THE DIELECTRIC RESPONSE FUNCTION FOR AN ANISOTROPIC PLASMA

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We demonstrate how approximations may be obtained for the longitudinal dielectric response function of the anisotropic plasma of Joyce and Montgomery by the simple additivity principle for the susceptibility and by a Galilean transformation of the Lindhard expression.

Joyce and Montgomery [1] investigated the plasma dielectric response function for the anisotropic electron distribution function:

$$f(v) = \left(\frac{m}{2\pi\theta}\right)^{3/2} \left[1 + \frac{\alpha}{2} \left(\frac{m}{\theta} (v \cdot \hat{a})^2 - 1\right)\right] \exp(-mv^2/2\theta) \quad (1)$$

where m is the electron mass, θ the temperature in energy unit and $\hat{a}$ is some unit vector. This distribution function is an approximation to the following function:

$$f(v) = \left(\frac{m}{2\pi\theta}\right)^{3/2} [(1-A) \exp(-mv^2/2\theta) + \frac{1}{2} A (\exp(-m(v-v_0)^2/2\theta) + \exp(-m(v+v_0)^2/2\theta))] \quad (2)$$

with the connection

$$\alpha = Amv_0^2/\theta \quad \text{and} \quad \hat{a} = v_0/v_0. \quad (3)$$

As is seen in expression (2), the plasma is essentially constituted of three parts, two of them moving with opposite mean velocities $\pm v_0$ with respect to the third part.

It is possible to derive good approximations for this anisotropic case by applying the additivity of the susceptibility, i.e., the dielectric function minus one. When applying this on the function $\epsilon(k, \omega)$ we have to take account on the Doppler shifts of the frequency ω of the streaming parts, which gives us the following result:

$$\epsilon(k, \omega) = 1 + (1-A)[\epsilon_L(k, \omega) - 1] +$$

$$+ \frac{A}{2} [\epsilon_L(k, \omega + k \cdot v_0) + \epsilon_L(k, \omega - k \cdot v_0) - 2]. \quad (4)$$

With ϵ_L we mean the well known Lindhard expression for the dielectric function of a Boltzman plasma [2]. The transformation of the frequencies may be understood from the fact that the Galilean transformation law $\mathbf{r}' = \mathbf{r} - \mathbf{v}t$ transforms a plane wave $\exp[i\mathbf{k} \cdot \mathbf{r} - i\omega t]$ into another plane wave $\exp[i\mathbf{k} \cdot \mathbf{r} - i(\omega + \mathbf{k} \cdot \mathbf{v})t]$.

In the low frequency limit, where

$$\epsilon(k, \omega) \approx 1 + \frac{m\omega_0^2}{k^2\theta} \left(1 - \frac{m\omega^2}{k^2\theta}\right)$$

with ω_0 the usual plasma frequency, we obtain the approximation

$$\epsilon(k, \omega) \approx 1 + \frac{m\omega_0^2}{k^2\theta} \left[1 - \frac{m\omega^2}{k^2\theta} - \frac{mA}{k^2\theta} (\mathbf{k} \cdot \mathbf{v}_0)^2\right] \quad (5)$$

which for $\omega = 0$ coincides with the result of Joyce and Montgomery. In the high frequency limit, where

$$\epsilon_L(k, \omega) \approx 1 - \omega_0^2/\omega^2$$

we get the approximation

$$\epsilon(k, \omega) = 1 - (\omega_0^2/\omega^2) [1 + (3A/\omega^2)(\mathbf{k} \cdot \mathbf{v}_0)^2]. \quad (6)$$

References

- [1] G. Joyce and D. Montgomery, Phys. Fluids 10 (1967) 2017.
- [2] J. Lindhard, Dan. Vid. Selk., Mat.-fys. Medd. 28, No. 8 (1954).