Basis of a finite-element simulation tool to predict the flexural behaviour of SFRC prisms

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Abstract. The fibre pull-out response of a steel fibre, both straight and end-hooked, embedded in a concrete matrix has been widely investigated in the past. Attention was given to the influence of the type of fibre, the concrete strength and the inclination of the pull-out force. Hooked-end fibres need more energy to be pulled out of the concrete and thus are preferably used in steel fibre reinforced concrete (SFRC) for structural purposes.

Based on this single fibre pull-out behaviour, numerous analytical and numerical methods were verified with experimental results of three- or four-point bending tests on small prisms. In this paper, a new approach to characterize the non-linear bending behaviour of steel fibre reinforced concrete is proposed. Finite element (FE) analyses were conducted on a small prism-model, containing randomly generated small embedded reinforcement elements, which material properties are based on the pull-out behaviour derived from earlier research activities. In this stage, attention was also given to the orientation of the fibres.

Eventually, a good correlation was found between the numerical and experimental results and the fine-tuned model was then used to perform a large number of numerical bending tests. In that way, the scatter of the parameters that describe the non-linear bending behaviour of small steel fibre reinforced concrete prisms was estimated in a much faster way than by experimental testing. These parameters are usually needed to evaluate the structural behaviour of SFRC elements according to current design guidelines for SFRC.

1. Introduction

In the past three decades, many researchers studied the single fibre pull-out response of a fibre embedded in a cementitious matrix. This research led to the development of several pull-out models to describe the pull-out response of both straight [1, 2] and hooked-end [3-8] fibres. Accurate predictions of the pull-out response served as a basis to develop computational models to characterize the behaviour of SFRC subjected to pure bending or uni-axial tension. This characteriza-

tion, in particular the bending behaviour of small prisms either by three- or four-point bending tests, is very important to derive parameters that are used in current design guidelines for SFRC [9].

In this paper a new approach to estimate flexural toughness parameters is explained. The pull-out response of a single fibre was used to generate different samples of a FE-model with randomly distributed fibres and results from both experimental and numerical analyses are compared and discussed.

2. Experimental testing

2.1 Test setup

Four small prisms with a span length l of 450 mm were subjected to a four-point bending test with the test setup in accordance with the Belgian standard NBN B15-238 (1992). The prisms have a square cross section with a height h of 150 mm and the total length is equal to 600 mm. 40 kg/m³ of DRAMIX[®] fibres, type ZC 60/.80 HC, were added to the concrete, which general characteristics are showed in Table 1. To achieve a load-deflection curve which can quantify the post-peak behaviour of the prism, a displacement controlled test was necessary.

Table 1. Main properties of the used concrete

Concrete characteristics	
Mean compression strength f_{cm}	90.3 N/mm ²
Mean tensile strength f_{ctm}	4.89 N/mm ²
Modulus of elasticity E	45 800 N/mm ²
Fracture Energy G_f	163 N/m
W/C ratio	0.30

2.2 Characterization of the flexural toughness

Flexural toughness can be quantified in two different ways as stated in the Belgian standard. According to the first method, the ratio D_n between the applied load F_n at a certain deflection l/n (i.e. $l/600$, $l/450$, $l/300$ and $l/150$) and the force F_r when cracking occurs, is calculated. The second method defines the flexural toughness as the surface B_n beneath the load-deflection curve until a prescribed deflection (i.e. $l/300$ and $l/150$) is reached. The conventional flexural strength is denoted as $f_{f,n}$ and is given by equation (1).

$$f_{f,n} = \frac{B_n n}{bh^2} \quad (1)$$

All values of D_n and $f_{f,n}$, derived from four experimental four-point bending tests, performed by Van Gysel, are summarized in Table 2. These values were derived from the load-deflection curves (solid black lines) displayed in Fig. 2.

Table 2. Parameters to characterize the flexural toughness of SFRC

Prism nr.	D_{600} [-]	D_{450} [-]	D_{300} [-]	D_{150} [-]	$f_{f,300}$ [MPa]	$f_{f,150}$ [MPa]
1	1.39	1.44	1.37	1.09	7.03	6.93
2 *	-	-	-	-	-	-
3	1.37	1.36	1.16	-	6.39	-
4	1.18	1.15	1.06	0.74	6.69	4.92
Mean	1.31	1.32	1.20	0.92	6.70	5.93

* failure at a central deflection of about 0.75 mm.

Regarding the values of Table 2, a general remark should be made: although all four-point bending were precisely executed as described in the Belgian standard, the results were not sufficient for all prisms to determine the flexural toughness indices. Without guarantee in advance of achieving useful test results, a significant number of tests is needed to determine the scatter of flexural toughness parameters of SFRC.

3. Numerical analysis

To avoid time-consuming test procedures and to assure a useful load-deflection curve, a solution of the problem stated earlier can be found through numerical simulations. By the continuous improvement of computer hardware, computation times of complex numerical problems are reduced in a very fast way and new approaches to elaborate a finite element simulation tool for SFRC has become feasible: it takes about 3 hours¹ to perform a complete non-linear analysis with the software program Atena Science².

3.1 FE-model

Due to the three dimensional distribution of fibres within a prescribed volume, an accurate simulation cannot be obtained by a much simpler two-dimensional numerical model and thus the whole prism was divided in tetrahedral (i.e. 3D solid) quadratic elements with a size of 15 mm, representing the coarseness of the concrete matrix. In the crack model, the option ‘rotating’ was chosen and a crack bandwidth of 15 mm was assumed. A simulated specimen without fibres was used to calibrate the input parameters for concrete. It followed that a small adaptation

¹ Duration of a single analysis with one central processing unit of 2,67 Ghz

² AtenaWin v4 Statics module, developed by Cervenka Consulting

of the input tensile strength and fracture energy, with respect to the values given in Table 1, was necessary.

3.2 Distribution of fibres

When adding 40 kg/m³ of DRAMIX[®] ZC 60/80 HC fibres to a prism with a volume of 150 x 150 x 600 mm³, about 2280 fibres should be drawn as a straight line. Considering this to be impossible to do by hand, a small software program was developed that reads all input data from the calculation file of Atena and adds an amount of fibres automatically, based on fibre properties and the shape of the volume (i.e. a rectangular prism). This volume is formed by a cross-section in a yz-plane that is extruded along the x-axis, in the longitudinal direction of the prism.

Although the main objective of the program is to add fibres in a volume in such way that the distribution of fibres approaches the real configuration, the presence of coarse aggregates was not taken into account at this stage. The orientation of a single fibre is represented by the angle θ with respect to the longitudinal axis x of the prism (i.e. the x-axis). Based on the experimental investigation of Laranjeira et al. [10], a Gaussian distribution was assumed for the orientation angle θ , which is defined by an orientation number η_θ , given by (2).

$$\eta_\theta = \frac{1}{N} \sum_{i=1}^N \cos \theta_i \quad (2)$$

The average θ_m and standard variation $\sigma_{(\theta)}$, both in radians, of the normal distribution can be calculated by equations (3) and (4).

$$\theta_m = \arccos(\eta_\theta) \quad (3)$$

$$\sigma(\theta) = \frac{\pi}{2} \eta_\theta (1 - \eta_\theta) \quad (4)$$

A second objective was to implement the pull-out model for hooked-end fibres developed by Van Gysel [8]. The pull-out response is mainly affected by the inclination of the pull-out force and the embedded length. So, for each fibre within the concrete volume a unique load-slip diagram exists, but implementing this would lead to a too complex model. To solve this problem, both the range of orientation angles and the embedded lengths are divided in discrete intervals.

In this way, the orientation angle θ is divided in six intervals from 0 to 90 degrees and each orientation angle interval is divided in another six sub-intervals going from 5 mm to half the fibre length, with an interval width of 5 mm. The discretization matrix for such a division is displayed in Table 3.

Table 3. Discretization matrix and corresponding material numbers

Orientation interval [°]	Embedded length l_f					
	5 mm	10 mm	15 mm	20 mm	25 mm	30 mm
[0;7.5]	1	2	3	4	5	6
[7.5;22.5]	7	8	9	10	11	12
[22.5;37.5]	13	14	15	16	17	18
[37.5;52.5]	19	20	21	22	23	24
[52.5;67.5]	25	26	27	28	29	30
[67.5;90]	31	32	33	34	35	36

Thus, when sampling a specimen that includes fibres, each fibre is linked to a material number between 1 and 36 within the FE-model, taking into account the normal distribution of the orientation angle θ and a uniform distribution of the embedded length l_f .

To model the pull-out response of a fibre, an equivalent stress-strain diagram was defined based on a load-slip diagram. The equivalent strain ε_{eq} is calculated as the ratio between the slip s and the total length of the fibre l_{tot} and the equivalent stress as the ratio between the pull-out-force F_u and the cross-section of the fibre A_f . As a result of this approach, cable elements were needed to assure that a constant stress would appear over the whole length of the fibre when a strain was locally induced due to crack formation. Fig. 1 shows a sampled FE-model with fibres.

Fig. 1. Sampled FE-model of a prism containing fibres

Three series of FE-models, each with different orientation number η_θ (0.80, 0.75 and 0.70), were analysed and the resulting load-deflection diagrams were then used to derive the flexural toughness parameters D_n and $f_{f,n}$.

4. Results

Thirty different SFRC prisms were sampled with the software tool and analysed with Atena. All load-deflection curves are displayed in Fig. 2.

Fig. 2. Load deflection curves as a function of the orientation number η_0

When an orientation number of 0.80 (5 simulations) was assumed, a systematic overestimating of residual strength occurred, while an orientation number of 0.70 (5 simulations) appears to be too conservative. A good correlation with the experimental results of the four-point bending tests by Van Gysel [8] was found with η_0 equal to 0.75. All results, according to an orientation number of 0.75 (20 simulations), are summarized in Table 4.

Table 4. Flexural toughness parameters for $\eta_0 = 0.75$

Prism nr.	D_{600} [-]	D_{450} [-]	D_{300} [-]	D_{150} [-]	$f_{f,300}$ [MPa]	$f_{f,150}$ [MPa]
1	1.52	1.42	1.26	1.07	6.79	6.16
2	1.24	1.16	1.12	0.88	5.77	5.28
3	1.43	1.30	1.12	0.88	6.36	5.52
4	1.43	1.42	1.32	*	6.52	*
5	1.37	1.37	1.21	*	6.21	*
6	1.49	1.43	1.27	1.12	6.73	6.32
7	1.21	1.19	1.10	0.98	5.87	5.35
8	1.21	1.15	1.08	0.87	5.77	5.15
9	1.30	1.27	1.21	0.96	6.11	5.65
10	1.36	1.34	1.21	1.07	6.27	5.9
11	0.13	0.13	1.08	*	5.81	*
12	1.31	1.19	1.06	0.84	5.92	5.25
13	1.57	1.46	1.32	*	7.04	*
14	1.22	1.19	1.10	0.94	5.8	5.23
15	1.46	1.31	1.09	*	6.42	*
16	1.63	1.51	1.37	1.18	7.29	6.46
17	1.56	1.57	1.43	1.18	7.33	6.81
18	1.46	1.37	1.27	1.03	6.66	6.13
19	1.33	1.32	1.21	1.00	6.47	5.89
20	1.35	1.23	1.12	1.02	6.01	5.6
Average	1.33	1.27	1.20	1.00	6.36	5.78
Std. Dev.	0.31	0.29	0.11	0.11	0.50	0.51
C.O.V.	23%	23%	9%	11%	8%	9%

The mean values of the experimental results (Table 2) and the simulated tests (Table 3) are in good agreement.

5 Conclusions

Finite-element simulations with randomly distributed embedded cable elements, representing steel fibres, were able to predict the bending behaviour of small steel fibre reinforced concrete prisms used to perform four-point bending tests according to the Belgian standard NBN B15-238 (1992).

Three different orientation numbers were compared to each other and the following conclusion can be made: the proper orientation number for a small prism subjected to bending should be in a range between 0.70 and 0.80.

Eventually, a good correlation was found for about twenty samples with an orientation number equal to 0.75 and the scatter of the flexural toughness parameter D_n and $f_{f,n}$ could be estimated accurately. For the values of D_{600} and D_{450} , a coefficient of variation equal to 23 % was found and for the values D_{300} , D_{150} , $f_{f,300}$ and $f_{f,150}$ the coefficient of variation was around 9 %. This scatter is inherent to the randomness of the fibre position and orientation.

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