Comprehensive parametrization of surface-layer transfer coefficients for use in atmospheric numerical models

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Abstract A new non-iterative bulk parametrization for surface-layer transfer coefficients for momentum and heat is presented. It is applicable for a wide range of aerodynamic and thermal roughness lengths, and includes the effect of the roughness sublayer. As a consequence, the non-iterative method is suitable for every surface type, especially for urban surfaces for which existing non-iterative parametrizations fail. The analytical approximation compares very well with an iterative approach. Our method can be easily implemented in atmospheric numerical models that already employ a non-iterative approach.

Keywords Monin-Obukhov Similarity Theory · Surface energy balance · Surface-layer transfer coefficients · Thermal roughness length · Turbulent fluxes

1 Introduction

Turbulent fluxes of momentum, sensible heat and latent heat from the surface to the atmosphere largely determine the interaction between the surface and the air above. Consequently, they have a substantial affect on wind, temperature, moisture and concentrations of other air constituents near the surface. A common approach to calculating the surface transfer coefficients and the consequential fluxes is by means of the Monin-Obukhov similarity theory (MOST). Within this framework, transfer coefficients generally need to be calculated iteratively. As iterative solutions for the transfer coefficients are computation-

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ally expensive, analytical approximations have been investigated for the past 30 years.

To simplify this problem, early analytical approximations such as Louis (1979), Byun (1990), Lee (1997) and Viterbo et al. (1999) assume $z_{0H} = z_0$ (z_0 and z_{0H} are the aerodynamic and thermal roughness lengths, respectively). Others assume a constant z_{0H}/z_0 ratio, or a fixed $kB^{-1} = \ln(z_0/z_{0H})$, as in Pleim (2006). More sophisticated methods such as Garratt (1992, see p. 243), Uno et al. (1995), Launiainen (1995), Holtslag and Ek (1996), Mascart et al. (1995), Song (1998), De Bruin et al. (2000), Blümel (2000) and Li et al. (2010) allow a varying kB^{-1} . A few of the above-mentioned analytical approaches have been compared in van den Hurk and Holtslag (1997). As stated by their respective authors, the analytical approaches above can reproduce the iterative procedure very well yet for limited ranges of z/z_0 (z is the reference height, or typically the height of the first vertical model level), kB^{-1} and/or Ri_B (bulk Richardson number). However, they fail to reproduce the full range $-0.5 \leq kB^{-1} \leq 30$, $10 \leq z/z_0 \leq 10^5$ and $-5.0 \leq Ri_B \leq 2.5$. In particular, the case of urban surfaces, for which very high kB^{-1} values of more than 10 (see Voogt and Grimmond, 2000; Sugawara and Narita, 2008) and very low z/z_0 values down to 10 are found, is problematic. As the horizontal resolution of atmospheric models increases, urban areas may well extend over several grid boxes. Therefore, a new analytical parametrization for turbulent fluxes applicable for this situation is necessary.

We present herein new bulk transfer coefficients for both unstable and stable surface-layer conditions as an approximation to the iterative procedure within the MOST framework. As opposed to existing parametrizations, they are applicable for $10 \leq z/z_0 \leq 10^5$ and $-0.5 \leq kB^{-1} \leq 30$ that capture most, if not all, surface types and model configurations. They also account for the roughness-sublayer (RSL) effect, which is important when z/z_0 becomes small (see Garratt, 1992, section 3.3.4). The advantage of an analytical form is easy implementation in models that already use an analytical approach. In addition, analytical formulations avoid slow or non-convergence that can occur with iterative procedures.

The structure of the paper is as follows: a summary of the MOST is given in Section 2 including a brief description of the iterative procedure. Section 3 describes the new analytical approximation, and in Section 4, the results of the new analytical procedure are shown. In Section 5, a comparison is made between existing analytical procedures, the new analytical procedure and the iterative approach. Finally, conclusions are given in Section 6.

2 Iterative solution for the transfer coefficients

The turbulent fluxes of momentum and sensible heat between the surface and the atmosphere are given by

$$(\overline{u'w'})_0 = -u_*^2 = -C_M u_a^2, \quad (1)$$

$$(\overline{w'\theta'})_0 = -u_* \theta_* = -C_H u_a (\theta - \theta_s), \quad (2)$$

where u_a and θ are the absolute wind speed and potential temperature at the reference height (or height of the first model level) z above the displacement height, θ_s is the surface potential temperature, C_M and C_H are the drag coefficients for momentum and sensible heat respectively, u_* is the friction velocity, and θ_* is the turbulent temperature scale. Based on MOST (Monin and Obukhov, 1954) and including RSL effects (e.g. Physick and Garratt, 1995; De Ridder, 2010), one assumes surface profiles for wind and temperature. Hereby, the lower boundary conditions, i.e. the zero wind and $\theta = \theta_s$, are considered at $z = z_0$ and $z = z_{0H}$, respectively. The transfer coefficients within this framework can be calculated by:

$$C_M = k^2 \left[\ln \frac{z}{z_0} - \Psi_M(\zeta) + \Psi_M\left(\frac{z_0}{z}\zeta\right) + \Psi_M^*\left(\zeta, \frac{z}{z_*}\right) \right]^{-2}, \quad (3)$$

$$C_H = k \sqrt{C_M} \left[\ln \frac{z}{z_{0H}} - \Psi_H(\zeta) + \Psi_H\left(\frac{z_{0H}}{z}\zeta\right) + \Psi_H^*\left(\zeta, \frac{z}{z_*}\right) \right]^{-1}, \quad (4)$$

where $\zeta = zk g \theta_* / (u_*^2 \theta_0)$ is the stability parameter, Ψ_M and Ψ_H are the (integrated) stability functions for momentum and heat, Ψ_M^* and Ψ_H^* are the profile corrections to account for the RSL, θ_0 is a reference potential temperature, $g = 9.81 \text{ ms}^{-2}$ is the gravitational acceleration, and z_* is the roughness sub-layer height. These surface-layer transfer coefficients can be calculated once ζ is known. One can find a relation between Ri_B and ζ (see e.g. Arya, 2001, Section 13.4), that can be solved for ζ iteratively using e.g. the root finding scheme of Ridders (1979) as described in Press et al. (1992). The ultimate challenge is to find an analytical expression for ζ that makes the iterative method dispensable.

To conclude this section, the flux-profile relationships used in the iterative solution and for calculating the transfer coefficients are given below. For unstable surface-layer conditions ($\zeta < 0$), the integrated profile functions of Paulson (1970) derived from the profile functions of Dyer (1967) or Businger (1966) are adopted:

$$\Psi_M = 2 \ln \frac{1+x}{2} + \ln \frac{1+x^2}{2} - 2 \tan^{-1} x + \pi/2, \quad (5)$$

$$\Psi_H = 2 \ln \frac{1+y}{2}, \quad (6)$$

where $x = (1 - \gamma_m \zeta)^{1/4}$ and $y = (1 - \gamma_h \zeta)^{1/2}$ and $\gamma_m = \gamma_h = 16$. When including RSL effects (see below), one needs the original non-integrated profile functions as well:

$$\phi_M = (1 - \gamma_m \zeta)^{-1/4}, \quad (7)$$

$$\phi_H = (1 - \gamma_h \zeta)^{-1/2}. \quad (8)$$

For stable conditions ($\zeta > 0$), integrated profile functions are taken from Cheng and Brutsaert (2005) as recommended by Guo and Zhang (2007):

$$\Psi_M(\zeta) = -a \ln \left[\zeta + (1 + \zeta^b)^{\frac{1}{b}} \right], \quad (9)$$

$$\Psi_H(\zeta) = -c \ln \left[\zeta + (1 + \zeta^d)^{\frac{1}{d}} \right], \quad (10)$$

with $a = 6.1$, $b = 2.5$, $c = 5.3$, $d = 1.1$. The corresponding non-integrated profile functions are:

$$\phi_M = 1 + a \frac{\zeta + \zeta^b (1 + \zeta^b)^{\frac{1-b}{b}}}{\zeta + (1 + \zeta^b)^{\frac{1}{b}}}, \quad (11)$$

$$\phi_H = 1 + c \frac{\zeta + \zeta^d (1 + \zeta^d)^{\frac{1-d}{d}}}{\zeta + (1 + \zeta^d)^{\frac{1}{d}}}. \quad (12)$$

To account for RSL effects, the following approximation for Ψ_M^* and Ψ_H^* proposed by De Ridder (2010) is adopted:

$$\Psi_i^* \left(\zeta, \frac{z}{z_*} \right) \simeq \phi_i \left[\left(1 + \frac{\nu}{\mu z / z_*} \right) \zeta \right] \frac{1}{\lambda} \ln \left(1 + \frac{\lambda}{\mu \frac{z}{z_*}} \right) e^{-\mu \frac{z}{z_*}} \quad (13)$$

where $\lambda \simeq 1.5$, $\mu = \mu_M \simeq 2.59$, $\mu = \mu_H \simeq 0.95$ and $\nu \simeq 0.5$. For both the iterative procedure and the analytical approximation presented below, a ratio of $z_*/z_0 = 16.7$ is adopted from Sarkar and De Ridder (2010).

3 The analytical expression for the transfer coefficients

The analytical approximation to the transfer coefficients is presented below. In essence, an approximate analytical expression for ζ is derived as a function of Ri_B , z/z_0 and the ratio z_0/z_{0H} for unstable and stable conditions, separately.

3.1 Unstable surface layer

The following analytical expression for the unstable surface layer ($Ri_B < 0$) that accounts for the RSL is proposed:

$$\zeta = (1 + pQ) \frac{(L_{0M}^*)^2}{L_{0H}^*} Ri_B, \quad (14)$$

where $p = \ln(1 - Ri_B)$,

$$\begin{aligned} Q = & -0.486 + 0.219p - 0.0331p^2 - 4.93e^{-L_{0H}} - 3.65 L_{0H}^{-1} \\ & + 0.38p L_{0H}^{-1} + 14.8 L_{0H}^{-2} - 0.946p L_{0H}^{-2} - 10.0 L_{0H}^{-3} \\ & + 0.392 L_{0M} L_{0H}^{-1} - 0.0840p L_{0M} L_{0H}^{-1} + 0.368 L_{0M} L_{0H}^{-2}, \end{aligned} \quad (15)$$

and

$$L_{0i} = \ln(z/z_{0i}), \quad (16)$$

$$L_{0i}^* = L_{0i} + \frac{1}{\lambda} \ln \left(1 + \frac{\lambda}{\mu_i \frac{z}{z_*}} \right) e^{-\mu_i \frac{z}{z_*}}. \quad (17)$$

The coefficients appearing in the expression for Q are found by multiple linear regression similar to Yang et al. (2001). Eq. 14 incorporates the correct limit behaviour for $\zeta \rightarrow 0$, for which the (integrated) flux-profile relationships can be linearized in which the slope at the origin $(Ri_B, \zeta) = (0, 0)$ is given by $(L_{0M}^*)^2 / L_{0H}^*$.

3.2 Stable surface layer

For the stable surface-layer ($Ri_B > 0$), the normalized transfer coefficients C_M / C_{Mn} and C_H / C_{Hn} have a remarkable behaviour (see Section 4). The values change drastically for a change from ‘small’ to ‘large’ values of kB^{-1} . Therefore, we distinguish between weakly and strongly stable conditions. The weakly stable solution for ζ is given in Section 3.2.1. Afterwards, it is connected to a strongly stable solution in Section 3.2.2.

3.2.1 The weakly stable solution

For weakly stable conditions (small positive ζ), we first linearize ϕ_i and Ψ_i . Afterwards, the relation between Ri_B and ζ is inverted analytically from which we find an expression for ζ .

As in Yang et al. (2001), a linear approximation for (9)-(10) can be made for weakly stable conditions by putting $\Psi_i(\zeta) \simeq -\beta_i \zeta$. The relation between Ri_B and ζ now reduces to:

$$Ri_B = \zeta \frac{L_{0H}^* + S_{0H}^* \beta_H \zeta}{(L_{0M}^* + S_{0M}^* \beta_M \zeta)^2}, \quad (18)$$

where

$$S_{0i}^* = 1 - z_{0i}/z + \left(1 + \frac{\nu}{\mu_i \frac{z}{z_*}} \right) \frac{1}{\lambda} \ln \left(1 + \frac{\lambda}{\mu_i \frac{z}{z_*}} \right) e^{-\mu_i \frac{z}{z_*}}. \quad (19)$$

It is now possible to obtain ζ from Eq. 18 analytically as in Byun (1990) for $z_0 = z_{0H}$ or Yang et al. (2001) for $z_0 \neq z_{0H}$:

$$\zeta = \frac{-L_{0M}^*}{S_{0M}^* \beta_M} - \frac{BC}{4(S_{0M}^* \beta_M)^3 (B^2 + |C r|)} + \frac{B - \sqrt{B^2 + C r} + \frac{BC r}{2(B^2 + |C r|)}}{2(S_{0M}^* \beta_M)^3 r}, \quad (20)$$

Fig. 1 The analytical approximation of ζ for the stable surface layer. The intercept of the strongly stable solution ($Ri_{B,t}$, ζ_t) that connects the weakly and the strongly stable regimes is shown. The location where $r = 0$ in (20) and for which we have to exclude the third term is indicated with an open circle. See text for further information.

where

$$r = Ri_B - S_{0H}^* \beta_H / (S_{0M}^* \beta_M)^2, \quad (21)$$

$$B = S_{0M}^* \beta_M L_{0H}^* - 2 S_{0H}^* \beta_H L_{0M}^*, \quad (22)$$

$$C = 4 (S_{0M}^* \beta_M)^2 L_{0M}^* (S_{0H}^* \beta_H L_{0M}^* - S_{0M}^* \beta_M L_{0H}^*). \quad (23)$$

The best agreement between the iterative solution and the analytical approximation is obtained with $\beta_M = 4.76 + 7.03 z_0/z + 0.24 z_{0H}/z_0$ and $\beta_H = 5$. Although this rarely happens, the last term of (20) has to be omitted when $r = 0$ to avoid division by zero. In that case (i.e. $Ri_B = S_{0H}^* \beta_H / (S_{0M}^* \beta_M)^2 \simeq 0.2$), the correct analytical limit for $r = 0$ is given by the first two terms.

3.2.2 The connection of the weakly stable solution with a strongly stable solution

As suggested by Li et al. (2010), a linear relation between Ri_B and ζ for strongly stable conditions (large positive ζ) can be assumed to approximate the iterative solution. Its constant slope $d\zeta/dRi_B$ and its intercept ($Ri_{B,t}$, ζ_t), at which the weakly stable solution ceases and the strongly stable solution commences, is now determined. Therefore, it is assumed that the slope of the strongly stable solution $d\zeta/dRi_B$ is equal to the slope of the weakly stable solution at the intercept $Ri_{B,t}$. The latter is expressed by:

$$D(\zeta_t) = \frac{(L_{0M}^* + S_{0M}^* \beta_M \zeta_t)^3}{L_{0M}^* L_{0H}^* + \zeta_t (2 S_{0H}^* \beta_H L_{0M}^* - S_{0M}^* \beta_M L_{0H}^*)}, \quad (24)$$

Fig. 2 A decision tree for calculating ζ and the consequential transfer coefficients

which can be obtained by taking the reciprocal of the derivative of (18). The following two criteria now have to be fulfilled simultaneously for the intercept $(Ri_{B,t}, \zeta_t)$ at which the weakly stable solution ceases and the strongly stable solution commences. Firstly, ζ_t should be lower than the maximum ζ for which the weakly stable solution ($\zeta < \zeta_t$) is a good approximation. Secondly, $D(\zeta_t)$ given by (24) should be a representative value for the slope of the strongly stable solution ($\zeta \geq \zeta_t$). The following expression for ζ_t meets these two criteria:

$$\zeta_t = -0.316 - 0.515e^{-L_{0H}} + 25.8e^{-2L_{0H}} + 4.36L_{0H}^{-1} - 6.39L_{0H}^{-2} + 0.834(\ln L_{0M}) - 0.0267(\ln L_{0M})^2. \quad (25)$$

Then, $Ri_{B,t}$ can be obtained by calculating (18) at ζ_t :

$$Ri_{B,t} = \zeta_t \frac{L_{0H}^* + S_{0H}^* \beta_H \zeta_t}{(L_{0M}^* + S_{0M}^* \beta_M \zeta_t)^2}. \quad (26)$$

The expression for the strongly stable solution ($Ri_B > Ri_{B,t}$) is now given by:

$$\zeta = \zeta_t + D(\zeta_t)(Ri_B - Ri_{B,t}). \quad (27)$$

For the weakly stable solution ($Ri_B \leq Ri_{B,t}$), the solution is given by (20).

An analytical solution for the whole stable range has now been constructed.

Figure 2 gives an overview of the presented analytical solution for both unstable and stable conditions for easy implementation.

4 Results

The normalized transfer coefficients $C_M/C_{M,n}$ and $C_H/C_{H,n}$, for which ‘n’ refers to neutral stability ($Ri_B = 0$), are compared for the approximate

Fig. 3 The approximate (apprx) and iterative (iter) transfer relations for momentum are demonstrated. Unstable and stable conditions are shown respectively on the left in panels (a)-(c)-(e)-(g)-(i)-(k) and on the right in panels (b)-(d)-(f)-(h)-(j)-(l). kB^{-1} is set to -0.5, 0, 2, 10, 20 and 30 in panels (a)-(b), (c)-(d), (e)-(f), (g)-(h), (i)-(j) and (k)-(l), respectively. In each panel, values for z/z_0 are shown in different colours.

Fig. 4 As in Fig. 3, but for the transfer relations for heat instead of momentum.

analytical and iterative solution for $kB^{-1} = \{-0.5, 0, 2, 10, 20, 30\}$, $z/z_0 = \{10, 10^3, 10^5\}$ and Ri_B between -5 and 2.5 by steps of 0.02 . The reference height (i.e. height of the first model level) is typically 10 to a few tens of metres, and consequently z_0 values up to 1.5 m - typical for urban surfaces - are captured. The resulting transfer relations are shown in Figs 3 and 4, and are discussed below.

For the stable region, the resulting analytical transfer relations follow the iterative solution very well, despite the fact that the functions change shape drastically when going from low to high kB^{-1} values. In particular, they decrease drastically from a certain Ri_B value and this value itself depends strongly on the ratio z/z_0 and kB^{-1} . This is captured very well, because the analytical approximation has a variable transition point $Ri_{B,t}$ between the weakly stable and the strongly stable regime that depends on z_0 and kB^{-1} . Because of the linear approximation for the strongly stable solution, there can be a substantial deviation between the analytical and iterative solutions for ζ as a function of Ri_B for strongly stable conditions. However, this has a minor effect on the final values of the transfer coefficients, because they are nearly constant for strongly stable conditions. The relative error is less than 10%, except at an Ri_B -range around $Ri_{B,t}$, at which the transfer coefficients fall abruptly with increasing Ri_B . For this narrow range, the relative error can be up to 40% for $kB^{-1} > 15$. Because of the dramatic drop around $Ri_{B,t}$ and the sensitivity of the position for this drop on z/z_0 and kB^{-1} , the error of the analytical approximation around $Ri_{B,t}$ is actually small compared to other uncertainties of either Ri_B , z/z_0 or kB^{-1} . Moreover, uncertainties in the flux-profile relationships are much larger than the error of the analytical solution for very stable conditions. This can be seen when comparing the iterative solution obtained with the flux-profile relationships of Beljaars and Holtslag (1991) with that obtained with the flux-profile relationships of Cheng and Brutsaert (2005), see Section 5. For unstable conditions, we obtain good agreement for C_M/C_{Mn} and C_H/C_{Hn} : the relative error of the transfer coefficients is always $< 10\%$ for $-5 \leq Ri_B < 0$.

5 Discussion: comparison with other non-iterative methods

A comparison is made between existing non-iterative parametrizations from Yang et al. (2001) and Li et al. (2010), the non-iterative parametrization presented herein and the iterative solution. This is done under conditions for which our parametrization was developed, i.e. urban surfaces. A typical value for the roughness length (z_0) for urban areas is 1 m and this leads to a ratio $z/z_0 = 15$ when assuming a height of the first model layer of $z = 15$ m. With this configuration, we discuss the normalized transfer coefficients C_H/C_{Hn} as a function of kB^{-1} for four stability conditions in Fig. 5 (similar results are found for C_M/C_{Mn}).

For the weakly unstable case ($Ri_B = -0.08$), Li et al. (2010) has a large deviation from the iterative solution for high kB^{-1} values. In contrast, Yang

Fig. 5 Normalized transfer coefficients for heat as a function of kB^{-1} between -0.5 and 30 for a fixed value of $z/z_0 = 15$. Weakly unstable ($Ri_B = -0.08$), weakly stable ($Ri_B = 0.08$), strongly unstable ($Ri_B = -3$) and strongly stable ($Ri_B = 1$) surface-layer conditions are demonstrated in panel (a), (b), (c) and (d) respectively. Results for the analytical approximations of Yang et al. (2001) (ya), Li et al. (2010) (li), the new parametrization (new) and the iterative solution (iter) are shown. For unstable stratification, the iterative solution (bd) uses (integrated) flux-profile relationships from Paulson (1970) for the unstable case. For the stable situations, the iterative solutions are shown using flux-profile relationships from Beljaars and Holtslag (1991) (bh) and from Cheng and Brutsaert (2005) (cb) respectively.

et al. (2001) and the current parametrization compare very well with the iterative solution. For the weakly stable case ($Ri_B = 0.08$), Yang et al. (2001) gives the best comparison with the iterative solution with flux-profile relationships from Beljaars and Holtslag (1991) and the new proposed analytical solution with the flux-profile relationships from Cheng and Brutsaert (2005). A large deviation is found for Li et al. (2010) compared to the iterative solution for $kB^{-1} > 4$: the ratio C_H/C_{Hn} becomes > 1 , i.e. the drag coefficient for the stable case becomes higher than that for the neutral case which is unrealistic.

tic. For the strongly unstable case ($Ri_B = -3$), similar results are obtained as in the weakly unstable case. In addition, Yang et al. (2001) has a substantial deviation for very large kB^{-1} values between 20 and 25, while it is invalid for $kB^{-1} > 25$. For the strongly stable case ($Ri_B = 1$), Yang et al. (2001) does not have a solution, but now Li et al. (2010) performs relatively well. Again here, the new parametrization performs well for all kB^{-1} values. As mentioned before, the new analytical approximation for stable stratification is constructed using flux-profile relationships from Cheng and Brutsaert (2005) for stable stratification. In contrast, the analytical approximation of Li et al. (2010) is constructed using those from Beljaars and Holtslag (1991). For the strongly stable case, a substantial difference occurs between the iterative solutions using the different flux-profile relationships, and consequently also in their respective analytical approximations. This results from a large discrepancy between the different flux-profile relationships for strongly stable stratification (Guo and Zhang, 2007). Given this uncertainty for strong stability, the errors made with the new analytical approximation over the iterative solution are negligible.

To conclude, both the analytical approximations of Li et al. (2010) and Yang et al. (2001) perform well for limited ranges of kB^{-1} and Ri_B values, but they fail to approximate the iterative solution(s) for the full range $-0.5 \leq kB^{-1} \leq 30$ and $-5 \leq Ri_B \leq 2.5$. In particular, large kB^{-1} values typical for urban surfaces are problematic. In contrast, the new non-iterative solution proposed herein captures the full range very well.

6 Conclusions

In order to determine turbulent fluxes momentum and sensible heat between the surface and the atmosphere in a mesoscale atmospheric model, a new analytical method is presented to approximate the iterative transfer coefficients within the MOST framework, including RSL effects. Essentially, an analytical expression for ζ as a function of Ri_B is found from which the transfer coefficients for momentum and sensible heat are obtained. The transfer coefficients for other scalar variables such as moisture (q) can be taken equal to the transfer coefficient of sensible heat as shown in e.g. Andreas and Murphy (1986). For the unstable case ($Ri_B < 0$), an ζ is obtained by multiple linear regression, in which the correct limit behaviour for $Ri_B \rightarrow 0$ is incorporated. For weakly stable conditions, the exact solution for ζ using linear stability functions is used, and for strongly stable conditions a linear relation between Ri_B and ζ is assumed. The weakly and strongly stable solutions are smoothly connected by a transition point ($Ri_{B,t}, \zeta_t$) that depends on kB^{-1} and the ratio z/z_0 .

The transfer coefficients resulting from this analytical approximation reproduce the iterative solution very well for $10 \leq z/z_0 \leq 10^5$ and $-0.5 \leq kB^{-1} \leq 30$ that represents most, if not all, existing surface types. This should lead to an overall better performance with regard to the momentum exchange and energy balance at the surface. As most existing global and regional at-

mospheric models use an analytical approach already, our new method will require only a small change to the model code¹. Especially, the presented bulk parametrization that avoids any iterative procedure can be used to simulate the contrasting behaviour among different land-surface types, for example to study the urban heat island effect as in Van Weverberg et al. (2008), Demuzere et al. (2008) and Sarkar and De Ridder (2010).

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¹ The Fortran code for the presented non-iterative approximation can be obtained from the authors on request.

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