Identification of axial forces in beam members by local vibration measurements^{☆,☆☆}

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Abstract

A method is presented to estimate the axial force in a bar which is part of a built-up structure. The estimation is based on the modal characteristics of the bar or, alternatively, on its operational deflection shape, as determined from a local vibration test. Timoshenko beam theory is assumed and, in addition, the rotational inertia of the bar and the mass of the sensors is accounted for. A major advantage of the proposed method is its generality. When data from five or more sensors along the length of the bar is available, no information on the connection of the bar to the remainder of the structure is required. The method can therefore be used for any beam or truss element. The method is first verified numerically using finite element simulations. Finally, a set of laboratory experiments is performed in order to ascertain the applicability and accuracy of the proposed method.

Keywords: axial force identification, Timoshenko beam, boundary condition, bending stiffness

1. Introduction

Slender beam members subjected to axial forces are widely used in civil structures, e.g. in truss girders or space truss structures. An experimental identification of the actual axial forces in these elements is of great importance to verify design calculations or to estimate the spare capacity of the elements composing the structure.

Forward methods can be used to estimate the axial forces in a bar, e.g. by finite element calculations. Very often, however, an accurate forward estimation of the axial forces can be cumbersome due to uncertainties regarding external loads, support conditions and beam end connections. For this reason, inverse methods have been proposed, based on the second order effect of the axial force on the transversal stiffness of a bar. Static methods were originally proposed for the determination of axial forces in tie-beams [1]. Extensions have been made to beam elements that are part of frame or truss structures [2]. Static methods make use of the measured deformation of the beam due to one or more known concentrated static loads. In practical situations, however, static

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loads of considerable size cannot be easily applied and the measurement of displacements is often difficult due to the absence of a stable and fixed reference position. Dynamic methods make use of vibration responses, measured at several points distributed along the entire structure [3], or a local vibration measurement along the bar of interest [4, 5]. Most of the existing dynamic methods are based on the identified modal characteristics of a bar, i.e. the natural frequencies and/or mode shapes. These modal characteristics depend on the stiffness of the structure and, consequently, are affected by the axial force in the bar. Several methods have been proposed to estimate the axial force in a bar from the natural frequency of a single mode [6, 7] or multiple modes [8, 9, 10]. These methods all rely on the assumption of fixed or hinged end support conditions of the beam member. In practice, however, boundary conditions are often complex and strongly influence the modal characteristics of the structure. As an alternative to analytical formulations, finite element model updating and system identification techniques have been proposed [11, 12, 13], but for these methods the accuracy of the identified axial force is to a large extent determined by the uncertainty on the model, which in most cases covers the entire structure. Tullini et al. [5] proposed a method, based on Euler-Bernoulli beam theory, that includes mode shape information to estimate the member axial force. The method makes use of a single natural frequency and of three displacement components of the corresponding mode shape. The axial force as well as the flexural stiffness of the end constraints can be identified under the assumption of infinite translational stiffness at the beam ends. This assumption is quite restrictive and, in practice, no accurate information on the the boundary conditions is available. Recently, a method was proposed by Li et al. [14], estimating the axial force in a beam member from a structural mode shape, determined with five or more sensors along the bar of interest, and its corresponding natural frequency. The method makes no assumptions on the boundary conditions of the bar and therefore can be generally applied. As mentioned in [14], the aforementioned methods cannot be used when the shear deformation and/or the rotational inertia of the bar becomes important. In addition, these methods do not account for the inertia of the measurement equipment, which often cannot be neglected [15].

The method proposed in this work focuses on slender beam members that are part of a larger structure. The method is derived analytically, based on Timoshenko beam theory. As the rotational inertia of the beam and the mass of the sensors affect the accuracy of the force estimation method, both effects are taken into account. The method is first verified using finite element simulations and validated subsequently by a set of laboratory experiments. Firstly, a bar with rectangular cross section is axially loaded in different steps. Secondly, the same bar is mounted in a three dimensional space truss.

2. Theoretical derivation

The transversal force and moment equilibrium for a free beam section are given by equation (1) and equation (2), respectively.

$$\frac{\partial V(x, t)}{\partial x} + N \frac{\partial^2 v(x, t)}{\partial x^2} = \rho A \frac{\partial^2 v(x, t)}{\partial t^2} \quad (1)$$

$$V(x, t) + \frac{\partial M(x, t)}{\partial x} = \rho I \frac{\partial^2 \beta_z(x, t)}{\partial t^2} \quad (2)$$

In these equations, $v(x, t)$ is the transverse displacement of the centerline, and $\beta_z(x, t)$ is the rotation of the beam cross-section. The shear force $V(x, t)$ and the bending moment $M(x, t)$ are

both defined along the deformed beam coordinate system. The term containing the axial force N corresponds to second order effects. The axial force N is positive in tension. The position along the centerline x and the time t occur as independent variables. The cross section A , geometric moment of inertia I , and material density ρ are assumed to be known.

Adopting Timoshenko beam theory [16], the shear force $V(x, t)$ and the bending moment $M(x, t)$, can be expressed as a function of $v(x, t)$ and $\beta_z(x, t)$:

$$V(x, t) = k_y GA \left(\frac{\partial v(x, t)}{\partial x} - \beta_z(x, t) \right) \quad (3)$$

$$M(x, t) = EI \frac{\partial \beta_z(x, t)}{\partial x} \quad (4)$$

In addition to the aforementioned geometric and material properties of the bar, the shear deformation coefficient $k_y (=A_s/A)$ and the moduli E and G are assumed to be known as well.

By eliminating $\beta_z(x, t)$ from equations (1) - (4), a partial differential equation is obtained, containing the transverse displacement $v(x, t)$ as a single dependent variable:

$$\begin{aligned} EI \frac{\partial^4 v}{\partial x^4} + \frac{EIN}{k_y GA} \frac{\partial^4 v}{\partial x^4} - N \frac{\partial^2 v}{\partial x^2} - \frac{EI\rho}{k_y G} \frac{\partial^4 v}{\partial t^2 \partial x^2} - \rho I \frac{\partial^4 v}{\partial t^2 \partial x^2} \\ - \frac{N\rho I}{k_y GA} \frac{\partial^4 v}{\partial t^2 \partial x^2} + \rho A \frac{\partial^2 v}{\partial t^2} + \frac{\rho^2 I}{k_y G} \frac{\partial^4 v}{\partial t^4} = 0 \end{aligned} \quad (5)$$

Equation (5) is the equation of motion under free vibration and can be transformed into an ordinary differential equation by performing a separation of variables, assuming the transverse displacement $v(x, t)$ to be harmonic at a frequency ω :

$$v(x, t) = \hat{v}(x) \sin(\omega t) \quad (6)$$

The following ordinary differential equation is obtained by introducing equation (6) in (5):

$$a \frac{d^4 \hat{v}(x)}{dx^4} + b \frac{d^2 \hat{v}(x)}{dx^2} + c \hat{v}(x) = 0 \quad (7)$$

where the parameters a , b , and c are defined as:

$$\begin{aligned} a &= EI \left(1 + \frac{N}{k_y GA} \right) \\ b &= -N + \frac{EI\rho\omega^2}{k_y G} + \rho I \omega^2 + \frac{N\rho I \omega^2}{k_y GA} \\ c &= -\rho A \omega^2 + \frac{\rho^2 I \omega^4}{k_y G} \end{aligned} \quad (8)$$

For a given value of the frequency ω , the parameters a , b and c only depend on the axial force N . The solution of equation (7), for a fixed value of N , is given by

$$\hat{v}(x) = \sum_{k=1}^4 C_k \exp(\beta_k x), \quad (9)$$

where

$$\begin{aligned}\beta_1 &= \sqrt{\frac{-b + \sqrt{b^2 - 4ac}}{2a}}, \quad \beta_2 = -\sqrt{\frac{-b + \sqrt{b^2 - 4ac}}{2a}}, \\ \beta_3 &= \sqrt{\frac{-b - \sqrt{b^2 - 4ac}}{2a}}, \quad \beta_4 = -\sqrt{\frac{-b - \sqrt{b^2 - 4ac}}{2a}}\end{aligned}\quad (10)$$

The parameters β_k ($k = 1, \dots, 4$) depend on the bar characteristics, which are assumed to be known, and on the unknown axial force N . The coefficients C_k depend on the boundary conditions, i.e. the connections of the bar to the rest of the structure. As no assumptions are made with respect to these connections, the coefficients C_k are identified from the measured vibration data, together with the axial force N . The total number of unknowns therefore equals five (N, C_1, C_2, C_3, C_4).

As the axial force enters equation (9) in a non-linear way, the axial force N as well as the coefficients C_k cannot be determined directly. The unknowns will therefore be determined in an iterative way from equation (9), assuming the left hand side of the equation, i.e. $\hat{v}(x)$, to be known at five or more positions x along the bar. From this assumption, n equations in 5 unknown are obtained, with n the number of sensors ($n \geq 5$).

For a fixed value of the axial force N , the number of unknown parameters is reduced to four and a linear system of equations is obtained. The resulting system of equations can be written as:

$$\mathbf{m} = \mathbf{A}\mathbf{c} \quad (11)$$

The vector $\mathbf{m}$ ($\in \mathbb{R}^{n \times 1}$) contains the value of $\hat{v}(x)$ at each of the sensor positions, calculated from the original vibration data at frequency ω . The vector $\mathbf{c}$ ($\in \mathbb{R}^{4 \times 1}$) contains the four coefficients C_k , to be determined. The coefficient matrix $\mathbf{A}$ ($\in \mathbb{R}^{n \times 4}$) is given by equation (12).

$$\mathbf{A} = \begin{bmatrix} \exp(\beta_1 x_1) & \exp(\beta_2 x_1) & \exp(\beta_3 x_1) & \exp(\beta_4 x_1) \\ \exp(\beta_1 x_2) & \exp(\beta_2 x_2) & \exp(\beta_3 x_2) & \exp(\beta_4 x_2) \\ \vdots & & & \\ \exp(\beta_1 x_n) & \exp(\beta_2 x_n) & \exp(\beta_3 x_n) & \exp(\beta_4 x_n) \end{bmatrix} \quad (12)$$

The obtained system of linear equations is over-determined for $n \geq 5$. Solving the system in a least squares sense yields the optimal value of $\mathbf{c}$:

$$\mathbf{c} = \mathbf{A}^\dagger \mathbf{m} \quad (13)$$

where $\mathbf{A}^\dagger$ is the pseudo inverse of the coefficient matrix $\mathbf{A}$.

The vector $\mathbf{c}$, calculated using equation (13), where $\mathbf{A}$ is constructed for an arbitrary value of the axial force N , generally does not satisfy equation (11). For the true value of the axial force N , however, the vector $\mathbf{c}$ obtained from equation (13) theoretically satisfies equation (11). Taking this consideration into account, an estimate of the true axial force value can be obtained by calculating the normalized error norm Δ_n for several feasible values of N . The normalized error norm Δ_n is defined as follows:

$$\Delta_n = \frac{\|\mathbf{A}\mathbf{c} - \mathbf{m}\|_2}{\sqrt{\|\mathbf{A}\mathbf{c}\|_2 \|\mathbf{m}\|_2}} \quad (14)$$

where $\|\cdot\|_2$ corresponds to the Euclidean vector norm.

The value of Δ_n reaches its minimal value, theoretically zero, for the true value of the axial force N . Measurement errors and modelling errors, however, give rise to errors in the measurement vector $\mathbf{m}$ and the coefficient matrix $\mathbf{A}$, respectively. In this way, minimizing Δ_n in presence of measurement errors and/or modelling errors does not always yield the true axial force value. This is illustrated in section 4.

The value of the axial force determines the two wave numbers, k_1 and k_2 , describing the bending behaviour of the bar at a frequency ω :

$$\hat{v}(x) = \sum_{n=1}^2 (P_n \exp(ik_n x) + Q_n \exp(-ik_n x)) \quad (15)$$

where $k_1 = i\beta_1 = -i\beta_2$, $k_2 = i\beta_3 = -i\beta_4$, and i is the imaginary unit. The description of the deflection shape $\hat{v}(x)$ in equation (15) is equivalent to its description in equation (9). McDaniel et al. [17, 18] have proposed a technique to estimate structural wavenumbers k_1 and k_2 for beam structures. In the present work it is not necessary to estimate the two wavenumbers independently, as there is only a single unknown, i.e. the axial force N . The method proposed by McDaniel et al. in [18] and the method proposed in this paper, however, are similar in the sense that in both methods an error function is minimized, such that the resulting wave form approximates the measured data.

The mass of the sensors, e.g. accelerometers, can be accounted for as follows. The bar length between the outer sensors is divided into $n - 1$ parts, with n being the number of sensors along the bar. For each of these parts, the general expression in equation (9) remains valid, with possibly different values for the coefficients C_k on every part. In this way, the total number of unknowns is augmented to $4(n - 1) + 1 = 4n - 3$. Equation (9) provides n equations and consequently does not suffice to uniquely determine each of the unknowns. The system of linear equations (11) is supplemented by $2(n - 2)$ equations that express the continuity of the transverse modal displacement and rotation at the inner sensor locations, and $2(n - 2)$ equations corresponding to the transverse force and moment equilibrium at these locations. In this way, the total number of equations becomes $5n - 8$. For $n \geq 5$, an overdetermined system of linear equations is obtained, for a given value of N . The axial force value can be determined in a similar way as explained before, by selecting the axial force value N corresponding to the minimum normalized error norm Δ_n for the augmented system.

Two approaches may be followed: a modal characteristics approach and a direct frequency domain approach. The modal characteristics approach considers each identified structural mode separately. In this case, $\hat{v}(x)$ corresponds to a mode shape $\phi_l(x)$ and ω to the corresponding natural frequency ω_l . For each of the identified modes, a single estimate of the axial force value is obtained. The direct frequency domain approach on the other hand accounts for the contribution of all structural modes. In this case, $\hat{v}(x)$ is the bar response due to harmonic loading at a single frequency ω , and so the operational deflection shape of the bar at that frequency. Without loss of generality, $\hat{v}(x)$ can also represent the Fourier transform of the bar response $v(x, t)$ due to broadband excitation. From the Fourier transform $\hat{v}(x)$, an estimate of the axial force value can be calculated for each of the frequencies ω independently. Since equation (9) is based upon the free vibration equilibrium equations, it is assumed that no forces are acting on the part of the bar which is covered by the sensors.

Although damping will be inherently present in practice, it is not taken into account in the frequency domain approach. It will be shown, however, that the influence of damping on the result is negligible, except at the anti-resonance frequencies (section 3.2).

A great advantage of the frequency domain approach is the fact that raw vibration data may be used, without any need for identifying the modal characteristics of the bar. In addition, the axial force value can be calculated for a wide range of frequencies ω , whereas for the modal characteristics approach, the number of estimated axial force values is limited to the number of identified modes.

3. Numerical verification

The axial force estimation procedure is illustrated for the case of a two-dimensional truss girder, shown in Fig. 1, using finite element (FE) simulations. The simulations correspond to an actual vibration measurement performed on an existing space truss supporting the roof of a concert hall.

Fig. 1: Two dimensional truss girder as considered for the numerical simulations (H: hammer impact location, S_k : position of sensor k).

The truss girder is composed of seven identical steel members with a length of 2.4 m, interconnected by stiff nodes. The bars have a hollow cylindrical section with an inner radius of 82.5 mm and an outer radius of 88.9 mm. The shear deformation coefficient is taken as 0.526. The Young's modulus, Poisson's ratio, and density are taken as 200 GPa, 0.3 and 7800 kg/m³, respectively. Each of the bars is discretized using 120 beam elements. Five sensors are uniformly distributed along the lower bar of the structure (Fig. 1, S_1 - S_5). The sensor mass and its mass moment of inertia correspond to the actual test setup, where the accelerometers were connected to the cylindrical bar by large magnets. In this way, a total mass of 1 kg and a mass moment of inertia of 5.33×10^{-3} kgm², with respect to the bar axis, are obtained for each sensor.

The structure is subjected to a static vertical force P of 100 kN, giving rise to an axial tensile force of 57.53 kN in the lower bar of the structure. For this simulated example, both axial force estimation approaches are applied.

3.1. Modal characteristics approach

The FE model is used to calculate the modal characteristics of the structure, taking second order effects into account. The mode shapes for the first six modes of the two-dimensional truss girder are shown in Fig. 2. For each of the calculated modes, the natural frequency and the vertical component of the corresponding global mode shape, calculated at the five sensor positions, are used to estimate the axial force in the lower bar of the structure. Fig. 3 shows the normalized error

norm Δ_n as a function of the axial force N assumed in the calculation, for the particular case of mode 1. The minimum value for Δ_n is obtained for N equal to the exact value 57.53 kN, as is also the case for each of the higher vibration modes.

Fig. 2: First six eigenmodes of the two dimensional truss girder.

Fig. 3: Axial force estimated using the modal characteristics approach for mode 1.

3.2. Frequency domain approach

For the application of the frequency domain approach, simulated response data are needed. A transient hammer impact is applied at 0.2 m from the leftmost sensor at the lower bar (Fig. 1). The time history and frequency content of the force are shown in Fig. 4.

The acceleration responses at the sensor locations are calculated at a sampling rate of 1000 Hz for a period of 4 s, assuming a modal damping ratio of 1% for all modes. The accelerations at the sensor positions are determined in the frequency domain. Second order effects are taken into account and modal superposition is used. When performing simulations, a large number of modes needs to be taken into account in order to obtain a good estimate of the axial force value for each of the frequencies considered. All transversal modes of the bar are contained in the operational deflection shape description, as given by equation (9), while only a limited number of modes can be taken into account when performing numerical simulations. Disregarding the higher modes leads to considerably more inaccurate estimations of the axial force value, especially at the antiresonance

Fig. 4: (a) Time history and (b) frequency content up to 300 Hz, of the applied force for 2D beam structure.

frequencies. For this reason, a detailed finite element model is used and all modes are accounted for in the simulations.

The Fourier transform of the accelerations obtained from the forward analysis is subsequently used to estimate the axial force in the lower bar of the structure. The Fourier transform of the acceleration signals for each of the sensors and the corresponding axial force estimate are shown for a frequency range up to 100 Hz in Fig. 5.

Fig. 5: (a) Narrow band frequency spectrum of the accelerations at the five sensor locations (S_1 to S_5 : grey to black) and (b) axial force estimated using the frequency domain approach for a 2D beam structure (identified axial force: black, true axial force: grey).

For nearly all frequencies considered in the analysis an excellent estimate of the axial force is obtained. The axial force value almost perfectly equals the true value of 57.53 kN, except for two small frequency intervals near anti-resonance. For these frequencies, the influence of the damping included in the FE model calculations, which is not accounted for in the derivation of the proposed method, becomes large, resulting in an error on the axial force up to 30.4%. For steel structures, the actual modal damping values are often lower than 1%, so the influence of damping will only lead to relatively small errors.

4. Error analysis

In practice the input needed for the axial force estimation will always be characterized by errors. Modelling errors result from the fact that the model provides an imperfect representation of the true structure and occur for example if an incorrect value of the model parameters is assumed for the calculations. Measurement errors are due to measurement noise and are inherently present when performing a vibration experiment.

The influence of modelling errors and measurement errors on the result of the axial force estimation procedure is investigated by means of an error analysis. For this purpose, the example of the 2D truss girder, as introduced in section 3, is reconsidered. An error is assumed on the geometrical properties and material properties of the considered bar. The sensor properties, such as the sensor mass, are varied as well and measurement noise is introduced.

4.1. Uncertain parameters

The following paragraphs summarize the assumptions made for the error analysis. For each parameter, only a single variation is considered. The sign of the parameter variations is chosen such that a worst case scenario is obtained. This choice is based on simulations considering each parameter variation independently. For clarity, the parameters used for the FE model calculations or the data signals obtained from the FE model calculations are denoted as $\square_{\text{true}}$. The parameters and data signals used as input to the axial force estimation procedure are denoted as $\square_{\text{inp}}$.

Geometrical properties - The geometrical properties of a bar are often not perfectly known. For instance, the outer diameter of a cylindrical hollow bar as considered in the simulations can be measured very accurately using a calipers. It is generally much harder to measure the inner diameter very accurately for an existing structure. In addition, both the inner diameter and outer diameter may vary along the bar and the bar section may not be perfectly cylindrical. In the calculations it is assumed that both the bar section A and the moment of inertia I are characterized by an error of 0.1%:

$$\begin{aligned} A_{\text{inp}} &= 1.001 A_{\text{true}} \\ I_{\text{inp}} &= 0.999 I_{\text{true}} \end{aligned}$$

Material properties - The material properties are often assumed to be known without performing a direct measurement. Since in the case of steel, the material properties are known quite accurately, it is assumed here that both the Young's modulus E and the material density ρ are characterized by a small error of 0.1%:

$$\begin{aligned} E_{\text{inp}} &= 0.999 E_{\text{true}} \\ \rho_{\text{inp}} &= 1.001 \rho_{\text{true}} \end{aligned}$$

Sensor properties - The mass of the sensors can be determined very accurately using a simple scale. Its mass moment of inertia is calculated from the mass and the shape of the sensors. The sensor shapes are often complex, not allowing for a straightforward and accurate determination of the mass moment of inertia. In addition, the mass of the wires is often neglected when cabled sensors are used. In the calculations it is assumed that the mass of each sensor m_s and its mass moment of inertia $I_{\text{zz},s}$ are both subjected to an error of 1%:

$$\begin{aligned} m_{\text{s,inp}} &= 1.01 m_{\text{s,true}} \\ I_{\text{zz,s,inp}} &= 1.01 I_{\text{zz,s,true}} \end{aligned}$$

The distance between the mounted sensors can be measured using a calipers, with an accuracy which generally falls below 1 mm. The error in sensor positions Δx_j as assumed for the error analysis is shown in table 1. The sensor positions assumed for the calculations are obtained as:

$$x_{j,\text{inp}} = x_{j,\text{true}} + \Delta x_j$$

Sensor noise is added in the time domain for each of the calculated response signals independently. The noise is assumed to be zero mean and white. The standard deviation of the measurement noise is assumed to be 10^{-5} ms^{-2} :

$$\ddot{v}_{\text{noise}}(x_j, t_l) = \ddot{v}_{\text{true}}(x_j, t_l) + 0.00001 r_{j,l}$$

In the equation above, $\ddot{v}_{\square}$ denotes an acceleration signal and $r_{j,l}$ ($j = 1, \dots, n$ and $l = 1, \dots, N$) is a random number drawn from a standard normal distribution. The subscript “noise” is introduced to indicate the polluted acceleration signals. An additional systematic error is introduced next.

The sensitivity of the sensors is a sensor characteristic used for the conversion of a sampled voltage signal to an acceleration signal. The sensitivity of a sensor is in practice often determined for a single, fixed calibration frequency. The corresponding calibration error can be assumed to be small. It has to be noted that an erroneous estimation of the sensor sensitivity results in a systematic measurement error. This error is comparable to an error which occurs if not all sensors are perfectly aligned. The error on the sensitivities δs_j assumed in the error analysis is shown in table 1. The response signal taking measurement noise and the error on the sensor sensitivities into account is calculated as:

$$\ddot{v}_{\text{inp}}(x_j) = \ddot{v}_{\text{noise}}(x_j) (1 + \delta s_j)$$

Table 1: The values assumed in the error analysis for sensor position and sensitivity.

	Sensor 1	Sensor 2	Sensor 3	Sensor 4	Sensor 5
Error position Δx [m]	5×10^{-4}	2×10^{-4}	-1×10^{-4}	-3×10^{-4}	4×10^{-4}
Error sensitivity δs [%]	-0.02	0.01	-0.03	-0.04	0.02

4.2. Modal characteristics approach

For the first six modes of the truss girder, the axial force is estimated independently, assuming the input values as summarized above. Sensor noise is not taken into account for the calculations. The results of the analysis are shown in table 2.

Table 2: Result error analysis, modal characteristics approach.

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Natural frequency f [Hz]	49.38	51.56	58.72	65.74	87.03	92.49
Identified axial force N_{id} [kN]	62.85	71.16	212.19	64.30	64.87	65.45
Error [kN]	5.32	13.63	154.66	6.77	7.34	7.92

From table 2, it is clear that the estimated axial force value is strongly affected by small errors on the input parameters. In addition, the error sensitivity depends on the mode considered. For all modes, except for modes 2 and 3, a reasonable estimate of the axial force value is obtained.

For mode 2 a larger error is obtained and for mode 3 the error becomes very large. For mode 3, the modal displacement of sensor 3 is very small, compared to the modal displacements at the other sensor locations (see Fig. 2). In all simulated examples, it is observed that when for a particular mode the modal displacement at one sensor location becomes small compared to the modal displacements at the other sensor locations, the corresponding axial force estimate becomes sensitive to small errors on the input parameters. There is no direct criterion which indicates for which of the modes an accurate estimate of the axial force is obtained. This can be seen as a general drawback of the modal characteristics approach.

4.3. Frequency domain approach

Fig. 6 shows the results of the error analysis for the case of the direct frequency domain approach.

Fig. 6: Result error analysis, (a) narrow band frequency spectrum of the accelerations at the five sensor locations (S_1 to S_5 : grey to black), noise included and (b) axial force estimated using the frequency domain approach for a 2D beam structure (identified axial force: black, true axial force: grey).

The estimated axial force value shows a large variation over the frequency range considered. Several observations can be made on this variation. These observations have been confirmed by several numerical simulations, not shown in this paper, considering each of the parameter variations independently.

Measurement errors result in a heavily alternating axial force estimate, mainly observed for those frequencies where the amplitude of the acceleration signals is low. Modelling errors on the other hand result in a systematic error on the axial force estimate, when plotted as a function of the corresponding frequency. Modelling errors due to an inaccurate estimation of the geometrical properties and/or material properties of the bar mainly affect the axial force estimate at high frequencies. Modelling errors due to an erroneous estimation of the sensor position and/or sensor characteristics mainly affect the axial force estimate at low frequencies.

Fig. 7 shows the deviation of the axial force value $\Delta N(f)$, as defined by equation (16), which is a measure for the first derivative of the curve shown in Fig. 6(b).

$$\Delta N(f_i) = |N(f_i + \Delta f) - N(f_i)| \quad (16)$$

where Δf is the frequency bin. From both Fig. 6(b) and Fig. 7, a frequency range can be identified where the variation becomes low, i.e. from 42 Hz to 54 Hz (indicated in grey in Fig. 7). An upper limit of 1 kN on the deviation $\Delta N(f)$ has been used. Other frequencies can be identified for which the axial force deviation becomes small as well. For the frequency range indicated, however, the estimated axial force is nearly constant over the entire frequency interval, which is preferably chosen as large as possible.

Averaging the axial force values over the frequency range from 42 Hz to 54 Hz yields a value of 62.82 kN, which is a reasonable estimate of the true axial force value: $N_{\text{fem}} = 57.53$ kN, error = 5.29 kN. A similar averaging procedure will be applied for two experimental case studies considered next.

Fig. 7: Result error analysis: deviation of the identified axial force value. In grey the frequency interval is indicated for which the averaging of the axial force values is performed.

5. Experimental validation

In this section, the effectiveness of the proposed algorithm is further demonstrated by means of two laboratory experiments. For the first laboratory test, an instrumented steel bar, with rectangular cross section, is axially loaded by a Schenck testing machine. For the second test, the same rectangular bar is mounted in a laboratory-scale space truss. For the two cases, both axial force estimation approaches are applied.

5.1. Test 1 - rectangular bar Schenck

A single steel bar with a rectangular cross section of 10 mm by 50 mm and a length of 0.9 m is axially loaded by a Schenck testing machine (Schenck Trebel 100 kN). The yield axial force of the bar is 117.5 kN, assuming a yield stress of 235 MPa. The Euler buckling load is 48.26 kN, assuming both ends of the bar to be fixed. The axial force applied by the testing machine is increased from 0 kN to 40 kN, in steps of 10 kN. The load steps are summarized in table 3. At each load step, a vibration measurement is performed.

A total of 14 accelerometers (PCB 333A, sensitivity ± 100 mV/g) is mounted along the bar to record its response to an impact force. The sensor configuration is shown in Fig. 8. Although in theory a total of five accelerometers is sufficient to apply the axial force estimation procedure, a

Table 3: Load steps considered for the experiment on the instrumented steel bar in the Schenck testing machine (target value axial force N_{target} , true value axial force applied by the Schenck testing machine N_{true}).

	Step 1	Step 2	Step 3	Step 4	Step 5
N_{target} [kN]	0	10	20	30	40
N_{true} [kN]	0.4	9.8	20.2	30.5	40.2

larger number of accelerometers is used. A series of impact forces is applied with an instrumented hammer (PCB 086C03, mass 0.136 kg) at one of the bar ends, near accelerometer S_{14} (Fig. 8). The bar is excited laterally along its weak axis, and the response is measured in this direction as well. Fig. 9 shows the measured acceleration response at sensor S_7 for the unloaded case (load step 1), both in time and frequency domain.

The bar properties have been determined very carefully to reduce the error on the estimated axial force due to errors on the input variables. Both the modal characteristics approach and the frequency domain approach are applied and discussed next.

Fig. 8: Experimental setup of the bar (white) with rectangular cross section (H: hammer impact location, S_k : position sensor k), loaded by a Schenck testing machine (grey).

Fig. 9: (a) Time history and (b) narrow band frequency spectrum up to 1000 Hz of the measured accelerations at sensor S_7 for the instrumented steel bar in the Schenck testing machine. Accelerations correspond to load step 1.

5.1.1. Modal characteristics approach

The modal parameters of the bar are identified for the frequency range up to 1000 Hz, using the data-driven combined deterministic-stochastic subspace identification (CSI-data) algorithm [19]. For the first four bending modes of the bar, the natural frequencies are shown in table 4. As expected, the natural frequencies increase with increasing value of the axial tensile force.

For each of the modes considered, the axial force is estimated using the method described in section 2. The results of the identification procedure are shown in table 5. Except for mode 1, the identified axial force N_{id} provides a reasonable estimate of the true axial force N_{true} .

5.1.2. Frequency domain approach

The Fourier transform of the measured acceleration data is determined by a Fast Fourier transform algorithm. From this Fourier transform, the axial force is estimated in the frequency range

Table 4: Identified natural frequencies for the first four bending modes of the instrumented steel bar in the Schenck testing machine.

Load step	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]	f_4 [Hz]
Step 1	66.32	179.78	350.11	583.16
Step 2	77.90	201.50	389.62	633.03
Step 3	89.87	224.47	428.45	689.16
Step 4	96.63	229.56	439.00	701.06
Step 5	102.88	251.17	450.89	713.99

Table 5: Result axial force identification using the modal characteristics approach, rectangular bar in the Schenck testing machine (true axial force value N_{true} , identified axial force value N_{id} and corresponding error).

Load step	N_{true} [kN]	Mode 1		Mode 2		Mode 3		Mode 4	
		N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]
Step 1	0.4	-4.54	-4.94	0.09	-0.31	-0.08	-0.48	-0.57	-0.97
Step 2	9.8	5.05	-4.75	7.92	-1.88	9.00	-0.80	8.59	-1.21
Step 3	20.5	17.11	-3.39	19.50	-1.00	20.33	-0.17	21.64	1.14
Step 4	30.5	29.40	-1.10	31.51	1.01	30.68	0.18	31.91	1.41
Step 5	40.2	41.57	1.37	40.95	0.75	42.17	1.97	43.50	3.30

from 0 Hz to 1000 Hz, without any need for prior data processing. A frequency step of $64T^{-1}$ is used, with T the sampling interval. The upper limit of the frequency range is determined by the working range of the accelerometers.

Fig. 10 shows the results of the axial force estimation procedure for the unloaded case (load step 1). The axial force values as shown in the figure are limited to the yield axial force value of 117.5 kN, both in tension and compression.

Fig. 10: Result axial force estimation using the frequency domain approach for load step 1, rectangular bar in the Schenck testing machine (identified axial force value: black, true axial force value: grey).

The estimated axial force value varies strongly with frequency. The observations made in section 4 may help to explain the results obtained in Fig. 10. At low frequencies, up to 200 Hz, the influence of measurement errors on the axial force estimate is large, as can be seen from the large variation of the axial force values. Apart from this large and heavily oscillating variation, a decreasing trend on the axial force value can be observed at low frequencies as well, which may be due to an error in the determination of the sensor positions and characteristics. At higher frequencies, from ca. 750 Hz on, the axial force values are characterized by a slightly increasing trend. This indicates that, despite the care that has been taken to accurately determine the bar characteristics, these characteristics are still subject to a small error.

Fig. 11 shows the deviation of the axial force value $\Delta N(f)$, as defined in equation (16). For a frequency range from 320 Hz to 427 Hz and from 511 Hz to 632 Hz, the deviation of the axial force becomes smaller than 1 kN. For both frequency ranges a good estimate of the axial force is obtained. The interval from 511 Hz to 632 Hz is retained because of its larger frequency span. Averaging the axial force values over this frequency range yields a value of -0.94 kN, which is a very good estimate of the true axial force value: $N_{\text{true}} = 0.4$ kN, error = -1.34 kN.

Fig. 11: Deviation of the identified axial force value for load step 1 (black), rectangular bar in the Schenck testing machine. In grey the frequency range is indicated for which the averaging of the axial force values is performed.

The same procedure is repeated for each of the other load steps. The results are summarized in table 6. The table also indicates the limits of the frequency range for which the averaged axial force value is calculated. For each of the load steps, the averaging frequency range contains one of the natural frequencies (table 4). This is expected, since for these frequencies the influence of measurement noise is small. As can be seen from table 6, an excellent estimate of the axial force value is obtained for each of the load steps. It is important to note that even small axial forces can be identified very accurately.

Table 6: Result axial force identification using the frequency domain approach, rectangular bar loaded by a Schenck testing machine (true axial force value N_{true} , identified axial force value N_{id} and corresponding error, lower and upper limit of the averaging frequency range $f_{\text{min,av}}$ and $f_{\text{max,av}}$, respectively).

Load step	N_{true} [kN]	N_{id} [kN]	Error [kN]	$f_{\text{min,av}}$ [Hz]	$f_{\text{max,av}}$ [Hz]
Step 1	0.4	-0.94	-1.34	511	632
Step 2	9.8	8.76	-1.05	610	646
Step 3	20.2	19.95	-0.22	407	455
Step 4	30.5	30.41	-0.09	414	453
Step 5	40.2	42.08	1.88	429	467

When comparing the results of the frequency domain approach and the modal characteristics approach it is clear that the frequency domain approach generally yields slightly better results and gives a better idea about the accuracy of the estimate obtained. By plotting the estimated axial force values as a function of the frequency, important information on the axial force estimate can be acquired and a frequency range can be identified for which a good estimate of the axial force can be expected.

When the number of sensors is decreased, the estimated axial force becomes more sensitive to errors on the input parameters. Therefore, the number of accelerometers along the bar was chosen as fourteen instead of five, which is the minimum number of sensors for the method to

be applicable. After performing the measurements using fourteen accelerometers, the axial force, however, can be calculated by retaining a limited number of measured acceleration signals.

By selecting five non consecutive sensors, e.g. Fig. 12, the inertia of the intermediate sensors (e.g. sensor 13, see Fig. 8) cannot be accounted for. This results in an error on the identified axial force, which becomes large when the mass of the sensors is relatively high. Table 7 and

Fig. 12: Experimental setup of the bar (white) with rectangular cross section (H: hammer impact location, S_k : position sensor k), loaded by a Schenck testing machine (grey). Five selected non consecutive sensors are labelled, the remaining sensors are indicated by a black dot without label.

Table 8 show a comparison of the axial force values identified using fourteen sensors (Fig. 8) and five sensors (Fig. 12), for the modal characteristics approach and the frequency domain approach, respectively. For both approaches, it is clear that the results obtained using fourteen sensors are in general more accurate than those obtained using five non consecutive sensors. Except for mode 1, the axial force values obtained using five sensors are lower than the values obtained using fourteen sensors. This indicates that a systematic error occurs, which is mostly due to the inertia of the intermediate accelerometers. For the experiments performed, however, lightweight accelerometers are used ($m_s = 7$ g) and still a good estimate of the axial force is obtained.

Table 7: Axial force identification using the modal characteristics approach, rectangular bar loaded by Schenck testing machine. The results of the axial force identification using fourteen sensors are compared to the results using five (of fourteen) non consecutive sensors (true axial force value N_{true} , identified axial force value N_{id} and corresponding error).

Load step	N_{true} [kN]	Mode 1				Mode 2			
		14 sensors		5 sensors		14 sensors		5 sensors	
		N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]
Step 1	0.4	-4.54	-4.94	8.32	7.92	0.09	-0.31	-0.28	-0.68
Step 2	9.8	5.05	-4.75	26.43	16.63	7.92	-1.88	8.09	-1.71
Step 3	20.5	17.11	-3.39	24.77	4.27	19.50	-1.00	19.46	-1.04
Step 4	30.5	29.40	-1.10	32.70	2.20	31.51	1.01	26.92	-3.58
Step 5	40.2	41.57	1.37	42.20	2.00	40.95	0.75	40.43	0.23
Load step	N_{true} [kN]	Mode 3				Mode 4			
		14 sensors		5 sensors		14 sensors		5 sensors	
		N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]
Step 1	0.4	-0.08	-0.48	-1.68	-2.08	-0.57	-0.97	-1.81	-2.21
Step 2	9.8	9.00	-0.80	7.13	-2.67	8.59	-1.21	7.71	-2.09
Step 3	20.5	20.33	-0.17	17.87	-2.63	21.64	1.14	15.40	-5.10
Step 4	30.5	30.68	0.18	28.04	-2.46	31.91	1.41	27.25	-3.25
Step 5	40.2	42.17	1.97	39.48	-0.72	43.50	3.30	30.71	-9.49

Table 8: Axial force identification using the frequency domain approach, rectangular bar loaded by a Schenck testing machine. The results of the axial force identification using fourteen sensors are compared to the results using five (of fourteen) non consecutive sensors (true axial force value N_{true} , identified axial force value N_{id} and corresponding error).

Load step	N_{true} [kN]	14 sensors		5 sensors	
		N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]
Step 1	0.4	-0.62	-1.02	-2.04	-2.44
Step 2	9.8	8.77	-1.03	7.25	-2.55
Step 3	20.2	20.15	-0.02	18.04	-2.13
Step 4	30.5	30.49	-0.01	28.11	-2.39
Step 5	40.2	42.02	1.82	39.29	-0.91

5.2. Test 2 - rectangular bar in a space truss

For the second test, the same bar as considered in the previous section is mounted in a laboratory-scale space truss. An overview of the test setup is given in Fig. 13.

Fig. 13: (a) Overview of the laboratory space truss and (b) accelerometer placement on the bar with rectangular cross section.

The truss structure is vertically loaded at the top by a hydraulic jack, as shown in Fig. 13(a). The vertical force exerted by the hydraulic jack is increased from 0 kN to 35 kN, in steps of 5 kN. The bar with rectangular cross section is mounted at the lower part of the structure and therefore loaded under tension (Fig. 13).

The axial force in the rectangular bar is measured using an axially mounted load cell - type Interface 1210ACK (50 kN). For the load steps considered, the axial force is gradually increased from 0 kN up to 14.13 kN (tension force). The load steps are summarized in table 9. For each load step, a vibration measurement is performed and the recorded vibration data are used to estimate the axial force in the rectangular bar.

Table 9: Load steps considered for the experiment on the instrumented steel bar with rectangular cross section as a part of the laboratory-scale space truss (load applied by the hydraulic jack P_{jack} , true axial force value measured by the axially mounted load cell N_{true}).

	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8
P_{jack} [kN]	0	5	10	15	20	25	30	35
N_{true} [kN]	0.14	2.11	4.12	6.27	8.35	10.36	12.34	14.13

Before mounting, the bar is shortened to a length of 0.72 m, measured between the bolts

connecting the bar to the remaining part of the structure. The measurement setup is shown in Fig. 14. As was the case for the first set of experiments, 14 accelerometers (PCB 333A, sensitivity ± 100 mV/g) are mounted along the bar. A series of impact forces is applied with an instrumented hammer (PCB 086C03, mass 0.136 kg) at one of the bar ends, near accelerometer S_1 (Fig. 14). The lateral accelerations of the bar along its weak axis are recorded. Fig. 15 shows the acceleration signal at sensor S_7 for the unloaded case (load step 1), both in time and frequency domain.

Fig. 14: Overview setup of the bar with rectangular cross section as a part of the laboratory-scale space truss. White part shows the rectangular bar (H: hammer impact location, S_k : position sensor k), grey part shows the connection of the bar to the surrounding structure and hatched part shows the axially mounted load cell.

Fig. 15: (a) Time history and (b) narrow band frequency spectrum up to 1000 Hz of the measured accelerations at sensor S_7 for the instrumented steel bar, mounted in a laboratory-scale space truss. Accelerations correspond to load step 1.

The yield axial force of the bar is 117.5 kN. Assuming both ends of the bar to be hinged, the Euler buckling load is 16.6 kN. The results of both the modal characteristics approach and the frequency domain approach are discussed next.

5.2.1. Modal characteristics approach

The modal parameters of the bar are identified for a frequency range up to 1000 Hz, using the data-driven combined deterministic stochastic subspace identification (CSI-data) algorithm [19]. As the bar is part of a truss structure, the identified modes are all global modes, which only have been measured along the rectangular bar. The identified natural frequencies are those of the whole structure, rather than those of a bar with fixed boundary conditions. Table 10 shows the natural frequencies of six of the identified modes, for each of the load steps considered.

For some of the load steps, not all of the modes could be retrieved, e.g. for load steps 1 and 2, mode 5 identified in load steps 3 to 8 is not found. For each of the first five modes, the natural frequency increases for an increasing vertical load and, therefore, for an increasing tensile force in the rectangular bar. This indicates that the modal characteristics for these modes are dominated by tensile forces in the lower part of the structure.

For each of the modes, the axial force is estimated using the method explained in section 2. The results of the identification procedure are shown in table 11.

The accuracy of the axial force estimate depends on the mode considered. For modes 4 and 5, a very good estimate of the axial force is obtained for each load step, whereas for the other modes the error becomes large. For all modes the errors obtained are quite small when compared to the yield axial force ($N_y = 117.5$ kN), but rather large compared to the Euler buckling load ($N_{\text{buckl}} = 16.6$ kN).

5.2.2. Frequency domain approach

The axial force is estimated in a frequency range from 0 Hz to 1000 Hz, using a frequency step of $64T^{-1}$, with T the sampling interval. Fig. 16 shows the resulting estimation of the axial force in the unloaded case (load step 1). For the figure, the axial force values are limited to the yield axial force value of 117.5 kN, both in tension and compression.

Fig. 16: Result axial force estimation using the frequency domain approach for load step 1, rectangular bar mounted in laboratory-scale space truss (identified axial force value: black, true axial force value: grey).

The same conclusions as drawn in section 5.1.2 hold for the variation of the axial force with frequency. Large and heavily alternating variations are probably due to measurement noise. The trend observed at low and high frequencies can be attributed to modelling errors. The number of natural frequencies in the frequency range of interest is increased with respect to the experiment for the bar loaded by the Schenck testing machine (section 5.1). This results in a higher number of peaks in the estimated axial force plotted as a function of frequency (Fig. 16).

Fig. 17 shows the deviation of the axial force value $\Delta N(f)$, as defined in equation (16). For a frequency range from 356 Hz to 382 Hz, the deviation of the axial force becomes smaller than 1 kN. This again corresponds to a nearly constant value of the axial force estimate. Averaging the axial force value over this frequency range yields a value of 0.03 kN, which is a very good estimate of the true axial force value: $N_{\text{true}} = 0.14$ kN, error = -0.11 kN.

Table 10: Identified natural frequencies for six bending modes of the instrumented steel bar, mounted in a laboratory-scale space truss. The number of halve wavelengths in bending is indicated between brackets.

Load step	f_1 [Hz] (1)	f_2 [Hz] (1)	f_3 [Hz] (2)	f_4 [Hz] (3)	f_5 [Hz] (3)	f_6 [Hz] (4)
Step 1	30.24	80.70	174.79	301.61	-	558.26
Step 2	35.90	84.37	179.02	-	-	553.50
Step 3	37.78	84.06	187.44	-	343.38	558.93
Step 4	39.53	85.92	190.05	304.42	344.53	555.59
Step 5	40.79	87.73	191.45	305.39	346.36	557.61
Step 6	41.89	89.96	192.63	305.84	348.71	559.26
Step 7	42.87	90.72	193.83	306.44	349.38	558.27
Step 8	43.73	91.52	195.25	306.34	351.04	557.85

Table 11: Result axial force identification using the modal characteristics approach, rectangular bar mounted in laboratory-scale space truss (true axial force value N_{true} measured by the axially mounted load cell, identified axial force value N_{id} and corresponding error).

Load step	N_{true} [kN]	Mode 1		Mode 2		Mode 3	
		N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]
Step 1	0.14	-6.93	-7.07	-3.90	-4.04	-2.49	-2.63
Step 2	2.11	-6.12	-8.23	-0.80	-2.91	0.24	-1.87
Step 3	4.12	-4.43	-8.55	-0.72	-4.84	0.09	-4.03
Step 4	6.27	-2.35	-8.62	0.72	-5.55	1.77	-4.50
Step 5	8.35	-1.39	-9.74	3.87	-4.48	3.98	-4.37
Step 6	10.36	1.67	-8.69	5.15	-5.21	5.91	-4.45
Step 7	12.34	2.90	-9.44	7.54	-4.80	8.43	-3.91
Step 8	14.13	5.40	-8.73	8.44	-5.69	8.75	-5.38

Load step	N_{true} [kN]	Mode 4		Mode 5		Mode 6	
		N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]	N_{id} [kN]	Error [kN]
Step 1	0.14	-1.63	-1.77	-	-	-7.27	-7.41
Step 2	2.11	-	-	-	-	-5.38	-7.49
Step 3	4.12	-	-	4.02	-0.10	-1.76	-5.88
Step 4	6.27	5.18	-1.09	5.06	-1.21	-2.42	-8.69
Step 5	8.35	7.38	-0.97	7.36	-0.99	-0.11	-8.46
Step 6	10.36	10.25	-0.11	10.30	-0.06	5.63	-4.73
Step 7	12.34	11.22	-1.12	11.79	-0.55	5.20	-7.14
Step 8	14.13	13.86	-0.27	14.28	0.15	10.55	-3.58

The same procedure is repeated for the remaining load steps. Table 12 shows that very good results are obtained for each of the load steps considered.

In this experiment, the bar is part of a structure and its boundary conditions cannot be represented by idealized support arrangements such as hinges or springs. Moreover, the effective boundary conditions for this case will be frequency dependent. This shows that the method succeeds in estimating the axial force in a bar very accurately, regardless of the boundary conditions of the considered bar.

When comparing the results of both the frequency domain approach and the modal characteristics approach it is clear that the frequency domain approach yields better results and gives more insight into the accuracy of the axial force estimate obtained. In addition, a frequency range can be determined for which the best axial force estimate is expected. For the modal characteristics approach it is not clear which of the modes provide a reasonable estimate of the axial force.

6. Conclusions

An analytical method has been proposed to estimate the axial force in a beam member. The method accounts for bending stiffness effects and for the rotational inertia and shear deformation of the beam member. The mass of the sensors used to record the response of the bar is accounted for as well. An important advantage of the proposed method is the fact that it does not require any information on the connections of the bar to the remaining part of the structure. The estimation is based on data from five or more sensors along the length of the bar and, therefore, can be used for any beam or truss element in a built-up structure.

Fig. 17: Deviation of the identified axial force value for load step 1 (black), rectangular bar mounted in a laboratory-scale space truss. In grey the frequency range is indicated for which the averaging of the axial force values is performed.

Table 12: Result axial force identification using the frequency domain approach, rectangular bar mounted in a laboratory-scale space truss (true axial force value N_{true} , identified axial force value N_{id} and corresponding error, lower and upper limit of the averaging frequency range $f_{\text{min,av}}$ and $f_{\text{max,av}}$, respectively).

Load step	N_{true} [kN]	N_{id} [kN]	Error [kN]	$f_{\text{min,av}}$ [Hz]	$f_{\text{max,av}}$ [Hz]
Step 1	0.14	0.03	-0.11	356	382
Step 2	2.11	1.76	-0.35	351	373
Step 3	4.12	3.60	-0.52	355	373
Step 4	6.27	4.88	-1.39	281	303
Step 5	8.35	7.44	-0.91	293	306
Step 6	10.36	9.48	-0.88	294	308
Step 7	12.34	11.25	-1.09	395	402
Step 8	14.13	13.50	-0.63	303	313

Two slightly different approaches are considered, i.e. the modal characteristics approach and the frequency domain approach. Both start from the equation of motion under free vibration and allow estimating the axial force in a bar from the response, known for at least five sensors distributed along the bar.

The modal characteristics approach estimates the axial force in a bar for each of the identified modes separately, hereby assuming the mode shapes to be extracted at each of the sensor positions. For this reason, a preliminary determination of the modal characteristics of the structure is required. The frequency domain approach makes use of the original vibration data. From the Fourier transform of the response, an estimate of the axial force is obtained for each of the frequencies considered. A great advantage of this approach is the fact that it does not require an intermediate modal parameter estimation step. Furthermore, by plotting the estimated axial force value as a function of the frequency, useful insight into the error on the axial force estimate can be acquired.

Numerical simulations were used to illustrate the proposed method as well as to investigate its practical applicability. The influence of both modelling errors and measurement errors is investigated. An experimental validation confirms the effectiveness of the proposed method. A bar with rectangular cross section was first axially loaded by a Schenck loading device. For a second test, the bar was mounted in a laboratory space truss. Both approaches yield satisfactory results. The results of the frequency domain approach are slightly better than those obtained using the modal characteristics approach. Moreover, from the second set of experiments, it was demonstrated that

the proposed method can be applied in practice for a bar making part of any structure. In order for the method to be applicable, the main requirement for the bar is that its characteristics are accurately known.

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