Lifshitz backgrounds from 10d supergravity

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Abstract

We investigate whether 4-dimensional static and cosmological Lifshitz solutions can be found from deforming the existing (A)dS$_4$ compactifications in IIA and IIA* supergravity. Using a well motivated compactification Ansatz on SU(3)-structure manifolds with 19 undetermined parameters we demonstrate that this is not the case in ordinary IIA supergravity, thereby generalising previous nogo results in different ways. On the other hand, for IIA* we construct explicit cosmological Lifshitz solutions. We also consider solutions with non-constant scalars and are able to find simple static and cosmological Lifshitz solutions in IIB* supergravity and a Euclidean Lifshitz solution in ordinary Euclidean IIB supergravity, which is similar to a non-extremal deformation of the D-instanton. The latter solutions have $z = -2$.

1 Introduction

Since the advent of the gravity/Lifshitz-QFT correspondence [1] there is the need for an embedding of Lifshitz spacetime (Li) in string theory. Such an embedding might allow for a microscopic understanding of the correspondence and allows one to define the correspondence for all values of the gravitational coupling since string theory is a UV complete gravity theory. There is also a more practical use if the embedding can be done in the supergravity limit. This would allow one to investigate possible supersymmetry preservation and hence stability issues.

Recently, reference [2] found some stringy constructions of Lifshitz spacetime but they seem to fail to be simple supergravity solutions, and it is not clear to us whether the solutions are really 10-dimensional solutions. A perhaps simpler approach would be to consider supergravity flux-compactifications to 4 dimensions for which the effective theories have the ingredients required for supporting Lifshitz spacetimes: massive vectors or massive tensors and a negative cosmological constant. The existence of these ingredients is not enough to guarantee a solution. This usually comes from the complicated scalar field interactions in the effective theory [3]. Insisting on fixed scalar fields, when vectors are turned on, is a non-trivial problem and resembles the issue of the attractor mechanism of black hole solutions in string theory. In this paper we will take the scalar fields constant and we comment in the end that this might be the reason behind the various nogo conditions we find.

Reference [4] was able to construct a Lifshitz solution from a flux compactification of IIB supergravity (based on earlier work [5]). This solution has a non-constant dilaton field, but a more pressing problem is the fact that the Lifshitz anisotropy is only possible in one direction. This is an unwanted feature in the correspondence as was later argued by the same authors in [6]. For that reason an investigation was performed for flux compactifications of 11- and 10-dimensional supergravity under specific Ansatze for which nogo theorems could be proven. In the present paper we continue this investigation but specify to the case of (massive) IIA compactifications on SU(3)-structure manifolds with fluxes and with the possibility of wrapped O6/D6 sources. These compactifications are known for its susy AdS$_4$ vacua with the possibility of stabilising all moduli at tree-level in the string coupling g_s and α', see [7, 8, 9] and references therein. The effective theories are expected to possess massive vectors, aside from the already present possibility of minimizing the potential at a negative value.

The SU(3)-structure manifolds that support the susy AdS$_4$ solutions have 2 of the 5 torsion classes non-zero. In this setup we make the most general Ansatz for the fluxes that can be studied in all generality without specifying to a specific model. This implies that the Ansatz consists out of the so-named 'canonical' forms, being the (would-be) Kähler form J, the complex holomorphic three-form Ω and the torsion forms W_1, W_2. A rationale for such an Ansatz is the fact the internal part of the Einstein equations seems to indicate that solutions are naturally carried by these forms (though not necessarily). This is because the geometry side of the Einstein equation is written entirely in terms of the canonical forms and the same must be true for the matter side. Indeed, the known susy AdS solutions are

captured by it, as well as the recently discovered non-susy AdS solutions in [10] and [11] (see also [12] and [13] for related work). In [10], the same Ansatz was also used to establish de Sitter solutions[1].

An investigation of this kind was carried out in [6] but with $W_2 = 0$ and a negative result was found. We will demonstrate that the Ansatz is much more general when W_2 is added and includes *19 undetermined parameters*. One would then think that it is much easier to find Lifshitz solutions, but, as we will demonstrate, a negative result is found again.

Just as the AdS/CFT correspondence can be generalised to the (more hypothetical) dS/ECFT correspondence [14], the Li/condensed matter (Li/CM) correspondence can be generalised to a *time-dependent* Li/ECM correspondence [15], where 'E' stands for Euclidean. These time-dependent Lifshitz spacetimes will be named *cosmological* Lifshitz spacetimes in this paper. It was shown in [15] that these spacetimes violate the null energy condition, and therefore the supporting matter content should be non-trivial. One possibility that was mentioned in [15] is the use of orientifolds as null-energy breaking ingredients. In this paper we allow for orientifolds in IIA supergravity and search for cosmological Lifshitz solutions without success. Nonetheless, these compactifications we consider are known to allow de Sitter critical points [16, 17, 10].

Reference [15] also suggested ghostlike matter as a possibility. Such matter can be introduced into 10d supergravity in two ways: 1) Euclidean IIB is known to have the flipped sign for the RR zero-form kinetic term. 2) The so-called II* theories are obtained from timelike T-duality of the ordinary type II theories and have the flipped sign for all RR kinetic terms [18, 19]. On of the properties of the star theories is that the natural vacua are de Sitter instead of Anti-de Sitter. Timelike T-duality is a contested concept for various reasons and one of them is exactly the ghostlike matter fields. But as explained in [15] this might be turned into a virtue since Euclidean CM theories can break unitarity. We demonstrate explicitly that the ghostlike terms in IIA* supergravity indeed allow cosmological Lifshitz solutions!

Finally we initiate in this paper the search for Lifshitz solutions carried by non-constant scalar fields that depend on the holographic coordinate. We demonstrate the existence of Euclidean Lifshitz solutions in Euclidean IIB and static and cosmological Lifshitz solutions in IIB*. The reason for this is exactly the flipped sign in the kinetic terms for the RR zero form. Note that, whereas IIB* might be contested, this is not the case for Euclidean IIB theory. It is known that the usual AdS/CFT correspondence can be described either in the Euclidean or the Lorentzian way [20].

The rest of this paper is organised as follows. In section 2 we describe some basic features of the Lifshitz geometries required for the rest of the paper. In section 3 we investigate the deformations of the $(A)dS_4$ flux compactifications on SU(3)-structure spaces of IIA and IIA* theory. In section 4 we consider Euclidean IIB and IIB* theory with running axion-dilaton scalars and present the Lifshitz solutions. Finally in section 5 we discuss the obtained results and further directions for research.

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[1]Although manifolds with the desired properties are still to be constructed.

2 Lifshitz spaces: static, cosmological and Euclidean

Consider the following three line-elements of 4-dimensional Lifshitz spaces (Li$_4$)

$$ds_s^2 = -\frac{dx_0^2}{r^{2a}} + \frac{dx_1^2}{r^{2b}} + \frac{dx_2^2}{r^{2c}} + \frac{dr^2}{r^2} \,, \tag{1}$$

$$ds_c^2 = +\frac{dx_0^2}{r^{2a}} + \frac{dx_1^2}{r^{2b}} + \frac{dx_2^2}{r^{2c}} - \frac{dr^2}{r^2} \,, \tag{2}$$

$$ds_e^2 = +\frac{dx_0^2}{r^{2a}} + \frac{dx_1^2}{r^{2b}} + \frac{dx_2^2}{r^{2c}} + \frac{dr^2}{r^2} \,, \tag{3}$$

where the subscript 's' means static, 'c' means cosmological and 'e' means Euclidean. In the static case x_0 is the time direction and in the cosmological case r is the time direction. For each example the holographic coordinate is denoted by r (the RG scale) and the dual field theory lives on the slice of constant r. When we rescale $r \to \lambda r$ then the metric is invariant if

$$x_0 \to \lambda^a x_0 \,, \qquad x_1 \to \lambda^b x_1 \,, \qquad x_2 \to \lambda^c x_2 \,. \tag{4}$$

In many cases it is desired to treat two directions on the holographic slice as "equal" by taking $b = c$. We can then find a coordinate transformation such that $b = c = -1$. If we furthermore denote $a = -z$, the line elements become

$$ds_s^2 = -r^{2z}dx_0^2 + r^2(dx_1^2 + dx_2^2) + \frac{dr^2}{r^2} \,, \tag{5}$$

$$ds_c^2 = +r^{2z}dx_0^2 + r^2(dx_1^2 + dx_2^2) - \frac{dr^2}{r^2} \,, \tag{6}$$

$$ds_e^2 = +r^{2z}dx_0^2 + r^2(dx_1^2 + dx_2^2) + \frac{dr^2}{r^2} \,. \tag{7}$$

z is the "anisotropy" parameter. When $z = 1$ we have (A)dS$_4$ space. For this reason we call $z \neq 1$ anisotropic, since time and space scale differently. For original work on Lifshitz spacetimes we refer to [21].

The Vielbein one-forms are denoted $\theta^0 = r^z dx_0, \theta^r = r^{-1}dr, \theta^{1,2} = rdx^{1,2}$ in accordance with the existing literature. They obey the following simple Cartan–Maurer equations

$$d\theta^r = 0 \,, \qquad d\theta^0 = z\theta^r \wedge \theta^0 \,, \qquad d\theta^{1,2} = \theta^r \wedge \theta^{1,2} \,. \tag{8}$$

For later use we present the Ricci curvatures in the general case

$$\begin{aligned}
R_{00}^s &= -R_{00}^e = +R_{00}^c = +a(a+b+c)r^{-2a} \,, \\
R_{11}^s &= +R_{11}^e = -R_{11}^c = -b(a+b+c)r^{-2b} \,, \\
R_{22}^s &= +R_{22}^e = -R_{22}^c = -c(a+b+c)r^{-2c} \,, \\
R_{rr}^s &= +R_{rr}^e = +R_{rr}^c = -(a^2+b^2+c^2)r^{-2} \,.
\end{aligned} \tag{9}$$

The generalisation of all the above to any dimension is straightforward.

4

3 A nogo for some massive IIA compactifications

3.1 Type IIA/IIA* supergravity

In IIA and IIA* theory the RR field strengths are F_0, F_2, F_4. The difference in the IIA* theory is that the RR field strengths are transformed according to $F_p \to iF_p$. We implement this by adding a sign, ϵ_*, such that $\epsilon_* = 1$ corresponds to type IIA and $\epsilon_* = -1$ is type IIA*. The form and dilaton equations of motion in Einstein frame are

$$
\begin{aligned}
&\mathrm{d}(\star \mathrm{e}^{3\phi/2} F_2) + \mathrm{e}^{\phi/2} \star F_4 \wedge H = 0\,,\\
&\mathrm{d}(\star \mathrm{e}^{\phi/2} F_4) - F_4 \wedge H = 0\,,\\
&\epsilon_* \mathrm{d}(\star \mathrm{e}^{-\phi} H) + \mathrm{e}^{\phi/2} \star F_4 \wedge F_2 - \tfrac{1}{2} F_4 \wedge F_4 + F_0 \mathrm{e}^{3\phi/2} \star F_2 = 0\,,\\
&\epsilon_* \mathrm{d} \star \mathrm{d}\phi - \tfrac{1}{4} \mathrm{e}^{\phi/2} \star F_4 \wedge F_4 + \epsilon_* \tfrac{1}{2} \mathrm{e}^{-\phi} \star H \wedge H - \tfrac{3}{4} \mathrm{e}^{3\phi/2} \star F_2 \wedge F_2 - \tfrac{5}{4} \mathrm{e}^{5\phi/2} \star F_0 \wedge F_0 = 0\,,
\end{aligned}
\tag{10}
$$

where F_0 is the Romans' mass m. The Bianchi identities read

$$
\mathrm{d}H_3 = 0\,, \qquad \mathrm{d}F_2 = F_0 H\,, \qquad \mathrm{d}F_4 = F_2 \wedge H_3\,.
\tag{11}
$$

The Einstein equation is given by

$$
\begin{aligned}
0 = {}& \epsilon_* \mathcal{R}_{ab} - \tfrac{1}{2}\epsilon_* \partial_a \phi \partial_b \phi - \tfrac{1}{12} \mathrm{e}^{\phi/2} F_{acde} F_b{}^{cde} + \tfrac{1}{128} \mathrm{e}^{\phi/2} g_{ab} F_4^2 - \epsilon_* \tfrac{1}{4} \mathrm{e}^{-\phi} H_{acd} H_b{}^{cd}\\
&+ \epsilon_* \tfrac{1}{48} \mathrm{e}^{-\phi} g_{ab} H^2 - \tfrac{1}{2} \mathrm{e}^{3\phi/2} F_{ac} F_b{}^c + \tfrac{1}{32} \mathrm{e}^{3\phi/2} g_{ab} F_2^2 - \tfrac{1}{16} g_{ab} \mathrm{e}^{5\phi/2} F_0^2\,.
\end{aligned}
\tag{12}
$$

3.2 Spaces with $SU(3)$-structure

A SU(3)-structure space is characterised by a real two form J and a complex three form $\Omega = \Omega_R + i\Omega_I$. The exterior derivatives are given by

$$
\begin{aligned}
\mathrm{d}J &= -\frac{3i}{2} W_1 \Omega_R + W_3 + W_4 \wedge J\,,\\
\mathrm{d}\Omega &= W_1 J \wedge J + W_2 \wedge J + W_5 \wedge \Omega\,,
\end{aligned}
\tag{13}
$$

where the W_i are complex forms whose rank can be deduced from the above equations. In the following we restrict to spaces for which $W_3 = W_4 = W_5 = 0$ and where W_1 is an imaginary zero-form and W_2 is an imaginary two-form for reasons explained in the introduction. It is expected that, as the moduli flow away from the susy AdS solution, other torsion classes can be turned on. Nonetheless we will consider the case were only these two torsion classes are non-zero.

These forms obey the following form identities

$$
\begin{aligned}
&\star_6 \Omega = -i\Omega\,, \qquad \star_6 J = \tfrac{1}{2} J \wedge J\,, \qquad \star_6 W_2 = -J \wedge W_2\,,\\
&\Omega \wedge \Omega^* = \tfrac{4i}{3} J \wedge J \wedge J\,, \qquad J \wedge J \wedge J = 6\epsilon_6\,,\\
&\Omega \wedge J = 0\,, \qquad W_2 \wedge J \wedge J = 0\,, \qquad W_2 \wedge \Omega = 0\,,
\end{aligned}
\tag{14}
$$

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and the following contractions[2]

$$J_{mn}W_2^{mn} = 0, \qquad J_m{}^n J_p{}^q (W_2)_{nq} = (W_2)_{mp},$$
$$(\Omega_R)_{ab}^2 = (\Omega_I)_{ab}^2 = 4g_{ab}, \qquad J_{ab}^2 = g_{ab}. \tag{15}$$

We furthermore assume that $\mathrm{d}W_2$ is proportional to Ω_R. The constant of proportionality is fixed by internal consistency

$$\mathrm{d}W_2 = -(i|W_2|^2/8)\,\Omega_R. \tag{16}$$

This condition is rather common for many explicit geometries [9] and is required for some $\mathrm{Ad}S_4$ [8] and dS_4 solutions [10].

For the sake of solving the equations of motion, we single out a special class of geometries which we call 'degenerate' since some tensors become linearly dependent on each other

$$W_2 \wedge W_2 = \frac{1}{12}|W_2|^2 J \wedge J - 2i\chi J \wedge W_2 \quad \Longrightarrow \quad (W_2^2)_{ij} = \frac{W_2^2}{6}g_{ij} + i\chi(JW_2)_{ij}, \tag{17}$$

with χ some real number different from zero. The degenerate condition is a necessary condition for having non-susy (A)dS solutions in this setup as described in [10, 11].

One can also express the curvature tensors in terms of the torsion classes [22, 23]

$$\mathcal{R}_{mn} = -\frac{3i}{4}(\Omega_R)_n{}^{ps}\partial_{[p}(W_2)_{sm]} - \frac{1}{4}W_1(W_2)_{mr}J_n{}^r - \frac{1}{2}(W_2)_{mq}(W_2)_n{}^q + \frac{5}{4}g_{mn}|W_1|^2. \tag{18}$$

3.3   The Ansatz

We consider Lifshitz spacetimes with just one anisotropy parameter (z) and we add the symbol ϵ to distinguish between the static ($\epsilon = +1$) and cosmological ($\epsilon = -1$) case

$$\mathrm{d}s^2 = -\epsilon r^{2z}\mathrm{d}x_0^2 + r^2(\mathrm{d}x_1^2 + \mathrm{d}x_2^2) + \epsilon\frac{\mathrm{d}r^2}{r^2}. \tag{19}$$

The Euclidean case will be considered later. This ϵ is not be confused with the ϵ_* that distinguishes between normal and star supergravity.

At this point we can make the most general Ansatz consistent with the 4-dimensional Lifshitz symmetries and which features fluxes along the canonical forms J, W_2, Ω and wedges thereof

$$F_2 = aJ + \alpha\theta^0 \wedge \theta^r + \eta\theta^1 \wedge \theta^2 + icW_2,$$
$$H_3 = \beta\theta^1 \wedge \theta^2 \wedge \theta^r + k\Omega_R,$$
$$F_4 = f\theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^r + g\theta^1 \wedge \theta^2 \wedge J + h\theta^0 \wedge \theta^r \wedge J + q\theta^0 \wedge \Omega_R$$
$$+ \frac{s}{2}J \wedge J + ie\theta^1 \wedge \theta^2 \wedge W_2 + il\theta^0 \wedge \theta^r \wedge W_2 + ipW_2 \wedge J. \tag{20}$$

---

[2]The notation we use for "squaring" a tensor $T_{i_1...i_n}$ is $T_{ij}^2 = T_{ii_2i_3...i_n}T_j{}^{i_2i_3...i_n}$.

We have eliminated terms from the most general Ansatz with canonical forms which obviously have to be zero from Bianchi identities and form equations of motion, such as $\theta^0 \wedge \Omega_I$.

When we take the Lorentz symmetry-breaking terms equal to zero

$$\alpha = \eta = \beta = g = h = q = e = l = 0\,, \tag{21}$$

we end up with the Ansatz used for (non)-susy (A)dS solutions. Hence, this is the natural Ansatz that is expected to "deform" the (A)dS solutions into Lifshitz solutions. As an example, the susy AdS solutions are given by

$$a = \tfrac{1}{4}iW_1\,, \qquad k = -\tfrac{2}{5}m\,, \qquad f = \tfrac{9}{4}iW_1\,, \qquad s = \tfrac{3}{5}m\,, \qquad c = 1\,, \tag{22}$$

with the extra conditions that

$$
\begin{aligned}
|W_2|^2 &= 3|W_1|^2 - \tfrac{16}{5}m^2\,, \\
6 &= \tfrac{27}{8}|W_1|^2 + \tfrac{6}{25}m^2\,.
\end{aligned}
\tag{23}
$$

Where the second line sets the value of the cosmological constant, which we have fixed to be $R_4 = -12$.

Furthermore, we take the smeared O6/D6 sources in the usual way for these compactifications which means [9, 10]

$$
\begin{aligned}
\mathrm{d}F_2 &= mH + \mu\Omega_R\,, \\
\mathrm{d} \star \mathrm{d}\Phi &= \ldots - 3\mu\epsilon_{10}\,, \\
R_{\mu\nu} &= \ldots + \frac{1}{4}\mu g_{\mu\nu}\,, \\
R_{ij} &= \ldots - \frac{3}{4}\mu g_{ij}\,,
\end{aligned}
\tag{24}
$$

where $\mu > 0$ implies net O6 charge and $\mu < 0$ implies net D6 charge. Especially in the cosmological case ($\epsilon = -1$) we are required to add orientifolds because we need to violate the null-energy condition, as explained in [15]. In the IIA* case ($\epsilon_* = -1$) there do not exist space-filling sources since the sources of type IIA* have Euclidean worldvolumes. Hence we take $\mu = 0$ when $\epsilon_* = -1$.

We have all the necessary information to investigate the equations of motion and we expect to end up with a system of many algebraic relations in the 19 *real* variables

$$z, iW_1, |W_2|^2, a, \alpha, \eta, c, \beta, k, f, g, h, q, s, e, l, p, m, \mu\,.$$

When we plug the Ansatz into the equations of motion we need to make a distinction between the two possible families of geometries. Let us first assume the non-degenerate case and later assume the degenerate case (17).

The non-degenerate case

The Bianchi identities give the following relations[3]

$$-mk - \tfrac{3}{2}iaW_1 + \tfrac{1}{8}c|W_2|^2 - \mu = 0 \,, \tag{25}$$

$$-\eta k + \tfrac{1}{8}e|W_2|^2 - \tfrac{3}{2}iW_1 g = 0 \,, \tag{26}$$

$$-\alpha k + \tfrac{1}{8}l|W_2|^2 - \tfrac{3}{2}iW_1 h - qz = 0 \,, \tag{27}$$

$$m\beta - 2\eta = 0 \,, \tag{28}$$

$$2g - a\beta = 0 \,, \tag{29}$$

$$2e - c\beta = 0 \,, \tag{30}$$

where equations (25,28) come from the F_2-field Bianchi identity and the others from the F_4-field Bianchi identity. The form equations of motion give

$$f\beta + 4\epsilon qk + 2\alpha = 0 \,, \tag{31}$$

$$2l - \epsilon q - p\beta = 0 \,, \tag{32}$$

$$\tfrac{1}{2}\beta s + h + i\epsilon qW_1 = 0 \,, \tag{33}$$

$$-\tfrac{3}{2}isW_1 - \tfrac{1}{8}p|W_2|^2 + \beta q - kf = 0 \,, \tag{34}$$

$$el + cp = 0 \,, \tag{35}$$

$$\epsilon\epsilon_*\beta z - \tfrac{1}{2}pl|W_2|^2 + 3hs - 3ga + \alpha f - \tfrac{1}{2}ec|W_2|^2 - m\eta = 0 \,, \tag{36}$$

$$f\eta + 3ha + \frac{1}{2}lc|W_2|^2 + 3gs - \frac{1}{2}pe|W_2|^2 + m\alpha = 0 \,, \tag{37}$$

$$\epsilon_* ikW_1 + \tfrac{1}{2}fs + gh - as - \tfrac{1}{2}g\eta - \tfrac{1}{2}ma + \tfrac{1}{2}h\alpha = 0 \,, \tag{38}$$

$$\epsilon_* k + gl + eh + fp + ap - cs - \alpha l + \eta e + mc = 0 \,, \tag{39}$$

where equation (31) comes from the F_2 eom, equations (32-34) come from the F_4 eom and all others come from the H eom. The dilaton equation of motion gives

$$0 = \tfrac{1}{4}f^2 - \tfrac{3}{4}s^2 - \tfrac{1}{8}p^2|W_2|^2 - \tfrac{3}{4}g^2 + \tfrac{3}{4}h^2 - \tfrac{1}{8}(e^2 - l^2)|W_2|^2 + \epsilon q^2 + \epsilon\epsilon_*\tfrac{1}{2}\beta^2 \,,$$
$$+ \epsilon_* 2k^2 - \tfrac{9}{4}a^2 - \tfrac{3}{8}c^2|W_2|^2 + \tfrac{3}{4}\alpha^2 - \tfrac{3}{4}\eta^2 - \tfrac{5}{4}m^2 + 3\mu \,. \tag{40}$$

To write the Einstein equations in a more compact form we introduce the number A

$$A = \tfrac{3}{16}f^2 - \tfrac{9}{16}s^2 - \tfrac{3}{32}p^2|W_2|^2 - \tfrac{9}{16}g^2 + \tfrac{9}{16}h^2 + \tfrac{3}{32}(l^2 - e^2)|W_2|^2 + \epsilon\tfrac{3}{4}q^2 \,,$$
$$-\epsilon\epsilon_*\tfrac{1}{8}\beta^2 - \epsilon_*\tfrac{1}{2}k^2 - \tfrac{3}{16}a^2 - \tfrac{1}{32}c^2|W_2|^2 + \tfrac{1}{16}\alpha^2 - \tfrac{1}{16}\eta^2 + \tfrac{1}{16}m^2 \,. \tag{41}$$

The external Einstein equations then read

$$(tt) \,: \quad \epsilon\epsilon_* z(z+2) = 2\epsilon q^2 + \tfrac{1}{2}\alpha^2 + \tfrac{1}{2}f^2 + \tfrac{3}{2}h^2 + \tfrac{l^2}{4}|W_2|^2 - A - \tfrac{1}{4}\mu \,, \tag{42}$$

$$(rr) + (tt) \,: \quad \epsilon_* 2(z-1) = 2q^2 + \epsilon_*\tfrac{1}{2}\beta^2 \,, \tag{43}$$

$$(xx) - \epsilon(rr) \,: \quad \epsilon_*\epsilon z(z-1) = \tfrac{1}{4}(e^2 + l^2)|W_2|^2 + \tfrac{3}{2}(g^2 + h^2) + \tfrac{1}{2}(\alpha^2 + \eta^2) \,. \tag{44}$$

---

[3]Here we have assumed that (16) holds. If one where to relax this condition one finds only special cases of the presented relations.

Taking the trace of the 10-dimensional Einstein equation and using the dilaton equation, we find

$$\epsilon 2(z^2 + 2z + 3) = \tfrac{15}{2}|W_1|^2 - \tfrac{1}{4}|W_2|^2 - \tfrac{1}{2}\epsilon\beta^2 - 2k^2 + 2\mu\,. \tag{45}$$

The traceless part of the internal Einstein then gives two more conditions

$$\epsilon_* iW_1 = 4sp - 4lh + 4eg + 4ac\,, \tag{46}$$

$$\epsilon_* = -p^2 - l^2 + e^2 + c^2\,. \tag{47}$$

It can be shown that the Einstein equations with mixed indices are automatically solved. The only mixed components arise in $(F_4^2)_{\mu i}$, and they are proportional to $\Omega_{ijk}^R J^{jk}$ and $\Omega_{ijk}^R W_2^{jk}$. These terms vanish since Ω is of type $(3,0)$ and J and W_2 are both of type $(1,1)$.

The degenerate case

In the degenerate case (17) we find that conditions (35), (46) and (47) do not exist anymore and that equations (38) and (39) are altered in the following way

$$\epsilon_* ikW_1 + \tfrac{1}{2}fs + gh - as - \tfrac{1}{2}g\eta - \tfrac{1}{2}ma + \tfrac{1}{2}h\alpha - \boxed{\tfrac{1}{12}(el + cp)|W_2|^2} = 0\,, \tag{48}$$

$$\epsilon_* k + gl + eh + fp + ap - cs - \alpha l + \eta e + mc + \boxed{2\chi(el + cp)} = 0\,. \tag{49}$$

Notice that we gained one variable χ this way. However this value gets determined by the internal Einstein equation (the traceless part) as follows

$$[\epsilon_* + p^2 + l^2 - e^2 - c^2]2\chi = iW_1\epsilon_* - 4sp + 4lh - 4eg - 4ac\,. \tag{50}$$

This equation fixes χ and replaces the two conditions (46) and (47). This implies that in total we have three equations less in the degenerate case (17). If the degenerate case does not allow solutions then the non-degenerate case doesn't allow solutions as well. For that reason we will assume the degenerate case.

3.4 Solving the equations

No sources and nogo for cosmological Li_4 in IIA

First we demonstrate that the sources have to vanish. If we compare equation (25) with equation (26) and (37), after substituting η, g, e, α, l and h using (28)-(30) and (31)-(33), we can deduce the following

$$\beta\mu = 0\,, \qquad q\mu = 0\,. \tag{51}$$

Hence we can either take $\beta = 0$ and $q = 0$, which from (43) gives $z = 1$, that is (A)dS solutions, or $\mu = 0$.

This means that the charge has to be zero in order to find Lifshitz solutions. This excludes the cosmological Lifshitz solutions in ordinary massive IIA, since there is no ingredient to break the null energy condition. Hence we discard the possibility of finding solutions for $(\epsilon, \epsilon_*) = (-1, 1)$.

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Nogo for static Li_4 in IIA*

Consider the following relation

$$6a^2 + 2f^2 + 6g^2 + 6h^2 + 2m^2 + 8q^2 + 6s^2 + c^2|W_2|^2 + e^2|W_2|^2 + l^2|W_2|^2$$
$$+ p^2|W_2|^2 + 16z + 8z^2 + 2\alpha^2 + 2\eta^2 = 0. \tag{52}$$

which is a combination of (40) and (42), with signs $(\epsilon, \epsilon_*) = (1, -1)$. This implies that $z \in (-2, 0)$, to have real solutions. Equation (44) implies $z \in (0, 1)$ and hence provides a contradiction.

Nogo for static Li_4 in IIA

In the case of $(\epsilon, \epsilon_*) = (1, 1)$ we can find the following relations

$$q\left(8\beta^2 + 32k^2 + 24|W_1|^2 + |W_2|^2\right) = 16qz, \tag{53}$$
$$8k^2 + |W_2|^2 + 24 + 16z + 8z^2 + 2\beta^2 = 30|W_1|^2, \tag{54}$$

which for $q \neq 0$ cannot be solved simultaneously. Furthermore we can prove after some algebra that $q = 0$ only gives imaginary solutions in any set of signs, $(1, 1)$ or $(-1, -1)$, with $z = 0, -4$. Hence we can deduce that the only possibility for a solution is for $q \neq 0$ and $(\epsilon, \epsilon_*) = (-1, -1)$.

Solutions for cosmological Li_4 in IIA*

Assuming $\beta = 0$, which emerges as a natural assumption when reducing the equations, one finds the following relation

$$12k^2 + 27|W_1|^2 + 4\left(3 + 4z + z^2\right) = 0. \tag{55}$$

This implies that $z \in [-3, -1]$ where $z = -3$ or $z = -1$ have $W_1 = k = 0$. The same equation therefore also excludes the possibility for dS solutions in massive IIA*.

Using this we are able to find infinite sets of solutions. One particular simple solutions is in the set-up where the following parameters are zero

$$W_1, \mu, \beta, k, c, p, \eta, g, e, \alpha, h \tag{56}$$

Notice that only two Lorentz-breaking parameters are present. Non-zero parameters are

$$|W_2|^2 = 48, \ z = -3, \ l = -1, \ q = 2, \tag{57}$$

and the rest are determined by the following equations

$$0 = fs - a(m + 2s), \tag{58}$$
$$0 = 9a^2 + 5m^2 + 3s^2 - 8f^2, \tag{59}$$
$$0 = 2f^2 + 3s^2 - 2 - m^2, \tag{60}$$

which have an infinite set of solutions. One instance is

$$f = -0.998921, \ s = 0.68206, \ a = -0.267852, \ m = 1.17954. \tag{61}$$

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4 Solutions with running scalars in IIB* and Euclidean IIB

Let us drop the assumption that the scalar fields have to be constant. For simplicity we consider the case where the vectors (and tensors) are also turned off, as was done in [15]. Instead of starting with a compactification Ansatz we first investigate whether the following Lagrangian in 4 dimensions

$$S = \int \mathrm{d}x^4 \sqrt{|g|}\left(R - \tfrac{1}{2}G_{ij}\partial\phi^i\partial\phi^j - \Lambda\right),$$
(62)

can support Lifshitz geometries. The equations of motion read

$$R_{\mu\nu} = \tfrac{1}{2}G_{ij}\partial_\mu\phi^i\partial_\nu\phi^j + \tfrac{1}{2}\Lambda g_{\mu\nu},$$
(63)

$$\frac{1}{\sqrt{|g|}}\partial_\mu(\sqrt{|g|}g^{\mu\nu}\partial_\nu\phi^i) + \Gamma^i_{jk}\partial^\mu\phi^j\partial_\mu\phi^k = 0.$$
(64)

We make the assumption that the scalars only depend on the holographic coordinate, $\phi^i = \phi^i(r)$. If we insist on having the anisotropy parameters a, b and c not coinciding (since that would correspond to (A)dS solutions), we infer from the Einstein equations in the x^i directions that

$$a + b + c = 0 = \Lambda, \qquad (z = -2).$$
(65)

In terms of the coordinate $\rho = \ln r$, the scalar field equation of motion becomes the equation of motion for a geodesic on the target space with metric G_{ij} and with ρ as affine parameter

$$\ddot{\phi}^k + \Gamma^k_{ij}\dot{\phi}^i\dot{\phi}^j = 0,$$
(66)

where a dot represents a derivative with respect to ρ. Denoting the constant affine velocity as v^2

$$G_{ij}\dot{\phi}^i\dot{\phi}^j = v^2,$$
(67)

we can rewrite the (rr) component of the reversed Einstein equation in the following way

$$-(a^2 + b^2 + c^2) = \tfrac{1}{2}v^2.$$
(68)

This shows that we need target spaces of indefinite signature such that the geodesic velocity can be negative. In the case of Euclidean field theories this can occur naturally. For instance, when IIB supergravity is Wick-rotated it is known [24] that the RR zero-form C_0 flips sign in the kinetic term, such that the axion-dilaton part of the action in Einstein frame reads

$$S = \int \sqrt{g}\left(-\tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}\mathrm{e}^{b\phi}(\partial C_O)^2\right),$$
(69)

where the number $b^2 = 4$ and represents the curvature of the $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(1,1)$ - sigma model. Upon dimensional reduction this number b grows and ϕ becomes a linear combination of the the string coupling and the radii of the internal dimensions. In any case, this

demonstrates that the Euclidean Lifshitz solutions exist in Euclideanised IIB supergravity or its dimensionally reduced children. This solution closely resembles the so-named *non-extremal D(-1) solution or instanton* [25, 26, 27]. The extremal D(-1) solution corresponds to the case with lightlike geodesics and the corresponding flat space metric [24].

Another example of indefinite kinetic terms is in II* theories as we already discussed. In the case of IIB* theory, we have the same sigma model (69) without having to Euclideanise the theory. This implies that we have the stationary and the cosmological Lifshitz solutions in (dimensionally reduced) IIB* theory.

Finally, we like to emphasize that these solutions have vanishing background cosmological constant. In case we add higher-derivatives on the axion-dilaton the background cc does not vanish [15] from the Einstein equations. In Euclidean IIB theory this background cc in $D = 5$ can be generated in the usual way from the Freund-Rubin compactification (aka the D3 brane near-horizon), also in the Euclidean case. Since the axion and dilaton do not couple to the cc in the latter case we are in the situation described by Nakayama [15], however in the Euclidean version of the theory. In the IIB* case we can generate the *positive* background cc from the near-horizon of the so-called E4 brane, or equivalently a Freund-Rubin compactification on a (compactified) 5-dimensional hyperboloid with F5 RR flux (where the F5 form has the opposite sign of the kinetic term). This implies that α' corrections to our solutions naturally allow a background cc.

5 Discussion

Let us summarize the results of this paper:

- We have shown that for a well-motivated and extended Ansatz one cannot find static Lifshitz solutions in IIA on SU(3)-structure manifolds (and orientifolds).

- However, the same Ansatz does allow explicit cosmological Lifshitz solutions in IIA* supergravity. For simplicity we presented a solution with $z = -3$, but there are other possible solutions with z-values between $[-3, -1)$.

- When we allow running scalar fields we can construct static and cosmological solutions with $z = -2$ in IIB* supergravity and a Euclidean Lifshitz solution in Euclidean IIB supergravity, also with $z = -2$, that can be interpreted as a non-extremal deformation of the D-instanton.

The nogo conditions we obtained for the static Lifshitz backgrounds needs some interpretation in order to be useful. We can think of two scenarios. Either we cannot find a solution of the Ansatz with constant scalars simply because the Ansatz is restricted in many ways. It assumes SU(3)-structures, and more importantly, it assumes fluxes along specific directions. Especially the Lorentz-violating terms have to be chosen with care since they correspond to the 4-dimensional massive tensors and vectors, see e.g. [12]. It could be that we have turned on those massive vectors and tensors that do not lead to a solution[4].

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[4]We thank Yu Nakayama for some explanations on that point.

An alternative explanation is that we do not find solutions to the Ansatz because of the assumption of constant scalar fields. We can schematically write the 4D effective theory in the following way:

$$S = \int \sqrt{g} \left\{ R - \tfrac{1}{2}(\partial\phi)^2 - f_1(\phi)F^2 - f_2(\phi)m^2 A^2 - V(\phi) \right\}, \tag{70}$$

where we pretended, for simplicity, that only one vector and one scalar is turned on. f_1 and f_2 are some functions of the scalar and V is the scalar potential. Since we consider AdS compactifications the scalar potential has a stationary point at a negative value

$$V(\phi_*) < 0 \quad \& \quad \partial V_{|\phi_*} = 0. \tag{71}$$

If the vector is non-zero, the scalar field equation of motion effectively feels a new scalar potential $\tilde{V}$, where

$$\tilde{V}(\phi) = V(\phi) + \alpha f_1(\phi) + \beta f_2(\phi) \tag{72}$$

with α, β some real numbers. There is no guarantee that $\tilde{V}$ has also a stationary point, which would imply that the scalar field has to run. The easiest way to check this possibility is by investigating the effective field theories in four dimensions directly.

For the cosmological Lifshitz solutions in IIA* theory we found that V has no stationary point, since there are no dS solutions in our model, but $\tilde{V}$ has since we did find the Lifshitz solutions. This implies that one does not necessarily have to consider AdS/dS compactifications in order to find static/cosmological Lifshitz solutions. In such cases there will not be a partner AdS/dS solution to the static/cosmological Lifshitz solution.

For both aforementioned reasons we anticipate on investigating the existence of static Lifshitz solutions in 4d effective field theories. One could even relax the requirement of knowing the 10d origin of the effective field theories and consider that question as a second step. A sensible set of theories to investigate would be gauged $\mathcal{N} = 2$ supergravities coupled to massive tensor multiplets. Such theories have the necessary ingredients for Lifshitz solutions and are still constrained enough to make the analysis tractable. These theories also are expected to originate from generalised Calabi-Yau flux compactifications of 10d supergravity.

Concerning the Euclidean Lifshitz solution with $z = -2$ we have found in IIB (as well as the static and cosmological IIB* solutions) we have not yet touched upon issues regarding the regularity of the solution. Generically the dilaton profile has a singularity in its derivative as well as the axion field [26, 27]. However, this cancels in the Einstein equation such that this singular point does not backreact on the geometry. Nonetheless, it might be an issue of concern when taking the solutions serious. We will not go into this discussion here but mention a possible way out of the problem. Upon going to Euclidean signature one might allow axionic fields different from C_0 to Wick-rotate as well. The effect of the multiple axion-dilaton pairs then removes the singularity [28, 29]. Another option is to look at more involved compactifications such that one ends up with $\mathcal{N} = 2$ Euclidean theories, where the singularity is also absent [30].

Acknowledgements

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