Time-Domain Finite-Element Homogenization for Laminated Iron Cores Using a Magnetic-Field Formulation

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***Abstract*—This paper deals with a time-domain finite-element homogenization technique for laminated iron cores of electromagnetic devices based on a magnetic-field formulation. The separate discretization of each lamination is avoided by using dedicated skin-effect basis functions.**

***Index Terms*—Homogenization techniques, finite-element methods, eddy currents, laminated ferromagnetic cores.**

I. INTRODUCTION

The eddy current effects in laminated iron cores may considerably alter the overall performance of electromagnetic AC devices and should be thus accounted for in early stages of the design. Finely discretizing each separate lamination in standard finite-element (FE) models is computationally prohibitive. More pragmatic techniques have been conceived (see overview in [1]).

An *ad hoc* time-domain homogenization technique for lamintated iron cores, based on the magnetic-vector-potential ($a-$)formulation, is developed in [2]. In this paper the method is extended to the dual magnetic-field ($h-$)formulation.

II. 1-D MODEL OF A LAMINATION WITH SKIN-EFFECT BASIS FUNCTIONS

Let us consider a homogeneous isotropic lamination (conductivity σ) of thickness d ($-d/2 \leq z \leq d/2$). The magnetic behaviour is characterised by the nonlinear constitutive law $b(z,t) = b_{\mathrm{fe}}(h(z,t))$, with $b(z,t)$ and $h(z,t)$ the x components of the induction and the magnetic field, respectively. The 1D eddy-current problem is governed by the following partial differential equation:

$$\partial_z^2 h(z,t) = \sigma\,\partial_t b(z,t) \quad \text{with} \quad b(z,t) = b_{fe}\left(h(z,t)\right). \qquad (1)$$

We introduce skin-effect polynomial basis fuctions to approximate the time-domain FE solution of (1) [2]. Key homogenization quantities are the induction averaged over the thickness, $b_0(t)$, and the surface magnetic field, $h_{\mathrm{s}}(t)$. We adopt the Legendre polynomials $\alpha_k(z) = \mathcal{P}_{2k}\left(\frac{2z}{d}\right)$ and the polynomials $\beta_k(z)$, satisfying $-d^2\partial_z^2\beta_k(z) = \alpha_k(z)$, $k = 0, \cdots, n$. The expansion of $h(z,t)$ is given by:

$$h(z,t) = h_{\mathrm{s}}(t) + \sum_{k=0}^{n} \beta_k(z)\; h_k(t)\,. \qquad (2)$$

Then from (1), the time derivative of $b(z,t)$ follows:

$$\partial_t b(z,t) = -\frac{1}{\sigma d^2}\sum_{k=0}^{n} \alpha_k(z)\, h_k(t)\,. \qquad (3)$$

Finally by weakly imposing the time derivative of the constitutive law, we obtain the system of differential equations to be coupled with the FE implementation of the problem at hand. It reads:

$$\partial_t\left[B_{\mathrm{s}}(t)\right] = \frac{1}{\sigma}\left[P\right]\left[H(t)\right] + \int_{-d/2}^{d/2} \partial_t b_{\mathrm{fe}}\Big(h(z,t)\Big)\left[A(z)\right]\mathrm{d}z\,. \qquad (4)$$

For the sake of validation, we compare the normalized analytical complex reluctivity ($\boldsymbol{\nu}/\nu_{\mathrm{fe}}$) [2], as a function of the relative lamination thickness d/δ (skin depth δ), with the one we can get from the linear frequency-domain version of (4) (with $b_{\mathrm{fe}} = \mu_{\mathrm{fe}}\,h$, μ_{fe} constant, $\nu_{\mathrm{fe}} = 1/\mu_{\mathrm{fe}}$). This system is (complex vectors in bold)

$$\left[\boldsymbol{H}_{\mathrm{s}}\right] = -\frac{\imath\delta^2}{2d^2}\left[P\right]\left[\boldsymbol{H}\right] + \left[Q\right]\left[\boldsymbol{H}\right]\,. \qquad (5)$$

In Fig. 1, we observe a fast convergence of the solution to the reference with increasing k.

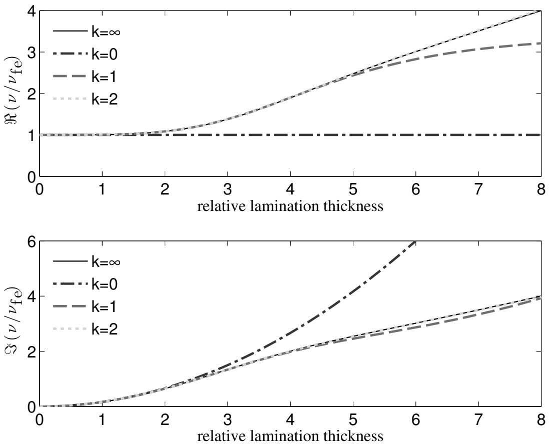

Fig. 1. Normalized complex reluctivity (real and imaginary part) as a function of the relative lamination thickness (d/δ).

In the extended paper, the proposed approach will be elaborated in detail and a nonlinear time-domain application considered.

REFERENCES

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