Comparison of Two Nonlinear Finite-Element Homogenization Methods for Laminated Iron Cores

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***Abstract*—In this paper, we compare qualitatively and quantitatively the performance and validity range of a so-called one-step homogenization technique with those of a multiscale computational homogenization method when applied to model a laminated iron core accounting for eddy currents and magnetic hysteresis. The analysis includes both global and local field quantities.**

***Index Terms*—Homogenization techniques, eddy currents, magnetic hysteresis, finite element methods.**

I. Introduction

The eddy current effects in laminated iron cores may considerably alter the overall performance of electromagnetic AC devices and should be thus accounted for in early stages of the design. Finely discretizing each separate lamination in standard finite-element (FE) models is computationally prohibitive. More pragmatic techniques have been conceived for dealing with these stacks: nonconducting and homogeneous stacked core with *a posteriori* loss estimation, anisotropic surrogate material laws,... (see overview in [1]). Homogenization methods accounting for hysteresis are mostly limited to time-harmonic analysis [2], [3].

An *ad hoc* time-domain homogenization technique is developed in [4] for tackling laminated iron cores with eddy currents. It consists in embedding extra eddy-current unknowns in the FE model of the homogenized stack. Herein, the lamination model is implicit and the solution of the problem is achieved in one step. The inclusion of hysteresis has further been presented [5].

In [6] a multiscale computational homogenization method is applied for studying these type of nonlinear magnetodynamic problems . It comprises a FE model at each of the considered scales: the macroscale (homogenized stack) and the mesoscale (portion of lamination plus isolation around the macroscale points of interest, e.g. Gauss points). The solution is then obtained by iterating between the macroscale problem and the mesoscale problems per time step.

In this paper, we aim at comparing the performance and validity range of these two approaches. Though the multiscale computational approach is substantially more expensive than the so-called one-step approach, it helps computing accurate local field values in critical regions, namely in the vicinity of edges and corners.

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II. FE-based homogenization

Let us consider a magnetodynamic problem in a bounded domain $\Omega = \Omega_c \cup \Omega_c^C \in \mathbb{R}^3$ with boundary Γ. The conductive and non-conductive parts of Ω are denoted by Ω_c and Ω_c^C, respectively. The classical magnetic vector potential ($\underline{a}$) formulation reads: find $\underline{a}$ such that

$$\left(\underline{h}(\underline{b}), \operatorname{curl}\underline{a}'\right)_\Omega + \left(\underline{\underline{\sigma}}\,\partial_t \underline{a}, \underline{a}'\right)_{\Omega_c} + \langle \hat{n}\wedge\underline{h}, \underline{a}'\rangle_\Gamma = (j_s, \underline{a}')_{\Omega_s}, \quad (1)$$

holds for all test functions $\underline{a}'$ in a suitable function space; $\underline{h}$ is the magnetic field; $\underline{b} = \operatorname{curl}\underline{a}$ is the magnetic flux density; $\underline{\underline{\sigma}}$ is the conductivity tensor; j_s is a prescribed current density; $\hat{n}$ is the outward unit normal vector on Γ; $(\cdot,\cdot)_\Omega$ and $\langle\cdot,\cdot\rangle_\Gamma$ denote a volume integral in Ω and a surface integral on Γ of the scalar product of their arguments. For the sake of simplicity, hereafter Ω_c comprises only the laminated domain with nonlinear irreversible material law $\underline{h}(\underline{b})$. Elsewhere the media is assumed linear and isotropic, $\underline{h} = \nu\,\underline{b}$, reluctivity ν.

Furthermore, for hysteretic materials, given a state $(\underline{h}^-, \underline{b}^-)$ and $\underline{b}$, the corresponding $\underline{h}$ can be obtained by $\underline{h} = \underline{h}^- + \int_{b^-}^{b} \frac{\partial \underline{h}}{\partial \underline{b}}\left(\underline{b}, \underline{h}, \operatorname{sign}(h-h^-)\right)\mathrm{d}b$, with $\frac{\partial \underline{h}}{\partial \underline{b}}$ the differential reluctivity tensor and $f = |\underline{f}|$ the field modulus [7]. After space and time discretization of (1), the system of nonlinear algebraic equations is solved by means of the Newton-Raphson method.

A. One-step approach

This homogenization technique follows from a 1-D lamination model with isotropic conductivity σ [4]. The eddy-current effects in the laminations are accounted for by dedicated basis functions (even polynomials of order n) and associated degrees of freedom in the homogenized core. Indeed, considering a finite number of basis functions for $\underline{b}$ and $\underline{h}$ and weakly imposing the constitutive law, we get a nonlinear system of $1 + n/2$ equations to be coupled with (1) [4].

From (1) and taking into account the history of the material, we can write the full system for e.g. $n = 2$:
Find $\underline{a}$ and $\underline{b}_2$ so that

$$\begin{aligned}
&\left(\nu\operatorname{curl}\underline{a}, \operatorname{curl}\underline{a}'\right)_{\Omega_c^C} + \left(\underline{h}(\underline{h}^-, \operatorname{curl}\underline{a}^-, \underline{b}_2^-, \operatorname{curl}\underline{a}, \underline{b}_2), \operatorname{curl}\underline{a}'\right)_{\Omega_c}\\
&+ \left(\sigma d^2 q_{00}\,\partial_t \operatorname{curl}\underline{a}, \operatorname{curl}\underline{a}'\right)_{\Omega_c} + \left(\sigma d^2 q_{02}\,\partial_t \underline{b}_2, \operatorname{curl}\underline{a}'\right)_{\Omega_c}\\
&= (j_s, \underline{a}')_{\Omega_s}, \forall \underline{a}'\\
&\left(\underline{h}(\underline{h}^-, \operatorname{curl}\underline{a}^-, \underline{b}_2^-, \operatorname{curl}\underline{a}, \underline{b}_2), \underline{b}_2'\right)_{\Omega_c}\\
&+ \left(\sigma d^2 q_{02}\,\partial_t \operatorname{curl}\underline{a}, \underline{b}_2'\right)_{\Omega_c} + \left(\sigma d^2 q_{22}\,\partial_t \underline{b}_2^+, \underline{b}_2'\right)_{\Omega_c} = 0, \forall \underline{b}_2' \quad (2)
\end{aligned}$$

with q_{kl}, $k, l = 0, 2$ the elements of a triangular matrix determined by the 1-D lamination model [4].

B. *Multiscale computational approach*

The multiscale approach we consider is built on the heterogeneous multiscale method framework [8]. It couples: 1) a macroscale problem that captures the slow variations of the overall solution on a coarsely discretized domain (Fig. 1, left, right half); 2) many mesoscale problems that allow determining the macroscale constitutive law by means of finely discretized representative domains around some points of interest of the macroscale mesh (Fig. 1, right) [6].

The macroscale problem is governed by (1) with $\underline{\underline{\sigma}}$ and $\underline{h}(\underline{b})$ to be *upscaled* from the associated mesoscale ones (subscript m). Indeed, we compute: 1) $\underline{\underline{\sigma}}$ from $\underline{\underline{\sigma}}^{\varepsilon}_m$ by applying the asymptotic expansion method [9]; 2) $\underline{h}$ from $\underline{h}^{\varepsilon}_m$ by means of the two-scale convergence method [10] (superscript ε refers to quantities with rapid spatial variations). Further, the differential reluctivity tensor $\frac{\partial \underline{h}}{\partial \underline{b}}$ needed by the Newton-Raphson scheme at the macroscale is obtained by finite differences [6].

The mesoscale problems have as source the macroscale fields, $\underline{e}$ and $\underline{b}$, that must be *downscaled*. The two-scale convergence theory allows also expanding $\underline{e}^{\varepsilon}_m$ and $\underline{b}^{\varepsilon}_m$ as [6]:

$$\underline{e}^{\varepsilon}_m = -\partial_t \underline{a}^{\varepsilon}_c + \underline{e} - \kappa \partial_t \underline{b} \wedge \boldsymbol{y} \,, \qquad \underline{b}^{\varepsilon}_m = \operatorname{curl} \underline{a}^{\varepsilon}_c + \underline{b} \,, \quad (3)$$

where $\underline{a}^{\varepsilon}_c$ is a correction magnetic vector potential; $\boldsymbol{y}$ is the mesoscale spatial position and $\kappa = 1, 1/2$ for 2-D and 3-D problems, respectively. Assuming, $\underline{e} = \langle \underline{e}^{\varepsilon}_m \rangle_{\Omega_m}$ and $\underline{b} = \langle \underline{b}^{\varepsilon}_m \rangle_{\Omega_m}$, the average of $\underline{e}^{\varepsilon}_m$ and $\underline{b}^{\varepsilon}_m$ over Ω_m, respectively, the periodic boundary conditions for the mesoscale problems are fully determined [6]. The weak formulation governing the mesoscale problems reads:
find $\underline{a}^{\varepsilon}_c$ such that

$$\left(\underline{h}^{\varepsilon}_m (\operatorname{curl} \underline{a}^{\varepsilon}_c + \underline{b}), \operatorname{curl} \underline{a}^{\varepsilon'}_c \right)_{\Omega_m} + \left(\underline{\underline{\sigma}}^{\varepsilon}_m \partial_t \underline{a}^{\varepsilon}_c, \underline{a}^{\varepsilon'}_c \right)_{\Omega_{m_c}}$$
$$= \left(\underline{\underline{\sigma}}^{\varepsilon}_m (\underline{e} - \kappa \partial_t \underline{b} \wedge \boldsymbol{y}), \underline{a}^{\varepsilon'}_c \right)_{\Omega_{m_c}}, \forall \underline{a}^{\varepsilon'}_c \,. \quad (4)$$

III. Application Example

As validation test case, we study a stacked ring core (20 laminations, thickness 0.5 mm, $\sigma = 5$ MS/m, separated by 0.02 mm thick airgaps) surrounded by an inductor [4]. The parameter values of the J-A model are those of ferrosilicon in [5], [6]. A brute-force FE approach with a sufficiently fine discretization of each lamination (10 layers of elements) produces a reference solution. The meshes of the fine reference model, the homogenized or macroscale model (exploiting symmetry) and the mesoscale model (one lamination plus half an isolation layer up and down) are depicted in Fig. 1.

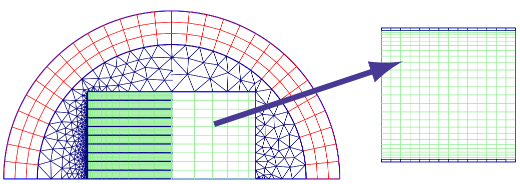

Figure 1. Meshes of the reference fine model (left, left half), the homogenized or macroscale model (left, right half) and the mesoscale model (right).

Hereafter, only results obtained by the averaging-type homogenization method are shown. The actual comparison will be performed in the full paper.

Time-stepping simulations with imposed sinusoidal current of same amplitude but different frequencies are carried out. The normalized flux linkage versus time (left) and imposed current (right) at 500 Hz is shown in Figs. 2. The overall losses versus frequency are represented in Fig. 3. A good convergence towards the reference results ("fine") is observed when increasing n.

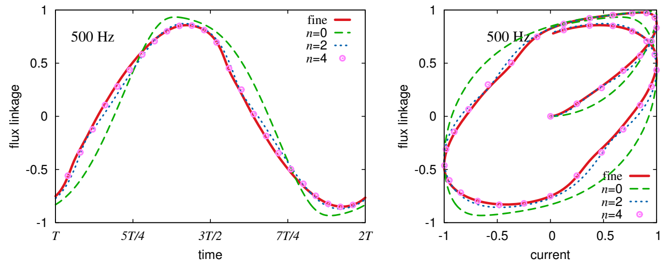

Figure 2. Normalized flux linkage versus time (left) and imposed current at 500 Hz (right) in steady state (second period).

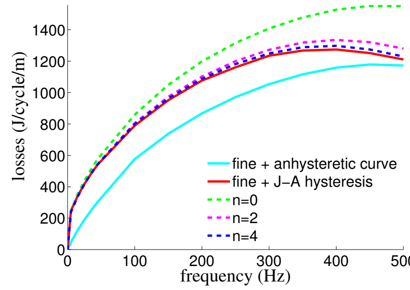

Figure 3. Iron losses: results obtained with reference fine model and averaging homogenization method for different values of n.

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