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Commuting in a federation:
*Horizontal and vertical tax externalities revisited**

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Abstract

In this paper, commuting is introduced to a federal setting where an ad valorem *residence* based tax on labour income is decentralised. Under full decentralisation, this has lower-level (state) governments set inefficiently low taxes, even when households as a whole do not migrate. The motive of state governments is not to attract more workers, but to boost labour supply of own residents and hamper labour supplied by non-residents. When the labour tax base is co-occupied by the federal and state governments furthermore, either public under- or overtaxation may occur. Our model identifies clear conditions for states to overprovide, i.e. for the overall fiscal externality to be negative. Moreover, such a negative externality may arise even when the vertical as well as horizontal externalities are positive in isolation, and one would rather expect *under*provision. Lastly, when states differ in terms of preferences and technology, an inflow of commuters makes it more likely for states to set taxes inefficiently low.

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1 Introduction

When more than one (level of) government has taxing authority in a federation, taxes will be imposed on the same, or at least interdependent tax bases. Tax policies of one lower-level government [1] thus have an impact on tax revenues raised by other governments, as well as on the welfare of residents living in other states. Now, when state governments disregard these effects of own taxation on other states, *tax externalities* will distort regional fiscal decision making leading to under- or overprovision of public services.

In the case of tax base mobility between states, positive *horizontal* externalities drive the outcome where regional taxation as well as public provision is set inefficiently low [2]. Tax competition leading to a 'race to the bottom' scenario is often given as the textbook example here, although other forms of tax exporting exist. When tax bases are co-occupied by federal and state governments moreover, *vertical* externalities enter the fray. Here the externality works through the effect on the *shared* tax base, which may contract due to increased state taxation so that federal tax revenues decrease as well. This negative effect is not taken into account by state governments, resulting in inefficiently *high* regional tax rates compared to the unitary country second-best optimum [3]. Also, when taxation is ad valorem these vertical externalities may have a positive sign, as was shown by Dahlby (2003). Which kind of externality gains the upper hand when both horizontal and vertical externalities are at work lastly, was looked into by Keen and Kotsogiannis for capital taxation in a theoretical framework (2002), and empirically by Brulhart and Janetti (2006).

In this paper we set up a theoretical model sizing up vertical as well as horizontal externalities, with labour income as the tax base through which the externalities will work. Given the recent reforms agreed on by the Belgian federal government, decentralising 25% of the personal income tax, and not in the least because of the often unique characteristics of the Belgian federation, the model is tailored as much as possible to the Belgian setting. Being a small federation of only three states, marked by significant inter-state disparities in preferences and productivity, and enjoying a very high degree of inter-state commuting [4] Belgium indeed makes for an interesting case. Also, zooming in on these characteristics brings out a blind spot in the existing literature, where the impact of commuting is usually downplayed.

Most models dealing with labour income as a source of externalities, are based on wage formation following from *non-integrated* regional labour markets. In other words, the standard assumption is that people only supply labour in their state of residence[5]. The only way to introduce externalities in such a setting consequently, is to allow entire households to migrate across states. But when households are perfectly mobile in this sense, horizontal externalities cancel out altogether because of 'incentive equivalence' as pointed out by Myers (1990, [6]). In this case state governments will in fact maximise the welfare of households living in the federation *as a whole*, since they take household

[1] Throughout the paper, lower-level jurisdictions will be referred to as 'states'.

[2] See e.g. Wilson (1986), Zodrow and Mieszkowski (1986). For interesting empirical work on horizontal externalities, see Brueckner (2003).

[3] See e.g. Boadway and Keen (1996), Boadway et al. (1998), Keen and Kotsogiannis (2002, 2004). Hayashi and Boadway (2001) or Andersson et al. (2004) deliver insightful empirical work.

[4] E.g., 400.000 high earning workers commute daily from Wallonia and Flanders to the Brussels region (out of a total labour force of only 4.5 million)

[5] A notable exception can be found in the last section of Boadway (1996), yet also here clearing regional labour markets drive wage formation which delivers different results compared to ours.

[6] See also Boadway (1996, 2004) or Wellisch (2000). For an interesting theoretical analysis of the inefficiencies occurring in a setting of imperfect household mobility, see Sato (2000).

migration into account. As a result, the Samuelson condition will always be met as long as the labour tax base is not co-occupied by the federal government (Boadway, 1996). The only welfare losses then follow from an inefficient *household* allocation across regions, which lies beyond our interest here since labour market-induced migration in Belgium is almost nonexistent.

For a federation with Belgian features, where at least 10% of the workforce commutes between states, the assumption of non-integrated regional labour markets becomes difficult to maintain. In our model we therefore make use of a *common* labour market, where wages are endogenously determined as commuting flows equilibrate wages across all states [7]. Policy changes in one state will consequently be felt throughout the entire federal system, even when household *migration* does not occur. We thus model a situation where horizontal externalities are re-introduced to the analysis through commuting effects. Further tailoring our model to the Belgian case, labour taxes will be *residence* based, so that workers commute until they receive the same *gross* wage in every state. Following Dahlby (2003) and Kotsogiannis and Martinez (2008) we employ an ad-valorem tax, enriching the model when it comes to vertical externalities. An inter-state commuting cost lastly, would not be vital to our Belgian focus here. Commuting from one Belgian state to another will often be as costly as commuting from one city to another within the same state.

The paper proceeds as follows. Following the detail of the model in section 2, section 3 considers the benchmark case of a unitary country where only the federal government raises taxes. Since no externalities can occur in such a setting, the second-best optimum is attained. Section 4 subsequently, sees fiscal decision making (partially) decentralised so that the externalities come to the fore. In a fully decentralised case only the state governments will levy taxes and decide on public provision. In a shared tax base case secondly, the federal government as well as state governments tax the same labour income tax base to finance their respective part of public provision. Lastly, section 5 redoes the first exercise but introduces regional heterogeneity to the analysis. Section 6 concludes.

2 The structure of the model

The federation consists of a limited number (n) of states, where the population of representative households per state is normalised to 1. Both state and federal governments are benevolent, maximising the welfare of citizens living within their respectiveborders.

Preferences of the representative household living in state i are defined by utility of the form $U_i(c_i, L_i, G_i, G_i^F) = c_i + l_i(L_i) + g_i(G_i) + g_i^F(G_i^F)$, with c_i consumption of a composite (numeraire) private good and L_i labour supplied by this household. G_i will be the *publicly provided private* good in state i provided by state i itself, whilst G_i^F marks the publicly provided private good provided by the federal government in state i [8]. Sub-utility $l_i(L_i)$ is concave and decreasing in L_i, whilst $g_i(G_i)$ and $g_i^F(G_i^F)$ are concave and increasing in G_i and G_i^F respectively. The assumption of separability in the utility function implies public provision does not affect the leisure-consumption decisions of households, and also omits income effects.

[7] A similar approach was followed by Keen and Kotsogiannis (2002), where unit taxes on capital drive the externalities and capital is fully mobile across regions.

[8] National public goods could be introduced, but this would complicate the analysis with little additional insight to the effects of decentralisation examined in what follows. Our focus lies with the inefficiency of state policies, and national public goods have no particular role to play in that regard (Boadway et al., 1998). Also, since we work with a representative consumer here, public provision can also be seen as a state public good.

Public provision is financed by an ad valorem **residence** based tax on labour, denoted by τ_i for the states and τ_0 for the federal government ($\tau_i + \tau_0 = \tau$). Profit taxes $\theta_{(i)}$, when included, are exogenously fixed and profits are taxed away entirely by the government(s).

State aggregate output x_i is produced by applying labour to a fixed factor such as land, $x_i = F_i(L_{D_i})$, with the usual properties of $F' > 0 > F''$ [9]. Firms are immobile and maximise profits, given by $\pi_i = F_i(L_{D_i}) - w_i L_{D_i}$, and choose labour demand L_{D_i} that satisfies $F'(L_{D_i}) = w_i$, with w_i the gross wage. Labour demand will thus decrease in gross wages, $L'_{D_i}(w) < 0$. Production is used for private as well as public consumption, with the marginal rate of transformation between the publicly provided private and fully private good equal to 1.

The representative household maximises $U_i(c_i, L_i, G_i, G_i^F)$ subject to its budget constraint $c_i = \bar{w}_i L_i + \pi_i$, with $\bar{w}_i = (1-\tau_i)w_i$ the net wage. Labour supply $L_i(\bar{w}_i)$, is implicitly defined by $U_{c_i}(.)\bar{w}_i + U_{L_i} = 0$. It increases with net wage, $L'_{S_i}(\bar{w}_i) > 0$, and is assumed to be perfectly mobile. As said before, households as a whole will never migrate. We have thus modeled commuting through a common inter-state labour market, where wages are endogenously determined as commuting flows equilibrate *gross* wages in all states. Indirect utility V_i then becomes $V_i(\bar{w}_i, \pi_i, G_i, G_i^F)$.

To derive the wage effects of a *state* tax increase, we start from the common labour market clearing condition:

$$\sum_i^n L_{S_i}(\bar{w}_i) = \sum_i^n L_{D_i}(w) \tag{1}$$

Taking the total differential with respect to τ_i of this condition gives us (see appendix A.1):

$$\frac{\partial w}{\partial \tau_i} = \frac{w\eta}{n(1-\tau)(\eta - \varepsilon)} > 0 \tag{2}$$

With $\eta > 0$ the labour supply elasticity, $\varepsilon < 0$ labour demand elasticity and$\tau = \tau_i + \tau_0$. As usual, tax incidence will be shifted onto labour supply and demand according to the relevant elasticities. The net wage effect will then be (see appendix A.1):

$$\frac{\partial \bar{w}}{\partial \tau_i} = \frac{w\,(n\varepsilon - (n-1)\eta)}{n\,(\eta - \varepsilon)} < 0 \tag{3}$$

Since *federal* taxation has a direct impact on labour supplied in all states, the wage responses to a marginal tax increase at the federal level will be different. To see this, we again take the total differential of the same common labour market condition (1), but now with respect to a federal tax τ_0 levied in all states (see appendix). This gives us:

$$\frac{\partial w}{\partial \tau_0} = \frac{w\eta}{(1-\tau)\,(\eta - \varepsilon)} > 0 \tag{4}$$

And for the net wage effect:

$$\frac{\partial \bar{w}}{\partial \tau_0} = \frac{w\varepsilon}{(\eta - \varepsilon)} < 0 \tag{5}$$

We see that the gross wage effects of state taxation are smaller, whilst the net wage effects are larger. This should not be surprising, since a state tax increase is expected to

[9] A subscript denotes the derivative of a function of several variables whereas a prime denotes the derivative of a function of one variable.

have a smaller impact on wages as a uniform, national tax increase. As the gross wage starts to rise in the state raising its taxes, more and more workers from other states will flock to this region, mitigating the gross wage increase. When n goes to infinity, the gross wage effect will be fully countered by the commuting response, as can be seen in expression (2), and thus be equal to zero. The marginal tax burden will then fall entirely on the local workers and weigh in at $-w$, since $\frac{\partial \bar{w}_i}{\partial \tau_i} = (1 - \tau_i - \tau_0) \frac{\partial w}{\partial \tau_i} - w$.

What remains is the effect of taxation on profits, which is the same for state as well as federal taxation:

$$\frac{\partial \pi_i}{\partial \tau_{i,0}} = \frac{\partial \left(F_i(L_{D_i}) - wL_{D_i} \right)}{\partial \tau_{i,0}} = -L_{D_i} \frac{\partial w}{\partial \tau_{i,0}} \tag{6}$$

3 Second-best optimum in a 'unitary' country

We start with the benchmark case of a unitary country, where the states are given no taxing or spending powers and the federal level makes all the calls. The federal government thus sets a *uniform* tax rate τ to finance consolidated public provision. Since taxation in this setting is uniformly federal, no tax externalities can arise. Gross wages, as well as net wages respectively, are identical across states for the same reason.

The federal government will tailor regional provision (G_i and G_i^F) in each state i to the preferences of the representative household living there[10]. We thus assume regional preferences are known by the federal government, so that the inefficiencies working through federal *policy uniformity* are ruled out. In other words, the federal government need not set the same level of public good provision in each state. Expenditure externalities will also be excluded since inter-state spillovers of public provision itself are assumed away in the model. All of these restrictions to keep the focus strictly on *tax* externalities and the resulting inefficiencies.

To keep matters simple, we start out by assuming that states are identical in every way. Also, profits are assumed to accrue entirely to the representative household living in the state where the rents are realised. In section 5 of the paper these assumptions will be relaxed. Since we do not deal with redistributional issues in this paper lastly, the federal government simply maximises a Utilitarian welfare function subject to its budget constraint:

$$Max_{G_i, G_i^F, \tau}\, n \left\{ V_i(\bar{w}, \pi_i, G_i, G_i^F) \right\}$$

$$s.t.\, n \left(G_i + G_i^F \right) = \tau n L_i w \tag{7}$$

With the values of τ, G_i^F and G_i to be chosen by the government. The first order conditions readily reduce to (see appendix B):

$$MRS = \frac{\frac{\partial V_i}{\partial G_i^F}}{\lambda_i} = \frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = \frac{1}{\left(1 - \frac{\tau \eta \varepsilon}{(1-\tau)\varepsilon - \eta}\right)} = MCPF_u \quad for\, all\, i = 1, .., n \tag{8}$$

With λ_i the marginal utility of income in region i. This expression, together with the budget constraint (7), characterizes the second-best optimum denoted by $(\tau^*, G_i^*, G_i^{F*})$. It simply states that at the unitary optimum the distortionary tax τ is set such that the marginal rate of substitution (MRS) between both the publicly provided good and the

[10] With G_i being the public provision to be provided by the states in the shared tax case.

private good must be equal to the Marginal Cost of Public Funds (MCPF). As is well known, the MCPF is the efficiency cost of raising revenue with a distortionary tax.

4 Decentralised taxation

Before we move on to the case where both federal as well as regional governments levy taxes on a shared labour income tax base, and by consequence where both horizontal and vertical externalities occur, we focus on the other side of the spectrum: the fully decentralised case. This will serve as a stepping-stone to the more complex shared tax base case.

4.1 Fully decentralised case

When fiscal decision making is fully decentralised to the state level, each state i levies a labour tax τ_i to finance the publicly provided goods G_i and G_i^F. As a result, horizontal externalities may distort tax and spending decisions. We assume that all states take decisions made by other states as given, and thus behave as Nash competitors. We keep on assuming that states are identical in every way, and calculate the symmetric equilibrium. The government of state i then maximises the indirect utility of the representative household living within its borders, subject to its budget constraint:

$$Max_{G_i,G_i^F,\tau_i}\, V_i(\bar{w}_i, \pi_i, G_i, G_i^F)$$

$$s.t.\, G_i + G_i^F = \tau_i L_i w \tag{9}$$

The first order conditions of this optimisation problem give us (see appendix C):

$$\frac{\frac{\partial V_i}{\partial G_i^F}}{\lambda_i} = \frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = \frac{1}{\left(1 - \frac{\tau_i\eta(n\varepsilon-(n-1)\eta)}{(1-\tau_i)(n\varepsilon-(n-1)\eta)-\eta}\right)} = MCPF_i \tag{10}$$

This expression, together with the budget constraint (8), characterizes the Nash equilibrium, denoted by (τ_i^*, G_i^*, G_i^F). Also in the fully decentralised case public provision continues until the marginal rate of substitution (MRS) between both the publicly provided good and the private good is equal to the state Marginal Cost of Public Funds (MCPF). However, this efficiency cost overlooks all effects of own taxation on other states. It will therefore generally be biased. The cost of raising revenue as it is perceived by the state government in other words, will weigh in more or less than the socially relevant cost derived in section 3. To verify the sign of this bias, a logical move would be to compare the unitary MCPF which was unaffected by externalities ($MCPF_u$), to the regional MCPF ($MCPF_i$) derived here. This gives us the following (evaluated at the Nash equilibrium $\tau_i^* = \tau$):

$$MCPF_i = \frac{1}{\left(1 - \frac{\tau_i^*\eta(n\varepsilon-(n-1)\eta)}{(1-\tau_i^*)(n\varepsilon-(n-1)\eta)-\eta}\right)} \lesseqqgtr \frac{1}{\left(1 - \frac{\tau\eta\varepsilon}{(1-\tau)\varepsilon-\eta}\right)} = MCPF_u \tag{11}$$

Looking at the denominators in (11), we find that when the LHS outweighs the RHS in (12) the denominator of the regional MCPF will be smaller than its unitary counterpart, so that the state MCPF will be perceived as larger than the unitary MCPF.

$$\frac{\tau\eta\left(n\varepsilon-(n-1)\eta\right)}{(1-\tau)\left(n\varepsilon-(n-1)\eta\right)-\eta}>\frac{\tau\eta\varepsilon}{(1-\tau)\varepsilon-\eta} \tag{12}$$

When (12) holds in other words, the state MCPF will be biased upwards because of a horizontal positive externality. The positive effect of regional taxation on other states is not included in the state welfare cost, so that state governments will perceive the cost of distortionary taxation to be *higher* than it actually is. As a result, state taxation as well as public provision are set at inefficiently low levels compared to the second-best outcome derived in section 3. As shown by (12), this will always be the case in our simplified setting here[11]. In section 4, where profit taxation and regional heterogeneity will be introduced, we will see that *negative* horizontal externalities may also appear. Lastly, if the state government were to internalise the positive externality in its welfare cost calculations, the cost would weigh in precisely at its socially relevant level (to see this formally, we refer to appendix C.2).

Now, as can be seen formally in (53), a positive externality stems from the positive effect of a tax increase in region i on tax revenues and consumer welfare in other states. Here, the effects on *tax revenues* in other states are twofold. Due to the gross wage increase, which will be identical across states because of commuting, the tax base in other states logically rises and so will collected tax revenues. Yet because of the higher gross wage, overall labour demand decreases, leading to a fall in total labour supply. Now, since taxes are left unaltered in other states, net wages will also be pulled up in these states. Labour supply follows suit, which partially compensates for the decrease in state i where the net wage falls because of the tax increase. Partially, since as we said total labour supply will fall as well. In any case, this labour supply increase gives a second boost to tax revenues raised in other states.

Both effects are strengthened by a third, and *direct* effect on non-resident welfare. A higher net wage in other regions not only improves non-resident welfare through increased public provision, but also because purchasing power comes out reinforced. Furthermore, all three effects combined appear to dominate the negative direct effect on non-resident welfare, which is the drop in collected profits due to the higher gross wage. Lastly, the higher the amount of regions in the federation the stronger the positive externality and subsequent welfare losses, a result reminiscent of Hoyt's (1991) capital tax model. This can also easily be seen in (12), where a higher n nudges up the LHS compared to the RHS.

Looking at this result from a more strategic point of view, it would seem that states set taxes at inefficiently low levels not to attract more workers, but to boost labour supply of their own residents and discourage labour supplied by non-residents. Indeed, if a state would decrease its taxes the net wage of its own residents would increase, whilst the net wage of non-residents would go down (because of the gross wage decrease). Of course, since all states are symmetric and will follow the same strategy, we arrive at the familiar sub-optimal welfare level where all taxes are set too low.

[11] Keeping in mind that $\varepsilon<0$ and $\eta>0$, the term $(n\varepsilon-(n-1)\eta)$ on the LHS of expression (12) will be larger in absolute value compared to its counterpart in the RHS which is simply ε. It then suffices to take the derivative of the RHS with respect to ε, which is negative, and thus positive in absolute value, to prove that the inequality always holds.

4.2 Shared tax base case

We now move on to the case where both the federal and regional governments levy taxes on labour, giving rise to horizontal *and* vertical tax externalities. The federal government will again tailor regional provision (G_i^F) in each region i to the preferences of the representative household living there. On top of this, each region will raise additional regional taxes to finance own public provision (G_i). We continue assuming that all governments behave as Nash competitors[12] and that states are identical in every way. We furthermore assume that the federal government takes into account all the effects of its taxation policy on state budgets, so that its MCPF will be the same as in the unitary case. We can therefore jump straight to the regional government's problem.

The government of state i again optimises the following welfare function subject to its budget constraint:

$$Max_{G_i,\tau_i}\, v_i(\bar{w}_i, \pi_i) + g_i^F(G_i^F) + g_i(G_i)$$

$$s.t.\; G_i = \tau_i L_{S_i} w_i \tag{13}$$

The first order conditions of this optimisation problem give us (see appendix D):

$$\frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = \frac{1}{\left(1 - \frac{\tau_i \eta (n\varepsilon - (n-1)\eta) - \tau_0 \eta}{(1-\tau)(n\varepsilon - (n-1)\eta) - \eta}\right)} = MCPF_i^{Shared} \tag{14}$$

This expression, together with the budget constraint (13), characterizes the Nash equilibrium, denoted by (τ_i^*, G_i^*). Again, public provision will continue until the marginal rate of substitution (MRS) between both the publicly provided good and the private good is equal to the state Marginal Cost of Public Funds (MCPF). Moreover, not only does this efficiency cost overlook all (positive) horizontal effects of own taxation on other states, it will also fail to internalise the vertical effects. Increased state taxation will have an impact on tax revenues raised by the federal government, an overlooked vertical effect which will be twofold. The higher gross wage will increase federal revenue, whilst the drop in total labour supplied will have a negative effect on the federal budget since the tax base shrinks. To find out which effect will come out on top and thus to determine the sign of the overall externality, we again compare the unitary MCPF ($MCPF_u$) with the MCPF obtained here. We evaluate at the Nash equilibrium $\tau_i^* + \tau_0^* = \tau$, where the share of both state and federal tax rates in the total tax will depend on the relative strengths of federal and state preferences for public provision. We then get:

$$MCPF_i^{Shared} = \frac{1}{\left(1 - \frac{\tau_i^* \eta (n\varepsilon - (n-1)\eta) - \tau_0^* \eta}{(1-\tau)(n\varepsilon - (n-1)\eta) - \eta}\right)} \lesseqqgtr \frac{1}{\left(1 - \frac{\tau \eta \varepsilon}{(1-\tau)\varepsilon - \eta}\right)} = MCPF_u \tag{15}$$

If the efficiency cost under the shared tax base case turns out to be smaller, the overall externality will be negative and state overprovision ensues. In this case the upward

[12] In most models of fiscal interaction between regional and federal governments, the federal government is modeled as a Stackelberg leader (See, Boadway (1996) Sato (2000) or Kotsogiannis and Martinez (2007)). Whether federal governments deliberately manipulate states' fiscal decisions remains an, however interestingly, empirical question to be answered. We follow Dahlby's (2008) position which questions the fact that in a democracy, where voters have limited knowledge about the interactions between the various levels of governments, Stackelberg leadership behaviour would emerge.

pressure on the perceived state efficiency cost caused by the positive horizontal externality is more than undone by the vertical effects. If, on the other hand, the regional MCPF remains larger than the second-best efficiency cost, the positive effects (horizontal as well as vertical) on non-resident welfare will have prevailed. Zooming in on (15), we extract the following condition for such a scenario to occur :

$$\frac{\tau_i^* \left(n\varepsilon - (n-1)\eta\right) - \tau_0^*}{(1-\tau)\left(n\varepsilon - (n-1)\eta\right) - \eta} > \frac{\tau\varepsilon}{(1-\tau)\varepsilon - \eta} \tag{16}$$

As (16) clearly shows, it becomes more likely for the regional MCPF to be biased upwards as the state share τ_i^* in the total tax rate τ increases in equilibrium. This should not be too surprising, since such an increase would bring us closer to the fully decentralised case where the only externalities are horizontal and positive, as shown in section 4.1 above. It is therefore interesting to investigate the forces at hand which drive down the state MCPF when the state tax share is smaller, which has us taking a closer look at the vertical interaction. The effect of a marginally increased state tax rate τ_i on the *federal* budget, which will be ignored by the state in question, can be written as (see appendix D.2):

$$\frac{\partial R_0}{\partial \tau_i} = \left\{(n-1)\left(\tau_0 L \frac{\partial w}{\partial \tau_i} + \tau_0 w(1-\tau_j)\frac{\partial L_j}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau_i}\right)\right\} + \tau_0\left(L\frac{\partial w}{\partial \tau_i} + w\frac{\partial L}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau_i}\right) \tag{17}$$

We see that the vertical effect indeed follows from both the gross wage increase as the labour supply response. Splitting up the vertical interaction into two parts, a first part between large brackets in (17) concerns the other states and will be positive [13]. The remaining, second effect concerns state i itself, and could turn either way depending on the sign of:

$$\tau_0\left(L\frac{\partial w}{\partial \tau_i} + w\frac{\partial L}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau_i}\right) \tag{18}$$

Now, zooming in on the first term between large brackets in (17), we can verify under which conditions (18) will be negative. This first term of (17) in fact represents a fraction of the *total effect* of increased taxation in state i on revenue collected in other states (the federal as well as state level), which is (again evaluated at the shared tax case Nash equilibrium):

$$\left\{(n-1)\left(\left(\tau_0^* + \tau_i^*\right)L\frac{\partial w}{\partial \tau_i} + \left(\tau_0^* + \tau_i^*\right)w(1-\tau_j)\frac{\partial L_j}{\partial \bar{w}}\frac{\partial w}{\partial \tau_i}\right)\right\} \tag{19}$$

Indeed, and keeping in mind that $\tau_0 + \tau_i = \tau$, (19) is equal to the last term in the denominator of (53) which was the effect of increased state taxation on the revenues of all other states in the decentralised case. Overlooking this effect, the MCPF obtained in section 4.1 ($MCPF_i$) thus also overlooked the first effect of vertical interaction as expressed by the first term of (17). If we then compare this $MCPF_i$ to the efficiency cost derived here which ignores both the first and second effect of (17), we can determine the sign of (18). If $MCPF_i^{Shared}$ is smaller than $MCPF_i$, (18) will be negative since it has driven the perceived state MCPF down compared to the decentralised case. The necessary and sufficient condition for this to occur is the following (see appendix D.2.2):

[13] Keeping in mind that $\frac{\partial \bar{w}_j}{\partial \tau_i} = \frac{\partial\left((1-\tau_j)w\right)}{\partial \tau_i} = (1-\tau_j)\frac{\partial w}{\partial \tau_i} > 0$ (see also appendix D.2).

$$(n\varepsilon - (n-1)\eta) < -1 \tag{20}$$

A result which can also easily be obtained by plugging in the wage effects (2) and (3) in (18). As labour demand becomes more elastic, (18) will become more negative, and chances overall of having a negative externality will rise. This is a logical result, as the positive effect on federal revenue due to the increasing gross wage will be mitigated the more elastic is labour demand. Secondly, as (2) also shows, a higher number of states in the federation would only strengthen this mitigating effect. A higher labour supply elasticity lastly, will result in (18) to turn negative as well, and thus also puts a downwards bias on the state efficiency cost. Again this is logical, since highly elastic labour supply will result in a more pronounced drop in total labour supplied across the federation.

Turning our attention back to (17), we can easily see why such a negative externality would gain in momentum as the federal tax rate accounts for a larger share of the total tax rate τ. Assuming (18) is negative (which is readily the case under realistic assumptions), we can see that a larger federal tax rate will bolster this negative effect. On the other hand, the positive effect of the vertical interaction which was the first term between large brackets in (17), will remain unchanged. This for the simple reason that as the federal tax share increases, the regional share would go down. The positive revenue effects would thus be exactly the same as before as can be seen in (19). We summarise these first findings as follows:

> **Proposition 1.** *When workers can commute costlessly between identical states of a federation, and when the labour income tax base is co-occupied by the federal and state governments, state taxes will more likely be set inefficiently high when*:
>
> (a) Labour demand as well as supply are more elastic
>
> (b) There are more states in the federation
>
> (c) The federal tax rate accounts for a larger share in the total tax rate

As proposition 1 shows, determining the extent of inefficient public provision in our shared tax setting is rather different than one would expect. It does not boil down to first identifying the sign of the horizontal and vertical externality in isolation, to ascertain in a second step which externality will dominate the other in a shared tax case as done by Keen and Kotsogiannis (2002; 2004). In isolation here would then mean a case without federal taxation as for the horizontal externality, and a case where tax bases are immobile between states for the vertical externality. Indeed, one of the main findings of this paper is that the overall externality may be negative even when both the horizontal as well as vertical externality evaluated in isolation are positive.

We delve deeper into this issue by considering what would happen to the state MCPF when we look at the vertical externality in isolation. By internalising all *horizontal* effects of increased state taxation in state i, the perceived efficiency cost only overlooks vertical effects of own taxation as it would without . We thus include the effects of own taxation on consumer welfare in other states to the numerator of $MCPF_i^{Shared}$, and the effects on revenue collected by other states to the denominator (effects included between large brackets):

$$MCPF_i = -\frac{\left(L_{S_i}\frac{\partial \bar{w}_i}{\partial \tau_i} + \frac{\partial \pi_i}{\partial \tau_i}\right) + \left\{\sum_{j\neq i}^{n}\left(L_{S_j}\frac{\partial \bar{w}_j}{\partial \tau_i} + \frac{\partial \pi_j}{\partial \tau_i}\right)\right\}}{\left(L_{S_i}w + \tau_i L_{S_i}\frac{\partial w}{\partial \tau_i} + \tau_i w \frac{\partial L_{S_i}}{\partial \bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i}\right) + \left\{\sum_{j\neq i}^{n}\left(\tau_j L_{S_j}\frac{\partial w}{\partial \tau_i} + \tau_j w \frac{\partial L_{S_j}}{\partial \bar{w}_j}\frac{\partial \bar{w}_j}{\partial \tau_i}\right)\right\}} \tag{21}$$

Now, this efficiency cost still overlooks the impact of own taxation on the federal budget, yet it takes into account the horizontal effect of own taxation on other states. Since we have proven in section 4.1 that ignoring this effect pushes the perceived state MCPF upwards, we would expect the same here. If the vertical externality would already be positive consequently, leaving out the positive horizontal effects may then be expected to strengthen the upward bias on the state MCPF. In other words, we would expect (21) to increase when the effects between large brackets are excluded. The idea that vertical and horizontal externalities of the same sign (when considered in isolation) reinforce each other when put together, would then be a valid one. Yet as we will show, this is far from always the case in our setting. Rewriting (21) gives us [14]:

$$MCPF_i = -\frac{\left(L_{S_i}\frac{\partial \bar{w}_i}{\partial \tau_i} - L\frac{\partial w}{\partial \tau_i}\right) - \left\{(n-1)L\frac{\partial w}{\partial \tau_i}\right\}}{\left(L_{S_i}w + \tau_i L_{S_i}\frac{\partial w}{\partial \tau_i} + \tau_i w \frac{\partial L_{S_i}}{\partial \bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i}\right) + \left\{(n-1)\left(\tau_i L\frac{\partial w}{\partial \tau_i} + \tau_i w(1-\tau_j)\frac{\partial L_j}{\partial \bar{w}}\frac{\partial w}{\partial \tau_i}\right)\right\}} \tag{22}$$

Looking at the addition between large brackets in the denominator of (22), which is the positive effect of own taxation on the tax revenue raised in other states, we see that this represents only a part of the total effect expressed by (19). Not too surprising, since the effect on other states' revenues working through the impact of own taxation on *federal* revenues is still excluded here. However, the full direct effect on non-resident utility is taken into account in the numerator, which is negative because of the decreasing profits in other states. Therefore, when the regional tax share in the total tax is low, the negative profit effects may outweigh the positive budgetary effects, causing (22) to *decrease* if we leave out both effects between large brackets [15].

The attentive reader may have noticed that this line of reasoning runs entirely parallel to our approach set out earlier arriving at proposition 1. There we started by considering the horizontal externality in isolation, and compared the efficiency cost under this decentralised case to the cost derived when taxation is shared. The former is indeed none other than the latter, if we internalise those vertical effects which do not concern other states. To see this, we add the vertical effects that only concern state i to the denominator of $MCPF_i^{Shared}$:

$$MCPF_i = -\frac{\left(L_{S_i}\frac{\partial \bar{w}_i}{\partial \tau_i} + \frac{\partial \pi_i}{\partial \tau_i}\right)}{\left(L_{S_i}w + \tau_i L_{S_i}\frac{\partial w}{\partial \tau_i} + \tau_i w \frac{\partial L_{S_i}}{\partial \bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i}\right) + \left\{\tau_0\left(L_{S_i}\frac{\partial w}{\partial \tau_i} + w\frac{\partial L_{S_i}}{\partial \bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i}\right)\right\}} \tag{23}$$

We thus arrive at the efficiency cost derived under section 4.1, which was biased upwards. Now, even when the total vertical interaction (17) is positive, leaving out the addition between large brackets will not necessarily have (23) increase. On the contrary,

[14] Taking into account that states are identical and thus also that $\frac{\partial \bar{w}_j}{\partial \tau_i} = \frac{\partial\left((1-\tau_j)w\right)}{\partial \tau_i} = (1-\tau_j)\frac{\partial w}{\partial \tau_i}$, as well as (6).

[15] Keeping in mind that the numerator of (22), otherwise negative, will turn positive because of the minus sign in front of the fraction.

(23) will decrease when (19) is negative, an effect strengthened by a larger share of federal taxation as shown before. Again not too surprising, since the positive effect on *other* states' revenues working through the federal budget is still omitted from the denominator.

Both approaches thus represent two opposite sides of the same coin, which is why the end result is the same. Since part of the effect of the vertical (horizontal) externality evaluated in isolation will always be part of the horizontal (vertical) externality evaluated in isolation, combining both in a shared tax case scenario may lead to unexpected results. The extent of which depends on the share of federal taxation in the total tax, and thus on the outcome of the Nash equilibrium. We summarise in proposition 2:

Proposition 2. *When workers can commute costlessly between identical states of a federation, and when the labour income tax base is co-occupied by the federal and state governments, the overall externality may be negative even when the vertical and horizontal externalities are positive when evaluated in isolation. There is a critical equilibrium value of federal taxation above which this will be the case.*

Proof: see appendix D.3

5 Heterogeneous regions

We now let go of the assumption that regions are identical in every way. Preferences, as well as technology across regions may differ in this setting. Also, all rents are assumed to accrue to the public sector since they represent an efficient source of tax revenues ($\theta_{(i)} = 1$). We again start with defining the second-best optimum, and will then limit ourselves to a brief analysis of the fully decentralised case. Since the shared tax case is not considered here, we need not distinguish between state and federal public provision.

5.1 Second-best optimum in a 'unitary' country

The federal government will again maximise a Utilitarian welfare function subject to its budget constraint:

$$Max_{G,\tau} \sum_{i}^{n} \{V_i(\bar{w}) + g_i(G_i)\} \tag{24}$$

$$s.t. \sum_{i}^{n} G_i = \tau \sum_{i}^{n} L_i w + \sum_{i}^{n} \theta \pi_i \tag{25}$$

With the values of τ and G_i to be chosen by the government. The first order conditions will readily reduce to (see appendix E.1 for intermediary steps):

$$\frac{\frac{\partial V_i}{\partial G_i}}{\sum_i^n (s_i \lambda_i)} = \left(\frac{1}{1 - \frac{\tau}{L(1-\tau)} \sum_i^n \eta_i L_i} \right) \quad for\, i = 1,..,n \tag{26}$$

With λ_i the marginal utility of income in region i. This expression, together with the budget constraint (25), characterizes the second-best optimum, denoted by (τ^*, G_i^*) for all $i = 1,..,n$. It simply states that at the unitary optimum the distortionary tax τ is set such that the marginal rate of substitution (MRS) between the publicly provided good in state i and a weighted average of private consumption must be equal to the Marginal

Cost of Public Funds (MCPF). This again is the efficiency cost of raising revenue with a distortionary tax, which will be identical across all states at the unitary optimum.

5.2 Fully decentralised case

We keep on assuming that all regions take tax and expenditure decisions of other regions as given, and play a simultaneous game resulting in a Nash equilibrium. Since regional taxation need not be symmetric in this setting, regional net wages may differ whilst the common labour market will still clear at an identical gross wage. A regional government again maximises the indirect utility of the representative household living within its borders, subject to its budget constraint:

$$Max_{G_i,\tau_i}\, V_i(\bar{w}_i, G_i)$$

$$s.t.\, G_i = \tau_i L_i w + \theta\pi_i \tag{27}$$

The first order conditions of this optimisation problem give us (see appendix E.2):

$$\frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = \frac{1}{1 - \frac{\tau_i\eta_i}{(1-\tau_i)} - \left(1 - \frac{L_{D_i}}{L_{s_i}}\right)\left(\frac{\eta_i \frac{L_{s_i}}{\bar{w}_i}}{\left(\left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}\right) - \sum_{j\neq i}^n \left((1-\tau_j)\eta_j \frac{L_{s_j}}{\bar{w}_j}\right)\right)}\right)} \tag{28}$$

Again this efficiency cost will fail to internalise all horizontal effects of own taxation on other states. But unlike our previous result, this horizontal externality can be of any sign. To see this, we follow the familiar approach employed above to identify the sign of the externality. Comparing the denominator of the state efficiency cost in (28) to its counterpart under the second-best outcome, we get that the externality will be positive when:

$$\left(\frac{\tau_i^*\eta_i}{(1-\tau_i)}\right) + \left(1 - \frac{L_{D_i}}{L_{s_i}}\right)\left(\frac{\eta_i \frac{L_i}{\bar{w}_i}}{\left(\left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}\right) - \sum_{j\neq i}^n \left((1-\tau_j)\eta_j \frac{L_{s_j}}{\bar{w}_j}\right)\right)}\right) > \frac{\tau}{L(1-\tau)} \sum_i^n \eta_i L_i \tag{29}$$

Again evaluated at the Nash equilibrium, where we set $\tau = \tau_i^*$. The main result we wish to stress here is the effect of commuting, i.e. the regional out-or inflow of commuters ($\frac{L_{D_i}}{L_{s_i}}$). Since the third term between brackets on the LHS of expression (29) is negative, this LHS will more likely outweigh the RHS when less people work in region i than is demanded, i.e. $L_{D_i} > L_{S_i}$. In other words, an inflow of commuters makes it more likely for the horizontal externality to be positive. This is a logical result, since the positive effect on tax revenues in other regions due to higher gross wages will be stronger the higher the level of labour supplied in these regions, and thus the more region i relies on labour flowing in from other regions. But there is more to the story. We zoom in on the total budgetary effect of marginally increased taxation in state i on other states to clarify:

$$\sum_{j_{i\neq j}}^n \frac{\partial R_j}{\partial \tau_i} = \sum_{j_{i\neq j}}^n \left(\tau_j L_j \frac{\partial w}{\partial \tau_i} + \tau_j w \frac{\partial L_j}{\partial \bar{w}_j}\frac{\partial \bar{w}_j}{\partial \tau_i} - L_{D_j}\frac{\partial w}{\partial \tau_i}\right) \tag{30}$$

Since profits are taxed in our setting, there will also be a negative budgetary effect due to the gross wage increase which is expressed by the third term on the RHS of (30).

It then becomes clear that a higher labour supply in other regions combined with a low labour demand, is what drives the positive budgetary effects. The more workers region j has to tax at a higher gross wage, as expressed by the first terms on the RHS of (30), the higher the chance for the positive effect on state j's tax revenue to outweigh the negative effect working through the profit tax. A negative effect which will itself be lower when less labour is demanded in state j. Hence, an outflow of workers out of all states to state i, makes it more likely for the externality to turn positive. We summarise in proposition 3:

> **Proposition 3.** *When states in a federation differ in terms of preferences and technology, a commuting inflow of workers makes it more likely for states to set taxes inefficiently low.*

6 Summary and concluding remarks

As one of the smallest federations in Europe, Belgium makes for an interesting case study. Counting only three sub-central entities (states), each with diverging preferences and productivity, it enjoys a very high degree of cross-state commuting. Zooming in on these commuting flows brings out a blind spot in the fiscal federalism literature on tax externalities, where models have workers supply labour *only* in their state of residence. For a federation with Belgian features, where at least 10% of the workforce commutes between regions, this assumption of non-integrated regional labour markets becomes difficult to maintain.

We therefore presented a theoretical model based on a *common* labour market, where wages are endogenously determined as commuting flows equilibrate wages across all states of a federation. Policy changes in one state will consequently be felt throughout the entire federal system, even when household *migration* does not occur. We thus model a situation where horizontal externalities are re-introduced to the analysis through commuting effects. To allow for positive as well as negative *vertical* externalities furthermore, the effects of a (partial) decentralisation of an *ad valorem* tax on labour income were studied, a tax which was *residence* based following the Belgian setting.

When taxation was fully decentralised first of all, state governments would set inefficiently low taxes not to attract more workers, but to boost labour supply of own residents and hamper labour supplied by non-residents. In other words, when considered in isolation, the horizontal externality was shown to be positive but different in nature compared to the familiar capital tax competition models. When the labour tax base was co-occupied by the federal and state governments secondly, either public under- or over-provision would occur. Our model identified clear conditions for states to overprovide, i.e. for the overall fiscal externality to be negative. An elastic labour supply as well as demand, the number of states and the share of federal spending in total public provision, were crucial elements here. Also, and quite the interesting find, such a negative fiscal externality could arise even when the vertical as well as horizontal externalities were positive in isolation, and one would expect underprovision. Lastly, in a an attempt to have the model capture the Belgian setting to the fullest, we allowed for regional heterogeneity. When states differ in terms of preferences and technology, we showed how an inflow of commuters will make it more likely for states to set taxes inefficiently low.

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Appendix

A Tax shifting formulas

A.1 Regional taxation

We obtain the tax shifting formula from the equilibrium in the labour market:

$$\sum_{i}^{n} L_{S_i}(\bar{w}_i) = \sum_{i}^{n} L_{D_i}(w)$$

d

Taking the total differential with respect to τ_i of the labour market equilibrium condition then yields:

$$\sum_{j\neq i}^{n} \left(\frac{\partial \left(L_{S_j}(\bar{w}_j) \right)}{\partial \bar{w}_j} \frac{\partial \bar{w}_j}{\partial \tau_i} \right) + \frac{\partial L_{S_i}(w_i)}{\partial \bar{w}_i} \frac{\partial \bar{w}_i}{\partial \tau_i} = \sum_{i}^{n} \left(\frac{\partial \left(L_{D_i}(w) \right)}{\partial w} \right) \frac{\partial w}{\partial \tau_i}$$

Rewriting net wages in terms of the gross wage and solving further gives:

$$\sum_{j\neq i}^{n} \left(L'_{S_j} \frac{\partial \left(w - \tau_j - \tau_0 \right)}{\partial \tau_i} \right) + L'_{S_i} \frac{\partial \left(w - \tau_i - \tau_0 \right)}{\partial \tau_i} = \left(\sum_{i}^{n} L'_{D_i} \right) \frac{\partial w}{\partial \tau_i} \tag{31}$$

Now, since we know that:

$$\frac{\partial \left(w - \tau_i - \tau_0 \right)}{\partial \tau_i} = \frac{\partial w}{\partial \tau_i} - 1$$

We also know that:

$$\partial \left(w - \tau_i - \tau_0 \right) = \partial w - \partial \tau_i \tag{32}$$

And:

$$\frac{\partial \left(w - \tau_j - \tau_0 \right)}{\partial \tau_i} = \frac{\partial w}{\partial \tau_i}$$

So that:

$$\partial \left(w - \tau_j - \tau_0 \right) = \partial w \tag{33}$$

Plugging (32) and (33) into (31) then gives us:

$$\sum_{j\neq i}^{n} \left(L'_{S_j} \partial w \right) + L'_{S_i} \left(\partial w - \partial \tau_i \right) = \left(\sum_{i}^{n} L'_{D_i} \right) \partial w$$

$$\left(\sum_{i}^{n} \left(L'_{S_i} \right) - \sum_{i}^{n} L'_{D_i} \right) \partial w = +L'_{S_i} \partial \tau_i$$

$$\frac{\partial w}{\partial \tau_i} = \frac{L'_{S_i}}{\left(\sum_{i}^{n} \left(L'_{S_i} \right) - \sum_{i}^{n} L'_{D_i} \right)} > 0 \tag{34}$$

Rewriting (35) in terms of regional labour supply elasticity $\eta_i = L'_{S_i}\frac{\bar{w}_i}{L'_{S_i}}$ and labour demand elasticity $\varepsilon_i = L'_{D_i}\frac{w}{L'_{D_i}}$ now yields:

$$\frac{\partial w}{\partial \tau_i} = \frac{w\eta_i \frac{L_{s_i}}{\bar{w}_i}}{\left(\sum_i^n \left((1-\tau_i-\tau_0)\eta_i \frac{L_{s_i}}{\bar{w}_i}\right) - \left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}\right)\right)} \tag{35}$$

We know that, in equilibrium, $\frac{\partial \bar{w}_i}{\partial \tau_i} = (1-\tau_i-\tau_0)\frac{\partial w}{\partial \tau_i} - w$, so plugging in (35) then gives us:

$$\frac{\partial \bar{w}_i}{\partial \tau_i} = \frac{(1-\tau_i-\tau_0)\, wL'_{S_i} - w\left(\sum_i^n \left(L'_{S_i}(1-\tau_i-\tau_0)\right) - \left(\sum_i^n L'_{D_i}\right)\right)}{\left(\sum_i^n \left(L'_{S_i}(1-\tau_i-\tau_0)\right) - \left(\sum_i^n L'_{D_i}\right)\right)}$$

$$\frac{\partial \bar{w}_i}{\partial \tau_i} = \frac{w\left(\left(\sum_i^n L'_{D_i}\right) - \sum_{j\neq i}^n \left(L'_{S_j}(1-\tau_j-\tau_0)\right)\right)}{\left(\sum_i^n \left(L'_{S_i}(1-\tau_i-\tau_0)\right) - \left(\sum_i^n L'_{D_i}\right)\right)} < 0 \tag{36}$$

For later purposes, we rewrite (37) in terms of regional labour supply and demand:

$$\frac{\partial \bar{w}_i}{\partial \tau_i} = \frac{w\left(\left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}\right) - \sum_{j\neq i}^n \left((1-\tau_j-\tau_0)\eta_j \frac{L_{s_j}}{\bar{w}_j}\right)\right)}{\left(\sum_i^n \left((1-\tau_i-\tau_0)\eta_i \frac{L_{s_i}}{\bar{w}_i}\right) - \left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}\right)\right)} \tag{37}$$

Under the assumption of homogeneous regions, (35) and (37) reduce to:

$$\frac{\partial w}{\partial \tau_i} = \frac{w\eta}{n(1-\tau)\,(\eta-\varepsilon)} \tag{38}$$

$$\frac{\partial \bar{w}}{\partial \tau_i} = \frac{w\,(n\varepsilon - (n-1)\eta)}{n\,(\eta-\varepsilon)} \tag{39}$$

A.2 Federal taxation

Taking the total differential with respect to τ_0 yields:

$$\sum_i^n \left(\frac{\partial L_{S_i}(\bar{w})}{\partial(\bar{w})}\frac{\partial \bar{w}}{\partial \tau_0}\right) = \sum_i^n \left(\frac{\partial L_{D_i}(w)}{\partial w}\frac{\partial w}{\partial \tau_0}\right)$$

Rewriting net wages in terms of the gross wage and solving further finally gives us:

$$\sum_i^n \left(L'_{S_i}\frac{\partial\left((1-\tau_i-\tau_0)w)\right)}{\partial \tau_0}\right) = \left(\sum_i^n L'_{D_i}\right)\frac{\partial w}{\partial \tau_0} \tag{40}$$

Now, since we know that:

$$\frac{\partial\left((1-\tau_i-\tau_0)w)\right)}{\partial \tau_0} = (1-\tau_i-\tau_0)\frac{\partial w}{\partial \tau_0} - w$$

We also know that:

$$\partial\left((1-\tau_i-\tau_0)w)\right) = (1-\tau_i-\tau_0)\partial w - w\partial \tau_0 \tag{41}$$

Plugging (41) into (40) then gives us:

$$\sum_i^n \left(L'_{S_i}\left((1-\tau_i-\tau_0)\partial w - w\partial\tau_0\right)\right) = \left(\sum_i^n L'_{D_i}\right)\partial w$$

$$\left(\sum_i^n \left(L'_{S_i}(1-\tau_i-\tau_0)\right) - \left(\sum_i^n L'_{D_i}\right)\right)\partial w = \sum_i^n \left(L'_{S_i} w\partial\tau_0\right)$$

$$\frac{\partial w}{\partial \tau_0} = \frac{w\sum_i^n L'_{S_i}}{\left(\sum_i^n \left(L'_{S_i}(1-\tau_i-\tau_0)\right) - \left(\sum_i^n L'_{D_i}\right)\right)} \tag{42}$$

For later purposes, we rewrite (42) in terms of regional labour supply and demand:

$$\frac{\partial w}{\partial \tau_0} = \frac{w\sum_i^n \eta_i \frac{L_{s_i}}{\bar{w}_i}}{\left((1-\tau_i-\tau_0)\sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i}\right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w}\right)\right)} > 0 \tag{43}$$

Moving on to the effect on net wages, we know that, in equilibrium, $\frac{\partial \bar{w}}{\partial \tau} = (1-\tau)\frac{\partial w}{\partial \tau} - w$. Plugging in (43) then gives us:

$$\frac{\partial \bar{w}}{\partial \tau_0} = \frac{(1-\tau_i-\tau_0)\, w\sum_i^n L'_{S_i} - w\left((1-\tau_i-\tau_0)\sum_i^n L'_{S_i} - \sum_i^n L'_{D_i}\right)}{\left((1-\tau_i-\tau_0)\sum_i^n L'_{S_i} - \sum_i^n L'_{D_i}\right)}$$

$$\frac{\partial \bar{w}}{\partial \tau} = \frac{w\sum_i^n L'_{D_i}}{\left((1-\tau_i-\tau_0)\sum_i^n L'_{S_i} - \sum_i^n L'_{D_i}\right)} < 0 \tag{44}$$

For later purposes, we rewrite (45) in terms of regional labour supply and demand:

$$\frac{\partial \bar{w}}{\partial \tau_0} = \frac{w\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}}{\left((1-\tau_i-\tau_0)\sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i}\right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w}\right)\right)} \tag{45}$$

Under the assumption of homogeneous regions, (43) and (45) reduce to the well known expressions:

$$\frac{\partial w}{\partial \tau_0} = \frac{w\eta}{(1-\tau)(\eta-\varepsilon)} \tag{46}$$

$$\frac{\partial \bar{w}}{\partial \tau_0} = \frac{w\varepsilon}{(\eta-\varepsilon)} \tag{47}$$

B Calculations second-best optimum in a 'unitary' country

The second best optimization problem is expressed by the Lagrangian:

$$\mathcal{L} = n\left\{V_i(\bar{w}, \pi_i, G_i, G_i^F)\right\} - \gamma\left\{n\left(G_i + G_i^F\right) - \tau n L_i w\right\}$$

Leading to the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial \tau} = n\left\{\frac{\partial V_i}{\bar{w}}\frac{\partial \bar{w}}{\partial \tau} + \frac{\partial V_i}{\pi_i}\frac{\partial \pi_i}{\partial \tau}\right\} + \gamma\left(\frac{\partial(\tau n L_i w)}{\partial \tau}\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial G_i} = \frac{\partial V_i}{\partial G_i} - \gamma = 0 \quad for\, i = 1, .., n$$

$$\frac{\partial \mathcal{L}}{\partial G_i^F} = \frac{\partial V_i}{\partial G_i^F} - \gamma = 0 \quad for\, i = 1, .., n$$

Solving both conditions yields:

$$n\left\{\frac{\partial V_i}{\bar{w}}\frac{\partial \bar{w}}{\partial \tau} + \frac{\partial V_i}{\pi_i}\frac{\partial \pi_i}{\partial \tau}\right\} + \gamma\left(nL_i w + \tau n L_i \frac{\partial w}{\partial \tau} + \tau n w \frac{\partial L_i}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau}\right) = 0 \ (\tau)$$

$$\frac{\partial V_i}{\partial G_i} - \gamma = 0 \quad for\, i = 1, .., n \quad (G_i)$$

$$\frac{\partial V_i}{\partial G_i^F} - \gamma = 0 \quad for\, i = 1, .., n \quad (G_i^F)$$

With $nL_i w + \tau n L_i \frac{\partial w}{\partial \tau} + \tau n w \frac{\partial L_i}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau}$ equal to $\frac{\partial R}{\partial \tau}$, being the marginal effect on federal revenue by raising the labour tax. Substituting for γ then yields:

$$\frac{\partial V_i}{\partial G_i^F} = \frac{\partial V_i}{\partial G_i} = -\frac{n\left\{\frac{\partial V_i}{\bar{w}}\frac{\partial \bar{w}}{\partial \tau} + \frac{\partial V_i}{\pi_i}\frac{\partial \pi_i}{\partial \tau}\right\}}{\frac{\partial R}{\partial \tau}} \quad for\, i = 1, .., n \tag{48}$$

Public provision in each region will thus continue until its marginal benefits equal its marginal cost, and this in terms of welfare cost as well as actual provision cost (see also Dahlby, 2008). The RHS of the equation thus expresses the marginal welfare cost in utility terms of raising an additional euro of revenue to finance public provision in region i, multiplied by the marginal cost of actual provision (1 in our case). This expression can be reformulated to arrive at the conventional MCPF expression using Roy's identity:

$$\frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = -\frac{\left(L_i \frac{\partial \bar{w}}{\partial \tau} + \frac{\partial \pi_i}{\partial \tau_i}\right)}{L_i w + \tau L_i \frac{\partial w}{\partial \tau} + \tau w \frac{\partial L_i}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau}} \quad for\, i = 1, .., n \tag{49}$$

Expression (49) simply states that at the unitary optimum the ad valorem tax τ is set such that the marginal rate of substitution (MRS) between both the public and the private good must be equal to the MCPF. The conventional MCPF formula is thus extracted, now in monetary terms as is usual:

$$MCPF_C = -\frac{\left(L_i \frac{\partial \bar{w}}{\partial \tau} + \frac{\partial \pi_i}{\partial \tau_i}\right)}{L_i w + \tau L_i \frac{\partial w}{\partial \tau} + \tau w \frac{\partial L_i}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau}}$$

With subscript C standing for Centralised case. Rewriting the third term of the RHS denominator as:

$$\tau w \frac{\partial L_i}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau} = \tau w \frac{\partial L_i}{\partial \bar{w}}\frac{\bar{w}}{L_i}\frac{L_i}{\bar{w}}\frac{\partial \bar{w}}{\partial \tau} = \tau w \eta_i \frac{L_i}{w(1-\tau_i)}\frac{\partial \bar{w}}{\partial \tau} = \frac{\tau L_i \eta_i}{(1-\tau_i)}\frac{\partial \bar{w}}{\partial \tau}$$

And plugging in the profit effect (6), we get:

$$MCPF_C = -\frac{\left(L_S \frac{\partial \bar{w}}{\partial \tau} - L_D \frac{\partial w}{\partial \tau}\right)}{\left(L_S w + \tau L_S \frac{\partial w}{\partial \tau} + \tau w \frac{\partial L_S}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \tau}\right)}$$

Plugging in the wage effects (4) and (5), with wages and labour supply/demand dropping out (homogeneous regions), then gives us

$$MCPF_C = -\frac{\frac{\varepsilon}{(\eta-\varepsilon)} - \frac{\eta}{(1-\tau)(\eta-\varepsilon)}}{\left(1 + \tau \frac{\eta}{(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)} \frac{\varepsilon}{(\eta-\varepsilon)}\right)} \tag{50}$$

$$MCPF_C = -\frac{\frac{(1-\tau)\varepsilon-\eta}{(1-\tau)(\eta-\varepsilon)}}{\left(1 + \tau \frac{\eta}{(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)} \frac{\varepsilon}{(\eta-\varepsilon)}\right)}$$

$$MCPF_C = -\frac{1}{\left(\frac{(1-\tau)(\eta-\varepsilon)}{(1-\tau)\varepsilon-\eta} + \tau \frac{\eta}{(1-\tau)\varepsilon-\eta} + \frac{\tau\eta\varepsilon}{(1-\tau)\varepsilon-\eta}\right)}$$

$$MCPF_C = \frac{1}{\left(1 + \frac{\tau\eta}{(1-\tau)\varepsilon-\eta} - \tau \frac{\eta}{(1-\tau)\varepsilon-\eta} - \frac{\tau\eta\varepsilon}{(1-\tau)\varepsilon-\eta}\right)}$$

$$MCPF_C = \frac{1}{\left(1 - \frac{\tau\eta\varepsilon}{(1-\tau)\varepsilon-\eta}\right)}$$

So that:

$$\frac{\frac{\partial V_i}{\partial G_i^F}}{\lambda_i} = \frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = \frac{1}{\left(1 - \frac{\tau\eta\varepsilon}{(1-\tau)\varepsilon-\eta}\right)} \tag{51}$$

C Calculations fully decentralised case

C.1 Optimisation problem

The decentralised optimization problem is expressed by the Lagrangian:

$$\mathcal{L} = V_i(\bar{w}, \pi_i, G_i, G_i^F) - \gamma \left\{ \left(G_i + G_i^F\right) - \tau L_i w \right\}$$

Giving us the following FOC's:

$$\frac{\partial \mathcal{L}}{\partial \tau_i} = \frac{\partial V_i}{\bar{w}_i} \frac{\partial \bar{w}_i}{\partial \tau_i} + \frac{\partial V_i}{\pi_i} \frac{\partial \pi_i}{\partial \tau_i} + \mu \left(L_{S_i} w + \tau_i L_{S_i} \frac{\partial w}{\partial \tau_i} + \tau_i w \frac{\partial L_{S_i}}{\partial \bar{w}_i} \frac{\partial \bar{w}_i}{\partial \tau_i} \right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial G_i} = \frac{\partial V_i}{\partial G_i} - \gamma = 0$$

$$\frac{\partial \mathcal{L}}{\partial G_i^F} = \frac{\partial V_i}{\partial G_i^F} - \gamma = 0$$

Therefore:

$$\frac{\partial V_i}{\partial G_i^F} = \frac{\partial V_i}{\partial G_i} = -\frac{\frac{\partial V_i}{\bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i} + \frac{\partial V_i}{\pi_i}\frac{\partial \pi_i}{\partial \tau_i}}{\left(L_{S_i} w + \tau_i L_{S_i}\frac{\partial w}{\partial \tau_i} + \tau_i w \frac{\partial L_{S_i}}{\partial \bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i}\right)}$$

Using Roy's identity, with λ_i marginal utility of income, and since $\frac{\partial V_i}{\pi_i} = \frac{\partial V_i}{\partial c_i}\frac{\partial c_i}{\pi_i} = \lambda_i \times 1$, we get:

$$MRS_i = \frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = -\frac{L_{S_i}\frac{\partial \bar{w}_i}{\partial \tau_i} + \frac{\partial \pi_i}{\partial \tau_i}}{\left(L_{S_i} w + \tau_i L_{S_i}\frac{\partial w}{\partial \tau_i} + \tau_i w \frac{\partial L_{S_i}}{\partial \bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i}\right)}$$

Since we are dealing with homogeneous regions, and after plugging in the profit effect (6), we can write the regional welfare cost here as:

$$MCPF_i = -\frac{\left(L_S \frac{\partial \bar{w}}{\partial \tau} - L_D \frac{\partial w}{\partial \tau}\right)}{\left(L_S w + \tau L_S \frac{\partial w}{\partial \tau} + \tau w \frac{\partial L_S}{\partial \bar{w}}\frac{\partial \bar{w}}{\partial \tau}\right)}$$

Rewriting as before, and plugging in the wage effects (2) and (3), with wages and labour supply/demand dropping out:

$$MCPF_i = -\frac{\left(\frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)} - \frac{\eta}{n(1-\tau)(\eta-\varepsilon)}\right)}{\left(1 + \tau\frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)}\frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)}\right)}$$

Rewriting yields:

$$MCPF_i = -\frac{\frac{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}{n(1-\tau)(\eta-\varepsilon)}}{\left(1 + \tau\frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)}\frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)}\right)}$$

$$MCPF_i = \frac{1}{\left(\frac{n(1-\tau)(\varepsilon-\eta)}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta} - \frac{\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta} + \frac{\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta} - \tau\frac{\eta}{((1-\tau)(n\varepsilon-(n-1)\eta)-\eta)} - \frac{\tau\eta(n\varepsilon-(n-1)\eta)}{((1-\tau)(n\varepsilon-(n-1)\eta)-\eta)}\right)}$$

$$MCPF_i = \frac{1}{\left(1 + \frac{-(1-\tau)\eta+\eta-\tau\eta-\tau\eta(n\varepsilon-(n-1)\eta)}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)}$$

$$MCPF_i = \frac{1}{\left(1 - \frac{\tau\eta(n\varepsilon-(n-1)\eta)}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)}$$

So that:

$$\frac{\frac{\partial V_i}{\partial G_i^F}}{\lambda_i} = \frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = \frac{1}{\left(1 - \frac{\tau\eta(n\varepsilon-(n-1)\eta)}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)} \tag{52}$$

C.2 Internalising the horizontal externalities

Expression (54) gives us the MCPF of region i when all externalities are internalised. The denominator thus has the effect of a tax increase on other regions' tax revenues incorporated, whilst the numerator integrates the effect on the welfare of non-residents:

$$MCPF_{I_i} = -\left(\frac{L_{S_i}\frac{\partial \bar{w}_i}{\partial \tau_i} + \frac{\partial \pi_i}{\partial \tau_i} + \sum_{j\neq i}^{n}\left(L_{S_j}\frac{\partial \bar{w}_j}{\partial \tau_i} + \frac{\partial \pi_i}{\partial \tau_i}\right)}{\frac{\partial R_i}{\partial \tau_i} + \sum_{j\neq i}^{n}\frac{\partial R_j}{\partial \tau_i}}\right) \tag{53}$$

Or, rewriting:

$$MCPF_{I_i} = -\left(\frac{L_{S_i}\frac{\partial \bar{w}_i}{\partial \tau_i} + \frac{\partial \pi_i}{\partial \tau_i} + \sum_{j\neq i}^{n}\left(L_{S_j}\frac{\partial \bar{w}_j}{\partial \tau_i} + \frac{\partial \pi_i}{\partial \tau_i}\right)}{L_{S_i}w + \tau_i L_{S_i}\frac{\partial w}{\partial \tau_i} + \tau_i w\frac{\partial L_{S_i}}{\partial \bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i} + \sum_{j\neq i}^{n}\left(\tau_j L_{S_j}\frac{\partial w}{\partial \tau_i} + \tau_j w\frac{\partial L_{S_j}}{\partial \bar{w}_j}\frac{\partial \bar{w}_j}{\partial \tau_i}\right)}\right)$$

Plugging in the profit effect (6) gives us:

$$MCPF_{I_i} = -\left(\frac{L_{S_i}\frac{\partial \bar{w}_i}{\partial \tau_i} - L_{D_i}\frac{\partial w}{\partial \tau_i} + \sum_{j\neq i}^{n}\left(L_{S_j}\frac{\partial \bar{w}_j}{\partial \tau_i} - L_{D_j}\frac{\partial w}{\partial \tau_i}\right)}{L_{S_i}w + \tau_i L_{S_i}\frac{\partial w}{\partial \tau_i} + \tau_i w\frac{\partial L_{S_i}}{\partial \bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i} + \sum_{j\neq i}^{n}\left(\tau_j L_{S_j}\frac{\partial w}{\partial \tau_i} + \tau_j w\frac{\partial L_{S_j}}{\partial \bar{w}_j}\frac{\partial \bar{w}_j}{\partial \tau_i}\right)}\right)$$

Since $\frac{\partial \bar{w}_j}{\partial \tau_i} = \frac{\partial((1-\tau_j)w)}{\partial \tau_i} = (1-\tau_j)\frac{\partial w}{\partial \tau_i}$, and regions are homogeneous, this becomes:

$$MCPF_{I_i} = -\left(\frac{L\frac{\partial \bar{w}}{\partial \tau_i} - L\frac{\partial w}{\partial \tau_i} + (n-1)L\left((1-\tau)\frac{\partial w}{\partial \tau_i} - \frac{\partial w}{\partial \tau_i}\right)}{Lw + \tau L\frac{\partial w}{\partial \tau_i} + \tau w\frac{\partial L}{\partial \bar{w}}\frac{\partial \bar{w}_i}{\partial \tau} + (n-1)\left(\tau L\frac{\partial w}{\partial \tau_i} + \tau w\frac{\partial L}{\partial \bar{w}}(1-\tau)\frac{\partial w}{\partial \tau}\right)}\right) \tag{54}$$

Or, since $\tau w\frac{\partial L}{\partial \bar{w}} = \tau w\frac{\partial L}{\partial \bar{w}}\frac{\bar{w}}{L}\frac{L}{\bar{w}} = \tau w\eta\frac{L}{w(1-\tau)} = \frac{\tau L\eta}{(1-\tau)}$:

$$MCPF_{I_i} = -\left(\frac{L\frac{\partial \bar{w}}{\partial \tau_i} - L\frac{\partial w}{\partial \tau_i} + (n-1)L\left((1-\tau)\frac{\partial w}{\partial \tau_i} - \frac{\partial w}{\partial \tau_i}\right)}{Lw + \tau L\frac{\partial w}{\partial \tau_i} + \frac{\tau L\eta}{(1-\tau)}\frac{\partial \bar{w}_i}{\partial \tau} + (n-1)\left(\tau L\frac{\partial w}{\partial \tau_i} + \frac{\tau L\eta}{(1-\tau)}(1-\tau)\frac{\partial w}{\partial \tau}\right)}\right)$$

Plugging in the regional wage effects (2) and (3), and with labour supply and demand canceling out:

$$MCPF_{I_i} = -\left(\frac{\frac{w(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)} - \frac{w\eta}{n(1-\tau)(\eta-\varepsilon)} + (n-1)\left((1-\tau)\frac{w\eta}{n(1-\tau)(\eta-\varepsilon)} - \frac{w\eta}{n(1-\tau)(\eta-\varepsilon)}\right)}{w + \tau\frac{w\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)}\frac{w(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)} + (n-1)\left(\tau\frac{w\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)}(1-\tau)\frac{w\eta}{n(1-\tau)(\eta-\varepsilon)}\right)}\right)$$

Wages drop out as well:

$$MCPF_{I_i} = -\left(\frac{\frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)} - \frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + (n-1)\left(\frac{\eta}{n(\eta-\varepsilon)} - \frac{\eta}{n(1-\tau)(\eta-\varepsilon)}\right)}{1 + \tau\frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)}\frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)} + (n-1)\left(\tau\frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)}\frac{\eta}{n(\eta-\varepsilon)}\right)}\right)$$

So that we get:

$$MCPF_{I_i} = -\left(\frac{n\frac{\varepsilon}{n(\eta-\varepsilon)} - n\frac{\eta}{n(1-\tau)(\eta-\varepsilon)}}{1 + n\tau\frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)}\frac{n\varepsilon}{n(\eta-\varepsilon)}}\right)$$

Or:

$$MCPF_{I_i} = -\left(\frac{\frac{\varepsilon}{(\eta-\varepsilon)} - \frac{\eta}{(1-\tau)(\eta-\varepsilon)}}{1 + \tau\frac{\eta}{(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)}\frac{\varepsilon}{(\eta-\varepsilon)}} \right)$$

Which is exactly equal to the welfare cost (50) derived under the second-best unitary case:

$$MCPF_C = -\frac{\frac{\varepsilon}{(\eta-\varepsilon)} - \frac{\eta}{(1-\tau)(\eta-\varepsilon)}}{\left(1 + \tau\frac{\eta}{(1-\tau)(\eta-\varepsilon)} + \frac{\tau\eta}{(1-\tau)}\frac{\varepsilon}{(\eta-\varepsilon)}\right)} \tag{55}$$

D Calculations shared tax base case

D.1 Optimisation problem

The optimisation problem yields the same marginal rate of substitution as before :

$$MRS_i = \frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = -\frac{L_{S_i}\frac{\partial \bar{w}_i}{\partial \tau_i} + \frac{\partial \pi_i}{\partial \tau_i}}{\left(L_{S_i}w + \tau_i L_{S_i}\frac{\partial w}{\partial \tau_i} + \tau_i w\frac{\partial L_{S_i}}{\partial \bar{w}_i}\frac{\partial \bar{w}_i}{\partial \tau_i}\right)}$$

Plugging in the wage effects (2), (3) and profit effect (6), with wages and labour supply and demand dropping out as before:

$$MCPF_i = -\frac{\left(\frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)} - \frac{\eta}{n(1-\tau)(\eta-\varepsilon)}\right)}{\left(1 + \tau_i\frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{\tau_i\eta}{(1-\tau)}\frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)}\right)}$$

With $\tau = \tau_i + \tau_0$

Rewriting yields:

$$MCPF_i = -\frac{\frac{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}{n(1-\tau)(\eta-\varepsilon)}}{\left(1 + \tau_i\frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{\tau_i\eta}{(1-\tau)}\frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)}\right)}$$

$$MCPF_i = \frac{1}{\left(\frac{n(1-\tau)(\varepsilon-\eta)}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta} - \frac{\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta} + \frac{\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta} - \tau_i\frac{\eta}{((1-\tau)(n\varepsilon-(n-1)\eta)-\eta)} - \frac{\tau_i\eta(n\varepsilon-(n-1)\eta)}{((1-\tau)(n\varepsilon-(n-1)\eta)-\eta)}\right)}$$

$$MCPF_i = \frac{1}{\left(1 + \frac{-(1-\tau_i-\tau_0)\eta+\eta-\tau_i\eta-\tau_i\eta(n\varepsilon-(n-1)\eta)}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)}$$

$$MCPF_i = \frac{1}{\left(1 + \frac{\tau_0\eta-\tau_i\eta(n\varepsilon-(n-1)\eta)}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)}$$

$$MCPF_i = \frac{1}{\left(1 - \frac{\tau_i\eta(n\varepsilon-(n-1)\eta)-\tau_0\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)}$$

So that:

$$\frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = \frac{1}{\left(1 - \frac{\tau_i\eta(n\varepsilon-(n-1)\eta)-\tau_0\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)} \tag{56}$$

D.2 Calculations vertical interaction

D.2.1 Federal revenue effect

The effect of a marginally increased regional tax rate on the federal budget can in general be written as:

$$\frac{\partial R_0}{\partial \tau_i} = \frac{\partial \left(\tau_0 \sum_i^n L_i w\right)}{\partial \tau_i} \tag{57}$$

$$\frac{\partial R_0}{\partial \tau_i} = \tau_0 \sum_i^n \left(L_i \frac{\partial w}{\partial \tau_i}\right) + \tau_0 \sum_{j \neq i}^n \left(w \frac{\partial L_j}{\partial \bar{w}_j} \frac{\partial \bar{w}_j}{\partial \tau_i}\right) + \tau_0 w \frac{\partial L_i}{\partial \bar{w}_i} \frac{\partial \bar{w}_i}{\partial \tau_i} \tag{58}$$

Plugging in $\frac{\partial \bar{w}_j}{\partial \tau_i} = \frac{\partial((1-\tau_j)w)}{\partial \tau_i} = (1-\tau_j)\frac{\partial w}{\partial \tau_i}$ then yields:

$$\frac{\partial R_0}{\partial \tau_i} = \tau_0 \sum_i^n \left(L_i \frac{\partial w}{\partial \tau_i}\right) + \tau_0 w \sum_{j \neq i}^n \left((1-\tau_j) \frac{\partial L_j}{\partial \bar{w}_j} \frac{\partial w}{\partial \tau_i}\right) + \tau_0 w \frac{\partial L_i}{\partial \bar{w}_i} \frac{\partial \bar{w}_i}{\partial \tau_i} \tag{59}$$

Or:

$$\frac{\partial R_0}{\partial \tau_i} = \left\{\tau_0 \sum_{j \neq i}^n \left(L_j \frac{\partial w}{\partial \tau_i}\right) + \tau_0 w \sum_{j \neq i}^n \left((1-\tau_j) \frac{\partial L_j}{\partial \bar{w}_j} \frac{\partial w}{\partial \tau_i}\right)\right\} + \tau_0 L_i \frac{\partial w}{\partial \tau_i} + \tau_0 w \frac{\partial L_i}{\partial \bar{w}_i} \frac{\partial \bar{w}_i}{\partial \tau_i} \tag{60}$$

Which, for homogeneous regions becomes:

$$\frac{\partial R_0}{\partial \tau_i} = \left\{(n-1)\left(\tau_0 L \frac{\partial w}{\partial \tau_i} + \tau_0 w (1-\tau_j) \frac{\partial L_j}{\partial \bar{w}} \frac{\partial w}{\partial \tau_i}\right)\right\} + \tau_0 L \frac{\partial w}{\partial \tau_i} + \tau_0 w \frac{\partial L}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \tau_i} \tag{61}$$

D.2.2 Sign second effect vertical interaction

Comparing the MCPF obtained in section 4.1 ($MCPF_i$) with the efficiency cost derived under the shared tax base case in section 4.2 ($MCPF_i^{Shared}$), yields:

$$MCPF_i^{Shared} = \frac{1}{\left(1 - \frac{\tau_i^* \eta (n\varepsilon - (n-1)\eta) - \tau_0^* \eta}{(1-\tau)(n\varepsilon - (n-1)\eta) - \eta}\right)} \lesseqqgtr \frac{1}{\left(1 - \frac{\tau \eta (n\varepsilon - (n-1)\eta)}{(1-\tau)(n\varepsilon - (n-1)\eta) - \eta}\right)} = MCPF_i \tag{62}$$

Again evaluated at the Nash equilibrium $\tau_i^* + \tau_0^* = \tau$ of the shared tax base case. Now, we know that $MCPF_i^{Shared} < MCPF_i$ when:

$$\frac{\tau_i^* \eta \left(n\varepsilon - (n-1)\eta\right) - \tau_0^* \eta}{(1-\tau)\left(n\varepsilon - (n-1)\eta\right) - \eta} < \frac{\tau \eta \left(n\varepsilon - (n-1)\eta\right)}{(1-\tau)\left(n\varepsilon - (n-1)\eta\right) - \eta} \tag{63}$$

Which boils down to:

$$\tau_i^* \left(n\varepsilon - (n-1)\eta\right) - \tau_0^* > \tau \left(n\varepsilon - (n-1)\eta\right) \tag{64}$$

Which will hold if and only if:

$$\left(n\varepsilon - (n-1)\eta\right) < -1 \tag{65}$$

D.3 Proof proposition 2

Let us consider the isolated case where only vertical externalities can occur. The region would then solve the exact same optimisation problem as in section 4.2, but then facing the same wage effects (5) and (6) as the federal government would. One readily verifies that this set-up would lead to the following equilibrium condition:

$$\frac{\frac{\partial V_i}{\partial G_i}}{\lambda_i} = \frac{1}{\left(1 - \frac{\tau_i \eta \varepsilon - \tau_0 \eta}{(1-\tau)\varepsilon - \eta}\right)} = MCPF_i^{nohorizontal} \tag{66}$$

Now, we know that this efficiency cost will be biased upwards when:

$$MCPF_i^{nohorizontal} = \frac{1}{\left(1 - \frac{\tau_i^* \eta \varepsilon - \tau_0^* \eta}{(1-\tau)\varepsilon - \eta}\right)} > \frac{1}{\left(1 - \frac{\tau \eta \varepsilon}{(1-\tau)\varepsilon - \eta}\right)} = MCPF_C$$

Again evaluated at the Nash equilibrium $\tau_i^* + \tau_0^* = \tau$. This comes down to the following:

$$\frac{\tau_i^* \eta \varepsilon - \tau_0^* \eta}{(1-\tau)\varepsilon - \eta} > \frac{\tau \eta \varepsilon}{(1-\tau)\varepsilon - \eta}$$

Or:

$$\tau_i^* \varepsilon - \tau_0^* < \tau \varepsilon$$

Which gives us the necessary and sufficient condition for the vertical externality to be positive:

$$MCPF_i^{nohorizontal} > MCPF_C \Longleftrightarrow \varepsilon > -1 \tag{67}$$

A result which is also found by Dahlby (2003). Now, to prove a Nash equilibrium (τ_i^*, τ_0^*) exists at which the MCPF under the shared tax base case (where both vertical and horizontal interaction is combined) is lower than the second-best efficiency cost ($MCPF_C$), we have as before the following condition:

$$MCPF_i^{Shared} = \frac{1}{\left(1 - \frac{\tau_i^* \eta (n\varepsilon - (n-1)\eta) - \tau_0^* \eta}{(1-\tau)(n\varepsilon - (n-1)\eta) - \eta}\right)} < \frac{1}{\left(1 - \frac{\tau \eta \varepsilon}{(1-\tau)\varepsilon - \eta}\right)} = MCPF_C$$

Or, rewritten:

$$\frac{\tau_i \left(n\varepsilon - (n-1)\eta\right) - \tau_0}{(1-\tau)\varepsilon - \eta + \tau\eta(1 - \frac{1}{n})} < \frac{n\tau\varepsilon}{(1-\tau)\varepsilon - \eta} \tag{68}$$

Evaluating this condition for $\varepsilon > -1$, it is straightforward to see that a critical value of τ_0^* will exist so that (68) will begin to hold.

We have thus shown that above this critical value:

$$MCPF_i^{Shared} < MCPF_C < MCPF_i^{nohorizontal} \tag{69}$$

Which, together with the fact that the horizontal externality when evaluated in isolation will always be positive (see section 4.1), proves the proposition.

E Heterogeneous regions

E.1 Optimisation problem second-best optimum

The second best optimization problem is expressed by the Lagrangian:

$$\mathcal{L} = S - \gamma(\sum_i^n G_i - \tau \sum_i^n L_i w - \sum_i^n \pi_i) \tag{70}$$

With $\sum_i^n \{V_i(\bar{w}, G_i\} = S$. Leading to the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial \tau} = \frac{\partial S}{\partial \tau} + \gamma \left(\frac{\partial(\tau \sum_i^n L_i w + \sum_i^n \pi_i)}{\partial \tau} \right) = 0 \tag{71}$$

$$\frac{\partial \mathcal{L}}{\partial G_i} = \frac{\partial S}{\partial G_i} - \gamma = 0 \quad for\, i = 1, .., n \tag{72}$$

Solving both conditions yields:

$$\sum_i^n \frac{\partial V_i}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \tau} + \gamma \left(\sum_i^n L_i w + \tau \sum_i^n L_i \frac{\partial w}{\partial \tau} + \tau \sum_i^n w \frac{\partial L_i}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \tau} + \sum \frac{\partial \pi_i}{\partial \tau} \right) = 0 \; (\tau) \tag{73}$$

$$\frac{\partial V_i}{\partial G_i} - \gamma = 0 \quad for\, i = 1, .., n \quad (G_i) \tag{74}$$

With $\sum_i^n L_i w + \tau \sum_i^n L_i \frac{\partial w}{\partial \tau} + \tau \sum_i^n w \frac{\partial L_i}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \tau} + \sum_i^n \frac{\partial \pi_i}{\partial \tau}$ equal to $\frac{\partial R}{\partial \tau}$, being the marginal effect on federal revenue by raising the labour tax. Substituting for γ then yields:

$$\frac{\partial V_i}{\partial G_i} = -\frac{\sum_i^n \frac{\partial V_i}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \tau}}{\frac{\partial R}{\partial \tau}} \quad for\, i = 1, .., n \tag{75}$$

Public provision in each region will again continue until its marginal benefits equal its marginal cost, and this in terms of welfare cost as well as actual provision cost (see also Dahlby, 2008). This expression can be reformulated to arrive at the conventional MCPF expression using Roy's identity:

$$\frac{\partial V_i}{\partial G_i} = -\frac{\sum_i^n \left(L_i \lambda_i \frac{\partial \bar{w}}{\partial \tau} \right)}{\frac{\partial R}{\partial \tau}} \quad for\, i = 1, .., n \tag{76}$$

Which we can rewrite as:

$$\frac{\frac{\partial V_i}{\partial G_i}}{\sum_i^n (s_i \lambda_i)} = -\frac{L \frac{\partial \bar{w}}{\partial \tau}}{\frac{\partial R}{\partial \tau}} \quad for\, i = 1, .., n \tag{77}$$

With $L = \sum_i^n L_i$, λ_i the marginal utility of income of a household residing in region i, and s_i this household's share ($\frac{L_i}{L}$) in total labour supplied in the federation as a whole. The conventional MCPF formula is thus once again extracted:

$$MCPF_C = -\frac{L \frac{\partial \bar{w}}{\partial \tau}}{\sum_i^n L_i w + \tau \sum_i^n L_i \frac{\partial w}{\partial \tau} + \tau \sum_i^n w \frac{\partial L_i}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \tau} + \sum_i^n \frac{\partial \pi_i}{\partial \tau}} \tag{78}$$

With the third term of the denominator to be rewritten as:

$$\tau \sum_{i}^{n} w \frac{\partial L_i}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \tau} = \tau \sum_{i}^{n} w \frac{\partial L_i}{\partial \bar{w}} \frac{\bar{w}}{L_i} \frac{L_i}{\bar{w}} \frac{\partial \bar{w}}{\partial \tau} = \frac{w\tau}{\bar{w}} \frac{\partial \bar{w}}{\partial \tau} \sum_{i}^{n} \eta_i L_i = \frac{\tau}{(1-\tau)} \frac{\partial \bar{w}}{\partial \tau} \sum_{i}^{n} \eta_i L_i \quad (79)$$

Plugging in (43), (45) and (6), we get:
$MCPF_C =$

$$-\left(\frac{\frac{Lw \sum_i^n \varepsilon_i \frac{L_{D_i}}{w}}{\left((1-\tau) \sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w} \right) \right)}}{\sum_i^n L_i w + \frac{\tau \sum_i^n L_i w \sum_i^n \eta_i \frac{L_{s_i}}{\bar{w}_i}}{\left((1-\tau) \sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w} \right) \right)} + \frac{\frac{\tau}{(1-\tau)} Lw \sum_i^n \varepsilon_i \frac{L_{D_i}}{w} \sum_i^n \eta_i L_i}{\left((1-\tau) \sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w} \right) \right)} - \frac{nL_{D_i} w \sum_i^n \eta_i \frac{L_{s_i}}{\bar{w}_i}}{\left((1-\tau) \sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w} \right) \right)}} \right)$$

Factoring out Lw and solving further gives us:

$$MCPF_C = -\left(\frac{\frac{\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}}{\left((1-\tau) \sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w} \right) \right)}}{1 + \frac{\tau \sum_i^n \eta_i \frac{L_{s_i}}{\bar{w}_i}}{\left((1-\tau) \sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w} \right) \right)} + \frac{\frac{\tau}{(1-\tau)} \sum_i^n \varepsilon_i \frac{L_{D_i}}{w} \sum_i^n \eta_i L_i}{\left((1-\tau) \sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w} \right) \right)} - \frac{\sum_i^n \eta_i \frac{L_{s_i}}{\bar{w}_i}}{\left((1-\tau) \sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w} \right) \right)}} \right) \quad (80)$$

$$MCPF_C = -\left(\frac{1}{\frac{\left((1-\tau) \sum_i^n \left(\eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \sum_i^n \left(\varepsilon_i \frac{L_{D_i}}{w} \right) \right)}{\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}} + \tau \frac{\sum_i^n \eta_i \frac{L_{s_i}}{\bar{w}}}{\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}} + \frac{\tau}{L(1-\tau)} \sum_i^n \eta_i L_i - \frac{\sum_i^n \eta_i \frac{L_{s_i}}{\bar{w}}}{\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}}} \right) \quad (81)$$

$$MCPF_C = \left(\frac{1}{1 - \frac{\tau}{L(1-\tau)} \sum_i^n \eta_i L_i} \right) \quad (82)$$

So that:

$$\frac{\frac{\partial V_i}{\partial G_i}}{\sum_i^n (s_i \lambda_i)} = \left(\frac{1}{1 - \frac{\tau}{L(1-\tau)} \sum_i^n \eta_i L_i} \right) \qquad for\, i = 1, .., n \quad (83)$$

E.2 Optimisation problem fully decentralised case

It is straightforward to derive the following condition describing the optimum of the problem:

$$\frac{\frac{\partial g_i}{\partial G_i}}{\lambda_i} = -\frac{L_i \frac{\partial \bar{w}_i}{\partial \tau_i}}{L_i w + \tau_i L_i \frac{\partial w}{\partial \tau_i} + \tau_i w \frac{\partial L_i}{\partial \bar{w}_i} \frac{\partial \bar{w}_i}{\partial \tau_i} + \frac{\partial \pi_i}{\partial \tau_i}} \quad (84)$$

Plugging in (35), (37) and rewriting as before, we get:

$$MCPF_i = -\frac{\left(L_i \frac{w\left(\left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w} \right) - \sum_{j \neq i}^n \left((1-\tau_j) \eta_j \frac{L_{s_j}}{\bar{w}_j} \right) \right)}{\left(\sum_i^n \left((1-\tau_i) \eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w} \right) \right)} \right)}{L_i w_i + \tau_i L_i \left(\frac{w \eta_i \frac{L_{s_i}}{\bar{w}_i}}{\left(\sum_i^n \left((1-\tau_i) \eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w} \right) \right)} \right) + \frac{\tau_i L_i \eta_i}{(1-\tau_i)} \frac{w\left(\left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w} \right) - \sum_{j \neq i}^n \left((1-\tau_j) \eta_j \frac{L_{s_j}}{\bar{w}_j} \right) \right)}{\left(\sum_i^n \left((1-\tau_i) \eta_i \frac{L_{s_i}}{\bar{w}_i} \right) - \left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w} \right) \right)}} \quad (85)$$

$$
-L_{D_i}\left(\frac{w\eta_i\frac{L_{s_i}}{\bar{w_i}}}{\left(\sum_i^n\left((1-\tau_i)\eta_i\frac{L_{s_i}}{\bar{w_i}}\right)-\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)\right)}\right)
$$

Factoring out $L_i w$ and solving further gives us:

$$
MCPF_i=-\frac{\left(\frac{\left(\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)-\sum_{j\neq i}^n\left((1-\tau_j)\eta_j\frac{L_{s_j}}{\bar{w_j}}\right)\right)}{\left(\sum_i^n\left((1-\tau_i)\eta_i\frac{L_{s_i}}{\bar{w_i}}\right)-\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)\right)}\right)}{1+\tau_i\left(\frac{\eta_i\frac{L_{s_i}}{\bar{w_i}}}{\left(\sum_i^n\left((1-\tau_i)\eta_i\frac{L_{s_i}}{\bar{w_i}}\right)-\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)\right)}\right)+\frac{\tau_i\eta_i}{(1-\tau_i)}\frac{\left(\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)-\sum_{j\neq i}^n\left((1-\tau_j)\eta_j\frac{L_{s_j}}{\bar{w_j}}\right)\right)}{\left(\sum_i^n\left((1-\tau_i)\eta_i\frac{L_{s_i}}{\bar{w_i}}\right)-\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)\right)}} \tag{86}
$$

$$
-\frac{L_{D_i}}{L_{s_i}}\left(\frac{\eta_i\frac{L_{s_i}}{\bar{w_i}}}{\left(\sum_i^n\left((1-\tau_i)\eta_i\frac{L_{s_i}}{\bar{w_i}}\right)-\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)\right)}\right)
$$

$$
MCPF_i=-\frac{1}{\frac{\left(\sum_i^n\left((1-\tau_i)\eta_i\frac{L_{s_i}}{\bar{w_i}}\right)-\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)\right)}{\left(\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)-\sum_{j\neq i}^n\left((1-\tau_j)\eta_j\frac{L_{s_j}}{\bar{w_j}}\right)\right)}+\frac{\tau_i\eta_i\frac{L_{s_i}}{\bar{w_i}}}{\left(\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)-\sum_{j\neq i}^n\left((1-\tau_j)\eta_j\frac{L_{s_j}}{\bar{w_j}}\right)\right)}+\frac{\tau_i\eta_i}{(1-\tau_i)}-\frac{\frac{L_D}{L_{s_i}}\eta_i\frac{L_{s_i}}{\bar{w_i}}}{\left(\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)-\sum_{j\neq i}^n\left((1-\tau_j)\eta_j\frac{L_{s_j}}{\bar{w_j}}\right)\right)}} \tag{87}
$$

$$
MCPF_i=-\frac{1}{\frac{\left(\sum_{j\neq i}^n\left((1-\tau_j)\eta_j\frac{L_{s_j}}{\bar{w_j}}\right)+\eta_i\frac{L_{s_i}}{\bar{w_i}}-\left(\sum_i^n\varepsilon\frac{L_D}{w}\right)\right)}{\left(\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)-\sum_{j\neq i}^n\left((1-\tau_j)\eta_j\frac{L_{s_j}}{\bar{w_j}}\right)\right)}+\frac{\tau_i\eta_i}{(1-\tau_i)}-\frac{\frac{L_D}{L_{s_i}}\eta_i\frac{L_{s_i}}{\bar{w_i}}}{\left(\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)-\sum_{j\neq i}^n\left((1-\tau_j)\eta_j\frac{L_{s_j}}{\bar{w_j}}\right)\right)}} \tag{88}
$$

$$
MCPF_i=\frac{1}{1-\frac{\tau_i\eta_i}{(1-\tau_i)}-\left(1-\frac{L_{D_i}}{L_{s_i}}\right)\left(\frac{\eta_i\frac{L_{s_i}}{\bar{w_i}}}{\left(\left(\sum_i^n\varepsilon_i\frac{L_{D_i}}{w}\right)-\sum_{j\neq i}^n\left((1-\tau_j)\eta_j\frac{L_{s_j}}{\bar{w_j}}\right)\right)}\right)} \tag{89}
$$