Optimization and allocation of spinning reserves in a low-carbon framework

Kenneth Bruninx, *Student Member, IEEE*, Kenneth Van den Bergh, *Student Member, IEEE*, Erik Delarue, *Member, IEEE*, and William D'haeseleer

***Abstract*—Low-carbon electric power systems are often characterized by high shares of renewables, such as wind power. The variable nature and limited predictability of some renewables will require novel system operation methods to properly size and cost-efficiently allocate the required reserves. The current state-of-the-art stochastic unit commitment models internalize this sizing and allocation process by considering a set of scenarios representing the stochastic input during the unit commitment optimization. This results in a cost-efficient scheduling of reserves, while maintaining the reliability of the system. However, calculation times are typically high. Therefore, in this paper, we merge a state-of-the-art probabilistic reserve sizing technique and stochastic unit commitment model with a limited number of scenarios in order to reduce the computational cost. Results obtained for a power system with a 30% wind energy penetration show that this hybrid approach allows to approximate the expected operational costs and reliability of the resulting unit commitment of the stochastic model at roughly one thirtieth of the computational cost. The presented hybrid unit commitment model can be used by researchers to assess the impact of uncertainty on power systems or by independent system operators to optimize their unit commitment decisions taking into account the uncertainty in their system.**

***Index Terms*—Stochastic unit commitment, Probabilistic reserves, Wind power.**

I. Introduction

In low-carbon power systems with high shares of renewables (RES) ensuring reliability will become an increasingly critical issue. Some forms of RES, notably wind and solar PV, have a stochastic character, i.e., they are variable (not or only limitedly dispatchable) and to some extent unpredictable. Deviations from what is expected – e.g. forecast errors – need to be overcome with up- or downward regulation of dispatchable generation, load or storage. Moreover, the variability of these RES injections requires this reserve capacity to be sufficiently flexible. In this regard, novel power system operation methods will be needed to properly size and allocate operational reserves, in order to ensure a reliable and cost-efficient operation of the power system.

One of these novel power system operation methods is the so-called stochastic unit commitment model (SUCM). As shown below and discussed in detail in [1]–[5], the direct representation of the uncertainty via a set of scenarios in the unit commitment model leads to an optimal trade-off between reliability and operational system cost. The sizing and allocation of reserves is internalized in a SUCM. However, these SUCMs are not devoid of disadvantages [5]. First, the computational cost of solving such a SUCM is high and strongly increases with the number of scenarios one considers. In addition, solution stability[1] requirements impose a lower limit on the number of scenarios one can use to ensure a meaningful solution of the SUCM. A modeler thus continuously strives for a trade-off between optimality and computational cost. Second, capturing a continuous stochastic variable, such as e.g. the wind power forecast error (WPFE), in a (limited) set of discrete scenarios requires advanced scenario generation and reduction techniques. Third, in real-life power systems, one needs to consider multiple sources of uncertainty and multiple regions, drastically increasing the complexity of the problem at hand.

To limit the computational burden, modelers often resort to deterministic unit commitment models (DUCMs) with reserve requirements or they try to speed up the convergence of the SUCM. In the former category, especially probabilistic reserve requirements – i.e., reserve sizing based on the probability that an error of a certain size occurs – have gained attention over the last years [8], [9]. E.g., Wang et al. [8] show that a probabilistic reserve requirement (based on a so-called quantile forecast) outperforms other reserve rules in a DUCM when dealing with uncertainty on wind power. In the latter category, one can distinguish multiple approaches, such as, but not limited to, improved model formulations [10], decomposition methods [10]–[12], advanced scenario reduction methods [3], [13], relaxations of the problem [14] and the addition of reserve constraints in SUCM [1], [2], [8].

The last option – i.e., the addition of reserve constraints in SUCM – was first attempted by Ruiz et al. [1]. The addition of a static – i.e., constant over the time horizon of the optimization – reserve requirement to a SUCM considering 12 scenarios leads to lower expected costs compared to a DUCM when dealing with load and generation uncertainty. Tuohy et al. [2] employ the well-known WILMAR model, in which a reserve constraint based on the largest unit in the system and a fixed percentage of the forecasted wind power

K. Bruninx, K. Van den Bergh, E. Delarue and W. D'haeseleer are with the University of Leuven (KU Leuven) Energy Institute, TME Branch (Energy Conversion), Celestijnenlaan 300A box 2421, B-3001 Leuven, Belgium.

Corresponding author: William D'haeseleer, tel: +3216/322510 , fax: +3216/322985, e-mail: william.dhaeseleer@mech.kuleuven.be.

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[1]*Solution stability* means that the objective value – obtained by solving the SUCM – does not change (too much) when the set of scenarios considered enlarges and that this value is close to the true objective function – obtained by solving the SUCM on the 'full' set of scenarios with fixed first stage optimization variables [6]. The notion of solution stability should not be confused with power system stabiltiy [7].

is added to the SUCM. Compared to a deterministic variant with a static reserve requirement, the SUCM can lead to cost savings up to 1%. Wang et al. [8] show that the addition of a reserve requirement equal to 5% of the load to a SUCM leads to decreased operational costs. Botterud and Zhou [3] demonstrate in a similar setting that a dynamic, probabilistic reserve requirement in a DUCM even (slightly) outperforms a SUCM without additional reserve constraints in terms of operational costs[2].

In the hybrid unit commitment model or HUCM proposed in this paper, we combine a state-of-the-art scenario reduction method [13], [15] and a probabilistic reserve constraint [3], [8], [9] in a SUCM. Although the literature thus shows that a probabilistic reserve requirement outperforms any other reserve rule in a DUCM, the addition of such a probabilistic reserve requirement to a SUCM has – to the best of our knowledge – not yet been attempted. A HUCM requires a limited number of scenarios – lower than the number of scenarios needed to reach a stable solution in a full SUCM – to approximate the results of a SUCM in terms of operational costs, RES curtailment and lost load, but at a fraction of the computational cost of a stochastic unit commitment model. The probabilistic reserve requirement will impose a lower limit on the scheduled reserves, while the considered scenarios will ensure that (1) additional reserves are scheduled if required; (2) the reserves are scheduled as cost efficiently as possible, considering the cost of allocation and activation; (3) the scheduled reserves are sufficiently flexible to overcome the highly variable forecast errors.

The added value of this paper is twofold. First, we present a new design of a HUCM. A state-of-the-art probabilistic reserve constraint is combined with a SUCM. This model allows scheduling spinning and non-spinning reserves. Second, as demonstrated below, the presented HUCM approximates the stable solution of SUCM at a significantly lower computational cost. This model can be used to assess the impact of uncertainty on reasonably large low-carbon electric power systems, where SUCMs models would become computationally intractable. Transmission system operators (TSO) can assess the adequacy of procured reserves. Similarly, independent system operators (ISO) could directly employ this model to perform their unit commitment optimization.

The remainder of this paper is organized as follows. In Section II the developed methodology is discussed. A description of the scenario generation technique and the HUCM/SUCM is presented. Section III contains the results of the application of the presented model to the Belgian power system with a wind power penetration of 30% (annual, energy basis). After a discussion on the design, the effect of a reliability constraint is analyzed. To test the performance of the presented HUCM, four representative weeks are studied. Lastly, we formulate a conclusion and some suggestions for future research in Section IV.

[2]Note that this is only possible if the stable solution of the full SUCM was not reached. Solving the full stochastic problem would yield a lower bound on the operational costs attainable.

II. Methodology

The starting point of the design of the HUCM will be the traditional SUCM, as will be discussed in Section II-A. In all SUCMs, one tries to find a unit commitment for which a feasible dispatch is possible for all possible realizations (scenarios) of an uncertain variable, in this case the WPFE, that minimizes the expected operational cost. A feasible dispatch here means that the demand for electricity at each time step is met in all of the considered WPFE scenarios, while respecting all techno-economic constraints of the power plants. The solution of this stochastic optimization problem – taking into account all possible scenarios – would represent the best possible trade-off between optimality – in terms of expected operational costs – and reliability – in terms of lost load, whereby one can impose constraints on the optimization, thereby setting this at any desired level. However, as this full stochastic problem is intractable, the best available proxy is used: the stable solution of the SUCM, obtained on a reduced set of scenarios (see Section II-A). This stable solution still comes at high computational cost, as discussed in [5] and in Section III. However, as we will show below, this optimal solution can be approximated by adding additional reserve constraints on the forecast scenario. This reserve constraint is probabilistic in nature and can be seen as a state-of-the-art reserve sizing technique [5], [9].

The stochastic and hybrid model – incorporating scenario generation, scenario reduction and a SUCM – are discussed in Section II-A. Section II-B contains a description of the probabilistic reserve sizing methodology. As outlined in Section II-C, the Belgian power system will be studied for a high share of RES. To evaluate the performance of the unit commitment models, we look toward the so-called second stage optimization[3] results, i.e. the dispatch, while fixing the first stage optimization variables, i.e. the unit commitment of the power plants. Note that fast-starting units may be scheduled as non-spinning reserves, which requires a scenario-dependent unit commitment status (see below). In the second stage optimization problem, all scenario-dependent variables (such as the output of the power plants and the pumped hydro storage power plant, as well as the curtailment of wind power) are optimized, given the first stage variables, for each scenario individually and without uncertainty on the realization of wind power. This dispatch is performed for a large set of scenarios to gain statistical significance.

A full description of the scenario generation technique, model and data used in this paper can be found online [15], [16]. The model is implemented in GAMS 24.2 and MATLAB® 2011b. CPLEX 12.6 is used as solver. Calculations are run on the ThinKing HPC cluster of the KU Leuven, using a 2.8GHz machine with 20 cores and 64GB of RAM. The optimality gap was set to 0.5%.

[3]The so-called *first stage* variables are the optimization variables that are common to all scenarios. All other optimization variables are the so-called *second stage* variables – i.e. the variable(s) that take on different values in each scenario.

A. Model

First, as good WPFE scenarios are essential to obtain meaningful results, we have developed a WPFE scenario generation and reduction tool. This tool is based on the statistical characterization of the WPFE described in [9], a scenario generation technique based on Pinson et al. [17] and a modified version of the *fast forward scenario reduction algorithm* [13], [15], [18]. In [15], we evaluate the quality of the scenarios in terms of the probability density function of the WPFE and for a number of critical events, based on the method described in [19]. A set of 500 scenarios provides satisfactory results. To avoid intractability, a scenario reduction technique is employed. The aim of this method is to select a set of scenarios with a predefined, low cardinality, that will yield a stable solution of the SUCM. The proposed algorithm is a *forward selection* scenario reduction technique, in which a probability distance metric based on the operational cost of a scenario in a deterministic unit commitment model between the original and reduced set of scenarios is minimized [13], [15], [18].

Second, a **SUCM** is formulated using Mixed Integer Linear Programming. The first stage variables are the unit commitment status of the power plants ($z_{i,j}$). Only power plants that are considered 'fast-starting' units (subset $I^{FAST} \subset I$, the set of power plants) can have a scenario-dependent commitment status, indicated by the binary variable $z^*_{i,j,s}$. This and all other optimization variables, such as the output of the power plants, curtailment of RES and the output of the pumped hydro storage, are second stage variables. In a SUCM, the power plants are scheduled and dispatched in such a way that the overall cost of generating the demanded electricity over the simulated time period is minimized. This cost consists of fuel costs $fc_{i,j,s}$, start-up costs $sc_{i,j,s}$, ramping costs $rc_{i,j,s}$ and CO_2-emission costs $co_2t_{i,j,s}$. The objective function reads

$$\begin{aligned} min \sum_i \sum_j \sum_s P_s \cdot [sc_{i,j,s} + fc_{i,j,s} + co_2t_{i,j,s} + rc_{i,j,s}] \\ + \sum_j \sum_s P_s \cdot TP \cdot (VOLL \cdot \phi_{j,s} + VOC \cdot \chi_{j,s}) \end{aligned} \quad (1)$$

where P_s is the probability of a scenario s (set S). TP is the considered time step in the optimization. I is the set of power plants present in the model (index i) and J is the set of time steps (index j). $VOLL$ is the value of lost load $\phi_{j,s}$, while TP stands for the length of the time step. The fuel cost ($fc_{i,j,s}$) is determined by the fuel price and the efficiency of the power plant and depends on the scenario s, as the output of the power plant is scenario dependent:

$$\begin{aligned} \forall i, \forall j, \forall s: \ fc_{i,j,s} = TP \cdot \big[C_i \cdot (z_{i,j} + z^*_{i,j,s}) \\ + MA_i \cdot \big(g_{i,j,s} - P_i^{MIN} \cdot (z_{i,j} + z^*_{i,j,s})\big)\big] \end{aligned} \quad (2)$$

where C_i is the fuel cost when running the plant at its minimum power level and the binary variables $z_{i,j}$ and $z^*_{i,j,s}$ represent the commitment status of plant i. $z^*_{i,j,s}$ can only take non-zero values for fast-starting units when $z_{i,j}$ equals zero (Eq. (15)-(19)). MA_i is the marginal cost for the additional generation $g_{i,j,s}$ above its minimum power level ($g_{i,j,s} - P_i^{MIN} \cdot (z_{i,j} + z^*_{i,j,s})$). This is a linear approximation of the quadratic cost curve of a power plant. Note that the binary variable $z_{i,j}$, representing the commitment status of plant i, is independent of the scenarios s. The CO_2 cost $co_2t_{i,j,s}$ is based on the emissions, the load level and a fixed CO_2 price per ton CO_2 (CO_2P):

$$\begin{aligned} \forall i, \forall j, \forall s: \ co_2t_{i,j,s} = CO_2P \cdot TP \cdot \big[B_i \cdot (z_{i,j} + z^*_{i,j,s}) \\ + MB_i \cdot \big(g_{i,j,s} - P_i^{MIN} \cdot (z_{i,j} + z^*_{i,j,s})\big)\big] \end{aligned} \quad (3)$$

Similar as the fuel costs, the CO_2 cost consists of a fixed part (the emissions when a plant is running at its minimum power level B_i) and a term accounting for the marginal emissions of different generation levels (MB_i). The start-up cost $sc_{i,j}$ is calculated as

$$\forall i, \forall j, \forall s: \quad sc_{i,j,s} = STC_i \cdot (v_{i,j} + v^*_{i,j,s}) \quad (4)$$

with a fixed start-up cost, different per power plant and fuel, STC_i. The binary variable $v_{i,j}$ or $v^*_{i,j,s}$ is equal to 1 at time step j if the plant starts up at that time step j. In this model, we have not differentiated between hot and cold start-ups. The ramping costs are calculated based on a ramping cost per power plant RCP_i as

$$\begin{aligned} \forall i, \forall j, \forall s: \quad rc_{i,j,s} \geq RCP_i \cdot \big[g_{i,j,s} - g_{i,j-1,s} \\ -P_i^{MIN} \cdot (v_{i,j} + v^*_{i,j,s})\big] \end{aligned} \quad (5)$$

$$\begin{aligned} \forall i, \forall j, \forall s: \quad rc_{i,j,s} \geq RCP_i \cdot \big[g_{i,j-1,s} - g_{i,j,s} \\ -P_i^{MIN} \cdot (w_{i,j} + w^*_{i,j,s})\big] \end{aligned} \quad (6)$$

Note that ramping costs are only associated with changes in output during normal operation, not during start-ups (binaries $v_{i,j}$ and $v^*_{i,j,s}$) or shut-downs (binaries $w_{i,j}$ and $w^*_{i,j,s}$).

This optimization is subjected to a number of constraints. First, the supply and demand for electricity must be equal at all time steps j in every scenario s. The so-called market clearing condition reads:

$$\begin{aligned} \forall j, \forall s: \ D_j - \phi_{j,s} = \sum_i g_{i,j,s} + G_j^{MR} + G_j^F \\ + FE_{j,s} - \chi_{j,s} + \sum_r \left(g_{r,j,s}^{PHS,T} - g_{r,j,s}^{PHS,P}\right) \end{aligned} \quad (7)$$

The demand D_j on each time step j is assumed to be known and fixed. This demand must be met by (1) electricity generated from dispatchable power plants $g_{i,j,s}$; (2) generation from must-run systems with a known output (including electricity generation from RES, except wind) G_j^{MR}; (3) the forecast of the uncertain wind power, G_j^F, which can be curtailed ($\chi_{j,s}$), and the wind power forecast error $FE_{j,s}$; (4) the net injection of power from pumped hydro storage plants (index r), calculated as the difference between the injection of power $g_{r,j,s}^{PHS,T}$ and the withdrawal of power $g_{r,j,s}^{PHS,P}$ and (5) the shedding of load $\phi_{j,s}$.

Second, the power plants have several technical constraints, different per fuel and technology. The output of each power plant is restricted to a minimum (P_i^{MIN}) and maximum level (P_i^{MAX}) if the plant is online. Similarly, ramping constraints limit the variation of the output of the power plants. Shut-down and start-up ramping rates equal the minimum operating point of each plant. This means that at start-up and shut-down a power plant will run at its minimum operating point for one time step. For all power plants, the following constraints must hold:

$$\forall i, \forall j, \forall s: \quad g_{i,j,s} \leq P_i^{MAX} \cdot (z_{i,j} + z^*_{i,j,s}) \tag{8}$$
$$\forall i, \forall j, \forall s: \quad g_{i,j,s} \geq P_i^{MIN} \cdot (z_{i,j} + z^*_{i,j,s}) \tag{9}$$
$$\forall i, \forall j, \forall s: \quad g_{i,j,s} \geq 0 \tag{10}$$

Fast-starting units are assumed to be able to ramp up to their full capacity within one time step. Ramping constraints are thus only enforced on the other power plants:

$$\forall i \notin I^{FAST}, \forall j, \forall s: \quad g_{i,j,s} \leq g_{i,j-1,s} + P_i^{MIN} \cdot v_{i,j} + \Delta P_i^{MAX,+} \cdot (z_{i,j} - v_{i,j}) \tag{11}$$

$$\forall i \notin I^{FAST}, \forall j, \forall s: \quad g_{i,j,s} \geq g_{i,j-1,s} - P_i^{MIN} \cdot w_{i,j} - \Delta P_i^{MAX,-} \cdot z_{i,j} \tag{12}$$

The minimum up- and down-times (MUT and MDT) have been included in the model as in Rajan and Takriti [20]:

$$\forall i \notin I^{FAST}, \forall j: \quad \sum_{k=1}^{MUT-1} v_{i,j-k} \leq z_{i,j} \tag{13}$$
$$\forall i \notin I^{FAST}, \forall j: \quad \sum_{k=1}^{MDT-1} w_{i,j-k} \leq 1 - z_{i,j} \tag{14}$$

Fast-starting units are assumed to have minimum up and down-times equal to one time step. The binary variables $z_{i,j}$, $v_{i,j}$, $w_{i,j}$ and $z^*_{i,j,s}$, $v^*_{i,j,s}$, $w^*_{i,j,s}$ (fast-starting units) are linked as follows:

$$\forall i, \forall j: \quad z_{i,j} - z_{i,j-1} - v_{i,j} + w_{i,j} = 0 \tag{15}$$
$$\forall i, \forall j, \forall s: \quad z^*_{i,j,s} - z^*_{i,j-1,s} - v^*_{i,j,s} + w^*_{i,j,s} = 0 \tag{16}$$
$$\forall i \in I^{FAST}, \forall j, \forall s: \quad z_{i,j} + z^*_{i,j,s} \leq 1 \tag{17}$$
$$\forall i \notin I^{FAST}, \forall j, \forall s: \quad z^*_{i,j,s} = 0 \tag{18}$$
$$\forall i, \forall j: \quad z^*_{i,j,s^F} = 0 \tag{19}$$

In the forecast scenario s^F, the demand must be covered by spinning units (Eq. (19)).

Third, the pumped hydro storage power plants (set R, index r) are included in the model formulation. The energy content of the reservoir $e_{r,j,s}$ for each PHS r can be written as

$$\forall r, \forall j, \forall s: \quad e_{r,j,s} = e_{r,j-1,s} + TP \cdot \left(g^{PHS,P}_{r,j,s} \cdot \sqrt{\epsilon_r^{PHS}} - \frac{g^{PHS,T}_{r,j,s}}{\sqrt{\epsilon_r^{PHS}}} \right) \tag{20}$$

with ϵ_r^{PHS} the round-trip efficiency of the PHS. The energy levels of the hydro storage are limited to a minimum (E_r^{MIN}) and maximum (E_r^{MAX}) level, while the output of the pumped hydro storage power plant can vary freely between zero and the capacity of the plant (P_r^{MAX}).

$$\forall r, \forall j, \forall s: \quad E_r^{MIN} \leq e_{r,j,s} \leq E_r^{MAX} \tag{21}$$
$$\forall r, \forall j, \forall s: \quad 0 \leq g^{PHS,T}_{r,j,s} \leq P_r^{MAX} \tag{22}$$
$$\forall r, \forall j, \forall s: \quad 0 \leq g^{PHS,P}_{r,j,s} \leq P_r^{MAX} \tag{23}$$

Eq. (1) – (23) form the SUCM. If one considers sufficient scenarios, this model allows to calculate a cost-optimal trade-off between cheap spinning flexibility, expensive non-spinning flexibility, curtailment and load shedding.

In the **hybrid model**, a demand for upward reserves D_j^+, calculated via a probabilistic method (see Section II-B), is added in the forecast scenario s^F:

$$\forall j: \quad D_j^+ = \sum_i \left(sr^+_{i,j} + nsr^+_{i,j} \right) + \chi_{j,s^F} + s_j^+ \tag{24}$$

At each time step j, the demand for upward reserves must be met by free online capacity (spinning reserves, $sr^+_{i,j}$), fast-starting units that are not committed in the forecast scenario (non-spinning reserves, $nsr^+_{i,j}$) and scheduled wind power curtailment χ_{j,s^F}. If a shortage of supply occurs in the forecast scenario, the demand for reserve has to be relaxed before load shedding occurs. The introduction of a slack variable s_j^+, restricted to positive values and penalized at a high cost (less than $VOLL$) in the objective function, allows this. Note that we only impose a demand for reserves on the forecast scenario.

A power plant is only allowed to be scheduled as spinning reserves if (1) $z_{i,j} = 1$, (2) the sum of the output of that plant in the forecast scenario and the scheduled spinning reserves does not exceed the maximum output of the power plant and (3) the ramping constraints of that power plant are not violated if that reserve would be activated:

$$\forall i, \forall j: \quad g_{i,j,s^F} + sr^+_{i,j} \leq P_i^{MAX} \cdot z_{i,j} \tag{25}$$

$$\forall i, \forall j: \quad g_{i,j,s^F} + sr^+_{i,j} \leq g_{i,j-1,s^F} + P_i^{MIN} \cdot v_{i,j} + \Delta P_i^{MAX,+} \cdot (z_{i,j} - v_{i,j}) \tag{26}$$

Fast-starting units may be scheduled as non-spinning reserves if they are (1) $z_{i,j} = 0$ (Eq. (17)) and (2) dispatched in at least one scenario:

$$\forall i \in I^{FAST}, \forall j: \quad nsr^+_{i,j} \leq P_i^{MAX} \cdot (1 - z_{i,j}) \tag{27}$$
$$\forall i \in I^{FAST}, \forall j: \quad nsr^+_{i,j} \leq P_i^{MAX} \cdot \sum_s z^*_{i,j,s} \tag{28}$$
$$\forall i \in I^{FAST}, \forall j: \quad nsr^+_{i,j} \geq 0 \tag{29}$$
$$\forall i \notin I^{FAST}, \forall j: \quad nsr^+_{i,j} = 0 \tag{30}$$

The minimum operating point of these fast-starting units has not been considered if they are scheduled as non-spinning reserves. During dispatch, the minimum operating point of these units is enforced.

Pumped hydro storage is not scheduled to satisfy the demand for reserves, as the availability of these reserves would not be guaranteed. Indeed, the amount of stored energy in the upper basin of the pumped hydro unit, thus the availability of the plant, is dependent on the realization of the uncertain variable. Therefore, in this paper, we exclude them from the reserve requirement. Although this is a conservative approach, this does yield a reliable unit commitment policy. Note that the output of the pumped hydro unit can still be adapted during dispatch in order to overcome forecast errors. Other, more optimal scheduling approaches may exist and merit future research.

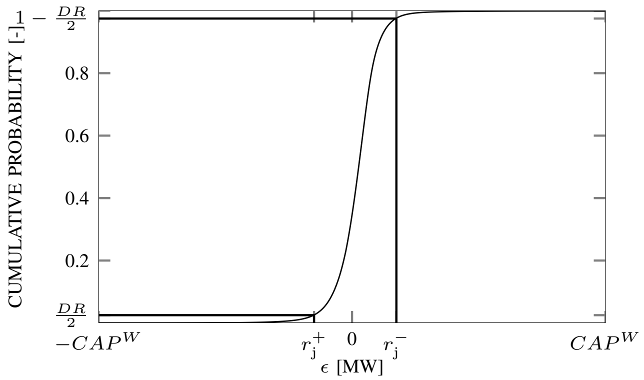


Fig. 1. Conceptual example of the calculation of the required reserves. Given the design reliability DR and the cumulative probability density function of the WPFE ϵ for forecast G_j^F, the upward r_j^+ and downward reserves r_j^- can be calculated [9]. CAP^W is the installed wind power capacity.

As wind power is the only source of uncertainty considered in this paper and can only be regulated downward – i.e. curtailment of excess wind power –, it is assumed that all downward reserves can be ensured by curtailment. There is no additional constraint imposed on the downward flexibility of the conventional power plants. However, during dispatch online power plants may ramp down to provide downward flexibility.

The HUCM thus consists of a SUCM (Eq. (1)–(23)), complemented with a demand for upward reserves and all constraints imposed on the reserves that can be scheduled to satisfy this demand (Eq. (24)–(30)). Furthermore, an additional term is added to the objective function (Eq. (1)) that allows the violation of the reserve constraint in case of a shortage of supply. This term is of the form $VOR \cdot TP \cdot \sum_j s_j^+$, in which VOR represents the value of not-scheduled reserves. This value should not exceed the value of lost load in order to curtail reserves before load is shed.

B. Probabilistic reserve sizing

Upward and downward reserves are calculated per forecast interval, based on the cumulative probability density function (cdf) of the forecast error, as described in detail in [9]. The reserves are sized to cover a certain percentage – i.e. a certain cumulative probability – of the WPFE. This percentage will be referred to as the *design reliability* or DR. The cumulative probability of errors that are not covered by the reserves is equally distributed over the upward and downward reserves. For each forecast G_j^F at time step j, upward r_j^+ and downward r_j^- reserves are thus determined as the smallest amounts of reserves that satisfy the following inequalities

$$\forall j: \quad F_j(r_j^+) \leq \frac{DR}{2} \tag{31}$$

$$\forall j: \quad F_j(r_j^-) \geq 1 - \frac{DR}{2} \tag{32}$$

with $F_j(\epsilon)$ the cdf of the forecast error ϵ for that forecast interval i, as illustrated in Fig. 1. The resulting demand for upward reserves D_j^+, as used in Eq. (24), is calculated as the minimum of $|r_j^+|$ and the demand at that time step D_j:

$$\forall j: \quad D_j^+ = \min(|r_j^+|, D_j) \tag{33}$$

Note that we here postulate a design reliability, equal for each forecast interval, without regard for the cost of ensuring that reliability. However, the reserve requirement will only serve as a lower limit on the amount of reserves that need to be scheduled in the HUCM. Extreme forecast errors will be dealt with via scenarios.

C. Data & assumptions

The simulations are run for a power system inspired on the Belgian power system, assuming a 30% wind power penetration (annually, energy basis). The peak demand in this system typically occurs in winter time and equals about 14 GW, while the lowest demand – around 6 GW – typically occurs during daytime in the summer. The annual Belgian demand amounts to about 82 TWh [21]. Electrical energy from RES other than wind (7% of annual electric energy demand) is treated as a demand correction and cannot be curtailed. The demand profile (2011) and wind power data (2012–2013) are obtained from Elia, the Belgian TSO [22]. The Belgian conventional generation system, consisting of 71 power plants and combined-heat-and-power plants, in total 13,920 MW of dispatchable capacity, has been taken from Elia [22]. The nominal efficiency of the plants is based on the type, the fuel and the age of the power plant. The other technical characteristics of the power plants are based on ENTSO-E [23]. Open cycle gas turbines and oil-fired units with a size of less than 100 MW, a minimum up- and down time of 1 time step and the capability to ramp from zero output to full capacity within a time step are considered as 'fast-starting units'. In total 35 fast-starting units (1,118 MW) are considered in this case study. One pumped hydro storage power plant has been included, with a maximum capacity of 1,308 MW, a round trip efficiency of 75% and a storage capacity of 3,924 MWh. The minimum energy content of the storage facility is set to 10% of the maximal capacity. The CO_2-price is set to 10 $\frac{\mathrm{EURO}}{\mathrm{tCO_2}}$. The value of lost load is set to 10,000 $\frac{\mathrm{EURO}}{\mathrm{MWh}}$, while VOR equals 5,000 $\frac{\mathrm{EURO}}{\mathrm{MWh}}$. Curtailment is assumed to be free.

The planning horizon considered in the optimization is 24 hours. The time step in the optimization is 15 minutes. To ensure continuity, each optimization takes into account the values of the optimization variables over the previous 24 hours, based on the dispatch taking into account the scenario that represents the scaled measured wind power output of the previous day[4]. The dispatch to evaluate the performance of the unit commitment models is performed for 500 scenarios per day to get a proxy of the expected performance – in terms of reliability and system cost – of the calculated unit commitment. During this dispatch, the unit commitment schedule ($z_{i,j}$) is fixed. Fast-starting units that were scheduled in at least one scenario, are allowed to start-up during dispatch if needed.

[4]This approach does not fully represent the structure of the decision problem a system operator is faced with. First, within the dispatch problem, there is no uncertainty. In reality, information is released as time progresses. As such, the dispatch of reserves is optimized and the flexibility of the power system might be overestimated. Second, adaptations of the unit commitment, except of fast-starting units, during the dispatch are not considered.

III. Results & Discussion

In Section III-A, we present the design of the proposed hybrid reserve rule. Based on simulations for the day with the demand closest to the average daily demand, we will show that the addition of a probabilistic reserve rule to a SUCM with a relatively small set of scenarios allows to approximate the expected costs (optimality) and expected loss of load (reliability) of the stable solution of that SUCM. Calculation times are, however, significantly lower. A detailed analysis of the scheduled reserves shows that the scheduled flexibility is similar. The effect of a constraint on the amount of load shedding during the unit commitment phase is analyzed in Section III-B. Results indicate that a higher reliability comes at a significant cost. Note that the 'average day' is merely used for the design of the HUCM and the analysis of the load shedding constraint. By no means should these results be interpreted as being representative for the performance of the HUCM under other circumstances. Therefore, and to gain statistical significance, in Section III-C the performance of the proposed reserve rule is investigated for four representative weeks of the year. Results confirm that the proposed HUCM (5 scenarios, probabilistic reserve requirement with a design reliability of 80%) approximates the stable solution of the corresponding SUCM, at roughly one thirtieth of the computational cost.

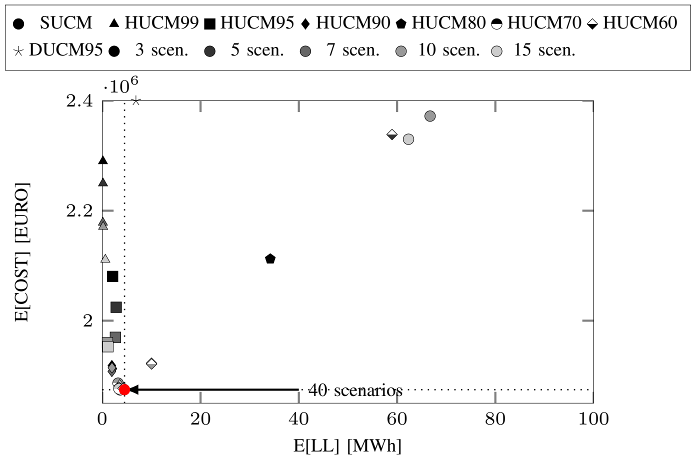
A. Design of a hybrid unit commitment policy

As a first step in the design of the hybrid reserve sizing and allocation, we analyze the performance of various probabilistic reserve rules in HUCMs for the day with a demand closest to the yearly average. The benchmark will be the stable solution of the corresponding SUCM. As explained in Section II, the stable solution is the best available proxy for the solution of the full SUCM, here approximated by the solution of the SUCM taking into account 40 scenarios[5]. This solution is indicated in red in Fig. 2a-2c. The solution of the full stochastic program – if it could be obtained – would be characterized by approximately the same expected cost (indicated by the horizontal line in Fig. 2a-2b).

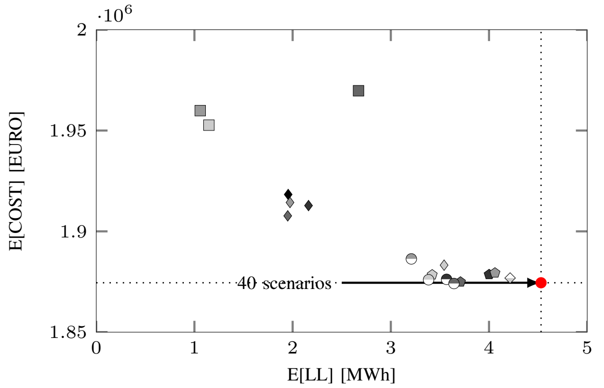
Reducing the number of scenarios in the SUCM leads to inferior results: expected costs increase due to less robust planning, which increases the amount of expected lost load and thus the expected costs. This explains the linear trend right of the dotted vertical line in Fig. 2a. The slope of this trend line is directly related to the value of lost load (10,000 $\frac{EUR}{MWh}$). As more scenarios are added to the optimization, costs in general decrease. As shown in Fig. 2a, this process of convergence[6] can be accelerated by adding a reserve requirement (Eq. (24)) to the optimization, e.g. HUCM80, considering 3 scenarios, already outperforms the SUCM solution considering 15 scenarios. These reserve requirements will ensure the availability of sufficient reserve capacity in the system in certain, not too

[5]Although one can never definitely prove that this solution is stable – as this would require solving the SUCM with an infinite set of scenarios – the results show that the addition of more scenarios does not further improve the quality of the solution. Hence, one can conclude that this solution is stable.

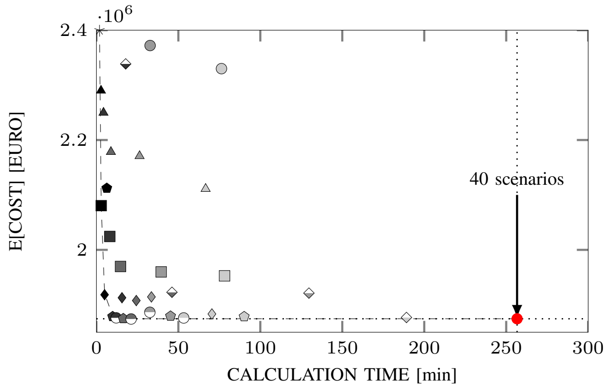
[6]One could interpret this as the evolution of the solution of the SUCM towards a stable solution.



(a) Expected cost vs. expected loss of load



(b) Expected cost vs. expected loss of load (detail)



(c) Expected cost vs. calculation time

Fig. 2. The proposed HUCM (reserves sized to capture 80% of the wind power forecast errors, complemented with 5 scenarios) results in costs and loss of load volumes comparable the solution of the SUCM considering 40 scenarios, while calculation time is reduced from over 4 hours to 10 minutes. The presented results are obtained from a simulation of the day with the demand closest to the average daily demand. 'HUCMX' indicates a HUCM with a probabilistic reserve sizing rule, designed to capture X% of the wind power forecast error. 'DUCM95'-results are obtained from a DUCM with a probabilistic reserve rule with a design reliability of 95% – the best possible DUCM solution. The grey shades indicate the number of scenarios in the optimization, ranging from 3 to 15. The dashed line in Fig. 2c represents an efficiency front: it connects the cost-optimal solutions as a function of the calculation time.

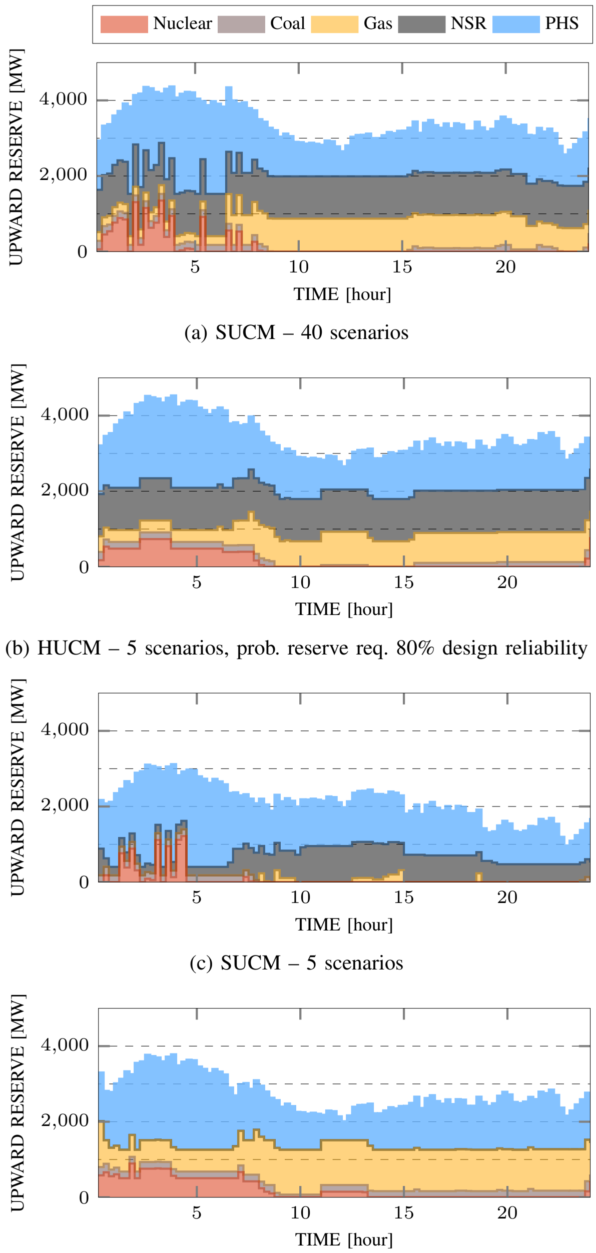
extreme scenarios, while the scenarios ensure that this capacity is sufficiently flexible (and available, in case of pumped hydro storage [5]). Furthermore, these scenarios cover certain extreme events or scenarios, which require more reserves than covered by the reserve constraint.

Alternatively, one could increase the demand for reserves in order to further accelerate the convergence of the solution or to completely eliminate the need to consider scenarios in the optimization. However, as indicated the Fig. 2a, from a certain level of reserves, this triggers too much online capacity. As a result, the expected loss of load decreases, even below the level of the stable solution of the SUCM, but operational costs rise significantly. As can be seen in Fig. 2a, this ‘overshoot’ in reserve capacity also means that the difference between adding a small number of scenarios (e.g. 3) or considering a large scenario set (e.g. 15) does not affect the solution as much as for a SUCM. However, the presence of these scenarios does impact the type and amount of reserves scheduled, thus the reliability and cost of the obtained unit commitment, as is evident from the comparison of the result of the best possible solution without considering any scenario (‘DUCM95’: DUCM with a probabilistic reserve rule designed to capture 95% of the wind power forecast errors)[7] and the equivalent results from HUCMs.

In general, as more scenarios are added to the optimization, calculation times increase (Fig. 2c). In the extreme case, when only the forecast is considered (no scenarios, ‘DUCM95’), calculation times drop to 1 minute. Adding scenarios steadily increases the calculation time. However, differences in calculation time are apparent between different hybrid strategies with the same number of scenarios. If the demand for reserves increases, fewer of the scenarios considered impose binding constraints on the optimization. Although the problem size is the same, calculation times are significantly lower than for hybrid (or pure stochastic) optimization problems with less stringent reserve constraints. The results connected by the dashed line in Fig. 2c can be seen as a Pareto front: for a specific calculation time, the dashed line indicates the lowest achievable expected operational cost. From that perspective, an efficient HUCM – weighing optimality, calculation time and reliability – is the combination of 5 scenarios and a probabilistic reserve rule that is designed to cover 80% of the forecast errors. Under forecast conditions, the SUCM & HUCM result in similar dispatches. Under the hybrid approach, the share of gas is increased, as this capacity is committed to satisfy the reserve constraint, while the output of pumped hydro storage is decreased.

In terms of the scheduled upward reserves (Fig. 3), differences are more evident. The upward flexibility is calculated as the scheduled reserves (spinning & non-spinning), curtailment of RES-based generation, the scheduled pumping power and the available turbine capacity of the pumped hydro storage plant under forecast conditions. Note that employing the last two options as upward flexibility – i.e. reducing the pumping power or increasing the turbine output – might lead to too little available water in the upper basin, thus energy, at a later stage. The upward flexibility from scheduled curtailment of wind power is in all cases non-existent and thus not shown.

The scheduled upward reserves as obtained from the SUCM (40 scenarios) (Fig. 3a) and the HUCM (Fig. 3b) are very similar. Throughout the day, around 2,000 MW of conventional capacity is scheduled as upward reserves. During the first hours of the day, when the infeed of RES-based generation is high, part of this flexibility stems from nuclear power plants. In addition, during these hours, additional flexibility



(a) SUCM – 40 scenarios

(b) HUCM – 5 scenarios, prob. reserve req. 80% design reliability

(c) SUCM – 5 scenarios

(d) DUCM – prob. reserve req. 80% design reliability

Fig. 3. The scheduled flexibility, per technology, offered by conventional capacity and the pumped hydro storage power plant as calculated with the SUCM, considering 40 or 5 scenarios, the HUCM, considering 5 scenarios and a probabilistic reserve constraint with an expected reliability of 80% and DUCM with the same probabilistic reserve requirement. NSR stands for ’non-spinning reserves’, PHS for pumped hydro storage.

[7]The DUCM does not have the option to schedule non-spinning units, as the probability of activating these units, and thus the cost of scheduling these units, is unknown. All reserves scheduled in the DUCM are thus spinning reserves (see further).

could be available from the PHS. Indeed, during these hours, the PHS is scheduled to pump under forecast conditions. During the remainder of the day, most of the spinning reserves (around 1,000 MW) are gas-fired units. Throughout the day, the SUCM and HUCM schedule around 1,000 MW of non-spinning reserves (oil- and gas-fired peaking units). In contrast, the SUCM considering 5 scenarios (Fig. 3c) and the DUCM (Fig. 3d) do not result in sufficient scheduled reserves, which triggers high levels of load shedding during dispatch (1,000 MWh and 135 MWh respectively). The SUCM considering 5 scenarios does not have sufficient information –i.e. scenarios– available to trigger spinning reserves. As a result, it relies on non-spining reserves, which are insufficient to mitigate the wind power forecast errors that occur during dispatch. In contrast, the DUCM by design only schedules spinning reserves. In order to schedule non-spinning reserves in a DUCM, one should ex-ante predict the cost associated with (1) excluding these power plants from the unit commitment schedule, i.e. a reservation cost and (2) activating these reserves. Typically, the last term dominates the cost of non-spinning reserves due to the high operational costs of the power plants providing these reserves. However, the cost of activation is an *expected* cost, depending on the operational cost of the power plant and the probability that this power plant will be activated. As this probability is ex-ante unknown, an optimal trade-off between spinning reserves (cheap, but online, leading to a less compressible power system) and non-spinning reserves (highly flexible, allowing a higher absorption of intermittent generation, but typically more expensive when activated) is difficult to attain, as it is highly sensitive to the estimated expected cost of non-spinning reserves. As a HUCM and SUCM contain scenarios in which these non-spinning reserves are dispatched, and the cost of activating these non-spinning reserves is thus calculated, such an optimal trade-off is possible.

In conclusion, we have shown that there exists a combination of a probabilistic reserve requirement with a certain design reliability and a reduced set of scenarios that, when combined in HUCM, allows to approximate the stable solution of the stochastic unit commitment model. Not only the operational cost, but also the scheduled spinning and non-spinning reserves and resulting reliability are similar. Solving such a hybrid model however takes significantly less time than a SUCM.

B. The cost of reliability

With the proposed formulation of the SUCM and HUCM one obtains a unit commitment schedule that reflects an optimal trade-off between reliability and the cost of ensuring that reliability. Indeed, load shedding is scheduled in extreme scenarios when the expected cost of load shedding is dominated by the cost reduction from committing less reserves, thus capacity[8]. As shown in Fig. 2a-2b, this optimum lies around 4.5 $\frac{MWh}{day}$ of expected load shedding.

[8]Note that the resulting level of load shedding is thus sensitive to the value of lost load ($VOLL$). A sensitivity analysis towards this value is however out of the scope of this paper.

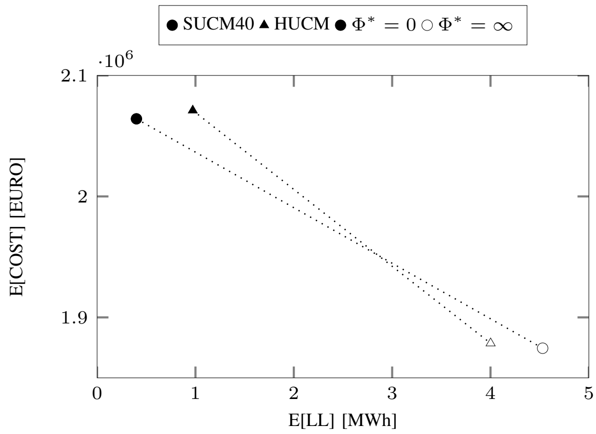


Fig. 4. The inclusion of a reliability constraint (Eq. (34)) increases the expected costs, but increases the reliability of the resulting unit commitment schedule if sufficient scenarios are considered in the unit commitment model (e.g. SUCM40). Expected costs include the cost of load shedding.

To investigate the cost of higher reliability levels, one could include a constraint on the amount of allowed load shedding during the unit commitment phase. Such a constraint limits the expected amount of load shedding over the planning period:

$$\forall j: \quad \sum_{s} P_s \cdot TP \cdot \phi_{j,s} \leq \Phi^* \tag{34}$$

Results for Φ^* equal to 0 and ∞ $\frac{MWh}{day}$, obtained with the SUCM (40 scenarios) and the proposed HUCM (5 scenarios, combined with a probabilistic reserve rule with a design reliability of 80%) for the day with the demand closest to the average daily demand are summarized in Fig. 4. Note that imposing constraint (34) on this optimization means the true solution of the stochastic optimization problem would result in an expected lost load $E[\phi]$ at most equal to the chosen Φ^*. Due to the relatively small set of scenarios[9] our stable solution for $\Phi^* = 0$ however results in a limited amount of lost load after re-evaluation on a large set of scenarios (Fig. 2a).

As results for $\Phi^* = 0$ and $\Phi^* = \infty$ indicate, increasing levels of reliability – imposed on the UCM – lead to increasing costs. According to the SUCM, the cost of reliability – i.e. the reduction of the expected lost load from 4.53 $\frac{MWh}{day}$ to 0.41 $\frac{MWh}{day}$ – amounts to 9% of the total operational cost. Fully relaxing Eq. (34) thus yields an expected operational cost reduction of 190,000 $\frac{EUR}{day}$ or 45,994 $\frac{EUR}{MWh}$. Note that the latter value exceeds the $VOLL$: the expected cost of load shedding, by definition equal to 10,000 $\frac{EUR}{MWh}$ is dominated by the cost reduction from committing less reserves if one does no enforce Eq. (34). In the HUCM, similar cost decreases can be observed. Relaxing the reliability constraint decreases the expected loss of load with 3.0 $\frac{MWh}{day}$, while costs increase by 192,490 $\frac{EUR}{day}$ or 10%.

The introduction of the reliability constraint, however, de-

[9]Although the probability of the scenarios considered in the optimization and the re-evaluation in both cases sums up to 1 – in other words, the probability is redistributed over the reduced set of scenarios in the unit commitment phase, it appears that a limited set of scenarios is not capable of capturing all events that will trigger load shedding. The probability weighted character of the reliability constraint is thus not the problem, but the number of scenarios, and thus the events considered, are.

creases the computational cost of the optimization in most cases. Calculation times for the SUCM considering 40 scenarios decreases from 256 to 118 minutes. The calculation time of the HUCM remains constant.

C. Performance of the proposed HUCM

To evaluate the performance of the hybrid reserve sizing and allocation methodology, four representative weeks were selected based on the residual demand[10]. The week with the residual demand closest to the average weekly demand for electrical energy (week 30), the week with the lowest residual energy demand (week 52), the week with the highest residual energy demand (week 9) and the week with the residual demand with the highest variability (week 39) were selected. For these weeks, expected costs, curtailment and loss of load were calculated based on the SUCM considering 40 and 5 scenarios, as well as with the proposed HUCM (5 scenarios, probabilistic reserve constraint designed to capture 80% of all forecast errors). The first result will serve as a benchmark[11], while the second allows us to assess the added value of a reserve constraint in a SUCM. A comparison to a SUCM considering the same number of scenarios as a HUCM is often found in the literature, the comparison to a solution obtained from a SUCM considering a large number of scenarios is seldom reported. In all simulations, the scenario reduction was performed considering the same probability metric. Results are summarized in Table I. In all simulations, the loss of load volume is unconstrained ($\Phi^* = \infty$).

During the 'average residual demand' week, wind energy covers approximately 11% of demand. The residual demand varies between 4,300 and 8,900 MW. The HUCM performs 48% better than the SUCM considering the same number of scenarios (SUCM5). This large cost difference stems from the reduced reliability and corresponding expected lost load in the SUCM5 solution. Remarkably, the HUCM outperforms the SUCM (40 scenarios) by 1.6%. However, the SUCM did not fully converge for all considered days within 12 hours.

Negative residual loads are experienced on a regular basis during the week with the lowest residual demand. The residual demand fluctuates between -2,500 and 7,400 MW. In this case, the HUCM outperforms the SUCM5 in terms of expected costs by a factor 10, which is due to the high amount of lost load in the SUCM5 solution. Note that increasing levels of (free) wind power will reduce operational costs in absolute terms, which will increase the relative differences between the results of the different UCMs – a remark that holds for the rest of this section. The computational cost of the SUCM5 is considerably lower in week 52 compared to the other weeks. Note the high curtailment volumes in the HUCM5 solution. The model schedules cheap, but rather inflexible power plants to absorb possible forecast errors. This keeps loss of load volumes low, but results in a incompressible power system and thus high curtailment volumes. The computational cost rises considerably for the SUCM40, no feasible solution within the optimality gap was found within 12 hours.

[10]This residual demand is calculated as the difference between the historical demand time series and the rescaled historical wind power time series.

[11]Note that we do not claim this solution to be the stable solution, as we have not tested this for all considered days. However, due to the high number of scenarios, one can regard this solution as a reference solution.

TABLE I
COMPARISON OF THE PROPOSED HUCM WITH THE SUCM EMPLOYING 40 (THE BENCHMARK) AND 5 SCENARIOS FOR FOUR REPRESENTATIVE WEEKS OF THE YEAR. ALL VALUES ARE EXPECTED VALUES (E[]), REPORTED FOR THE FULL WEEK, EXCEPT THE CALCULATION TIME, WHICH IS THE AVERAGE CALCULATION TIME PER UNIT COMMITMENT. SOLUTIONS THAT DID NOT FULLY CONVERGE ARE INDICATED WITH AN ASTERISK.

		SUCM40	HUCM5	SUCM5
E[cost]	[EUR]	13,500,652*	13,286,933	19,711,804
E[χ]	[MWh]	0.7950*	1.5	0.9
E[ϕ]	[MWh]	17.6*	1.8	660.3
E[wind]	[%]	10.6*	10.6	10.6
Calc. time	[min.]	655*	26	20

(a) Average residual demand (week 30).

		SUCM40	HUCM5	SUCM5
E[cost]	[EUR]	-	2,561,033	21,280,630
E[χ]	[MWh]	-	114,770	53,390
E[ϕ]	[MWh]	-	5.5	1,891
E[wind]	[%]	-	77.7	82.3
Calc. time	[min.]	-	14	6

(b) Min. residual demand (week 52).

		SUCM40	HUCM5	SUCM5
E[cost]	[EUR]	27,250,033	27,732,659	38,093,096
E[χ]	[MWh]	0	0	0
E[ϕ]	[MWh]	8.1	18.5	1,109
E[wind]	[%]	13.5	13.5	13.5
Calc. time	[min.]	447	14	13

(c) Max. residual demand (week 9).

		SUCM40	HUCM5	SUCM5
E[cost]	[EUR]	6,348,099	6,731,620	8,868,911
E[χ]	[MWh]	20,821	30,514	21,060
E[ϕ]	[MWh]	10.7	17.3	268
E[wind]	[%]	50.9	50.1	50.8
Calc. time	[min.]	449	15	12

(d) Max. var. residual demand (week 39).

In week 9 (highest residual demand), the residual demand varies between 5,250 and 11,500 MW. Similar to the results for the week with the average residual demand, the HUCM performs significantly better than the SUCM5 (expected costs are reduced by 32%), while the SUCM40 only performs 1.8% better than the HUCM.

To test the effect of variability, simulations were performed for week 39 (most variable residual demand), in which the residual demand exhibits ramps up to 1,650 $\frac{MW}{15\ min}$. Over 40% of the demand is covered by wind energy. Compared to the SUCM (40 scenarios), the HUCM leads to an increase of expected operational costs of 6%. These are partly related to the lower reliability level: the HUCM does not schedule sufficiently flexible capacity, which leads to load shedding. Note furthermore that the SUCM40 triggers less curtailment, as the resulting unit commitment schedule is more flexible than that obtained with the HUCM. Compared to the SUCM5, the HUCM reduces operational costs by 32% by significantly increasing reliability.

Throughout the simulations, calculation times for the SUCM5 and HUCM are comparable: 14 to 26 minutes for the HUCM, 6 to 20 minutes for the SUCM. Solving the SUCM50 takes at least thirty times longer.

IV. CONCLUDING REMARKS

As illustrated in our introduction, high shares of renewables (RES) will increasingly challenge power system operators to ensure a reliable and cost-efficient operation of the power system. Especially wind and solar PV have a stochastic character: they are variable (not or only limitedly dispatchable) and to some extent unpredictable. Although the accuracy of RES forecasts is increasing, deviations from what is expected need to be overcome with up- or downward regulation of dispatchable generation or load. Stochastic unit commitment models (SUCMs), with a direct representation of the uncertainty via a set of scenarios in the unit commitment model, lead to an optimal trade-off between reliability and system cost. However, these SUCMs are computationally costly. Furthermore, the literature and own research have shown that their performance can be improved by adding reserve requirements and that probabilistic reserve requirements outperform any other reserve requirement in a deterministic equivalent. However, the addition of such a probabilistic reserve requirement to a SUCM has not yet been attempted.

In this paper we therefore develop a hybrid unit commitment model or HUCM, combining a state-of-the-art probabilistic reserve rule and a SUCM. Such a HUCM combines a limited number of scenarios and a probabilistic reserve constraint to approximate the results of a SUCM in terms of operational costs, curtailment and lost load, but at a fraction of the computational cost. In a detailed design, we analyze the quantification and allocation of spinning and non-spinning reserves, with a specific focus on low-carbon systems with high RES penetration. As shown in Section III, the addition of probabilistic reserve constraints to a SUCM can speed up the 'convergence' of the stochastic problem to its stable solution – i.e. the benchmark in our analysis. However, adding too stringent reserve constraints will lead to an overshoot: reliability may be higher for these cases, but this comes at a significant cost. For the presented case study (i.e. the Belgian power system, with wind energy responsible for 30% of the annual energy demand), we propose a HUCM considering 5 scenarios and a probabilistic reserve requirement with a design reliability of 80%. This reduces calculation times by a factor 30 compared to a SUCM (40 scenarios), while reliability and costs are approximately the same. Notably, these observations were confirmed for various levels of required reliability (expressed by the reliability constraint Eq. (34)). The proposed HUCM was thoroughly tested for four representative weeks of the year. This confirmed that the HUCM outperforms the SUCM with the same number of scenarios in terms of cost, mainly due to an increased reliability. The SUCM considering a large set of scenarios yields lower expected operational costs (1.8 to 6%), but the computational cost is roughly a factor 30 higher.

The presented HUCM can be used to assess the impact of uncertainty on reasonably large low-carbon electric power systems where SUCMs models would become computationally intractable. Likewise, independent system operators (ISO) could use this model to optimize their unit commitment decisions taking into account the uncertainty in their system.

This work may be strengthened in the following fields. First, we currently only consider WPFE as a source of uncertainty. Considering multiple sources of uncertainty and studying their interaction may increase the added value of this work. Moreover, one could study for which sources of uncertainty scenarios are required, and for which traditional reserve constraints suffice. Second, the HUCM itself could be improved. Further detailing the reserve constraints used and allowing for the participation of pumped hydro storage units, may further improve performance. Last, the HUCM has been tested on the Belgian power system. Considering larger areas, such as Central West Europe, allows to study e.g. how this reserve rule allows pooling of reserves across areas, as well as the interaction of uncertainty in different areas.

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Kenneth Bruninx (S'15) received the M.S. degree in energy engineering in 2011 from the Univeristy of Leuven (KU Leuven).

Currently, he is working as a PhD-researcher at the University of Leuven Energy Institute, TME branch (energy conversion).

Kenneth Van den Bergh (S'15) received the M.S. degree in energy engineering in 2012 from the University of Leuven (KU Leuven), Belgium.

He is currently pursuing the Ph.D. degree at the Energy Institute of the University of Leuven, TME Branch (energy conversion).

Erik Delarue (M'15) received the M.S. degree in mechanical engineering in 2005 and the Ph.D. degree in mechanical engineering in 2009, both from the University of Leuven (KU Leuven), Belgium.

Currently, he is a postdoctoral research fellow of the Research Foundation – Flanders (F.W.O.), at the University of Leuven Energy Institute, TME branch (energy conversion).

William D'haeseleer received his M.S. degree in Electro-Mechanical Engineering and a Master in Nuclear Engineering from the University of Leuven (KU Leuven), Belgium, in 1980 and 1982, respectively. He obtained another M.S. degree in Electrical Engineering and the Ph.D. degree from the University of Wisconsin-Madison, USA, in 1983 and 1988, respectively.

Currently, he is a full-time Professor in the College of Engineering of the KU Leuven and Director of the University of Leuven Energy Institute.