necking regime was presented in [2]. The key point in this method is that the strain hardening behavior can be identified by minimizing the discrepancy between the internal and the external work in the region where the diffuse neck develops. The method was experimentally validated [3] beyond the point of maximum uniform strain using the tube expansion test [4]. Kim et al. [5] presented a similar approach to identify post-necking strain hardening behavior of sheet metal using the Virtual Fields Method (VFM). Clearly, both methods [2, 5] rely on the computation of (actual and virtual, respectively) internal work, and, consequently, need access to the stresses associated with the experimentally measured strains. The later requires a phenomenological material model and an appropriate stress updating algorithm. Unlike the work of Kim et al. [5], Coppieters and co-workers [2,3] incorporated in-plane plastic anisotropy in their identification procedure. The assumption made in [2] that an increase of the cost function caused by an error in the work hardening law is significantly larger than an increase caused by an error in the anisotropic yield criterion was scrutinized in [3, 6]. The key point to guarantee a sufficient accurate identification is that the adopted yield function enables to describe the material response within the diffuse neck. Theoretically, the stress state in the diffuse neck approaches plane strain which implies that a fairly large portion of the yield surface will be used in the identification of the post-necking hardening behavior. Although, in practice plane strain is not reached during diffuse necking, the accuracy of the yield function gains importance when biaxial stress states are probed in materials which exhibit strong plastic anisotropy. The identified post-necking hardening behavior in the rolling direction (RD) of the materials investigated in [2,3] did not strongly depend on the selected yield loci. This work is an attempt to extend the method to materials which exhibit strong in-plane anisotropy and differential work hardening.

## MATERIAL

To this purpose, a commercially pure titanium sheet with an initial thickness of 0.52 mm is studied in the present work. This material exhibits complex texture-induced anisotropy along with differential work hardening. Sumita and Kuwabara [7] identified the differential work hardening behavior of a pure titanium sheet up to an equivalent plastic strain of 0.3 using the servo-controlled combined tension-internal pressure testing machine developed by Kuwabara and Sugawara [4]. Figure 1 shows the measured stress points forming the contours of plastic work for different levels of $\varepsilon_0^{pl}$. It can be inferred that the work contours shows significant asymmetry with respect to the equibiaxial stress state, i.e. $\sigma_x = \sigma_y$. Moreover, the asymmetry of successive work contours decreases with an increase in $\varepsilon_0^{pl}$ exemplifying the so-called differential work hardening. This material behavior is quantitatively in line with other investigations on pure titanium, e.g. [8].

Standard uniaxial tensile tests (JIS 13 B-type) were conducted to determine work hardening properties and the r-values. The pre-necking strain hardening curve the rolling direction can be found in figure 2. This figure also shows the fitted Swift hardening law which reads as:

$$\sigma_{eq} = K(\varepsilon_0 + \varepsilon_{eq}^{pl}) \quad (1)$$

It must be noted that the Swift law cannot perfectly describe the strain hardening behavior in the pre-necking region. Indeed, the Swift law cannot accurately reproduce the initial yield stress and the yield stress at maximum uniform strain. The r-values can be found in table 1. It can be inferred from figure 2 that the Swift law cannot perfectly describe the material behavior. To be specific, the initial yield stress and the yield stress at maximum uniform strain $\varepsilon_{max} \approx 0.27$ cannot be accurately predicted by the Swift law. Figure 2 also shows the hardening behavior of the test material in the transverse direction (TD). It must be noted that the latter curve is extrapolated since the maximum uniform strain $\varepsilon_{max}$ in the TD is limited to $\varepsilon_{max} \approx 0.05$. Clearly, the strain hardening behavior in the TD significantly differs from the behavior in the RD. Figure 1 also shows selected yield loci associated with specific values of reference strain $\varepsilon_0^{pl}$. The stress based Hill48 yield function yields satisfactory results up to $\varepsilon_0^{pl} = 0.15$ . Beyond this point, however, Hill48-$\sigma$ tends to overestimate the work contours in certain stress states. The yld2000-2d yield function enables to accurately describe the work contours provided that the variations in $r_0$, $r_{90}$ and $r_b$ with respect to $\varepsilon_0^{pl}$ are taken into account in the parameter determination. Such material behavior requires the development of new constitutive material models. Yanaga et al. [9] proposed to reproduce DWH based on the Yld2000-2d yield function with the exponent and material parameters changing as functions of the amount of plastic work. Implementation of such advanced yield functions within finite element codes enables to optimize sheet metal forming processes [10]. It must be noted that accurate determination of differential work hardening requires a significant amount of experimental effort. In this contribution, we scrutinize the necessity of incorporating DWH into the material model when identifying the post-necking hardening through a tensile test as proposed by Coppieters et al. [2] and Coppieters and Kuwabara [3].

| Direction | r-value |
|---|---|
| **RD** | 1.39 |
| **45** | 2.74 |
| **TD** | 3.91 |

**Table 1** : r-values of pure titanium measured at an engineering strain of 0.15

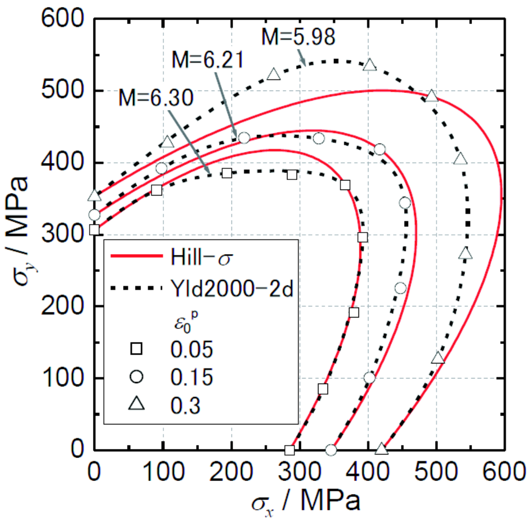


**Fig 1.** Stress points forming contours of plastic work compared with the theoretical yield loci calculated using the Yld2000-2d and Hill48 yield functions [7]

METHOD

The method presented in Coppieters et al. [2] was originally conceived from the observation that in a quasi-static tensile test the internal work equals the external work. As such, the key point in the method is the minimization of the discrepancy between the internal and the external work in the necking zone during a tensile test. Computation of the internal work requires access to the experimental strain field and the associated stress field. The experimental strain field is derived from the experimentally measured displacement field using stereo-DIC. To do so, an element mesh is fitted to the measured displacement field. The monitored zone in which the diffuse neck develops spanned a region of 12.5mm x 40 mm. This mesh contained 250 square elements (4 node bilinear elements) with an element size of 1.5 mm. The test material was subjected to the Post-Necking Tensile Experiment (PNTE) in the RD using a standard tensile specimen (JIS 13 type-B) and a regular tensile machine (Zwick/Roell) with a load capacity of 10kN. The experiment was displacement controlled using a constant cross-head speed of 0.05 $\frac{mm}{s}$. The tensile machine was equipped with a stereo-DIC system to capture the displacement fields at the surface of the specimen. Both cameras (8-bit AVT STINGGRAY F-201B 1/1.8) with a resolution of 1624x1232 $pixels^2$ were equipped with a Schneider 23 mm lens. The image resolution was approximately 0.08 mm/px. The images were synchronized with the tensile force and consequently each set of pictures corresponds to a different load step. All images were post-processed using MatchID-3D [11] via the so called subset-based method. During the PNTE full field information was captured in 95 load steps.

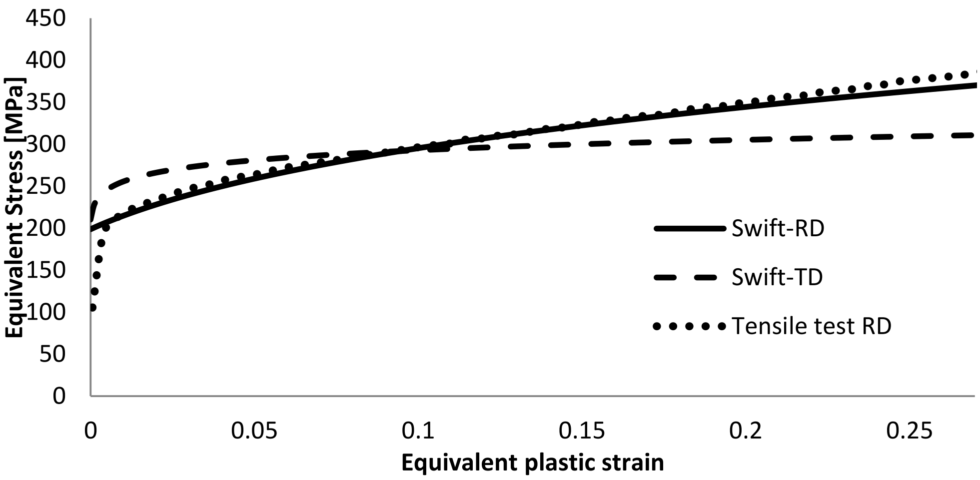


**Fig 2**. Pre-necking hardening behavior of pure titanium (strain rate$\approx 4 \times 10^{-4} \frac{1}{s}$). The Swift law in the TD was extrapolated beyond the point of maximum uniform strain $\varepsilon_{max} \approx 0.05$

VALDIDATION: PRE-NECKING HARDENING BEHAVIOUR

The aim of this paper is to identify the post-necking hardening behavior of pure titanium using the method presented in [2,3]. Before embarking on this, it should be noted that the pre-necking hardening behavior can be directly determined solely based on the measured force and the measured elongation using simple analytical formulas. Therefore, in a first step, the identification procedure is validated by identifying the pre-necking hardening behavior. Additionally, this step enables to tune the element mesh required to compute the strain fields from the measured displacement fields. In this section the experimentally acquired pre-necking data (55 load steps, i.e. the load step in which the maximum uniform strain $\varepsilon_{eq}^{pl} = 0.27$ is reached) are used to identify the Swift hardening law Eq.(1). Since the pre-necking hardening behavior is readily available from the tensile test, the accuracy of the identification procedure can be directly assessed. First, the potential in-plane anisotropy and DWH is ignored and the von Mises yield criterion is adopted to identify the pre-necking hardening behavior. Figure 3 shows the identified pre-necking hardening behavior using the von Mises yield criterion (labeled as *PNTE Swift-von Mises (1)*) along with the Swift law fitted to the pre-necking data.

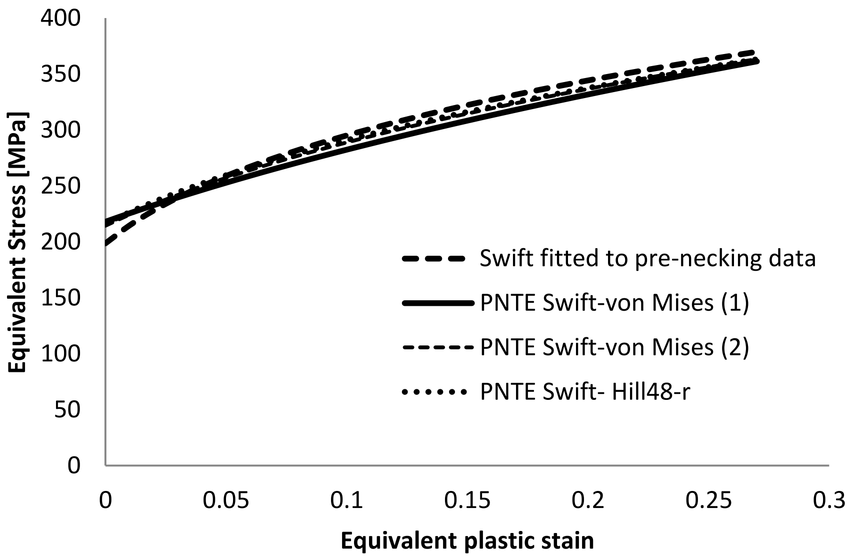


**Fig 3.** Identified pre-necking hardening behavior of pure titanium sheet in the RD

It can be observed that the pre-necking strain hardening behavior is not perfectly retrieved. It was found that this error originates from an inaccurate computation of the external work. Figure 4 shows the load-elongation curve of the tensile test in the RD. Although, the stereo-DIC system was synchronized with the tensile test machine, a deviation between the experimentally measured tensile forces occurred. The identification could be improved by using the corrected tensile force and the result (labeled as *PNTE Swift-von Mises (2)*) is shown in figure 3. Second, the in-plane anisotropy of the titanium sheet was included by using the r-based Hill48 yield criterion. It can be seen in figure 3 that this yields identical results as those obtained using von Mises yield criterion. Consequently, the identification procedure along with the adopted yield loci is validated in the pre-necking region.

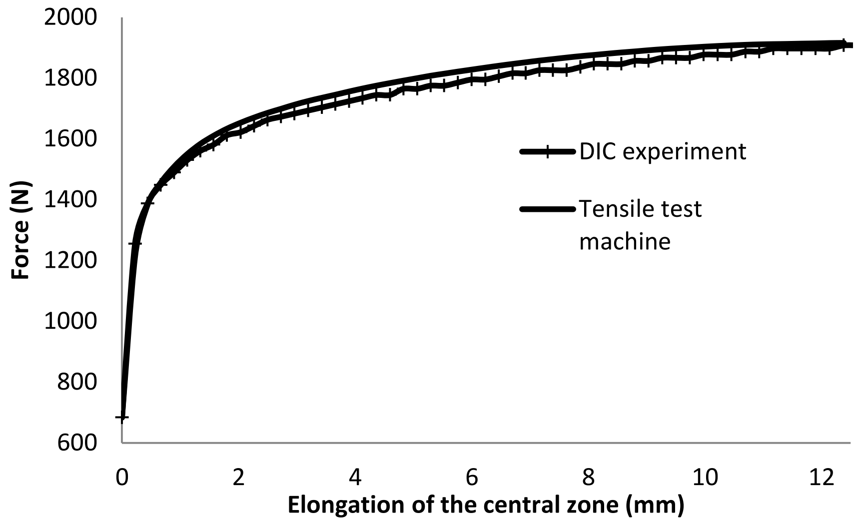


**Fig 4.** Force-elongation curve (pre-necking region, RD, initial length central zone=40mm)

RESULTS: POST-NECKING HARDENING BEHAVIOUR

In this section we embark on the post-necking strain hardening behavior of pure titanium. 95 load steps were used to identify the complete hardening behavior. The maximum equivalent strain probed in the experiment was $\varepsilon_{eq}^{pl} \approx 0.8$. Only diffuse necking is taken into account. Figure 5 shows the results. The black solid curve (labeled *EM Swift*) represents the extrapolated Swift law fitted to the pre-necking data. The same Swift law was also identified using the PNTE (labeled *PNTE Swift-von Mises*) and this yield a similar behavior as predicted by the extrapolation method (EM). The latter suggests that the Swift law is an appropriate hardening law to describe the hardening behavior of pure titanium in the RD. In order to disentangle pre- and post-necking hardening behavior the so-called p-model [3], which is based on successive phenomenological equations, can be used:

$$\sigma_{eq} = \begin{cases} K(\varepsilon_0 + \varepsilon_{eq}^{pl})^n, \ \varepsilon_{eq}^{pl} \leq \varepsilon_{max} \\ K(\varepsilon_0 + \varepsilon_{max})^n \ + Q\left[1 - e^{-p\left(\varepsilon_{eq}^{pl} - \varepsilon_{max}\right)}\right], \ \varepsilon_{eq}^{pl} > \varepsilon_{max} \end{cases} \quad (2)$$

In this phenomenological hardening model, the parameters K,$\varepsilon_0$ and n are identified through the available pre-necking data. $\varepsilon_{max}$ is the maximum uniform strain in the tensile test. In the post-necking regime the model switches to a post-necking hardening description. To guarantee a smooth transition a simple relation between *p* and *Q* can be found [3]. As a result, the only unknown in the post-necking regime is the post-necking hardening parameter *p*. In the case a von Mises material is assumed, the p-model (labeled *PNTE p-model – von Mises*) is in very good agreement with the extrapolated Swift law. If the p-model, however, is identified using the r-based Hill48 yield criterion then the post-necking parameter increases resulting in a slightly decreased strain hardening rate deep into the diffuse neck. Figure 6 shows the adopted yield loci used to identify the hardening behavior. Clearly, both yield loci cannot accurately describe the material response of pure titanium in the first quadrant of the stress space. However, they enable to describe the material response of titanium in the vicinity of uniaxial tension in the RD, i.e. the stress ratio $(\sigma_x:\sigma_y) = 1:0$.

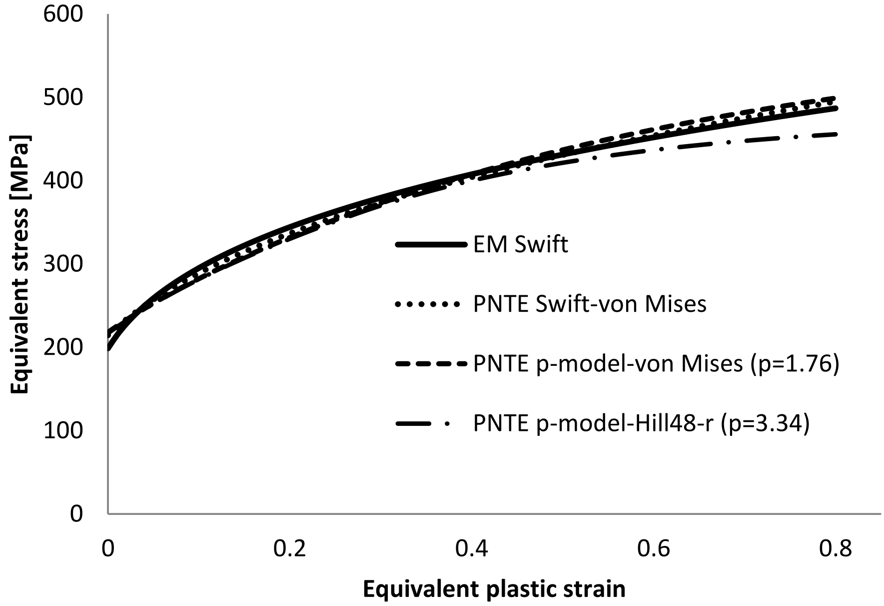


**Fig 5.** Identified pre- and post-necking hardening behavior of pure titanium in the RD

The later means that the yield loci can be adopted to retrieve the hardening behavior when the stress ratio in the diffuse neck remains close to $(\sigma_x:\sigma_y) = 1:0$. However, the deviation in the post-necking regime between the results obtained by the different yield loci indicates that the stress ratio deviates from $(\sigma_x:\sigma_y) = 1:0$. Indeed, in the diffuse neck transverse stresses develop, and, consequently, biaxial stress states are probed. It is inferable from figure 6 that the adopted yield loci differ significantly hence resulting in a different identified post-necking hardening behavior. Finally, it must be noted that the present work did not include the strain rate sensitivity of the material. Indeed, the strain rate in the diffuse neck increases when uniform straining ceases and deformation becomes concentrated in the necking zone. It was experimentally observed that some material points in the diffuse neck reach a strain rate of $\dot{\varepsilon}_{eq}^{pl} \approx 0.01\frac{1}{s}$. Given the fact that very few material point reach this value, however, it is currently assumed that the impact on the global identified hardening behavior will be small. Nevertheless, since pure titanium exhibits a significant strain-rate sensitivity [7], future work should try to include the strain rate as a state variable in the hardening law.

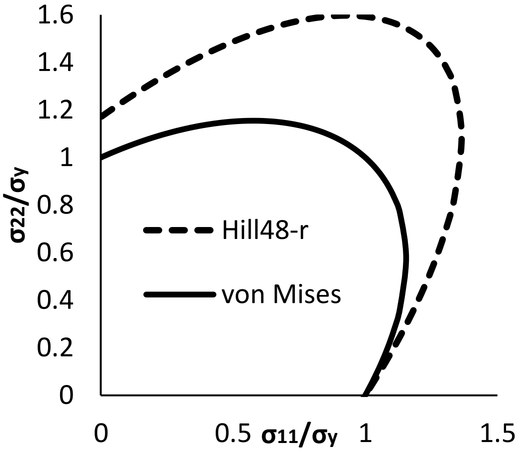


**Fig 6.** Adopted yield loci to identify the post-necking hardening behavior of pure titanium in the RD

## CONCLUSIONS

This work is an attempt to identify the post-necking hardening behavior of pure titanium sheet through a tensile test in the rolling direction. To this purpose, an alternative method is used which is based on the minimization of the external and internal work in the diffuse neck of a standard tensile specimen. It is well-known that the accuracy of this identification method depends on the adopted phenomenological yield function. Clearly, the yield function gains importance when the material exhibits strong in-plane anisotropy and biaxial stress states are probed in the diffuse neck. In this paper, the post-necking strain hardening behavior of pure titanium, which exhibits strong texture-induced anisotropy and differential work hardening, is under investigation. The aim of this paper was to scrutinize the necessity of using an advanced phenomenological yield function, which enables to describe such complex material behavior, to identify the post-necking hardening behavior. While inconclusive, the obtained results suggest that the post-necking hardening behavior of pure titanium can be determined without incorporating a highly advanced yield function provided that the post-necking tensile experiment is conducted in the rolling direction. A more profound analysis requires the implementation of an advanced material model which enables to describe differential work hardening. Additionally, strain rate should be included as a state variable in the hardening law. Research along these lines is currently conducted and will be published in a forthcoming paper.

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