Resonant and phase-mixed magnetohydrodynamic waves in the solar atmosphere*

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The magnetic field in the solar atmosphere is not uniformly distributed but organized in typical configurations: e.g., intense flux tubes in the photosphere, magnetic loops in the corona, plumes in the solar wind. Each of these magnetic configurations can support magnetohydrodynamic (MHD) waves and observations show that this is indeed the case. The intrinsic inhomogeneity of the magnetic configurations enables local (slow and) Alfvén waves to exist on individual magnetic surfaces. These local Alfvén waves provide a means for dissipating wave energy which is far more efficient in a weakly dissipative plasma than classical resistive or viscous MHD wave damping in a uniform plasma. This property has inspired a lot of work on the dissipation of driven Alfvén waves and wave heating in the solar atmosphere by resonant absorption and phase mixing. This review concentrates on the interaction between fast magnetosonic waves, local Alfvén waves and quasimodes and discusses recent results on the time evolution of phase mixing of resonant waves driven by footpoint motions. © 2001 American Institute of Physics. [DOI: 10.1063/1.1343090]

I. INTRODUCTION

Magnetohydrodynamic (MHD) waves transport energy and, when part of this energy is dissipated, they can heat plasmas. This property, combined with the fact they are observed in the solar atmosphere, makes MHD waves and, in particular, Alfvén waves a natural mechanism for heating (part of) the solar corona (see, e.g., Refs. 1–4). MHD waves are also important in their own right. They reflect the stable dynamic behavior of the plasma objects in which they reside and can be used as probes for investigating the structure and composition of the plasma objects in which they are observed (see, e.g., Ref. 5).

This review concentrates on resonant and phase-mixed Alfvén waves. The physical basis of resonant and phase-mixed Alfvén waves can be understood in the context of linear ideal MHD. In linear ideal MHD, each individual magnetic surface can oscillate at its own local Alfvén frequency without interaction with neighboring magnetic surfaces. Nonuniformity creates a range of local Alfvén frequencies known as the Alfvén continuum. In ideal MHD, these local Alfvén waves are confined to the resonant magnetic surfaces on which their dispersion relations are satisfied locally. Dissipative effects produce coupling to the neighboring surfaces, but the local Alfvén waves still have steep gradients across the magnetic surfaces. Because of these steep gradients, excitation of local Alfvén waves provides a means for dissipating wave energy which is far more efficient in weakly dissipative plasmas in the solar atmosphere than classical resistive or viscous MHD wave damping in a uniform plasma. This excitation can be direct by driving at the photospheric footpoints of the magnetic field lines (see, e.g., Ref. 6) or indirect by lateral driving (see, e.g., Refs. 7–13). Indirect driving involves a carrier wave (fast magnetosonic

wave) which transports energy across the magnetic surfaces and couples to local Alfvén oscillations at the resonant position x_A where $\omega_d = \omega_A(x_A)$. This review concentrates on the interaction between fast magnetosonic waves, local Alfvén waves, and quasimodes and discusses recent results on the time evolution of phase mixing of resonant waves driven by footpoint motions.

II. MHD WAVES IN LINEAR VISCORESISTIVE MHD

The study of driven Alfvén waves involves forced oscillations in dissipative MHD. This means that the time dependent nonlinear equations of dissipative MHD have to be integrated in the presence of a time varying force term. Most studies have used linear theory of wave motions superimposed on a static (or stationary) background. The simplest equilibrium state that still contains the basic physics is a one-dimensional planar slab of pressureless plasma with a constant vertical magnetic field and with a density ρ varying in one direction. Hence, in a system of Cartesian coordinates x, y, z : $\mathbf{B} = B_z \mathbf{1}_z$, $p = 0$ and $\rho(x)$. The assumption of a plasma without pressure removes the slow waves, the variable density introduces a variable Alfvén velocity $v_A^2(x) = [B^2/\mu\rho(x)]$. In our simple equilibrium state the magnetic surfaces are the planes $x = \text{constant}$ and the photosphere is at the positions $z = 0$ and $z = L$. y is an ignorable coordinate which enables us to Fourier analyze with respect to y and put the wave quantities proportional to $\exp(ik_y y)$ with k_y = the azimuthal wave number.

The basic time-dependent equations for linear waves in viscoresistive MHD are

$$\begin{aligned} \left(\frac{\partial^2}{\partial t^2} - v_A^2 \frac{\partial^2}{\partial z^2} - v_A^2 \frac{\partial^2}{\partial x^2} - (\eta + \nu) \frac{\partial^3}{\partial x^2 \partial t} \right) \xi_x &= ik_y v_A^2 \frac{\partial \xi_y}{\partial x}, \\ \left(\frac{\partial^2}{\partial t^2} - v_A^2 \frac{\partial^2}{\partial z^2} + k_y^2 v_A^2 - (\eta + \nu) \frac{\partial^3}{\partial x^2 \partial t} \right) \xi_y &= ik_y v_A^2 \frac{\partial \xi_x}{\partial x}, \end{aligned} \quad (1)$$

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ξ is the Lagrangian displacement. η is the coefficient of magnetic diffusivity and ν is the isotropic viscosity. ξ_y is the component in the magnetic surfaces and perpendicular to the magnetic field lines. It is the component that characterizes the Alfvén waves. ξ_x is the component normal to the magnetic surfaces and characterizes the fast waves. The azimuthal wave number k_y plays an important role as it controls the linear coupling between Alfvén waves and fast magnetosonic waves. For $k_y=0$ Eqs. (1) for ξ_x and ξ_y are decoupled so that the Alfvén waves and fast waves do not interact in linear MHD. However, for $k_y \neq 0$ these equations are coupled and, consequently, the Alfvén waves and fast waves do interact.

For the variation of the wave quantities along the loop we have to distinguish two cases. When viewed as an eigenvalue problem and also as a driven problem with lateral driving, the classic approach is to treat the loop as infinitely long. In that case z is an ignorable coordinate and the wave quantities can be put proportional to $\exp(ik_z z)$ with k_z the axial wave number. However, in the case of footpoint driving in closed loops there are boundary conditions in the axial direction and the axial dependence is now given by a Fourier series. Hence,

$$\exp(ik_z z) \Rightarrow \sum X_n \sin \frac{n\pi}{L} z \Rightarrow k_z = \frac{n\pi}{L}, \quad n \in \mathbb{N}.$$

When the interest is in the eigenmodes of the system or in the stationary state of a single frequency harmonically driven system, the time can be factored out as $\exp(-i\omega t)$ with ω unknown in the eigenvalue problem and prescribed and real in the stationary state of driven problem. When the aim is to study the temporal evolution of the system under given initial and boundary conditions, the only option is to solve the time dependent equations.

III. RESONANT ALFVÉN WAVES IN LINEAR IDEAL MHD

Dissipative MHD is only necessary to treat the behavior of the waves when they developed very short length scales and dissipative effects can no longer be ignored. This happens in a dissipative layer around the ideal resonance position. Since the viscous and magnetic Reynolds numbers are very large in the solar atmosphere, this dissipative layer is necessarily very thin. Part of the basic physics of resonant Alfvén waves can be understood in linear ideal MHD. Let us look at the eigenvalue problem or the stationary state of single frequency driven MHD waves. The governing equations are two first-order ordinary differential equations for ξ_x and the Eulerian perturbation of total pressure, P' and an algebraic equation for ξ_y :

$$\begin{aligned} \rho v_A^2 (\omega^2 - \omega_A^2) \frac{d\xi_x}{dx} &= [k_y^2 v_A^2 - (\omega^2 - \omega_A^2)] P', \\ \frac{dP'}{dx} &= \rho (\omega^2 - \omega_A^2) \xi_x, \\ \rho (\omega^2 - \omega_A^2) \xi_y &= ik_y P', \end{aligned} \quad (2)$$

ω_A is the local Alfvén frequency. Its square is defined as

$$\omega_A^2(x) = k_z^2 v_A^2(x). \quad (3)$$

The local Alfvén frequency is a function of position and defines a continuum of resonant Alfvén waves

$$\mathcal{AC} = [k_z \min v_A(x), k_z \max v_A(x)].$$

The variation of ω_A with position plays a key role for the resonant Alfvén waves. When there are no boundary conditions in the axial direction, there is one Alfvén continuum related to the specified k_z . For a line-tied loop the situation is more involved. Because of the Fourier sum in z , we have an infinite set of axial wave numbers $k_z = n\pi/L$, an infinite set of equations and an infinite number of Alfvén continua. The equations are singular and the waves show resonant behavior at the Alfvén resonance point $x = x_A$ where the local dispersion relation for Alfvén waves is satisfied $\omega^2 = \omega_A^2(x_A)$. For a frequency ω in the Alfvén continuum the solutions for ξ_x and ξ_y are singular. They are characterized, respectively, by a logarithmic singularity and a jump in ξ_x , and by a $1/s$ singularity and $\delta(s)$ contribution in ξ_y (see, for example, Ref. 14) where s is the distance to the ideal singularity $s = x - x_A$. The solutions have very steep (infinite) gradients in the x direction, i.e., normal to the magnetic surfaces and the dominant dynamics is in the y components, i.e., in the magnetic surfaces and perpendicular to the magnetic field lines, as expected for Alfvén waves.

The spectrum of eigenmodes consists of Alfvén and fast eigenmodes. The azimuthal wave number plays the role of coupling factor between Alfvén eigenmodes and fast eigenmodes. For $k_y=0$, the eigenmodes are decoupled into Alfvén continuum eigenmodes with $\xi_x=0, P'=0, \xi_y \neq 0$ and discrete fast eigenmodes with $\xi_x \neq 0, P' \neq 0, \xi_y=0$. The (real) eigenvalues of the fast eigenmodes can lie in the Alfvén continuum, but there is no coupling for $k_y=0$. However, for $k_y \neq 0$, the fast eigenmodes with an eigenfrequency in the Alfvén continuum couple to a local Alfvén continuum eigenmode and produce the famous quasimodes. These quasimodes are the natural wave modes of the system.^{15,16} They combine the properties of global and local oscillation modes. They are damped for a static equilibrium state and play a prominent role in MHD wave heating scenarios (see, e.g., Refs. 12, 13, 17, and 18).

IV. RESONANT ALFVÉN WAVES IN VISCORESISTIVE MHD

The singular solutions found in ideal MHD are transformed into well-behaved regular solutions when dissipation is included. Dissipation is important in a dissipative layer around the ideal resonant position with a thickness measured by the quantity δ_A (see, e.g., Refs. 12, 19, and 20): $\delta_A = (\omega(\eta + \nu)/|\Delta|)^{1/3} \propto (R)^{-1/3}$. $\Delta = d/dx[\omega^2 - \omega_A^2(x)]$ measures the variation of the local Alfvén frequency; R is the total Reynolds number. In the solar atmosphere $R \gg 1$ and

this results in very small length scales required for efficient damping. Let us look at the stationary state of driven oscillations. Close to the ideal resonance point(s) the solutions for the resonant contributions to ξ_x and ξ_y can be expressed in terms of the universal functions $G(\tau)$ and $F(\tau)$ (see Ref. 21)

$$G(\tau) = \int_0^\infty \frac{e^{-u^3/3}}{u} \{ \exp[iu\tau \text{sign}(\Delta)] - 1 \} du,$$

$$F(\tau) = \int_0^\infty \exp[iu\tau \text{sign}(\Delta) - u^3/3] du,$$

where $\tau = s/\delta_A$ is a scaled variable which is of order unity in the dissipative layer. $G(\tau)$ and $F(\tau)$ contain the remnants of the ideal singular behavior, but are finite everywhere. They can be used to compute the jumps of the physical variables over the dissipative layer and the amount of absorbed wave energy.²² The dominant dynamics is in the perpendicular component like in ideal MHD.

Let us now look at the eigenmodes. For $k_y \neq 0$, the discrete fast eigenmodes with an eigenfrequency in the Alfvén continuum couple to a local Alfvén eigenmode and become damped eigenmodes with a complex eigenfrequency $\omega = \omega_R + i\omega_I$, $\omega_I < 0$. Damped quasimodes were identified as normal modes of the dissipative MHD operator for fusion plasmas by Ref. 23 and by Ref. 24 for solar plasmas. Tirry and Goossens²⁴ obtained the solutions for ξ_x and ξ_y of these quasimodes close to the ideal resonance point(s) in terms of the modified $\tilde{F}(\tau)$ and $\tilde{G}(\tau)$ functions. Compared with $G(\tau)$ and $F(\tau)$, the integrands of $\tilde{F}(\tau)$, and $\tilde{G}(\tau)$ contain an extra factor $\exp(-u\Lambda)$ with $\Lambda = 2\omega_I\omega_A/\delta_A|\Delta|$.

V. FOOTPOINT DRIVING IN CLOSED LOOPS

Phase mixing of Alfvén waves in open and closed loops by footpoint motions was first studied as a possible source of coronal heating in a classic paper (Ref. 6). Since then, phase mixing has been studied in more detail (see, e.g., Ref. 25 and references therein). So far all work on phase mixing of Alfvén waves in open loops is for $k_y = 0$ (or $m = 0$ in the cylindrical case) and involves direct excitation of Alfvén waves without any interaction with fast waves. Here, we focus on Alfvén waves driven by footpoint motions in closed loops. Strauss and Lawson²⁶ were the first to set up a numerical simulation on the excitation of Alfvén waves in closed loops. They restricted their analysis to incompressible motions and considered both the stationary state and the initial value solutions.

A. Recapitulation of basic results

Results from the previous sections help us to understand basic properties of Alfvén waves driven by footpoint motions in closed loops. In our view the following results are essential:

There is an infinite number of Alfvén continua because of the Fourier sum for describing the axial dependence:

$$\forall n \in \mathbb{N}: \exists \mathcal{AC}_n = \left(\frac{\pi n}{L} \min v_A(x), \frac{\pi n}{L} \max v_A(x) \right).$$

The components ξ_x and ξ_y characterize the fast waves and the Alfvén waves, respectively.

For $k_y = 0$ the fast waves and the Alfvén waves are fully separated.

For $k_y \neq 0$ the fast waves and the Alfvén waves are coupled and a fast wave with a frequency in the Alfvén continuum excites a local resonant Alfvén wave. Also for $k_y \neq 0$ discrete fast eigenmodes with a frequency in the Alfvén continuum are transformed in quasimodes and these quasimodes are the natural oscillations modes of the system.

Driving normal to the magnetic surfaces excites fast waves. For $k_y = 0$ that is the end of the story. There are not any resonant waves excited in the system. For $k_y \neq 0$ the fast waves excite resonant Alfvén waves and also the quasimode. This can be summarized as follows (Channel I):

$$\begin{aligned} \xi_x : \text{Fast waves} &\Rightarrow \text{resonant Alfvén waves } \omega_d = \omega_A(x_A), \\ &\Rightarrow \text{quasimode } \omega_Q, \end{aligned} \quad (4)$$

where ω_d is the frequency present in the driving spectrum and ω_Q is the frequency of one of the quasimodes.

Driving in the magnetic surfaces perpendicular to the magnetic field lines excites Alfvén waves. For $k_y = 0$ these are torsional Alfvén waves which are resonant on the magnetic surface where their local dispersion is satisfied. There are no quasimodes excited. For $k_y \neq 0$, the Alfvén waves, in addition to being resonant on the magnetic surface where their local dispersion relation is satisfied, also excite fast waves. These fast waves excite resonant Alfvén waves and also the quasimode. This can be summarized as follows (Channel II):

$$\begin{aligned} \xi_y : \text{Alfvén waves} &\Rightarrow \text{Resonant Alfvén waves } \omega_d = \omega_A(x_A) \\ &\Rightarrow \text{Fast waves} \\ &\Rightarrow \begin{cases} \text{resonant Alfvén waves} \\ \text{quasimode } \omega_Q. \end{cases} \end{aligned} \quad (5)$$

B. Stationary state in viscoresistive MHD

The stationary state in viscoresistive MHD of driven resonant Alfvén waves for single frequency driving is described by Eq. (1) with the time derivative $\partial/\partial t$ replaced by $-i\omega_d$. For a closed loop, use of a Fourier series in z results in an infinite system of (uncoupled) equations. The footpoint motions are included as inhomogeneous boundary conditions. References 27–29 studied single frequency driven torsional Alfvén waves. Ruderman³⁰ showed that the steady state of directly driven axisymmetric Alfvén waves in coronal loops found in Ref. 28 is the asymptotic state of a non-stationary solution. In addition, Refs. 31 and 32 determined the stationary state solutions excited by footpoint motions normal to the magnetic surfaces having a nonzero divergence and a nonzero vorticity. Reference 33 considered asymmetric driving ($k_y \neq 0$) in the magnetic surfaces perpendicular to the magnetic field lines. The results in Ref. 33 are particularly interesting as they reveal a complicated dependence of absorption on the combination k_y, ω_d with a prominent role for the quasimodes. In addition to enhanced absorption for quasimode frequencies, they found an antiresonance line in

FIG. 1. ξ_y component as function of x at $z=L/2$ for $\omega_d \neq \omega_Q$ (left-hand side inset) and $\omega_d = \omega_Q$ (right-hand side inset).

the (k_y, ω_d) plane of reduced absorption. They related these results to the interaction of the direct Poynting flux associated with the directly driven Alfvén waves and the indirect Poynting flux due to the Alfvén waves driven by fast waves.

C. Temporal evolution in ideal MHD

In principle, the stationary state of driven MHD waves described by Eqs. (1) is attained for $t \rightarrow \infty$. The important question is how long it takes for the waves to create sufficiently short length scales. The time required for phase mixing to attain these short length scales should be shorter than the lifetime of the loops that are to be heated. The evolution towards short length scales can be studied in ideal MHD up to the point where the length scales become too short for dissipation to be further ignored. The equations that govern the temporal evolution of MHD waves in ideal MHD are obtained by dropping the dissipative terms in Eqs. (1). The temporal evolution in ideal MHD for $k_y = 0$ was studied in Refs. 34 and 35. We focus here on asymmetric driving with $k_y \neq 0$.

1. Monoperiodic driving normal to the magnetic surfaces

Here channel I applies. The fast wave with frequency ω_d excites a resonant Alfvén wave at the position x_A where $\omega_d = \omega_A(x_A)$ and also the quasimode at the position x_Q where $\omega_A(x_Q) = \omega_Q$. Figure 1 shows the temporal evolution of ξ_y for $\omega_d \neq \omega_Q$ (left-hand side inset) and for $\omega_A(x_Q) = \omega_Q$ (right-hand side inset). The left-hand side inset shows that initially the quasimode is more prominently present than the resonant Alfvén wave. The Alfvén wave that is excited is not a natural oscillation mode of the system. This implies that a persistent monoperiodic driving at the frequency ω_d is required for building up this resonant Alfvén wave with the creation of increasingly smaller length scales. The quasimode is initially clearly present but is phase-mixed away as time progresses. The right-hand side inset shows that there is

a resonant coupling between the excited quasimode and the local Alfvén continuum mode. The quasimode is excited right from the start with a much larger amplitude than in the previous case. Moreover the evolution towards increasingly larger amplitudes and smaller length scales is clearly much faster than in the previous case. Since the excited quasimode is a natural oscillation mode which combines the properties of a global fast eigenmode and a local Alfvén wave, there is also an enhanced excitation of ξ_x . The solution for ξ_y oscillates between a smoothed delta function and a smoothed $1/s$ function while the solution for ξ_x oscillates between a smoothed logarithm and a smoothed jump.³⁶ The length scale of the resonance defined as the full width at half maximum, decreases inversely proportional to time t for both cases, although the difference in reduction is clear.^{36,37}

According to a recent study on the transition to steady state in the approximation of a thin magnetic cavity,³⁸ two characteristic transitional times are relevant. The first one is inversely proportional to the decrement of the global mode; the second time, which often is the longest of the two, is proportional to $R^{1/3}$.

2. Random driving normal to the magnetic surfaces

Channel I applies again. De Groof *et al.*³⁹ have driven a loop by a series of 30 pulses with randomly distributed widths and randomly distributed time intervals in between the pulses. In contrast to monoperiodic driving, the driver contains a whole frequency interval (broad band driver) so that there is no need for fine tuning to hit a quasimode frequency. As a matter of fact, all the quasimodes lying in the frequency interval covered by the driver can be excited. As shown in Fig. 2 (left-hand side inset) where the Alfvén energy is plotted against x , indeed resonances are built up at the magnetic surfaces corresponding to these quasimodes. The modes corresponding to the first eigenfunctions and small values of k_z are the most dominant. As expected, resonant

FIG. 2. Left-hand side inset: Alfvén energy as function of x (integrated over z) for $t=10$ – 50 Alfvén crossing times after driving with a randomly varying pulse train. Right-hand side inset: comparison to monopерiodic driver [qualitatively comparable to ($t=40$) left-hand side inset].

peaks are growing in time at the magnetic surfaces where $\omega_A(x_A) = \omega_Q$ (vertical lines in the figure). De Groof *et al.*³⁹ also calculated the time scale in which the necessary small length scales are formed. They concluded that even for the slowest decrease, the time needed to generate a length scale of about 100 m is about 3 h.

In contrast to previous studies where only one dominant resonance peak was formed, this model shows resonant peaks that are packed rather closely together. Moreover, a variation of the pulse widths of the driving pulses or the consideration of a longer loop would result in even more quasimodes coupling to Alfvén modes and consequently the loop can be heated even more globally. In Fig. 2 (right-hand side inset), a corresponding result is shown for a monopерiodic driver. Only one peak results from a fast-Alfvén energy transfer. The second peak at $x=0.6$ is forced by the driver.

Apart from the random driver, a second important difference with previous studies is that all the possible values of k_z are taken into account. The classic assumption of a single k_z excludes all but the first harmonic of the fast modes in the driving spectrum.⁴⁰

3. Monopерiodic driving in the magnetic surfaces perpendicular to the magnetic field lines

Here the second channel applies with directly driven Alfvén waves and Alfvén waves driven by fast waves. These two types of Alfvén waves have different amplitudes and are usually out of phase, with the phase difference depending on the combination of k_y, ω_d . When $\omega_d = \omega_Q$ the two types of Alfvén waves are in phase providing the best conditions for the creation of large amplitudes and small length scales. It turns out that there are combinations of k_y, ω_d for which the directly driven Alfvén waves and the Alfvén waves driven by fast waves are exactly in antiphase. These combinations are the worst possible for creating resonant Alfvén waves and correspond in the stationary state to the antiresonance line.^{33,41}

The evolution of the amplitude of ξ_y in time depends on the frequency ω_d and position. As expected, the evolution towards larger amplitudes and smaller length scales is faster as the difference between ω_d and ω_Q decreases. On a resonant surface the increase of the amplitude is monotone; on non-resonant surfaces the time variation of ξ_y is characterized by beats.⁴¹

VI. CONCLUSIONS

The theoretical results collected in this paper give confidence that the excitation of resonant Alfvén waves may indeed play a key role in heating closed (and also open) loops in the solar corona. Small length scales required for efficient dissipation are attained on sufficiently short time spans. The inherent coupling of the fast waves and Alfvén waves for nonaxisymmetric motions ($k_y \neq 0, m \neq 0$) results in two channels for exciting resonant Alfvén waves. The quasimodes play a fundamental role for both channels as they provide (by far) the best conditions for excitation.

For a monopерiodic driver, the efficiency depends on the frequency in a complicated way, with optimal efficiency for a quasimode frequency and strongly reduced efficiency at frequencies where the directly driven and indirectly driven Alfvén waves are in antiphase. In addition to the need for fine tuning the frequency of the driver to the quasimode frequency, monopерiodic driving results in a single resonant layer. A random driver covering a broad band spectrum remedies these two weak points in addition to being a better approximation of physical reality. The random driving source is effectively filtered by the loop producing a disturbance that is essentially a superposition of all quasimodes with frequencies in the broad band spectrum. Therefore it is important to take into account all possible values of k_z to prevent from excluding all but the first harmonic of the fast modes in the driving spectrum. In this way, random driving results in multiple resonant peaks at the different magnetic surfaces where $\omega_A(x_A) = \omega_Q$. Consequently the loop is heated more globally. Moreover, if larger lengths are consid-

ered, more valuable quasimodes are present and the loop is heated even more globally. Thus we can conclude that random footpoint driving can act as an efficient dissipation mechanism to heat the loop globally!

Of course, this is not the end of the story. Within the present model, random driving in the magnetic surfaces needs to be explored. Furthermore, there is a need to go beyond the present model. The backreaction of the chromosphere, effects of flow, and effects of kinetic theory need to be considered.

- ¹J. A. Ionson, Sol. Phys. **100**, 289 (1985).
- ²J. V. Hollweg, Comput. Phys. Rep. **12**, 205 (1990).
- ³M. Goossens, in *Advances in Solar System MHD*, edited by E. R. Priest and A. W. Hood (Cambridge University Press, Cambridge, 1991), p. 137.
- ⁴M. Goossens, Space Sci. Rev. **68**, 51 (1994).
- ⁵B. R. Roberts, Sol. Phys. **193**, 139 (2000).
- ⁶J. Heyvaerts and E. R. Priest, Astron. Astrophys. **117**, 220 (1983).
- ⁷J. A. Ionson, Astrophys. J. **226**, 650 (1978).
- ⁸J. V. Hollweg, Astrophys. J. **312**, 880 (1987).
- ⁹J. V. Hollweg, Astrophys. J. **320**, 875 (1987).
- ¹⁰J. M. Davila, Astrophys. J. **317**, 514 (1987).
- ¹¹S. Poedts, M. Goossens, and W. Kerner, Sol. Phys. **123**, 83 (1989).
- ¹²S. Poedts, M. Goossens, and W. Kerner, Astrophys. J. **360**, 279 (1990).
- ¹³S. Poedts, M. Goossens, and W. Kerner, Comput. Phys. Commun. **59**, 95 (1990).
- ¹⁴J. P. Goedbloed, Rijnhuizen Rep. **83** (1983).
- ¹⁵B. Balet, K. Appert, and J. Vaclavik, Plasma Phys. **24**, 1005 (1982).
- ¹⁶R. S. Steinolfson and J. M. Davila, Astrophys. J. **415**, 354 (1993).
- ¹⁷A. N. Wright and G. J. Rickard, Astrophys. J. **444**, 458 (1995).
- ¹⁸L. Ofman, J. M. Davila, and R. S. Steinolfson, Astrophys. J. **444**, 471 (1995).
- ¹⁹J. M. Kappraff and J. A. Tataronis, J. Plasma Phys. **18**, 209 (1977).
- ²⁰J. V. Hollweg and G. Yang, J. Geophys. Res. **93**, 5423 (1988).
- ²¹M. Goossens, M. S. Ruderman, and J. V. Hollweg, Sol. Phys. **157**, 75 (1995).
- ²²M. Goossens and M. S. Ruderman, Phys. Scr. T **60**, 171 (1995).
- ²³S. Poedts and W. Kerner, Phys. Rev. Lett. **66**, 2871 (1991).
- ²⁴W. Tirry and M. Goossens, Astrophys. J. **471**, 501 (1996).
- ²⁵I. De Moortel, A. W. Hood, and T. D. Arber, *Proceedings of the Workshop on Waves in Dusty, Solar, and Space Plasmas* (AIP, New York, 2000), Vol. 537, p. 224.
- ²⁶H. R. Strauss and W. S. Lawson, Astrophys. J. **346**, 1035 (1989).
- ²⁷S. Poedts and G. C. Boynton, Astron. Astrophys. **306**, 610 (1996).
- ²⁸M. S. Ruderman, D. Berghmans, M. Goossens, and S. Poedts, Astron. Astrophys. **320**, 305 (1997).
- ²⁹M. S. Ruderman, M. Goossens, J. L. Ballester, and R. Oliver, Astron. Astrophys. **328**, 361 (1997).
- ³⁰M. S. Ruderman, Astrophys. J. **521**, 851 (1999).
- ³¹G. Halberstadt and J. P. Goedbloed, Astron. Astrophys. **301**, 559 (1995).
- ³²G. Halberstadt and J. P. Goedbloed, Astron. Astrophys. **301**, 577 (1995).
- ³³D. Berghmans and W. J. Tirry, Astron. Astrophys. **325**, 318 (1997).
- ³⁴D. Berghmans and P. De Bruyne, Astrophys. J. **453**, 495 (1995).
- ³⁵D. Berghmans, P. De Bruyne, and M. Goossens, Astrophys. J. **472**, 398 (1996).
- ³⁶W. J. Tirry, D. Berghmans, and M. Goossens, Astron. Astrophys. **322**, 329 (1997).
- ³⁷I. R. Mann, A. N. Wright, and P. S. Cally, J. Geophys. Res. **100**, 19441 (1995).
- ³⁸M. S. Ruderman and A. N. Wright, Phys. Plasmas **7**, 3515 (2000).
- ³⁹A. De Groof and M. Goossens, Astron. Astrophys. **356**, 724 (2000).
- ⁴⁰A. De Groof and M. Goossens, *Proceedings of the Workshop on Waves in Dusty, Solar, and Space Plasmas* (AIP, New York, 2000), Vol. 537, p. 208.
- ⁴¹W. J. Tirry and D. Berghmans, Astron. Astrophys. **325**, 329 (1997).