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Defining modern mathematics: Willy Servais (1913-1979) and mathematical curriculum reform in Belgium

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Abstract

The New Math reform which swept Europe in the 1960's was primarily instigated by the Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques. Since its foundation in 1952 the CIEAEM held annual meetings where mathematicians, logicians and psychologists discussed the direction of the modernization process. Several Belgian mathematicians played a prominent role in the CIEAEM, in particular Willy Servais and Georges Papy. In particular, Papy has been recognized as a leading, if not uncontested architect of the new mathematical curriculum. Much less is known about Willy Servais, who for more than twenty years acted as secretary of CIEAEM. In this paper we retrace the career of Servais against the background of the mathematical curriculum reform in Belgium. We reconstruct his views on the modernization of the mathematical curriculum, his work on mathematical models and his concern about the cultural role of mathematics in the modern world. Our analysis shows that the awareness of a need for an abstract, unified mathematics (as expressed in Papy's work) was not a dominant theme in the early 1950s debates in Belgium. Much more attention was given to the creation of teaching aids and the introduction of possible new topics such as probability theory, statistics or electrical technology. We further draw attention to issues concerning the social position of the mathematics teachers, the wider issues involved in the reform and the divergent views of mathematicians and school psychologists in Belgium.

Introduction

The New Math movement, or 'modern mathematics' as it was commonly called in Europe, refers to a rather brief, but dramatic and influential change in the way mathematics was taught in the U.S., in various European countries, and in some other parts of the developed world, from the end of the 1960's (Moon 1986; Stanic & Kilpatrick, 1992; Walmsley, 2003). The main feature of the movement was the introduction of new teaching contents, materials and practices in order to amend the generally perceived poor state of mathematics teaching after World War II. New Math emphasized insight in mathematical structure (rather than computational skill), often but not exclusively through the study of abstract concepts like sets, relations, graphs, algebraic structures, number bases other than 10, etc. Other characteristic changes were the replacement of traditional synthetic geometry based on Euclid by an algebraic approach, and the introduction of calculus through the concepts of continuity and limits, strictly defined in a topological environment. The New Math movement affected in various degrees mathematics in both primary and secondary schools, and often implied the replacement of Euclidean geometry and calculation skills.

The implementation of the New Math curricula generated a lot of controversy. In the U.S, Morris Kline became an outspoken opponent of the new curriculum, warning that it would have a negative impact on mathematics teaching by its overemphasis on abstract concepts and by its neglect of practical applications (Kline, 1973). In Europe, the situation was more complex. Although the New Math movement adhered to a common core, its implementation depended on national cultures and local educational systems. In 1975, Willy Servais presented an elaborate overview of the various national traditions and reform movements of mathematical teaching in continental Europe (Servais, 1975). In France, Belgium, the Netherlands, West and East Germany, Switzerland, Luxemburg, Spain, Italy, Poland and Hungary, new mathematical curricula had indeed been implemented in secondary schools, with, as their main common features, the replacement of synthetic geometry by an algebraic approach and the use of set theory. But Servais also noted many divergences between these national developments. The aims of the reform were not always the same, ranging from bringing the content of school mathematics closer to the current scientific level of the field, to renewing old fashioned teaching methods. In some countries the reform of mathematics was restricted to a limited number of experimental classes, driven by highly motivated individuals; in others the curriculum was defined by a central authority, leaving little room for teachers' own initiative. Servais

(1975) concluded that "continental Europe seems more homogeneous than it actually is. In the evolution of their mathematics education some countries have been bold, even rash. Others are advancing more cautiously, more patiently, more deeply" (p. 55). In this state of "permanent confusion", Servais concluded, "we must have confidence."

This diversity in the actual implementation of New Math curricula stands in sharp contrast to the very centralized and coherent preparation of the reform on an international scale. In particular, two organizations served as focal points for discussions and debates among a few dozen mathematicians, psychologists and educational scientists, who were responsible for giving the New Math movement its central orientation. The *International Commission on Mathematical Instruction* (ICMI), founded in 1908, convened the famous Royaumont conference (1959), chaired by its president Marshall Stone and supported by the *Organisation for European Economic Co-operation* (OEEC, later OECD). The proceedings of the conference were published in 1961, and became the first official manifesto of the New Math reform in Europe (Furinghetti, Menghini, Arzarello, & Giacardi, 2008; OEEC, 1961). But previously to the Royaumont conference, the debate had primarily been developed within the small group of members of the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM) / *International Commission for the Study and Improvement of Mathematics Teaching*. The CIEAEM was founded in 1952 on the initiative of Caleb Gattegno, a tireless and versatile researcher and reformer of mathematical teaching methods (Bernet & Jaquet, 1998; Félix, 1985). Gattegno gathered a group of mathematicians, logicians, psychologists and mathematics teachers, who in annual meetings explored the various aspects of the modernization of mathematics teaching. The Royaumont Conference was in effect the outcome of the work carried out by the CIEAEM.

The historiography of the New Math movement in Europe is still in its infancy. Only a few authors have attempted to describe in some detail the curriculum reform in their country (Charlot, 1984; De Bock, Janssens & Verschaffel, 2004; Matos, 2009; Noël, 1993). Others have focused on the main architects of the reform, either in the form of a tribute or as a commemorative text. Most of these accounts have been written by people directly involved or closely connected to the events described. Recently, historians have begun to reassess the reform from a critical distance, carefully retracing the various proposals and controversies which united and divided the small community of reformers (Coray, Furinghetti, Gispert, Hodgson, & Schubring, 2003; La Bastide-Van Gemert, 2006). These studies highlight the importance of differentiating between various approaches and doctrines, and of understanding the epistemological, educational and cultural context of the debates.

This paper will focus on the Belgian contributions to the CIEAEM, in particular on the work of Willy Servais, who served as its secretary from 1956 until 1979. Belgium played quite an important role in the elaboration and implementation of the curricular reform movement at the European and international level. The pivotal role of the Brussels mathematician Georges Papy is generally acknowledged, but at the same time it has tended to overshadow the contributions of his fellow countrymen, which were by no means negligible and did not coincide with Papy's emphasis on the teaching (from an early age on) of abstract mathematical structures derived from algebra and topology. Papy was irrefutably a main actor in the reform movement, but to understand the reception of the New Math reform in Belgium and abroad, it is necessary to look beyond Papy, not in the least in Papy's own country.

A career in mathematical education

As in many other European countries, debates on the improvement of education flared up in Belgium during the aftermath of World War II. One of the earliest initiatives was the foundation of the *Comité d'Initiatives pour la Rénovation de l'Enseignement* (CIREB) in 1945 by a group of supporters of the school system developed by Ovide Decroly. This *Comité*, under the presidency of the physicist Frans van den Dungen, consisted mainly of teachers of the *École Decroly* and professors of the *Université libre de Bruxelles*, among them Paul Libois, whose wife Lucie was director of the school. As professor of geometry and senator for the Belgian Communist Party, Libois was an important intellectual voice, strongly pleading for educational reform (Schandevyl, 1999). At the *École Decroly*, he experimented with new approaches to mathematics teaching on which he would later report during the annual conferences of CIEAEM. In (Libois, 1963) he dated his first attempts in this direction to "almost thirty years ago" (p. 56). In line with the views of Decroly (Libois, 1971), Libois started from the global, implicit notions of the pupils, as they were formed in real life. He also used shadows to show how geometrical figures could be transformed, thus enabling the students to grasp intuitively the idea of transformation. It is as yet difficult to assess the real importance of Libois in the ensuing discussions on the modernization of mathematics. But it seems probable that Libois was at the centre of a group of mathematicians at the *Université libre de Bruxelles*, who would become major proponents of the New Math movement in Belgium. In particular, Frédérique Lenger, the future wife of Georges Papy, worked as an assistant with Libois from 1947 until 1950, teaching at the same time at the *École Decroly*.

A key personality in the math education reform debates in Belgium from the early 1950's is Willy Servais. Unfortunately, there is hardly any reliable information on the early years and education of Servais. Félix (1985) provides some biographical elements based on a document written by Servais' wife Renée. Other information can be found in obituaries or commemorative texts, often without mentioning sources. Many uncertainties remain. Born in Nivelles on 1 February 1913, Servais studied at the *Athenée Royal* (secondary school) of his home town (Paulus, 2009), and went on to study mathematics at the *Université libre de Bruxelles*. He graduated in 1936 and also obtained his teacher's certificate (Paulus, 2009 erroneously mentions 1963). In 1937 he started to teach at the *Athenée du Centre* in Morlanwelz, a secondary school for boys with a secular, anti-clerical background. Félix (1985) mentions that besides mathematics, Servais had a broad interest in poetry, literature and painting. Servais was also president of a unnamed freethinkers' society (Gaulin, 1979).

At the outbreak of the war, Servais was enlisted in the Belgian army as an officer and deported with the majority of Belgian officers to the German prisoner of war camp Oflag III B, situated in Tibor (Cibórz) near the current German-Polish border. During his time as a prisoner, he acted as a mathematics teacher for his fellow prisoners, who wished to prepare for their university exams. He also organized small seminars on advanced mathematics and gave lectures. Although this may seem rather improbable, the extent of intellectual activities in Oflag III B can be imagined by reading the paper by Vandekerkhove (1941), who as a prisoner of war in Tibor constructed two rudimentary telescopes from simple magnifiers and ordinary glasses, and who was kept informed of other astronomical observations through correspondence with a Belgian astronomer. With the help of the Red Cross, Servais was able to obtain several works on mathematics, logic and methodology, in particular by the Swiss psychologists and philosophers Piaget and Gonseth. He also studied the first books of Bourbaki. Christine Keitel (2005) stated that Servais met Hans Freudenthal while in a German labour camp, but this is improbable as Freudenthal was only in 1944 and for a very short time imprisoned at the labour camp in Havelte (Netherlands) (La Bastide-Van Gemert, 2006). It shows, however, how the story of mathematical foundations being studied during the war in prison camps had become part of the collective memory.

Upon his return to Belgium in 1945, Servais resumed his work in Morlanwelz. In 1951, he also started to teach a course on logic at the *Institut Supérieur de Pédagogie*, a teacher training school equally at Morlanwelz. In 1958 he was appointed *préfet des études*. At a lecture given at Neuchâtel in 1961, Servais was introduced as the prefect of the *Athenée* and professor at the *Université libre de Bruxelles* (Séance, 1961, p. 192). He is further described as a specialist in mathematical logic and author of important works in that field. As far we know, Servais did not publish papers on mathematical logic and was never a professor at the University of Brussels, although Félix (1985) and Gaulin (1979) mention that he lectured at the *Université de Mons*, an affiliation which is confirmed (Unesco, 1973). Actually, Servais taught a course on logic at the *Institut Supérieur de Pédagogie* at Morlanwelz, which in 1965 was incorporated in the *Université de Mons*.

Servais entered the international arena in 1951, when he was invited to attend the second meeting of Gattegno's group, which took place in the Belgian town of Keerbergen. Servais rapidly assumed a central position in that group, collaborating closely with Gattegno in the organization of the meetings. In 1952, Servais was one of the founding members of the CIEAEM. Four years later, he was nominated as its secretary, a position he held until 1979. Bernet and Jaquet (1998) describe him as the "soul of the meetings" of the CIEAEM, providing "brilliant syntheses" at the end (p. 8). Servais' amiable character allowed him to remain untouched by the many controversies which divided the international community. Freudenthal wrote that "never in my life, with people I met, was friendship and profound disagreement more closely knitted than in my relation with Servais" (quoted in La Bastide-Van Gemert, 2006, p. 280).

In 1953, Servais founded the *Société belge des Professeurs de Mathématiques* (SBPM), of which he himself acted as president until 1969. The SBPM brought together a few hundred mathematics teachers from both linguistic regions (Dutch and French) and from all school types (in particular from state schools and

Catholic schools). It started its own journal *Mathematica & Paedagogia*. Servais served on the editorial board and was secretary from 1961 until his death (Miewis, 2003). The aims of the journal were reflected in the many (often quite traditional) papers by teachers on examples of successful didactical approaches, the presentation of teaching aids and the exploration of new applications of mathematics in physics or statistics. It also contained reports on and papers from CIEAEM meetings, making sure that the spirit of reform and reflection was transferred to the Belgian community of mathematics teachers.

In 1959 Servais was the Belgian delegate to the OEEC conference, held from 23 November until 4 December at the Centre Culturel de Royaumont in Asnière-sur-Oise (France). This conference would become the turning point in the European New Math movement, in particular due to the vehement attack of Jean Dieudonné on geometry teaching based on Euclid. Servais presented at Royaumont his views on the new curriculum to be constructed, and was appointed (with Paul Libois) as an international expert for the OEEC to prepare a modern syllabus in secondary school mathematics for students who were strong in scientific studies. The syllabus was adopted at the conference in Dubrovnik (1960), and subsequently widely distributed (OECD, 1961). Servais became recognized internationally as one of the main experts on mathematics education and was often invited to lecture and to present reports at conferences. In 1966, Servais inaugurated a training course for teachers in Montréal and Sherbrooke. He also became a member of the first editorial board of Educational Studies in Mathematics (founded by Freudenthal, its first volume appearing in 1968). Servais contributed to the debates in ICMI, wrote articles for ICMI's Unesco volumes on the teaching of mathematics and attended its new conference series ICME. A major achievement was the publication with Tamás Varga of *Teaching School Mathematics* (1971), which provided an often consulted guide to the field of mathematics education. Servais suddenly died in Budapest on 25 August 1979, only a few days after attending the CIEAEM meeting in Veszprem (Hungary).

The Papy era

During the 1950s, Servais started to collaborate with Frédérique Lenger, who was then teaching mathematics at the *Lycée royal* (secondary school for girls) in Arlon. As was quite common at the time, their interest was directed towards the use of mathematical models and concrete materials that could be used to stimulate students (and not only the best students) to discover mathematical structures in their everyday life (Lenger, 1953; Lenger & Servais, 1956). Much of Servais' early work was concerned with the problem of how to help children to make the transition from intuitive experiences towards abstract understanding. In 1955, the SBMP organized a special conference on the topic "Sources concrètes et intuitives de la mathématique", in particular with regard to the conceptualization of space. The use of cardboard models, light projections, Meccano constructions, geoplans, films, electrical circuits and the famous Cuisenaire rods, promoted by no one less than Caleb Cattegno himself, seemed to be the missing link between intuition and abstraction. These views were reflected in the structure of Servais' textbook on plane geometry (Servais & Jeronnez, 1959), following a path from an intuitive approach towards abstract and finally theoretical geometry.

The same inspiration was at the origin of Servais' attempt with Lenger to construct a new experimental program for mathematics teaching at the training school for nursery school teachers. As these teachers had very little first-hand knowledge of mathematics, the new program aimed at providing them with a number of everyday situations in which mathematical problems could be approached in a dynamical way. The curriculum was tested in schools in Arlon and Liège. Subsequently, they sought the advice of Georges Papy, professor of mathematics at the *Université libre de Bruxelles*. Their collaboration soon led to a deeper involvement of Papy in the debates on the modernization of mathematics teaching. Papy was interested in the pedagogical problems of teaching mathematics to young children. He devised a simple system based on Venn diagrams, arrow-graphs and colour conventions to bring out the mathematical structures underlying the real world situations. In 1959 he exposed his ideas during a meeting of the SBPM at Arlon.

Lenger married Papy in 1960 and moved to Brussels, where she worked as a teacher at the *Institut supérieur pédagogique "Berkendael"* in Uccle. Papy himself would also teach at the Froebel training school of the same institution. In 1961 Papy founded the *Centre Belge de Pédagogie de la Mathématique* (CBPM), in which leading Belgian mathematicians and authors of textbooks were brought together (Vazquez, 2008). The *Centre* was to work out, under the strict guidance of Papy, a new curriculum for the first years of secondary schools. With the support of the Belgian government, the curriculum was tested in a number of secondary schools during the year 1961-1962. As a result, Papy was able to publish in 1961 as first draft of his new mathematics curriculum for the first three years of secondary school (Miewis, 2003; Papy, 1961). In subsequent years, the curriculum was modified. Papy himself was not involved in the final version which was submitted to the government. The Belgian minister of education Henri Janne, made the new curriculum for the first three years optional for secondary (state) schools in 1964. After some further negotiations involving several commissions of university mathematicians, teachers and school inspector, and under a new minister, it finally became obligatory for the first year from 1 September 1968. Every year the program was extended for the next year of the curriculum. The Catholic schools followed suit and introduced a similar curriculum. In 1976, the reform was completed by the introduction of a new program for primary schools in Catholic schools and two years later also in state schools.

The reform put an end to the traditional conception of mathematics courses in subdisciplines (arithmetic, geometry, algebra). The foundation of mathematics was set theory, from which the exposition of other topics was deduced in a purely logical way. The main features of the curriculum were the unity of mathematics, the method of introducing in a progressive way fundamental mathematical structures, the need to teach the student how to find mathematical structures in (pseudo) concrete situations, the ability to make calculations and to construct a mathematical demonstration, and the familiarity with logical concepts (Noël, 1999). The aim of the mathematical curriculum was to attain a mathematics that was abstract, structural and algebraic. In the following years, Papy published several volumes of his textbook *Mathématique moderne* (Papy, 1964), which served as the basis for the preparation of school manuals.

The success of Papy's *Centre* in creating and implementing the new curriculum overshadowed some other initiatives. In 1961 the city of Brussels created a *bureau* to prepare the modernization of mathematics teaching. Paul Libois developed a program based on the teaching of geometry, which was used in the *École Decroly*, ut his views were not included in the official reform. There was also a reaction from teachers and inspectobrs involved with schools for technical education, who deplored the loss of geometrical representations and the emphasis on logic and abstract concepts (Smet, Vannecke & Baeten, 2002, p. 488-492). The new curriculum isolated mathematics from other courses as technical drawing, where the understanding of spatial forms was required. The opposition was headed by *Matec*, an organization of mathematics teachers in technical schools. But their protest was in vain. Technical schools had to adopt the new curriculum at most one year after the obligation had been imposed on other schools. *Matec* dissolved after 1969.

Although the new curriculum was prepared by a small group of mathematicians, its success depended on the adequate training of secondary school teachers. From 1960 until 1968, the SBPM and later the CBPM organized special three day workshops in Arlon, where hundreds of teachers studied the new concepts. Apart from these workshops, the CBPM organized weekly study groups in some fifteen cities. It is estimated that some 3000 mathematics teachers were involved in these groups. After the new program was made obligatory, the government itself attempted to organize some form of obligatory training for teachers, but it was not a success and soon abandoned (Noël, 1993). From 1968 until 1979, the CBPM published a journal *Nico* (a reference to Nicolas Bourbaki), in which the new syllabus was further explained to teachers. But Papy himself, together with his wife, turned his attention increasingly towards mathematics teaching at the primary school level, the use of the minicomputer for teaching and research on the mathematical education of young children. When in the late 1970's protesting voices were heard who asked for a reconsideration of some of the most extreme aspects of the New Math, Papy did not enter the debate.

The international renown of Papy reached its apogee in the 1960's. Papy succeeded in consolidating his position in the CIEAEM, of which he became vice-president in 1960, and president from 1963 until 1970. But his dogmatic, and according to some, unpleasant of even insulting behaviour, alienated many of the members. In 1970 he was forced to step down as president and he left the commission (Félix, 1985). Servais, who had assisted Papy for many years and at an early date had welcomed his views, did not leave with him.

Between content and methodology

During the 1960's Servais showed himself a loyal supporter of the curriculum changes proposed by Georges and Frédérique Papy. He wrote with enthusiasm about the choice of set theory as the foundation of mathematical education, and in particular lauded Papy's use of Venn diagrams and arrow-graphs, which were, in his opinion, ideally suited to the mind of young children. He also endorsed their attempts to bring the New Math to primary schools. According to Servais, the development of mathematical understanding in students was based on the experiences and notions acquired at an earlier age (Servais, 1959). It was therefore necessary to introduce ideas and elements for reflective activities "as soon as they can be made accessible to the child. It can be done in kindergarten." (Servais, 1968b, p. 797). But at a closer look, some differences emerge, perhaps obfuscated during the Papy era of the 1960's. Although Servais never directly criticised Papy, it is clear that his vision of the modernization of mathematics did not entirely coincide with those of Papy. In 1975, when discussing the New Math reform in various European countries, he expressed some reservations as to what had been done. He warned that, although "reform fights to get rid of the ancient and to improve radically and once and for all [...] it may lead us to build a new static stage and to deprive us of still valuable parts of our patrimony. What we want is to up-date teaching of mathematics both in content and in method and to keep it alive as a permanent activity" (Servais, 1975, p. 55). Already in 1964, Servais warned: "Let us make no mistake: any syllabus, however sensible and balanced it is, can degenerate into mere dogma in the hands of a dogmatic teacher" (Servais & Varga, 1971, p. 219). For Servais, every reform was but a temporary step, which needs to be continuously evaluated and improved upon in a next phase. The rigidness of any curriculum, including the one created by Papy, could not be left unchanged forever.

Servais' views on mathematical education were greatly influenced by the work of the Swiss mathematician and philosopher Ferdinand Gonseth. Gonseth's theory of idoneism postulated an epistemology based on an endless dialectical series of experiences and representations of objects. Knowledge about the world could only be attained in small steps which had to be reiterated. Even logic was not inborn or predetermined by the internal structure of the mind, but acquired through a chain of interactions with the world. Applied to mathematics, Gonseth pointed to the role of the scheme, a mental image or sketch of reality, which was not a faithful or 'true' representation of the objects but an abstract structure reflecting our experience of the objects. Servais often mentioned Gonseth in his writings and wrote several papers on his work (Servais, 1957, 1970).

Servais in particular emphasized the concept of a *pédagogie ouverte*, an open approach to the learning process. For this reason, he was critical of the use Euclid as a model for geometry courses to young learners. It was a book written for adults, aiming at building a stable and prestigious mathematical structure, but hardly suited to stimulate the mental activity of young children. Servais favoured a more active approach. "If we want our mathematics education to be a learning event, rather than a drill, it is indispensable that we should make ample space for the mathematizing activity of the student," by which he meant an "internal experience and an active intuition" (Servais, 1957, pp. 209 and 212). But the stage of intuition should be followed by a further step towards abstraction, which would allow understanding of the mathematical structures behind the experience.

> "From the point of logic, mathematics is evolving towards an axiomatic structure. At that stage, the intuitive support is reduced to the nominal form of axioms and rules of deductive logic. A mathematical culture should introduce [the student] to these questions. This initiation may be more or less advanced, but in a dialectical pedagogy, it can never be reduced to the dogmatic imposition of a ready-made system of axioms. The role of the system will be better understood if the axioms are obtained after an 'inductive synthesis' and if they are subsequently tested with regard to their deductive scope. […] All should be done to set the abstract mathematical model free from the intuitive relations, so that its structure can serve as the rational skeleton (*ossature*) of other sciences and technology." (Servais, 1957, pp. 212-213).

The practical application of this pedagogical program was provided by the "pedagogy of situations" which was developed by Caleb Gattegno. The idea was to confront the student with an intriguing phenomenon or object, that would stimulate a sustained investigation by the student.

> "The true involvement of students in mathematical work can only be assured by an adequate motivation at their level: pleasure of playing or of competition, interest for application, satisfaction of the appetite for discovery, the affirmation of themselves, a taste for mathematics itself. In order to learn mathematics in an active manner, it is best to present to the students a situation to be mathematized. So today's didactic is based, as far as possible, on mathematical initiations to situations easy to approach at the basic level and sufficiently interesting and problematic to create and sustain investigations by the students. They learn by experience to schematicize (*sic*), to untangle the structures, to define, to demonstrate, to apply themselves instead of listening to and memorizing ready-made results." (Servais, 1968b, p. 798)

It may seem that this sequence of mental activities would in the end lead to the most abstract understanding of mathematical structures, but for Servais this was not the ultimate goal. The same sequence could be found at different levels of mathematical knowledge, and it had to be repeated over and over again for every new step in the learning process. As in the open philosophy of Gonseth, any acquired knowledge was always the starting point of new investigations.

A direct consequence of this pedagogical view, was Servais' work on concrete models. These models were not simply toys nor every-day situations, but stylized objects which could provide a concrete form to mental activities and which allow for active manipulation aiming at a better understanding of mathematical ideas and structures. Typical examples of concrete models were the Cuisenaire rods, the Dienes multibase arithmetic blocks, the geoplan, mathematical films and projections, etc. Also the Venn diagrams and the arrow graphs utilized by Papy, were for Servais good examples of concrete models, since they acted as a middle step between actual class room situations (putting a rope around objects) and mental images. Apart from being useful, a concrete model also had to be attractive and appealing. Servais himself experimented from the late 1940's with electrical circuits to study the properties of conjunction and disjunction (Servais, 1969b). The repeated and investigative manipulations would lead to mental representations which subsequently will become independent of the model. "Every perception or action derived from the concrete duplicates itself in mental imagery; this becomes structured and can then be recalled in its own right" (Servais, 1970a).

The same pedagogy allowed Servais to interpret the role of applications of mathematics. Any application was a test for one's knowledge and an opportunity to learn. The inclusion of applications was from the start an important argument to legitimate the reform. Once the basic structural order was acquired in mathematics, it was easy to apply them to the understanding of other sciences. Servais dreamed of a better coordination and a dialogue between school programs for mathematics and physics. "Without physics and the other sciences, mathematics could be reduced to a formal game […]. Without mathematics, physics would regress to the level of an at most qualitative phenomenological description" (Servais, 1966b, p. 187). But Servais was aware that this was not easily done in practice. It depended too much on the willingness of the teachers, and he conceded that the mathematics teacher may not know enough physics, and vice versa. He had more hopes for the integration of probability theory and statistics in the curriculum, topics which could also be based on set theory and which had an enormous impact on modern natural and human sciences. But these topics were not accepted as essential parts of the reform (Servais & Varga, 1971).

Servais' pedagogical views do not seem to have changed much over the years, but from 1958 on, Servais became increasingly involved in the preparation of an actual new mathematical curriculum. In his first attempts with Lenger, he already included the basic concepts concerning sets, relations, elementary functions and topology. Over the next years, Servais collaborated with Papy and the CBPM to work out a comprehensive syllabus for the new curriculum. In 1964, Servais was able to present the basic guidelines of the new syllabus. The topics could be arranged, depending on the needs of the course, in either a logical, a practical or a psychological order. But, in any case, as a result of our improved understanding of mathematical ideas and theories, mathematics had recovered its unity through set theory. Servais was convinced that this basic unity provided the solution to the pedagogy that he had in mind: as the goal of mathematics teaching was to activate the mind of the child towards grasping the mathematical structures in the world around him, it was necessary to define these structures and to make them the backbone of the whole syllabus.

> "Teaching should proceed in the light of these findings and, using set theory as a basis, should build up a more unified construction, structured by homogeneous modern ideas. It should do this not only to present an authentic, albeit elementary, image of the science of mathematics, but also to develop the psychological ability to use mathematics as a tool in a broader, more deliberate and more effective way." (Servais & Varga, 1971, p. 217)

Although the preceding text was first published in 1964 in a OECD document, Servais left it unchanged in the publication in 1971. He also added a detailed syllabus based on several sources, including the work done at the CBPM. But Servais was careful to point out that any syllabus had to be implemented with great caution. The course could be adjusted to conditions in each particular country, but he added, "mathematical education of this kind depends not so much on the syllabus as on teaching method, for it is only good teaching that can make a syllabus meaningful" (Servais & Varga, 1971, p. 219).

Servais always maintained an open attitude towards the syllabus, and showed great willingness to compare and integrate the proposals originating from various countries. But he did not change his pedagogical commitment to his *pédagogie ouverte*. He summarized his views on the teaching of mathematics in 1967, during an ICMI conference organized by Hans Freudenthal in Utrecht on "How to teach mathematics as to useful". He sketched the process of mathematisation and the pedagogy of 'stimulating situations and the progressive genetic axiomatisation, for which he drew attention to what had been achieved in the Belgian reform (Servais, 1968a, p. 45). At the end he summed up the basic conditions for any reform of mathematics.

> "Let us formulate at least some general wishes: (1) that one teaches first and foremost the essential core of mathematics, 'its marrow substance'; (2) that one offers to every child the opportunity to acquire early on a structural thinking, as conscious, extended and organized as his intelligence will allow, (3) that one makes the pupils aware of the formal and real beauty which originates from the functional order of mathematics." (Servais, 1968a, p. 53)

In the end it was the beauty of mathematics and its crucial role for the emancipation of the human mind, which motivated Servais and many of his fellow mathematicians. As Servais wrote in the first editorial of *Mathematica & Paedagogia*,

> "Our time marks the beginning of the mathematical era. […] This fact, whatever the reactions, the opinions and the judgments it may provoke, increases the responsibility of every teacher, who, no matter on which level, teaches mathematics. […] If it befits to be worthy of a mathematical tradition, it is also important to allow the mathematization [of the world] to come. As much as it is true that he who devotes his life to teaching, accepts a mission of a world gone-by to build a world being born. The responsibility towards the future is greater than loyalty towards the past" (Servais, 1954, p. 89).

It is probably impossible to fully understand the motives and the efforts of the early reformers without taking this moral commitment into account. Whether legitimated by the fundamental advances in higher mathematics, or by the growing number of applications of mathematics in other sciences, the basic view was that mathematical thinking was a necessary tool for any modern citizen, and that it was the responsibility of mathematicians to educate their fellow citizens. Mathematics, Servais wrote, was "an activity through which structures were created that allowed people to grasp reality" (Servais, 1968a, p. 53). The modern world could only be understood and lived in through an understanding of modern mathematics.

Teachers and experts

The New Math movement was motivated by a desire to improve the existing mathematical instruction in schools. It was generally recognized and often repeated that for many students (and their parents) mathematics at school had become a nightmare. Of course, Servais (1957) observed, there were some very talented students who entered the realm of mathematics with great ease, but so many others only picked up just enough to pass their final exams. The result is that many adults, too are disgusted by mathematics, and, as Servais undoubtedly knew from his own experience in the *Athenée*, "the fear of mathematics is more hereditary than the gift for mathematics" (p. 208). Many teachers, Servais observed, felt that mathematics was only reserved for a small intellectual elite, and they were satisfied to make life difficult for the others.

But was there a real problem? Was it possible to identify the elements which had to be corrected? The Belgian reformers, including Servais, hardly ever mentioned empirical information on the current situation of mathematics teaching. Actually, according to international standards of that time, the quality of the teaching of mathematics in Belgian schools was not particularly problematic. In a comparative UNESCO study of the achievement of thirteen year old students in 12 European countries, carried out in 1959-1961, Belgium (actually only a small region in Hainaut was studied) stood out with the highest scores for mathematics, before France and Switzerland. The scores were "particularly high for items which require reasoning and the use of concepts" (Foshay, Thorndike, Hotyat, Pidgeon, & Walker, 1962, p. 49). Since the 1930s, Belgian primary education (ages 6-12) was considered a highly-esteemed model of child-centred pedagogy, as introduced by pedagogues such as Ovide Decroly. It was also intensively studied and monitored by school psychologists, who created empirical tests to measure the level attained by the pupils at different stages of their development. In particular with regard to mathematics, Fernand Hotyat, the director of the *Institut Supérieur de Pédagogie* in Morlanwelz, the same institution where Servais taught logic, had made detailed studies of levels of mathematical understanding and conceptual development (Hotyat, 1936, 1948, 1952). He concluded that generalized and symbolic reasoning was usually not attained by pupils before the age of 13. Younger pupils would still return to a more practical level to solve analogous problems with a trial and error approach (Hotyat, 1961). This would suggest that a deeper understanding of mathematics could only start in secondary school. Apparently, these empirical findings were ignored by the debating teachers in Servais' society. Servais and others maintained, but only based on their own experience, that it was possible to start that type of mathematical education from a very early age.

The lack of collaboration between the mathematicians and the education researchers can probably be understood by looking at the widely diverging interests of both groups. Whereas mathematicians studied the content and structure of mathematics and its reflection in the psychological development of children, the educationalists attempted to determine ways of measuring achievements and more in particular, the intellectual and social determinants of low achievers and the characteristics of learning difficulties. In the 1950's, most empirical researches in Belgium were directed towards problems on professional orientation, the length of obligatory schooling, the use and effect of homework, the social determinants of school achievements, etc. Furthermore, although there was a body of research on mathematical instruction, it was in the first place concerned with primary schools and arithmetical skills. But it was exactly the emphasis on these skills in teaching that the reformers were attempting to replace by other forms of abstract mathematical understanding.

Hotyat (1961) recognized that there was still much work to be done for secondary schools and he did see an opportunity for collaboration between teachers and educational researchers. He agreed that the reform initiatives, taken by individual schools or teachers, were symptoms of a "profound movement." Indeed, the first results were "encouraging". Yet, he clearly was not impressed by the work done by mathematicians and mathematics teachers.

> "[Mathematics] teaching in secondary schools has for a long time been limited to the selection and promotion of a purely academic elite; only to the extent that more and more students are enrolled, one has begun to look for more efficient teaching methods. On the other hand, the topics treated on this level involves thought processes on the nature of which mathematicians, logicians and psychologists have not yet reached a complete agreement. The mathematicians want to elaborate a program that is in harmony with the evolution of their science and with the needs of technology. Many of them are willing to break with the traditional teaching, but the proposals do not always agree among each other as to the extent of the new chapters, nor as to the old branches that should be sacrificed. The majority of secondary teachers do not welcome objective researchers on these matters." (Hotyat, 1961, p. 238).

There was indeed some distance between the educational researchers and the teachers wanting to rejuvenate their mathematical courses. Hotyat conceded that although psychologists did agree on the developmental model of intelligence, they did not have an understanding of how this would square with the required logical development of mathematical concepts. The relevance of empirical studies to the reforming debates of the mathematicians was not immediately clear. For the education researchers, the problem was to improve the efficiency of the existing teaching methods, and the understanding of the causes for failure. They often preferred to study the way children assimilated a doctrine that was taught to them, or concentrated on the mental operations of students in solving definite problems. Hotyat himself based his research on the somewhat outdated theory of the French logician Edmond Goblot, who put the nature of mathematical reasoning in the correct application of previously established rules to mental situations. This enabled the empirical study of performance and efficiency of teaching methods (e.g. whether or not to use visual aids in geometry classes), but had little to say on the desirability and relevance of the skills or knowledge acquired. In particular at the time when Servais and other were focusing on the selection of topics and basic structures to be included in the curriculum, they had little interest in learning about the shortcomings of the old curriculum.

From the side of the psychologists, Hotyat welcomed the invaluable expertise of school teachers, who better than anyone else understood the problems involved in teaching. He saw a common ground between teachers and educational researchers in the attention of teachers to the development of teaching aids and in the shared aim to increase the efficiency in the transformation process of intuitive understanding to abstract reasoning. Hotyat also acknowledged that psychological theories had still very little to offer that could be used to design an actual reform in teaching practices. On the other hand, to him, the efforts of teachers regarding innovating didactical approaches lacked the control of objective measurement, necessary to build a truly experimental science. Their personal experiences led to a form of "experiential" (or experience-based) psychology, but did not contribute to "experimental psychology". Their proposals remained mere hypotheses as long as they were not supported by objective verification.

"The possibility of progress [in the modernization of mathematics teaching] depend on a close collaboration between teachers and psycho-pedagogues; the first group, which is familiar with their programs and knows the difficulties of their students, is best qualified to suggest new venues of research; the second group, trained in research and experimental control, is in the best position to assess whether the new ideas are only simple illusions or truly stimulating actors towards a better performance." (Hotyat, 1961, p. 245)

Hotyat consciously did not include the (academic) mathematicians in his "close collaboration". Their motivation was to adapt the mathematics curriculum in secondary schools to the requirements of modern science. It was their concern to improve the level of mathematics, in particular with regard to abstraction and logical reasoning. Mathematicians should acquire a better understanding of the mental processes of logical thought, and accordingly prepare and assist the talented students. This appeared to be outside the concern of Hotyat's view on educational research, and he probably considered this to be of limited immediate importance for the reform of mathematical instruction at hand. Moreover, he showed himself rather critical of the "audacious experiment" currently being carried out by Papy, aimed at introducing set theory and intuitive topological notions in several teachers' training colleges. Hotyat did not discuss this aspect of the reform movement, but he ended with a clear message to all parties concerned:

"The whole problem of quantitative and qualitative objectives in the teaching of mathematics ought to be reconsidered from the point of view of the psychology of the pupils and without regard to doctrinal disagreements; if this cannot be achieved, the debates between teachers and psychologists will be lost in the mist of theoretical conflicts." (Hotyat, 1961, p. 245).

The distinction made by Hotyat between university mathematicians and school teachers may be revealing in reconstructing the environment of the reform efforts in the early years of which Papy and Servais were the leading figures. There were several types of (mathematics) school teachers, who followed different training paths. For primary schools, the aspiring teachers (who had to teach all subject matters, including mathematics) started a special training program of four years at the age of 15, after completing only the lower years of secondary school. Their final grade did not give entrance to a university education. Teachers for the lower years of secondary school had completed the full six years of secondary school and had subsequently followed a non-university course of two years. A third group of teachers had received a full four year university training (in mathematics) and had taken an additional pedagogical program, which allowed them to teach the higher level secondary school grades. They would also be qualified to teach courses at non-university teachers' colleges. In general, there was a latent desire for academic recognition and increased social status on the part of the non-university educated school teachers (Depaepe & Simon, 1997).

Probably all the main protagonists in the reform of mathematics in the 1950's and 1960's in Belgium belonged to the last category and, thus, were university trained mathematics teachers. Their view of mathematics was clearly defined by their being educated first as mathematicians. Mathematical arguments on the unity and clarity of mathematical structures would be positively welcomed, and mathematical understanding considered a valuable goal in itself. Their personal experience in secondary school teaching was confined to the higher level (16-18 years), where they only met students preparing themselves for further studies at the universities or non-university professional colleges. It is to this group of students that Servais referred in his 1953 editorial: "These students, our students, who are now working in our classrooms, will become, tomorrow, mathematicians, physicists, officers, technicians or, simply, representatives of hundreds of professions in which mathematics will play a more efficient role" (Servais, 1954, p. 89). But this was only a minority of students. At the time, the age of obligatory schooling in Belgium was set at 14 years (Mallinson, 1967). In general, only 40% of all students did indeed complete their secondary education. The teachers of the higher secondary school grades were thus primarily in contact with an intellectual elite of students, usually also from a more affluent social background.

In contrast, most of the changes brought about in the mathematical curriculum concerned the first three years of secondary education and primary education. The burden of teaching the new syllabus fell on non-university trained teachers, and the mathematical enthusiasm of the intended student audience was on average probably lower than the university trained teachers had imagined. But the non-university trained teachers were in no position to react. Their own teachers at the training colleges, their school inspectors, and their colleagues in the higher sections of secondary schools all defended the soundness of the reformed curriculum. Furthermore, it can be assumed that the New Math reform played an role in the emancipation of non-university teachers. The extra-curricular workshops they attended and the public attention given to the new mathematics may have increased their professional status, and made it more acceptable that a teacher's profession was not a simple affair. Whether these effects did indeed take place, should be further analysed.

Noël (2002) has argued that the debates on mathematical reform in Belgium after World War II should also be seen in the context of the emancipation of the mathematical and scientific sections in secondary schools, at a time when elite students were still largely directed towards humanistic studies. The introduction of the new mathematics, which, at least in the mind of the reformers, raised the intrinsic value of the courses and brought them nearer to the academic standards of the field, may have given mathematics a status comparable to the classical courses on Latin and Greek. That Latin-Greek remained for a long time the elite orientation in Catholic schools, whereas state schools were more inclined to emphasize science and mathematics, may also have played a role in the position taken by the New Math reform in the turbulent political atmosphere in Belgium in the 1950's. Although the documents show no political agenda and indeed point to a non-exclusive participation of teachers from both state and Catholic schools, it may not be a mere coincidence that enthusiasm for the New Math reform first originated in the circles around the *Université libre de Bruxelles*, a university devoted to secular and freethinking commitment.

Conclusion

The career of Willy Servais spanned almost three decades. Therefore, he may be taken to be a privileged witness of the early development of research in mathematics education, particularly marked by the New Math reform of the 1960's. During this period, the reform movement went through several phases (Bernet & Jaquet, 1998). From World War II until the Royaumont conference, the meetings of CIEAEM were dominated by foundational issues, confronting debates on mathematics, educational psychology and teaching methods. Although we are poorly informed about Servais' mathematical background, it is clear that his interest in these debates was primarily directed towards the formulation of active forms of teaching. Experimenting in his classroom with electrical circuits to explain the laws of logic, he discovered the work of Gonseth and his open approach to scientific methodology. Only rarely did he mention Choquet or Dieudonné, the leading mathematicians in CIEAEM, in his papers. His primary concern was always the invention of "stimulating situations" for the learner, by devising concrete models adapted to particular mathematical topics.

By the end of the 1950's, Servais became involved in the creation of an actual syllabus. His first attempts in collaboration with Lenger were positively received and he continued to work on this for several years. In this he was probably overwhelmed by the imposing personality of Georges Papy, who translated the Bourbaki view of a unified mathematics into a comprehensive syllabus. Although we need to know more about these crucial years of collaboration, it appears that Servais enthusiastically supported Papy, but without losing sight of his own pedagogical interest. Committed to a *pédagogie ouverte*, he appreciated the work of others and tried to integrate it into his own. This reflected the increased international outlook of the reform movement as exemplified by CIEAEM being absorbed by OECD and Unesco projects, and by the more prominent role of ICMI (which included U.S. mathematicians) in the field.

In the 1970's the reform movement was well under way in a number of countries. It was clear that not all countries followed the same Bourbaki line. The discussion on the 'ideal' syllabus lost much of its importance, and was replaced by a renewed interest in pedagogical modernization. Servais had no difficulty in adapting to the new agenda. Although his views, inspired by Gonseth, were by now superseded by a new generation of professional specialists in mathematics education, he remained a highly esteemed representative of the modernization movement. It remains to be seen how his ideas were received and whether they contributed to the development of modern pedagogical views.

Finally, the figure of Servais offers a particularly interesting case because of his long time involvement in both the international and the national movement. In Belgium the debate on the New Math reform was effectively monopolized by Georges Papy, Frédérique Lenger and Willy Servais. Once the basic choices had been made, all effort was directed towards the diffusion of the new curriculum and the training of teachers. There was also but little contact with education experts. But on the international level, debates remained much more open and diverse. It is not known how much of these debates filtered through to the Belgian teachers' community, or how much of the Belgian experience contributed to the positions taken by international educational researchers. Servais was probably right when he observed that Continental Europe did seem more homogenous than it really was.

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