Stress concentrations in hybrid unidirectional fibre-reinforced composites with random fibre packings

Yentl Swolfs^{*1}, Larissa Gorbatikh¹, Ignaas Verpoest¹

¹ Department Metaalkunde en Toegepaste Materiaalkunde, KU Leuven, Kasteelpark Arenberg 44 bus 2450, Belgium

^{*}Corresponding author. Tel.: +32 16 32 12 31; fax: +32 16 32 19 90.

E-mail address: yentl.swolfs@mtm.kuleuven.be (Y. Swolfs).

Abstract

Strength models for fibre-reinforced composites often rely on the calculation of the stress concentrations around a single broken fibre. This paper presents the first results for stress concentrations in unidirectional hybrid composites, more specifically around a broken carbon fibre. The centres of the carbon and hybridisation fibres are randomly placed in a two-dimensional packing. The common assumption that both fibre types have the same fibre radii, is proven to lead to significant errors. The relative ratio of the volume fraction of the two fibre types only has a minor influence on the stress redistribution. A small increase of the stress concentration factors on both fibre types is noted with decreasing carbon fibre content. The ineffective length, which is a measure of the length of the influenced zone, remains unaffected. A stiffer hybridisation fibre reduces the SCFs on the hybridisation fibre, while this influence on the SCFs on carbon fibres is much smaller. The influence of the hybridisation fibre on the ineffective length is again small. The differences with existing literature are explained based on the more realistic packings in this paper. These results should now be implemented in a model to predict the influence on the strength of hybrid composites.

Keywords: C. stress concentrations, A. hybrid composites, C. Finite element analysis, C. Modelling, random fibre packings

1 Introduction

Hybrid composites are composites in which two different types of fibres are combined. The most brittle fibre in hybrid composites is almost always carbon fibre, while the second fibre, called the hybridisation fibre, typically has a higher failure strain. Popular hybridisation fibres are glass and aramid fibres. By adding hybridisation fibres to carbon fibre composites, several improvements can be achieved [1-5]. If the inner layers of a thick carbon fibre reinforced composite are replaced by glass fibre composite, then the total price of the hybrid composite significantly decreases, while flexural and wear properties can remain unchanged. Other properties, like impact [4] or fatigue resistance [5], can even be improved. However, the most notable improvement is the hybrid effect, which is defined as the apparent failure strain enhancement of the carbon fibre.

Hayashi [6] discovered the hybrid effect in glass/carbon hybrids. This effect was attributed to the difference in thermal contraction of both fibres [1, 7]. Later, other authors Zweben [8] proved that the thermal contraction could only account for a hybrid

effect of +10%. To explain hybrid effects of up to +50% [1, 2], two additional effects were proposed: a statistical effect and a fracture mechanics effect. The statistical effect is caused by the improved dispersion and the decrease of the relative content of the carbon fibre [9]. The fracture mechanics effect is related to the hybridisation fibres bridging the crack tip formed by broken carbon fibres [8, 9]. Both effects also illustrate why the hybrid effect is more pronounced at low hybrid volume fractions [2], which is defined as the volume of carbon fibres over the total volume of fibres.

To understand and improve hybrid composites, there is a need for models that can predict this hybrid effect. The first models for fracture propagation in hybrid composites were based on the simple shear lag model developed by Hedgepeth [10]. Hedgepeth assumed that the fibres are the only component in unidirectional composites which carry axial load. The matrix is assumed to carry only shear loads. These assumptions allowed Hedgepeth to obtain an analytical solution for the stress redistribution after a single fibre failure in a 1D packing, which is a single row of parallel fibres [10]. Hedgepeth and Van Dyke later extended his approach to the more realistic, 2D packings, where the parallel fibres are arranged in square or hexagonal packings [11]. The stress redistribution is often simplified to two characteristics: stress concentration factor (SCF) and ineffective length [12, 13]. The SCF is the ratio between the stress on an intact fibre and the stress applied at infinity, while the ineffective length is a measure of the length over which the stress in the broken fibre is recovered. The latter is a crucial parameter as it relates to the extent over which the neighbouring fibres are subjected to stress concentrations.

Zweben [8] calculated both characteristics for hybrid unidirectional fibre-reinforced composites by extending Hedgepeth's shear lag model. Zweben assumed a one-dimensional packing of alternating carbon and hybridisation fibres and calculated the corresponding SCFs and ineffective lengths. With some additional assumptions, Zweben identified the three main parameters determining the hybrid effect: (1) the ratio of the failure strains of both fibres, (2) the ratio of the SCF in a composite with only carbon fibres over the SCF of a hybridisation fibre next to a broken carbon fibre, and (3) the ratio of the ineffective length in a composite with only carbon fibres to the ineffective length in a composite with both fibres. Fukuda, Chou and co-workers [9, 14-17] further developed this approach. Fukuda [15] made several improvements to Zweben's theories. A consequence is that for the ratio of the SCFs, the SCF of the nearest carbon fibre rather than the nearest hybridisation fibre is considered. Fukuda and Chou [9] demonstrated that the fracture propagation in hybrid composites occurs more gradually than in non-hybrid composites. In a subsequent paper, the same authors also revealed a decrease in the SCF on the carbon fibres, but an increase for the hybridisation fibres [14].

The failure strains of both fibres are measured experimentally by single fibre tests. The SCF and ineffective length have been measured experimentally using Raman spectroscopy [18-21], but are often calculated using shear lag models. The latter models, however, have some severe disadvantages when applied to hybrid composites. Firstly, they do not allow anisotropic fibres, which results in significant errors [22]. Secondly, they are unable to cope with random distribution of both fibre types. An alternative approach to calculate the SCF and ineffective length is the finite element

method. This approach results in a more accurate prediction of the stress redistribution [23]. Unfortunately, it is also computationally intensive. It cannot handle enough fibres to fully describe the statistical nature of composite failure. Therefore, the relevant data should be extracted from the FEM stress fields and put into a separate strength model [13, 24]. The results presented here are the first step in this procedure. This paper will describe how the SCF and ineffective length depend on the fibre radii used, the hybrid volume fraction and the type of hybridisation fibre. To allow for a proper comparison with the literature, all materials are assumed to be linearly elastic. A subsequent paper will demonstrate how these results are affected by matrix plasticity and how they can be incorporated into a strength model.

2 Model description

This section describes the finite element model which is used throughout this paper. This consists of a 3D model, with a 2D random fibre packing (see Fig. 1). The fibre in the centre of the cylindrical model is always a carbon fibre, while the other fibres can either be carbon fibre or hybridisation fibre. The hybridisation fibre is either a glass or aramid fibre and is assumed to remain intact, as their failure strain is typically higher than that of carbon fibre. The carbon fibre in the centre of the model is assumed to be the only broken fibre; all the other fibres are assumed to remain intact.

The model consists of three steps: the generation of random fibre packings, the creation of the finite element model and extraction of the data from the stress field. This procedure is based on the approach developed earlier in [22], but uses an adapted random fibre packing generator. The generator, which was developed by Melro et al. [25], was extended to work with different fibre radii.

As in the original generator, a three-step procedure was followed. The first step creates random fibre locations within the square representative volume element (RVE). The type and radius of the fibre are decided based on the hybrid volume fraction. If the current hybrid volume fraction is larger than the required fraction, then a hybridisation fibre is chosen. Otherwise, a carbon fibre is added. The newly generated fibre is added to the RVE, if it does not overlap with the other fibres. In the second step, the generator tries to move each fibre closer to its three nearest neighbours. This is done to create more open space to thereby increase the probability that a new fibre can be added in the first step. This step is explained in great detail in [25]. The third step moves the fibres at the edges of the RVE inward. This again creates more open space for the first step. The second and the third step remained unaltered compared to [25], except for the criterion which checks for overlapping of fibres. The overlapping criterion is used in all three steps and checks whether fibres overlap, taking into account the fibre radii.

These three steps are repeated until the required fibre volume fraction is reached. Since a more efficient packing is possible in packings with two distinct fibre radii [26], higher fibre volume fractions can be achieved. In all models presented in this paper, the overall fibre volume fraction was set to 70%. The hybrid volume fraction was varied between 0% and 100% of carbon fibre (CF): 0%CF, 20%CF, 50%CF, 80%CF and 100%CF.

Based on the two-dimensional packings, three-dimensional finite element models are created. To reduce edge effect, a circular model of 112 μm diameter is cut out of the square RVE of 200 μm by 200 μm . This was chosen large enough for the stresses around the broken fibre to be unaffected by the size of the model. The total amount of fibres included in each model depends on the hybrid volume fraction and varies between 60 and 180 fibres per model. A 3D view of the model is presented in Fig 1a and has a length of 210 μm , which is roughly six times the ineffective length. Fig.1b is a packing in which the hybridisation fibre is assumed to have the same fibre radius, while Fig. 1c uses two different fibre radii.

Figure 1: Illustration of the models with carbon fibres in black, glass fibres in white and matrix in purple: (a) 3D view of the entire model with the applied boundary conditions, (b) top view with the same fibre radii, and (c) with different fibre radii.

The properties of the matrix and fibres are summarised in Table 1. This includes the fibre radius R and the engineering constants: longitudinal Young's modulus E_L , transverse Young's modulus E_T , longitudinal Poisson's ratio ν_{LT} , longitudinal shear modulus G_{LT} , and transverse shear modulus G_{TT} . Glass and aramid fibres were chosen as they are the most common hybridisation fibres in literature. Due to the wide variety of aramid fibres, two types were chosen: Kevlar 29 and Kevlar 49. For both the aramid and glass fibres, a fibre radius of 12 μm was used. To allow a proper comparison with available literature, linear elasticity and perfect bonding were assumed for all materials.

Table 1: Engineering constants of the materials [27-30]

Type	Abbreviation	R (μm)	E_L (GPa)	E_T (GPa)	ν_{LT} (-)	G_{LT} (GPa)	G_{TT} (GPa)
Epoxy matrix	/	/	3	3	0.40	1.07	1.07
Carbon fibre	CF	3.5	230	15	0.25	13.7	6
Glass fibre	GF	3.5 or 6	70	70	0.22	28.7	28.7
Kevlar 29 fibre	K29	6	62	2.4	0.36	1.8	0.882
Kevlar 49 fibre	K49	6	124	6.9	0.36	2.8	2.8

The material properties of the fibres are assigned while the packing is being generated. The middle fibre, which represents the broken fibre, is always a carbon fibre. Five packings are generated for each of the five hybrid volume fractions. In these packings, fibre radii were used which are typical for carbon, glass and aramid fibres. Moreover, five packings are generated in which both the CF and GF are given a 3.5 μm fibre radius. Please note that 0%CF models contain exactly one carbon fibre as broken fibre.

The boundary conditions are applied as in [22] and are illustrated in Fig. 1a. The top plane is given a displacement equivalent to 0.1% strain, while the bottom surface was subjected to symmetric boundary conditions in the fibre direction. The latter condition is not applied to the bottom surface of the carbon fibre in the centre of the model, meaning this surface is traction-free. This represents the broken fibre, which is broken in the bottom plane. The lateral surface of the cylindrical model is traction-free.

Finite element models result in the full stress and strain field. From these models, two important characteristics, the SCF and ineffective length, are extracted, as previously described in [22]. The longitudinal fibre stresses are not constant over the fibre cross-section, but are higher near the broken fibre. Locally, the Weibull strength in a small volume would also be higher, which means the failure probability of that small volume remains low. This justifies averaging the stresses over cross-sections parallel to the crack plane. The ineffective length is then defined as twice the fibre length over which 90% of the longitudinal fibre stress is recovered. The SCF is defined as in [22] and is the maximum value of longitudinal fibre stress divided by the longitudinal fibre stress far away from the crack plane. The latter stress can be assumed to be equal to the stress at infinity, as the model length is about six times the ineffective length. For most of the intact fibres, this maximum value occurs slightly beneath the crack plane. Mesh verifications were performed to ensure that this artefact is not related to the mesh nor to the extraction procedure. This artefact is related to stress singularity around the broken fibre, but does not significantly influence the results.

3 Results

3.1 Influence of the fibre radius

In literature on hybrid composites, a common modelling assumption is that both fibres have the same radius. This simplifies many of the models. In this section, this assumption will be assessed by comparing the SCFs for packings with the same radii and two different fibre radii for GF and CF. An example of these packings can be found in Fig. 1b and c. All SCFs are plotted as a function of the normalised distance. This distance is defined as the surface-to-surface distance d between the broken and the intact fibre, divided by the radius R of the broken carbon fibre. This distance is illustrated in Fig. 2.

Figure 2: Schematic drawing of the normalised surface-to-surface distance d/R

Fig. 3a presents the SCF as a function of the relative distance. This is done for five realisations with the same radii and five with different radii. The results are split up into SCFs on GF and CF, both around a broken CF. For models with the same fibre radii, two distinct trend lines appear. The SCF on the glass fibres is about twice as high as for the carbon fibres. This can be understood from the following argument. The load that used to be carried by the broken fibre is now transferred onto the surrounding fibres. Since the amount of load that needs to be redistributed over the intact fibres remains the same, the load distribution can be assumed to be similar. Since both fibres have the same radius, they will also carry the same additional stress. As the glass fibres have a lower axial stiffness, however, the stress level before the fibre breakage was lower. Hence, the relative increase of the stress is larger, even though the additional stress is the same. The absolute stress increase is the same for carbon and glass fibres, but the higher relative increase in the glass fibres results in a larger SCF. In summary, the observed effect is partially caused by a different normalisation point.

Figure 3: Stress redistribution for 50%CF packings, with the same and different radii: (a) The stress concentration factor as a function of the normalised distance from the broken fibre, and (b) the ineffective length. Five realisations were calculated for both cases.

For the models with different fibre radii, the carbon and glass fibre data points seem to lie on the same trend line. This has two reasons. Firstly, if two fibres with the same surface-to-surface distance are considered, the centre of the larger fibre is farther away from the broken fibre than from the smaller fibre. Since Fig. 3 proves that the SCF is strongly distance-dependent, this tends to yield lower SCFs in the thicker fibres. Secondly, since the glass fibres now have a radius of $6\text{ }\mu\text{m}$, the cross-sectional area is increased by a factor of about 3. To carry the same load, this larger cross-section will be subjected to a three times lower SCF. Based on the latter argument, it can be expected that the glass fibre data points should be lower than the carbon fibre data points. This is not the case due to the influence of the anisotropy of the fibres. As proven in [22, 31], anisotropic fibres result in a higher SCF compared to isotropic fibres with the same stiffness. The complex interdependence between different material and model parameters makes the outcome difficult to predict. In conclusion, it can be stated that the nearly coinciding trend lines are a coincidence, which is determined by the combination of material properties and fibre radii.

Figure 3b demonstrates that ineffective length is also affected by the fibre radii, due to geometrical reasons. Fig. 1b and c visualise that packings with different fibre radii have less fibrous material nearby the broken fibre. Since the stress in the broken fibres is recovered by shear loads in the surrounding material, it is related to the homogenized shear stiffness. Less fibrous material nearby the broken fibre, results in a slower stress recovery and longer ineffective length for the models with different radii.

From this discussion, it is clear that the assumption of the same radii for both fibres introduces large errors. This should be avoided in future models for hybrid composites. Packings with different fibre radii will be used in the rest of this paper.

3.2 Influence of the hybrid volume fraction

The hybrid volume fraction, which is defined as the volume of carbon fibres over the total volume of fibres, is an essential parameter for hybrid composites. In literature, it is commonly stated that lower hybrid fibre volume fractions, which is equivalent to low carbon fibre content, result in a higher hybrid effect. To further understand this effect, carbon fibres will be hybridised with glass fibres in five different hybrid volume fractions: 0%CF, 20%CF, 50%CF, 80%CF and 100%CF. Five realisations of the microstructure are generated for each hybrid volume fraction and one of those realisations for each fraction is illustrated in Fig. 4. It should be noted that 0%CF contains one carbon fibre in the middle, which is broken and surrounded by glass fibres only.

Figure 4: Example of one of the five realisations for each hybrid volume fraction.

For the sake of clarity, the results are split up into glass fibre (see Fig. 5) and carbon fibre data points (see Fig. 6). Results are plotted for all five realisations of each of the five hybrid volume fractions. For glass fibres, the SCFs decrease slightly with increasing carbon fibre content. This is due to the increased longitudinal composite stiffness. The stiffer composite takes up more stress and hence reduces the stress carried by the glass fibres. Upon close investigation, a similar, but smaller decrease can be observed for CF in Fig. 6. This confirms the results in [22], in which a small influence of the fibre stiffness is observed for non-hybrid composites.

Figure 5: Stress concentration factors as a function of the distance from the broken fibre for packings with different fibre radii. The influence of hybrid volume fraction is shown for glass fibres.

Figure 6: Stress concentration factors as a function of the distance from the broken fibre for packings with different fibre radii. The influence of hybrid volume fraction is shown for carbon fibres.

The influence of the hybrid volume fraction on the ineffective length is illustrated in Fig. 7. The ineffective length is expressed relative to the radius of the broken carbon fibre. Similar to the SCF results, the ineffective length slightly decreases with increasing hybrid volume fraction. Two effects are counteracting each other in this case. The first effect is the lower shear modulus of the carbon fibre, resulting in a lower composite shear stiffness at higher fibre volume fractions. This results in slower stress recovery at higher hybrid volume fractions and hence a higher ineffective length. This trend is not observed, as the second effect appears to be stronger. In the 0%CF model, the small, broken carbon fibre is surrounded by larger glass fibres, resulting in a less efficient packing than in the 100%CF model. Hence, the latter model has more fibrous material in the vicinity of the broken fibre. Since the stress recovery is dominated by the material nearby the broken fibre, the 100%CF model locally has higher shear stiffness,

which results in faster stress recovery and lower ineffective length. This trend is actually observed in Fig. 7, but is small due to the two counteracting effects.

Figure 7: The ineffective length of carbon-glass hybrids for different hybrid volume fractions. The error bars indicates the 95% confidence interval based on five realisations.

3.3 Influence of the hybridisation fibre

Most literature on hybrid composites investigates carbon-glass hybrids. Nevertheless, the hybridisation fibre is not always glass, as aramid fibres are also a popular choice. These fibres have a wide range of possible mechanical properties, out of which two common grades were chosen: Kevlar 29 (K29) and Kevlar 49 (K49). The longitudinal stiffness E_L of K29 is two times lower than the E_L of K49 (see Table 1). The five 50%CF packings, which were used in section 3.2 for CF/GF hybrids, were copied and the engineering constants of aramid fibre were applied. This way, the mesh is exactly the same. The results are again split up into data points for glass or aramid fibres (see Fig. 8) and carbon fibres (see Fig. 9).

Fig. 8 illustrates the importance of an adequate choice of the hybridisation fibre, or more specifically its elastic properties. The higher longitudinal stiffness of K49 results in lower SCFs than in K29 and GF hybrids. Similar to the SCF decrease with increasing hybrid volume fraction, this is also caused by the increased longitudinal composite stiffness.

Figure 8: Stress concentration factors on the hybridisation fibres as a function of the distance from the broken fibre for 50%CF packings.

The influence of the choice of hybridisation fibre on the carbon fibre SCFs is smaller than the influence on the hybridisation fibre SCFs (see Fig. 9). The use of aramid fibres in a hybrid results in slightly higher SCF on the carbon fibres. This is related to the low shear stiffness of the aramid fibres, which results in more shear deformation of the fibres. This increased shear deformation transfers more stress onto the intact fibres, resulting in a higher SCF.

Figure 9: Stress concentration factors on the carbon fibres as a function of the distance from the broken fibre for 50%CF packings with different fibre radii.

Fig. 10 illustrates the influence of the hybridisation fibre on the ineffective length. Although the aramid fibres have a higher average ineffective length, the differences are not statistically significant. This confirms the findings in [22]: the shear moduli of the fibres do not significantly affect the ineffective length.

Figure 10: The ineffective length for different hybridisation fibres. The error bars indicates the 95% confidence interval based on five realisations.

4 Discussion

Parameters affecting the statistical part of the hybrid effect were analysed, while the smaller contribution of the thermal part was neglected. The early works of Zweben [8] and Fukuda [15] made a fundamental contribution in the understanding of the statistical

effect in hybrid composites. Both authors used shear lag models to analyse which parameters influenced the apparent failure strain enhancement of the carbon fibres. Analytical solutions for a regular 1D packing were obtained in both papers. This is a single row of alternating carbon and hybridisation fibres with a constant fibre spacing.

The packings used in the current paper are different in three aspects. Firstly, 2D packings are used instead of 1D packings, as they more accurately represent a real fibre-reinforced composite. Secondly, the fibre centres are randomly located in the packings, rather than at constant fibre spacing. Finally, the two fibre types are assigned in a random fashion, while Zweben and Fukuda used alternating hybridisation and carbon fibres. This means that the packings in the works of Zweben and Fukuda do not have a carbon fibre directly next to the broken carbon fibre.

Both Zweben [8] and Fukuda [15] obtained a similar analytic equation for the hybrid effect. Fukuda concluded that the hybrid effect for failure strain is proportional to three ratios:

- the ratio of failure strains of hybridisation and carbon fibres
- the ratio of the SCF on a carbon fibre in a carbon fibre-reinforced composite over the SCF on the nearest carbon fibre of a hybrid composite
- the ratio of the ineffective length in a carbon fibre-reinforced composite over the ineffective length in a hybrid composite.

These findings can be compared to the results presented in this paper. The first ratio is not assessed in the current paper, as it requires a full strength model. The second ratio, namely the ratio of the SCFs, is an interesting issue. The SCF on a carbon fibre in a carbon fibre-reinforced composite, corresponds to the 100%CF packing, while the SCF on the nearest carbon fibre of a hybrid composite corresponds to the 50%CF packing. The latter can be understood from the nature of the 1D packings used by Zweben and Fukuda. These packings consist of alternating carbon and hybridisation fibres with the same fibre radii, which results in a hybrid volume fraction of 50%CF. The results for both models are plotted in Fig. 6, which proves that both packings result in almost the same SCF. Hence, the ratio of SCFs, as mentioned by Fukuda, is equal to unity.

For the final ratio, namely the ratio of the ineffective lengths, the models that should be compared are again the 50%CF and 100%CF packings. As illustrated in Fig. 7, the ineffective lengths are almost the same in both cases. Hence, the third parameter also reduces to unity. Though surprising at first, this can be understood from the assumptions made in the models of Zweben and Fukuda. The broken fibre is always assumed to be a carbon fibre. In the 1D packings of Zweben and Fukuda, the nearest carbon fibre is separated from the broken fibre by a hybridisation fibre. Hence, this packing mixes up fibre spacing with the reduced amount of carbon fibres. In the current paper, this assumption is not needed and intact carbon fibres can still be the nearest neighbour of the broken carbon fibre. This results in more realistic results and also means that the parameters influencing the hybrid effect should be re-assessed. A more detailed analysis as well as a strength model are needed to list all those parameters. From the current analysis, the difference in fibre radius can be noted as a vital parameter for the strength of hybrid composites.

5 Conclusion

The stress redistribution after a single carbon fibre breakage in unidirectional hybrid fibre-reinforced composites was investigated using three dimensional finite element models. The assumption of equal fibre radii for both fibre types significantly affects the SCFs. The influence of the hybrid volume fraction on the stress redistribution in carbon/glass hybrids is analysed and found to be small. The ineffective length as well as the SCFs on glass and carbon fibres are slightly decreased with increasing carbon fibre content. The decrease in SCF is attributed to the increased longitudinal composite stiffness, while the decrease in ineffective length is related to the lower shear stiffness of carbon fibres compared to the hybridisation fibre.

Next, the influence of the choice of hybridisation fibre is analysed. The SCF on the hybridisation fibre strongly depends on the axial stiffness of the hybridisation fibre. Even though the SCF on the carbon fibre is also affected by the hybridisation fibre, the influence is limited. The hybridisation fibre did not significantly influence the ineffective length.

The obtained results were compared to literature and found to disagree. The differences were explained based on the different type of packings. The current paper presented a more realistic representation of hybrid composites and demonstrated that changes in the SCF in hybrid composites are limited compared to non-hybrid composites. This suggests that bridging of the broken carbon fibres by the intact hybridisation fibres is the major contribution to the hybrid effect. The results should now be implemented into a hybrid strength model to assess which parameters determine the hybrid effect.

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