# A Short Introduction to Probabilistic Soft Logic

Angelika Kimmig, Stephen H. Bach, Matthias Broecheler,
Bert Huang and Lise Getoor

NIPS Workshop on Probabilistic Programming 2012

http://psl.umiacs.umd.edu

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[Broecheler et al, UAI 10]

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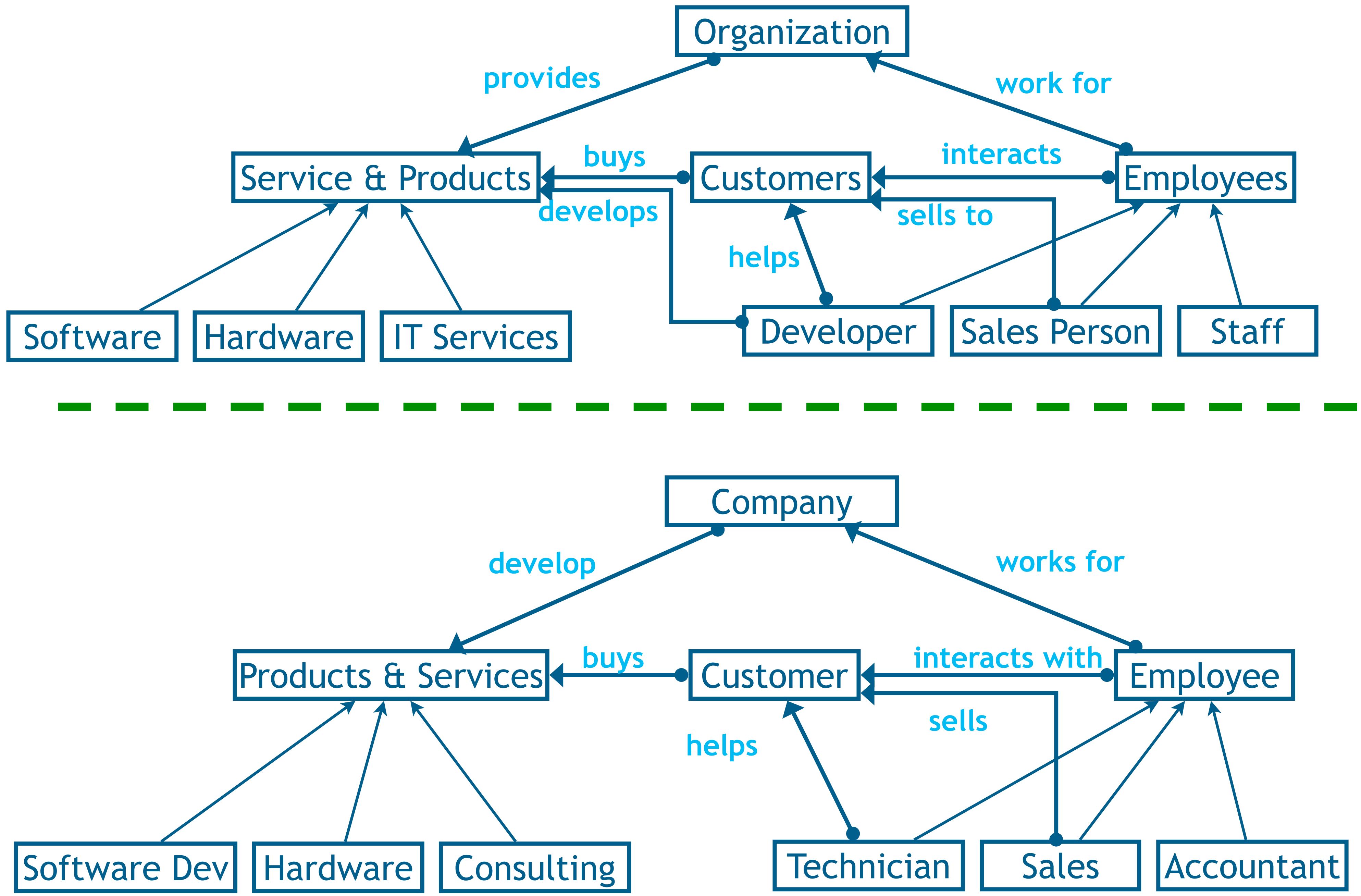
new approach
NIPS 12

Efficient MPE inference: continuous convex optimization

# Applications

- Collective classification
- Ontology alignment
- Entity resolution
- Link prediction
- Trust in social networks
- Social group modeling
- Personalized medicine
- ...

# Ontology Alignment



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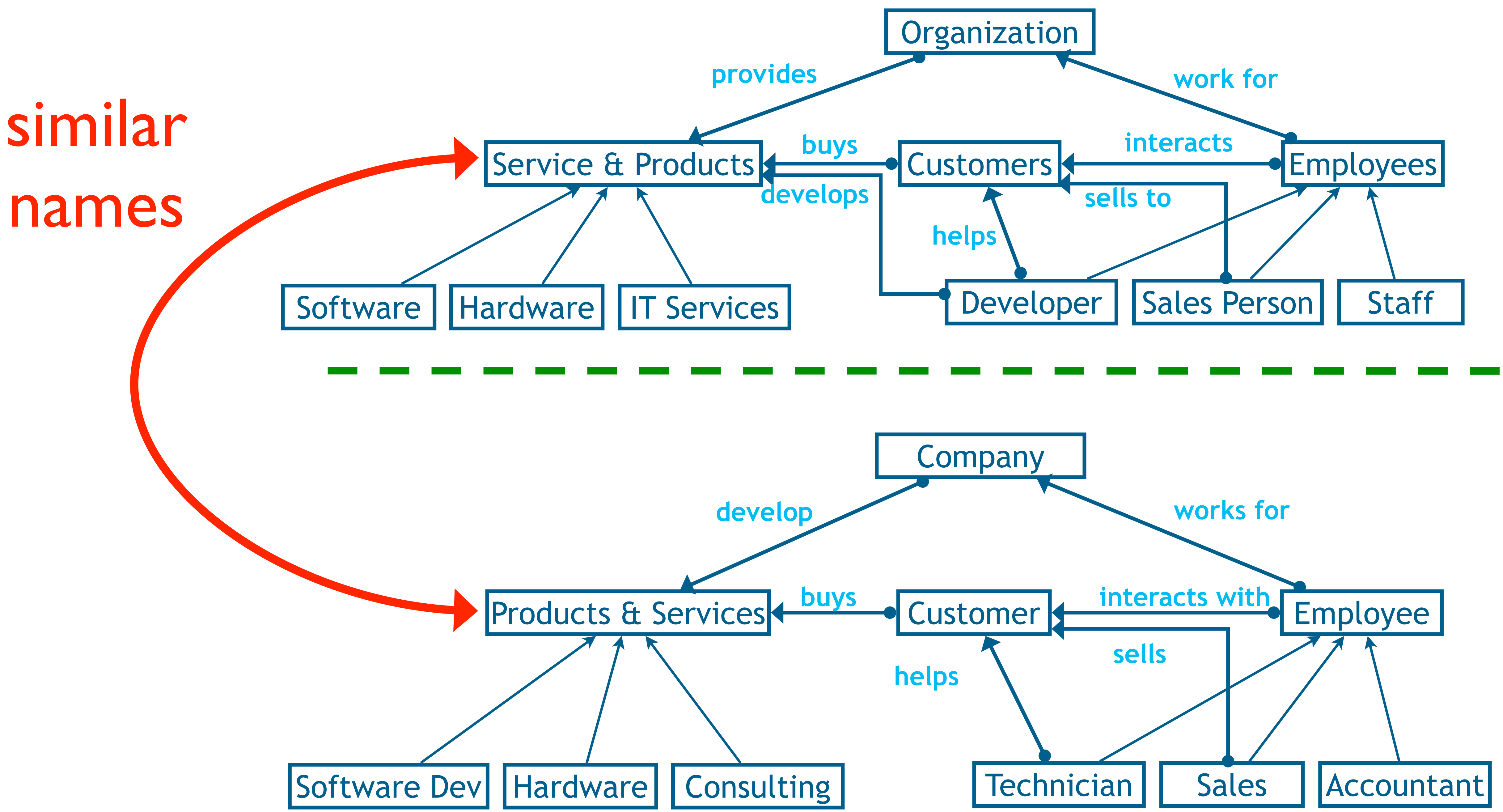

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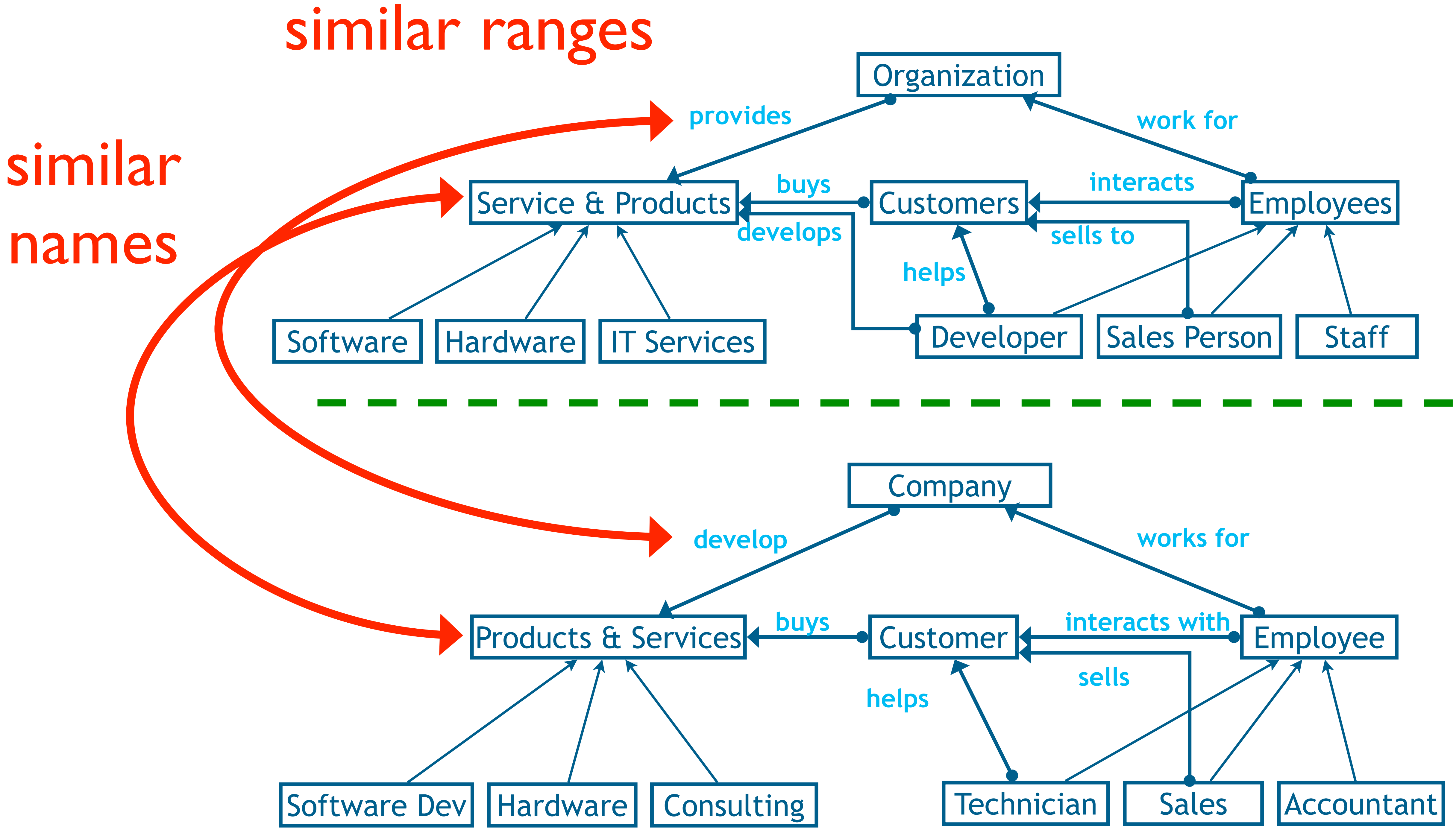

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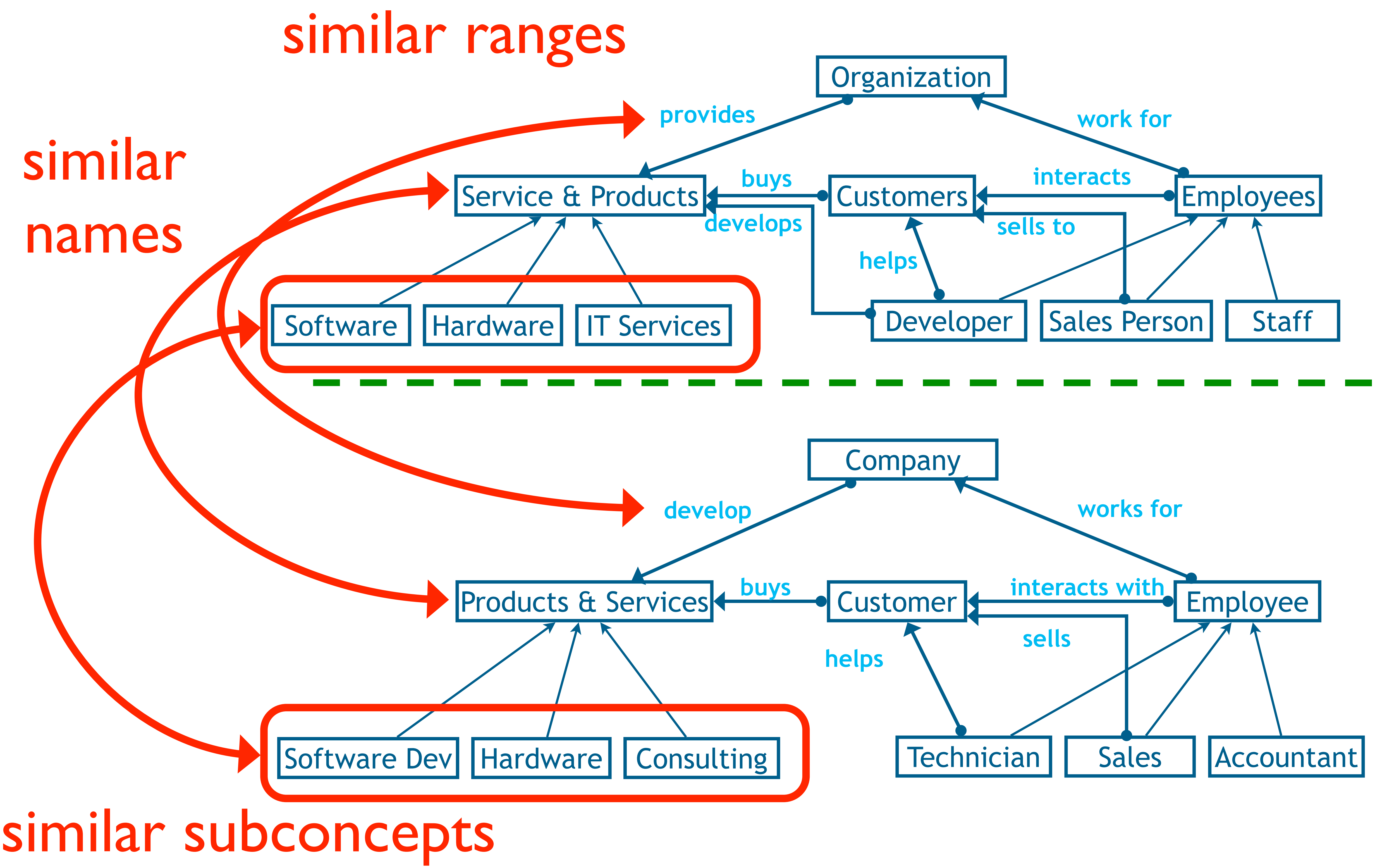
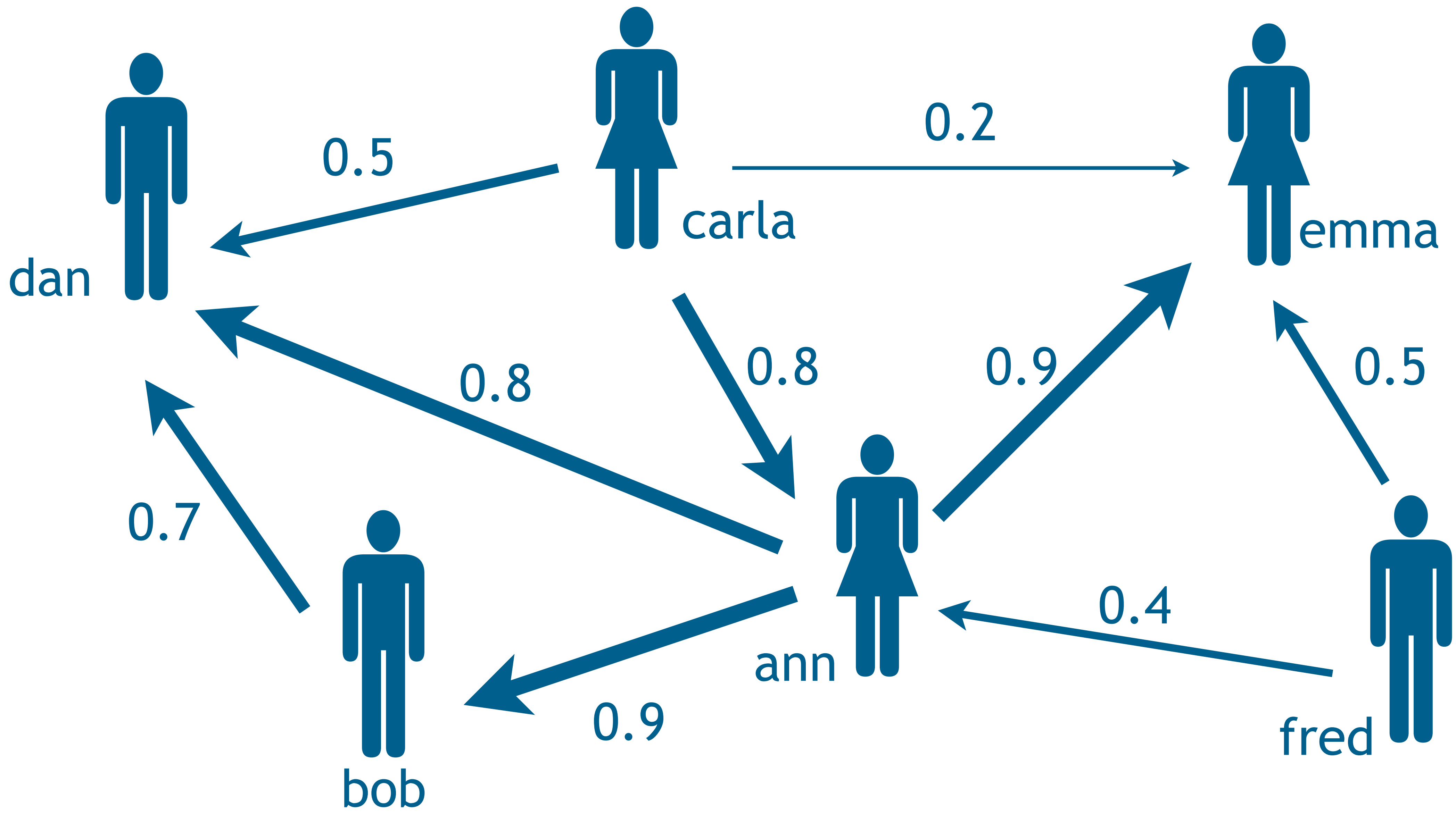


# Trust Modeling



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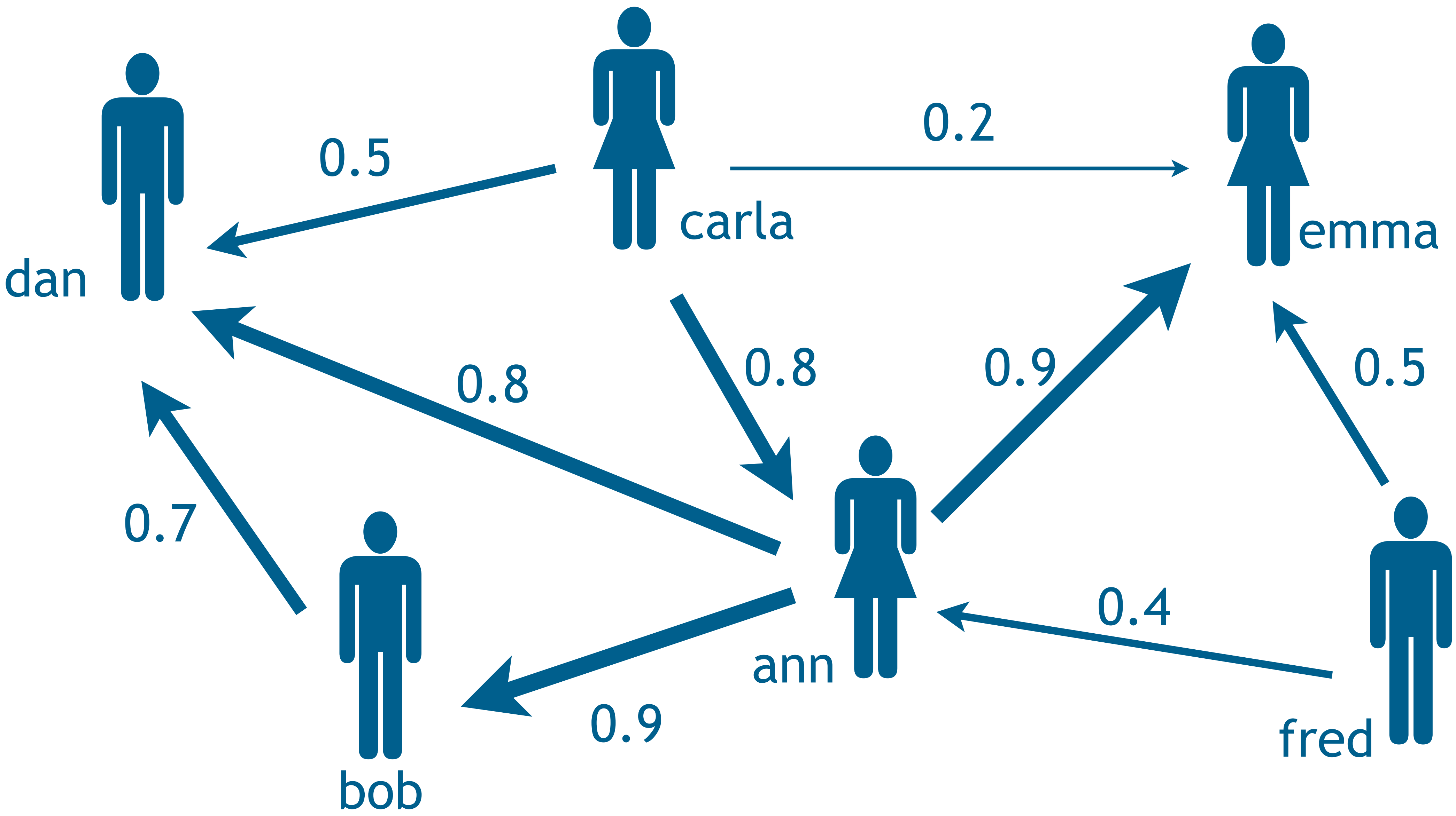


$$trusts(X,Y) \land trusts(Y,Z) \rightarrow trusts(X,Z)$$

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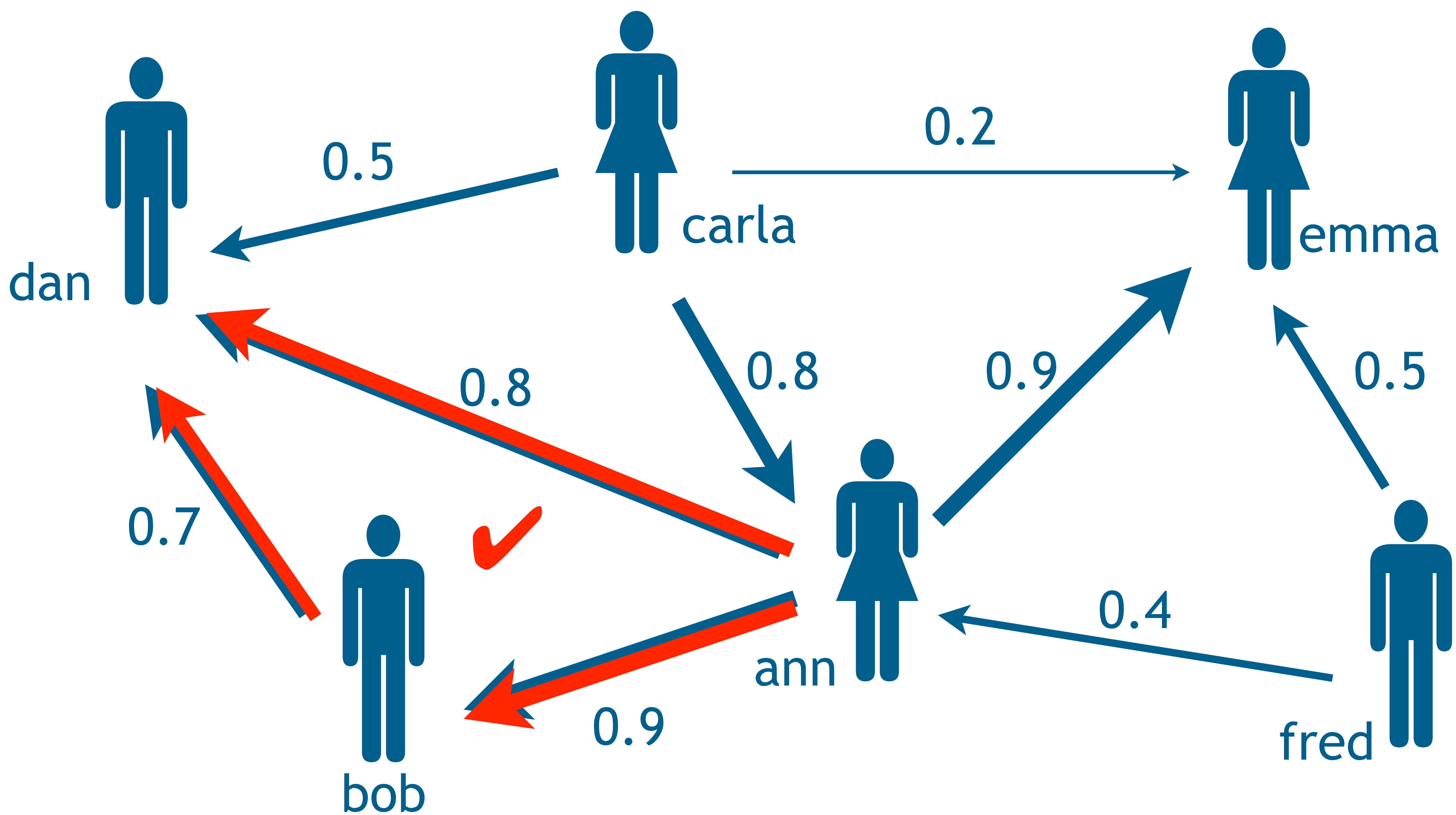


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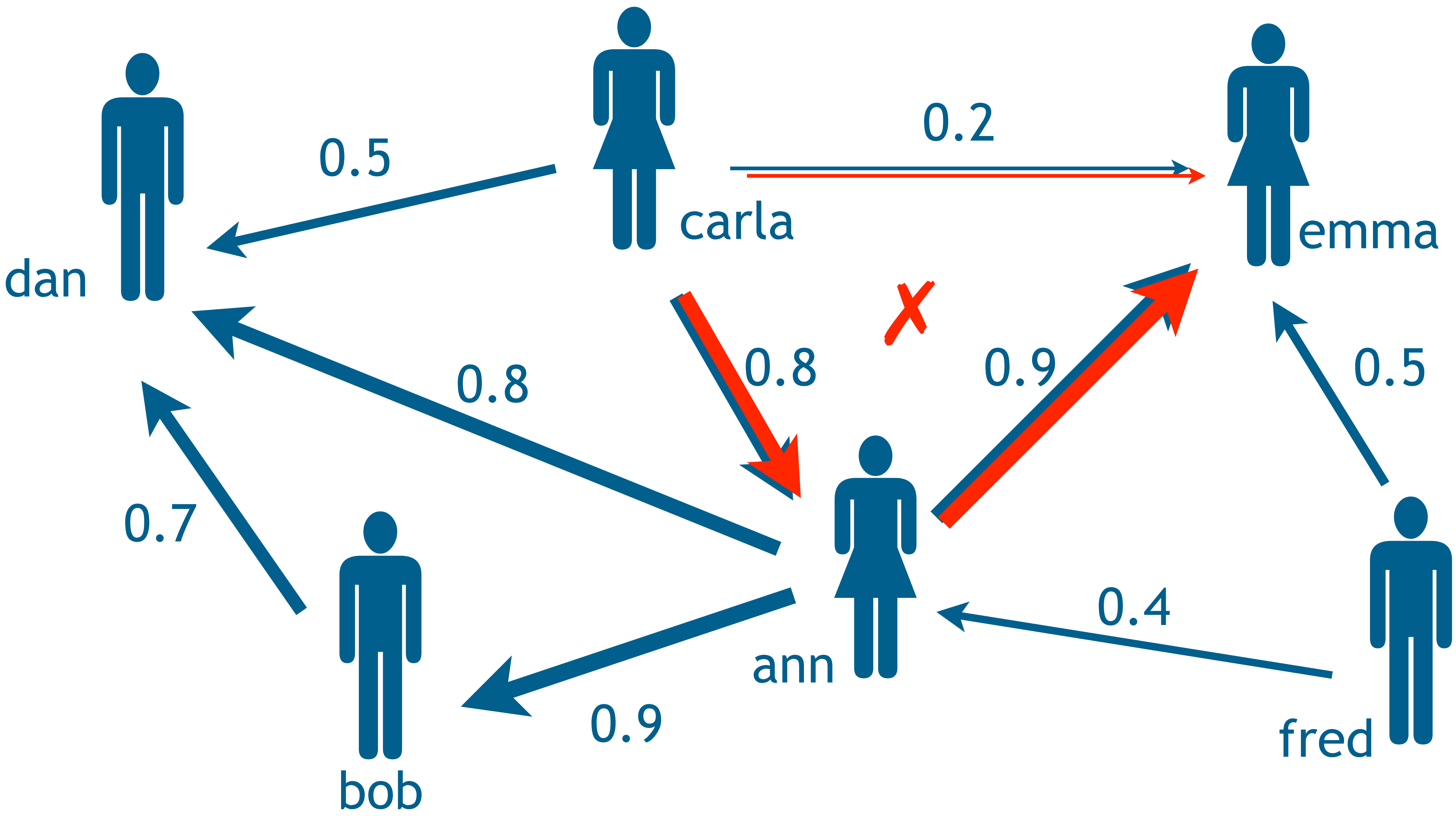


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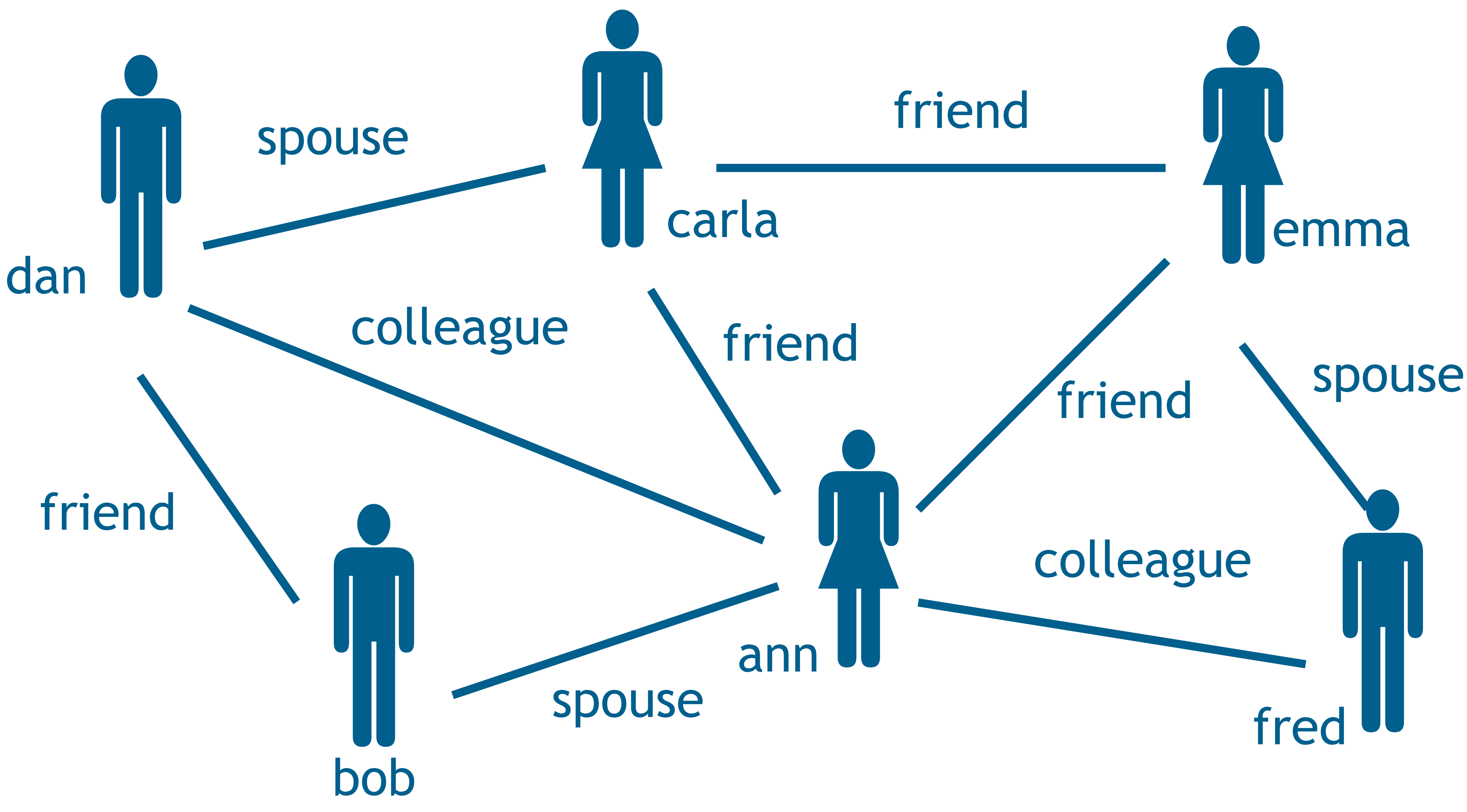
# Voter Opinion Modeling



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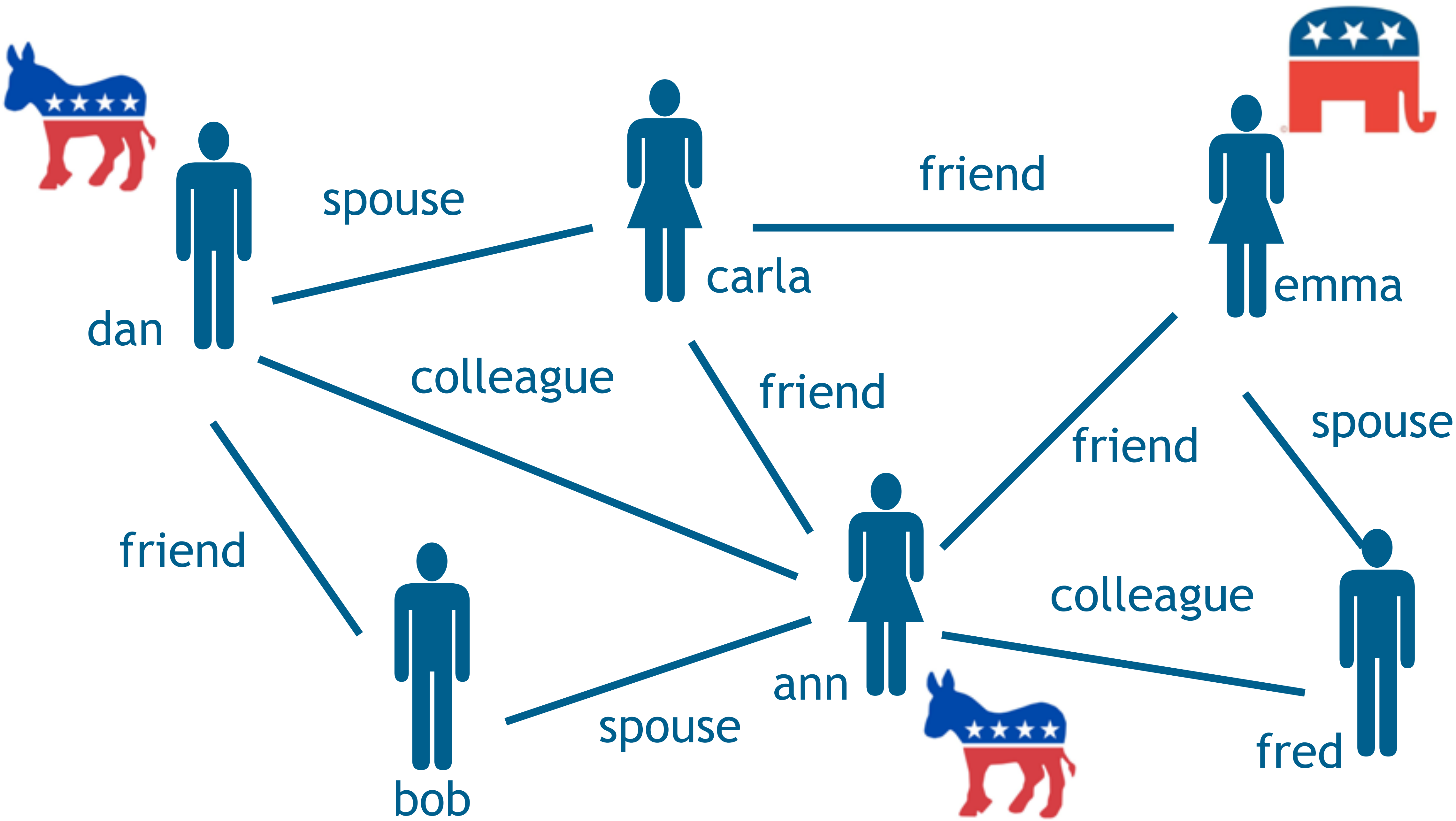

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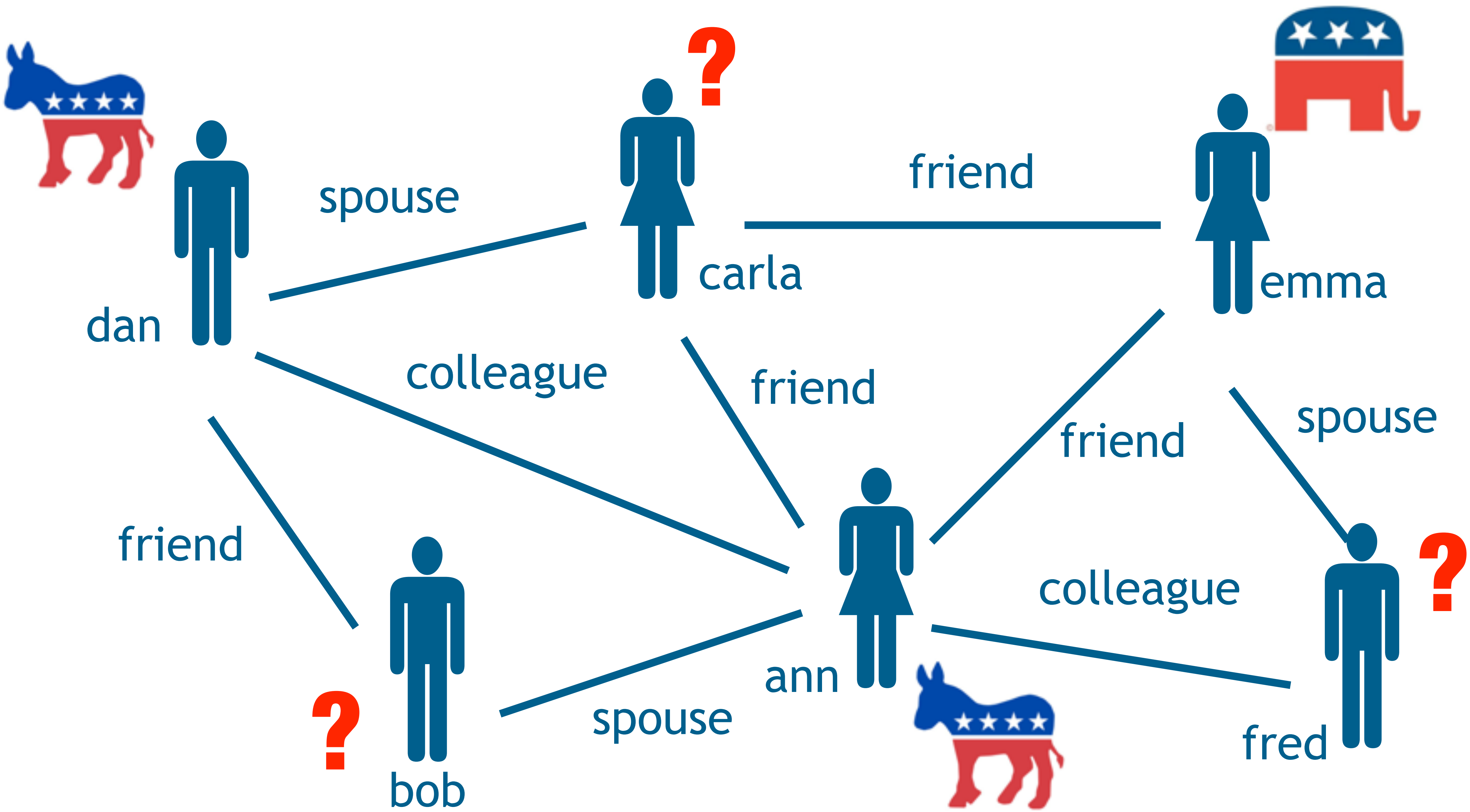
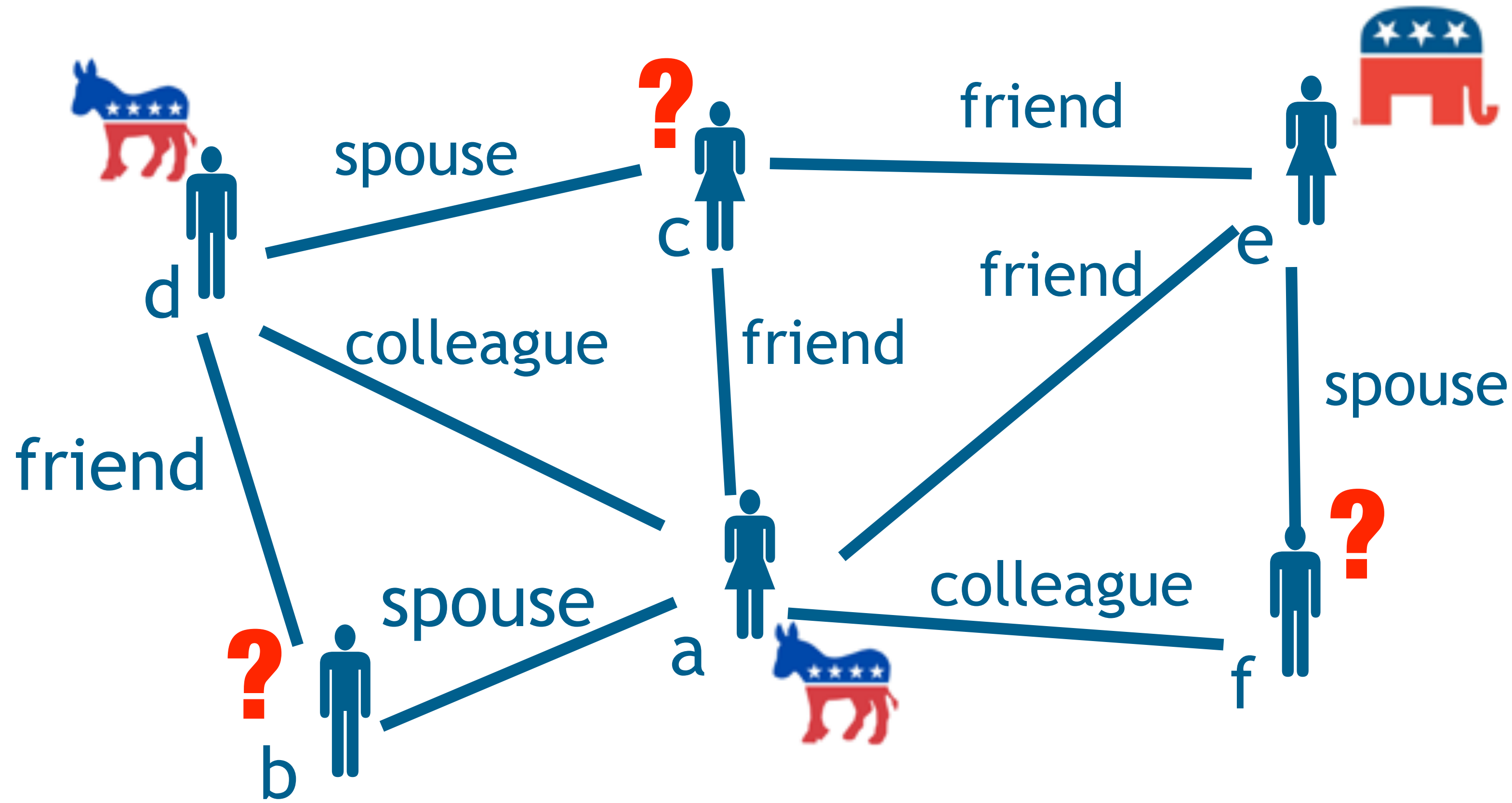


# PSL Program

friend(carla,emma)=0.9
friend(bob,dan)=0.4
spouse(ann,bob)=1.0
prefers(ann, )=0.8
...



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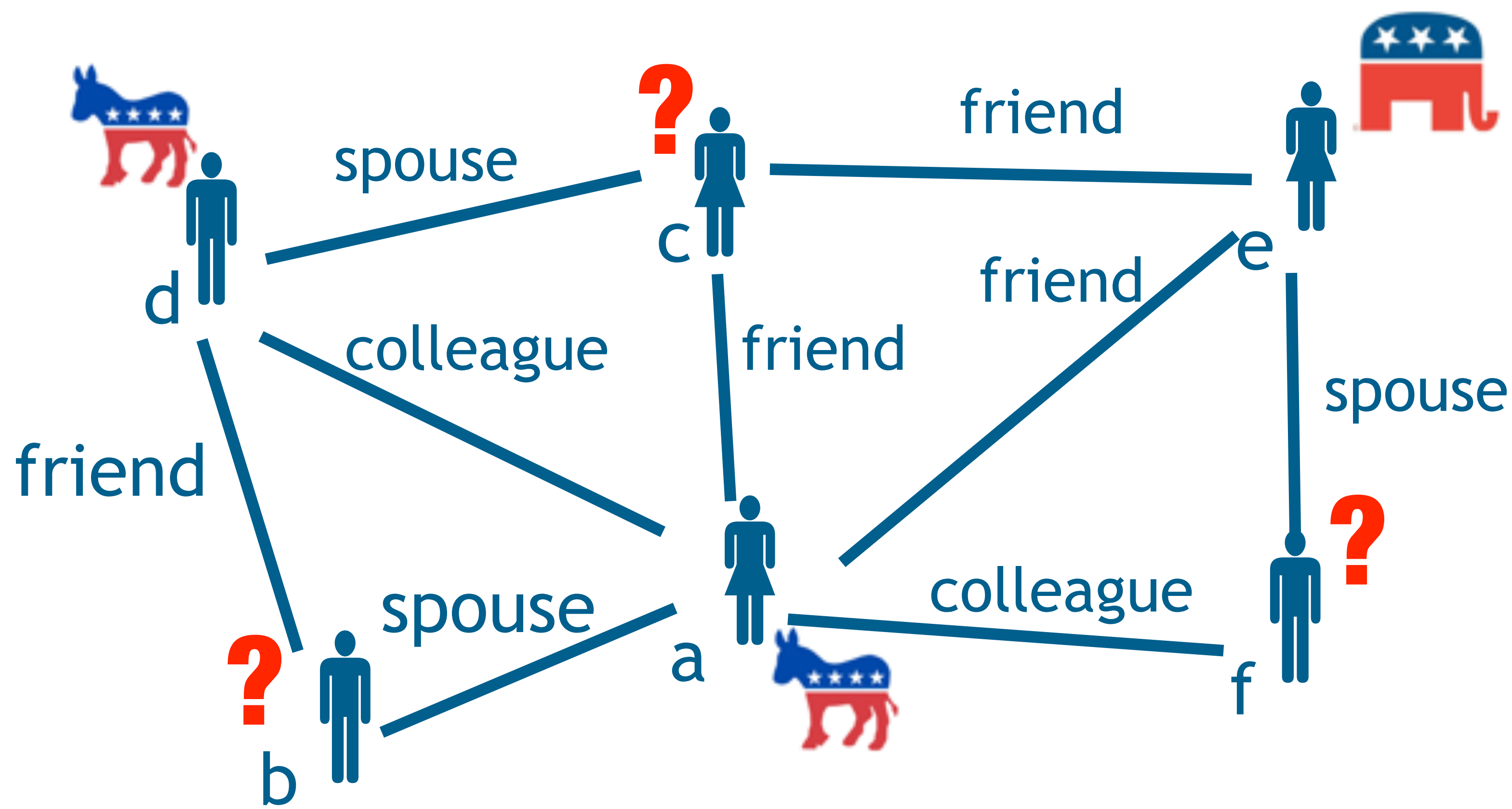
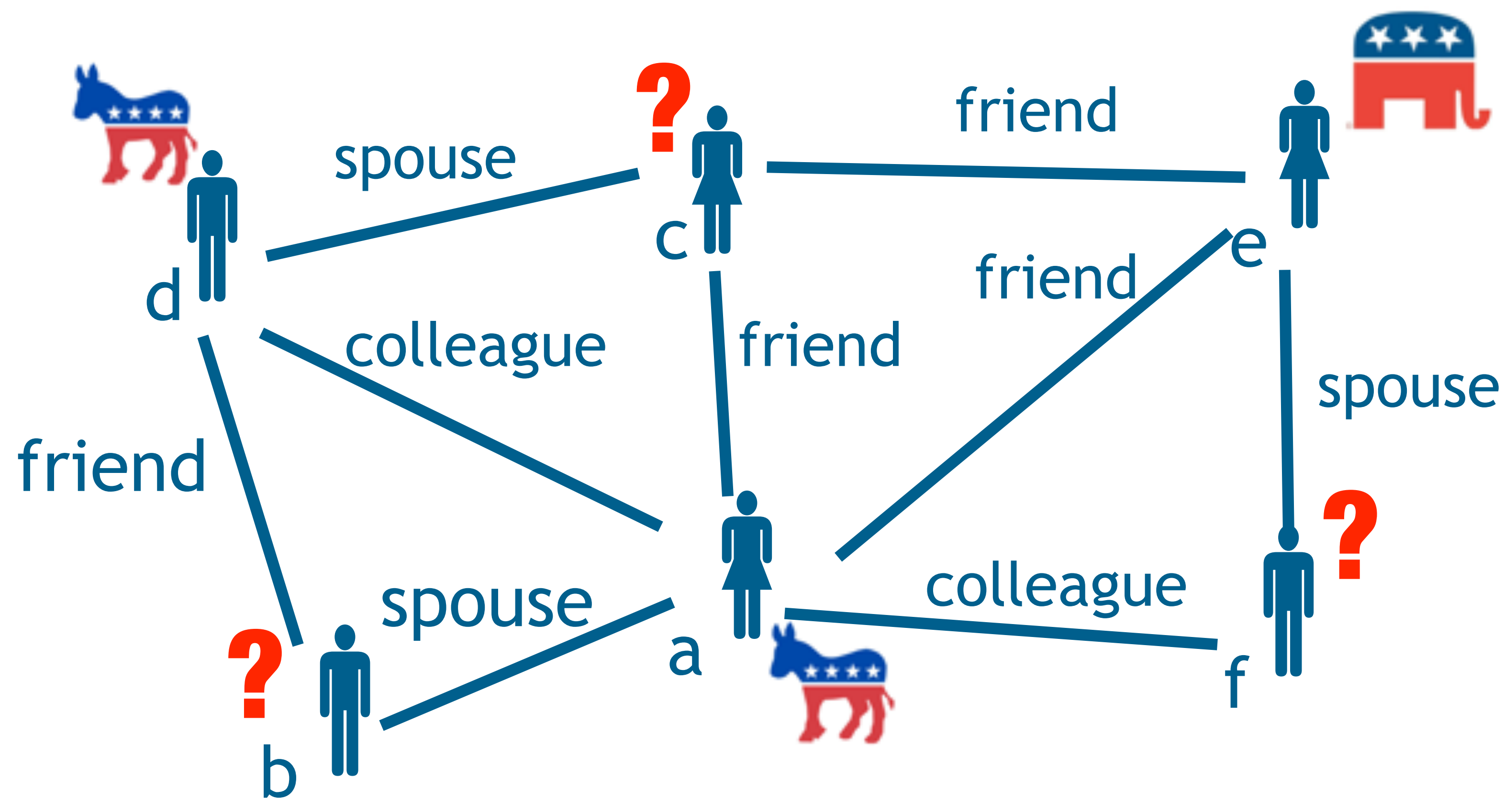



0.3: lives(A,S) ∧ majority(S,P) → prefers(A,P)
0.8: spouse(B,A) ∧ prefers(B,P) → prefers(A,P)
0.1: similarAge(B,A) ∧ prefers(B,P) → prefers(A,P)
0.4: prefers(A,P) → prefersAvg({A.friend},P)
partial-functional: prefers

# Constraints

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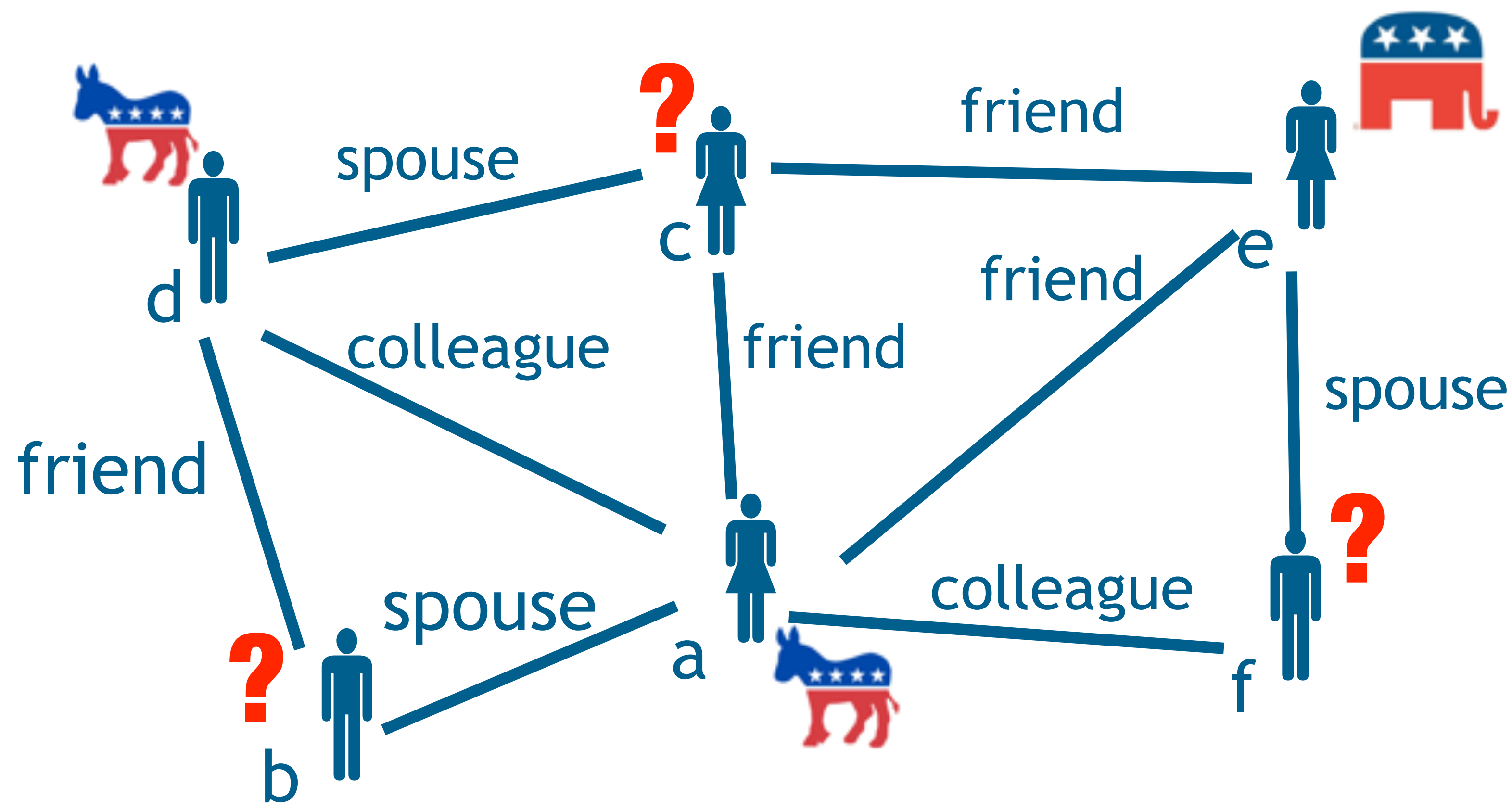
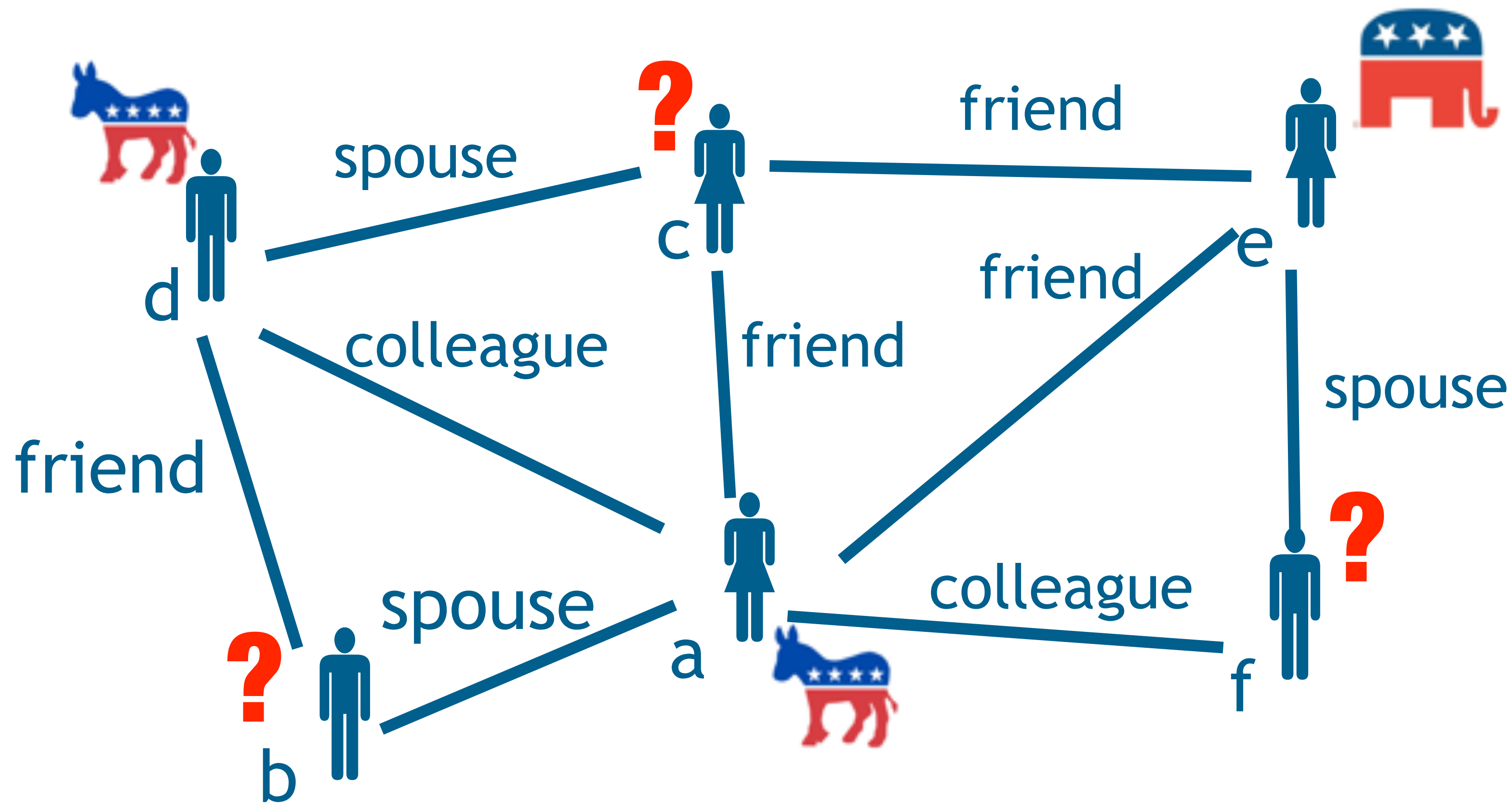



partial-functional: prefers

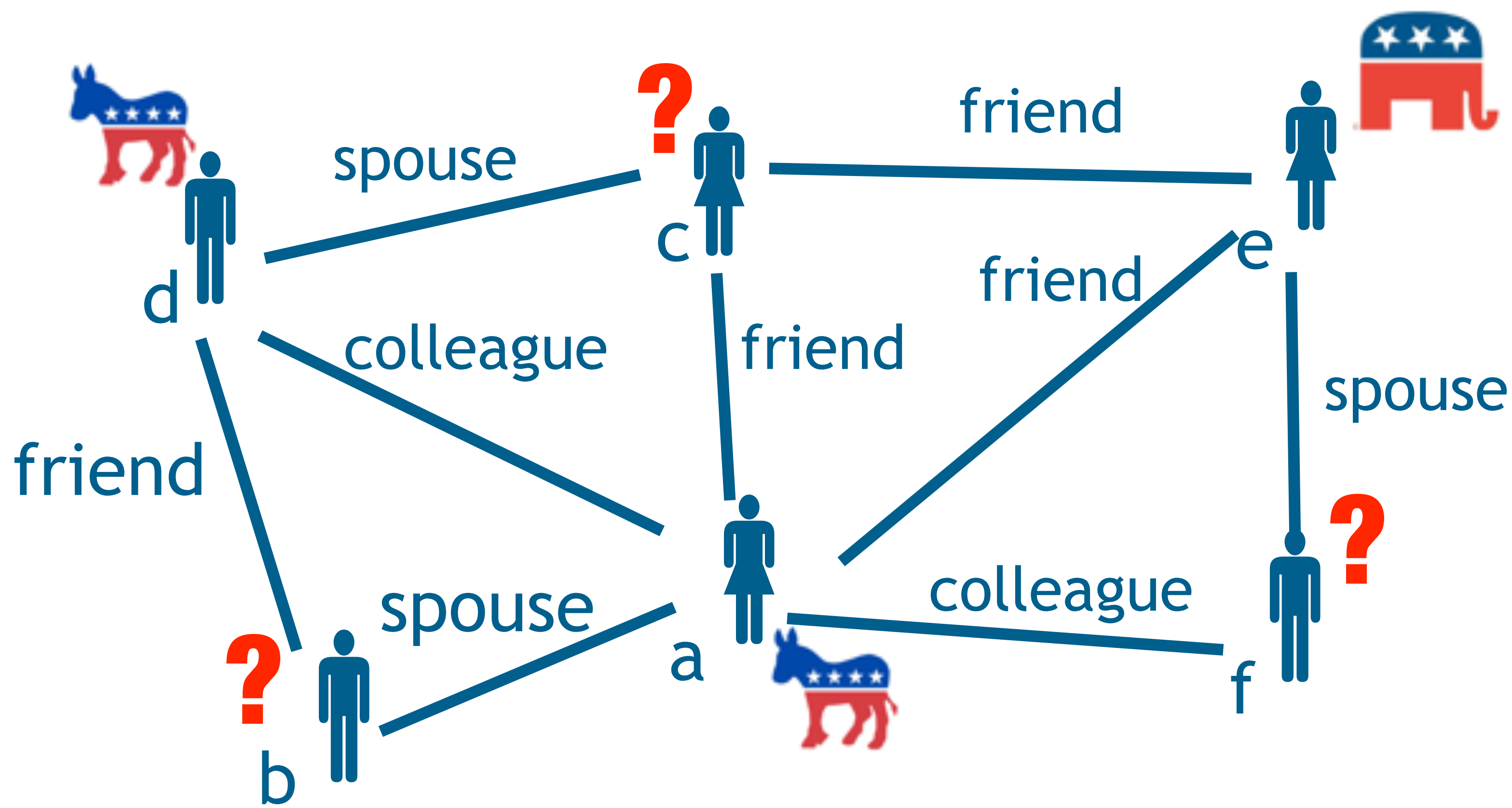
prefers(A, )+prefers(A, ) $\leq$ 1.0

# Local Rules



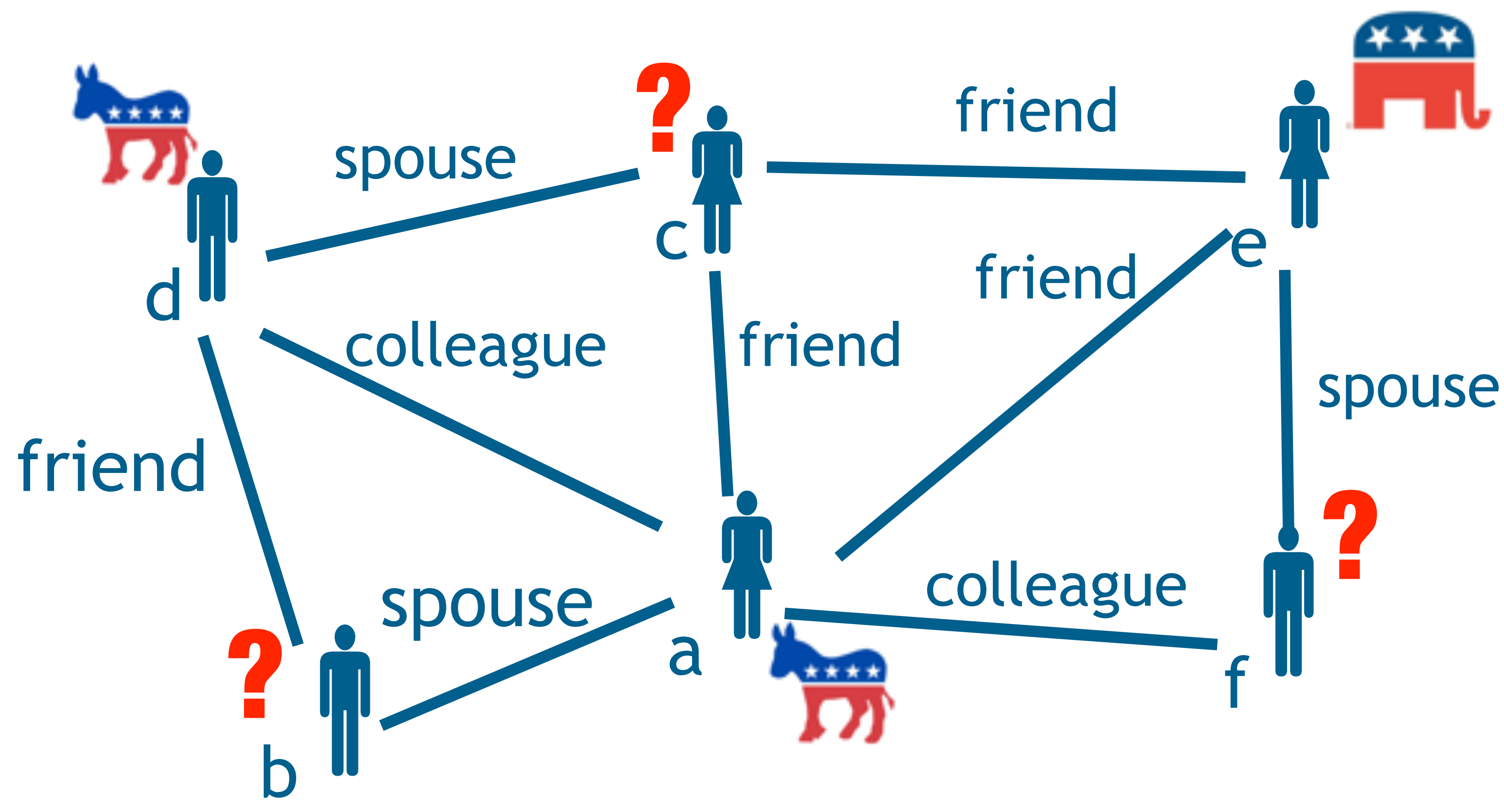

$$0.3: \text{lives}(A,S) \wedge \text{majority}(S,P) \rightarrow \text{prefers}(A,P)$$

# Propagation Rules




0.8: spouse(B,A) ∧ prefers(B,P) → prefers(A,P)

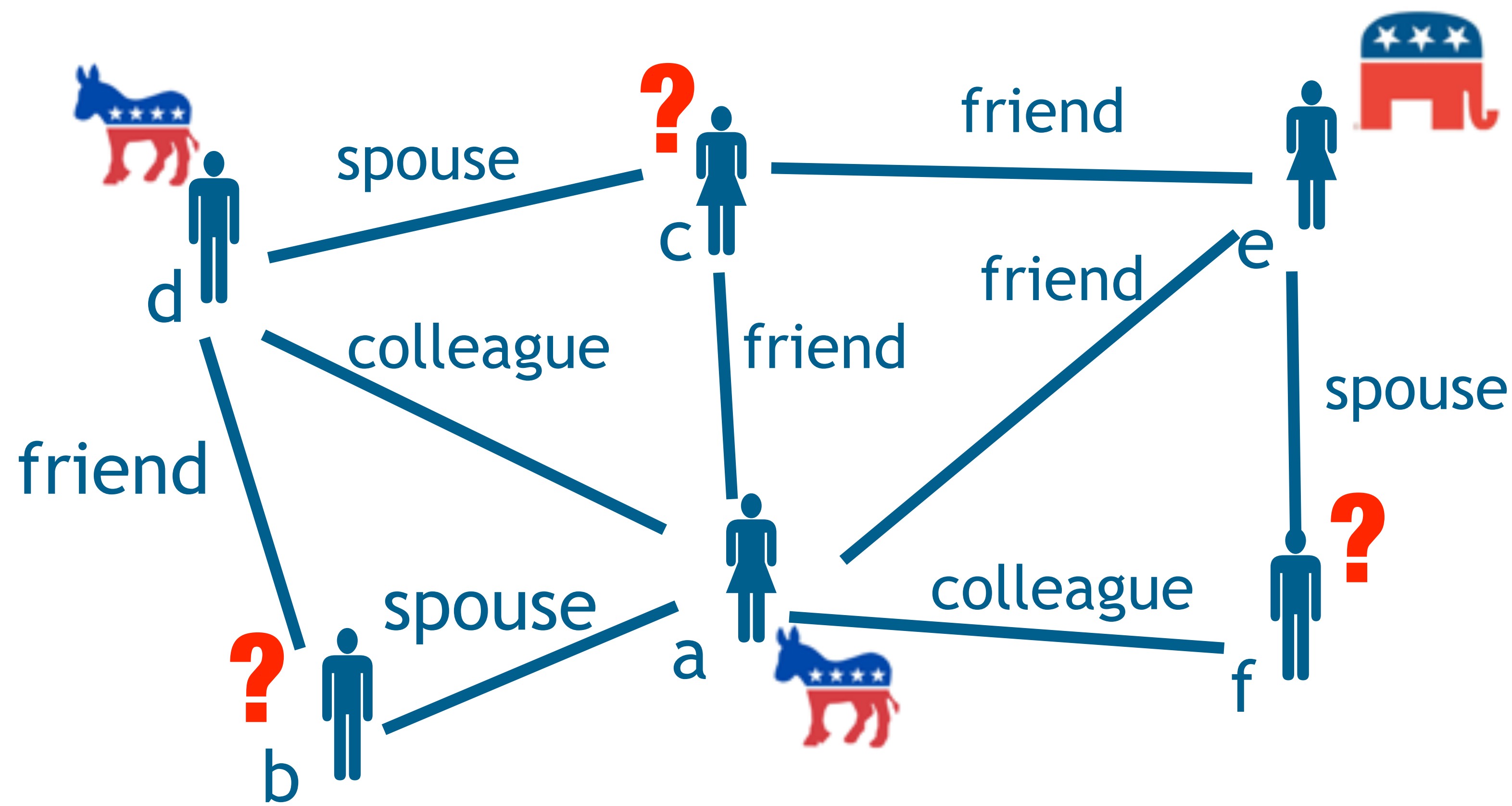
# Similarity Rules




0.1: similarAge(B,A) ∧ prefers(B,P) → prefers(A,P)

Similarity function with range [0,1]

# Sets

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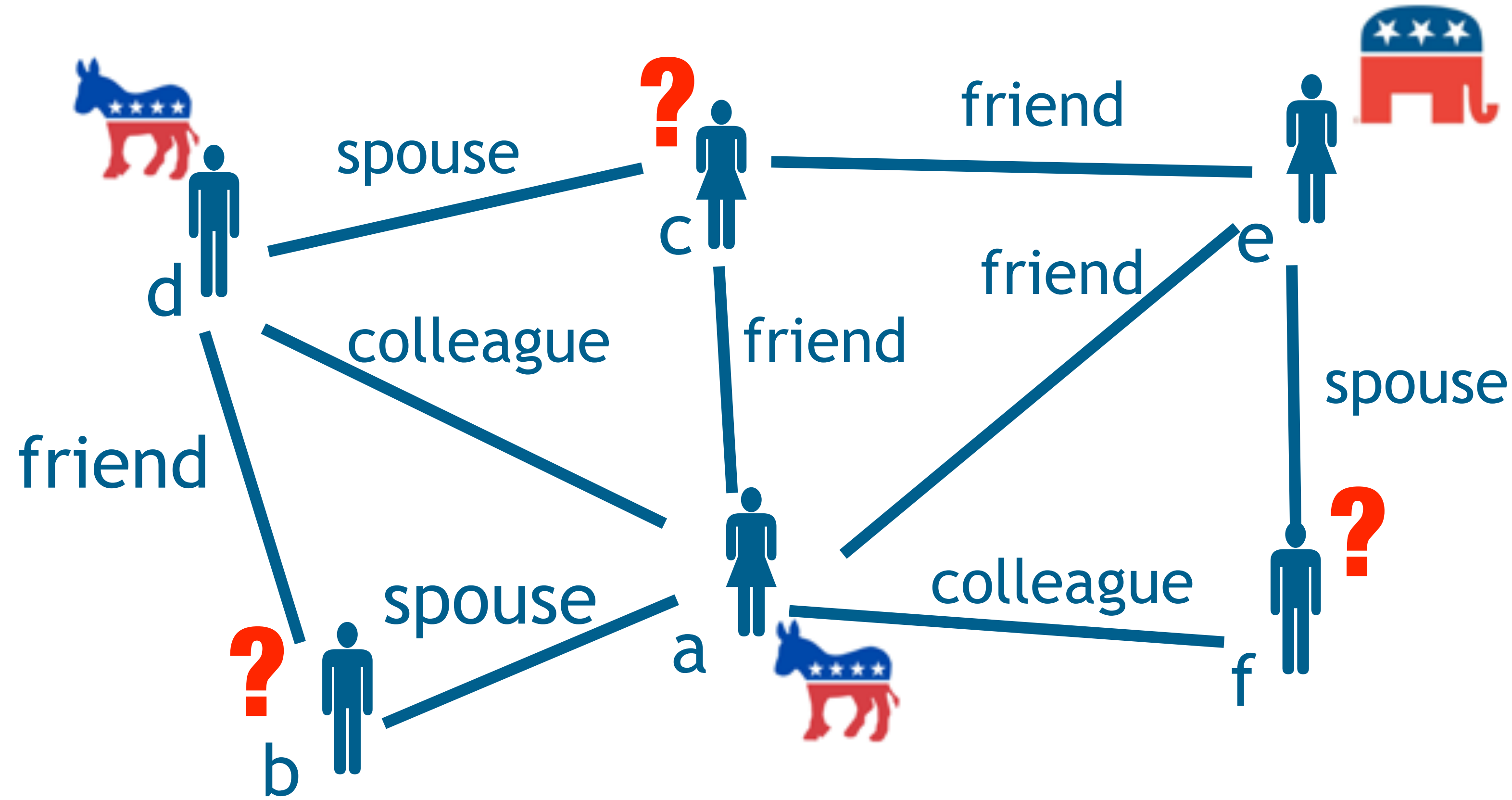


all X such that friend(A,X)

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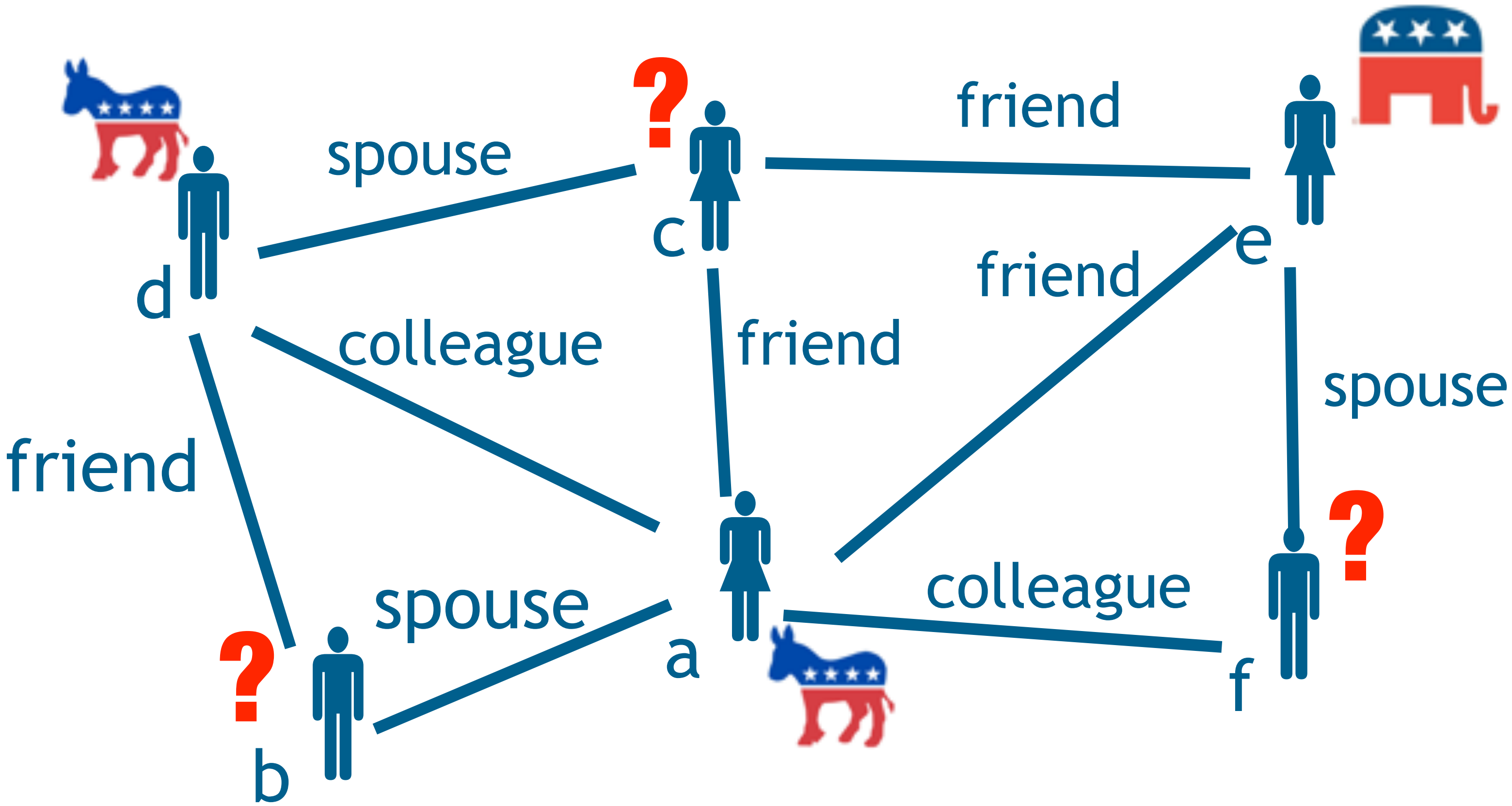


all X such that friend(A,X)

0.4: prefers(A,P) → prefersAvg({A.friend},P)

truth value := average truth value of prefers(X,P)

# PSL Program

- Ground atoms = random variables
- Soft truth value assignments
- Assignment satisfying more rules more likely
- Constraints to rule out unwanted assignments

# Probabilistic Model

$$f(I) = \frac{1}{Z} \exp\left( - \sum_{r \in P} \sum_{g \in G(r)} w_r (d_g(I))^k \right)$$

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Rule's weight

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# Probabilistic Model

Ground rule's distance from satisfaction given I

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Ground rule's distance from satisfaction given I

∈{1,2}

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Interpretation

Rule's weight

Set of rule groundings

Normalization constant

$$Z = \int_{J \in \mathcal{I}} \exp\left( - \sum_{r \in P} \sum_{g \in G(r)} w_r (d_g(J))^k \right)$$

# Distance from Satisfaction

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Lukasiewicz t-norm

$$I(v_1 \wedge v_2) = \max\{0, I(v_1) + I(v_2) - 1\}$$
$$I(v_1 \vee v_2) = \min\{I(v_1) + I(v_2), 1\}$$
$$I(\neg l_1) = 1 - I(v_1)$$

# Distance from Satisfaction

similarAge(bob,ann) $\wedge$ prefers(bob, ) $\rightarrow$ prefers(ann, )

0.8 0.5 0.5

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max{0, 0.8+0.5−1} ≤ 0.5 d=0.0

# Distance from Satisfaction

similarAge(bob,ann) $\wedge$ prefers(bob, ) $\rightarrow$ prefers(ann, )

0.8 0.5 0.2

$$d_r(I) = \max\{0, I(body) - I(head)\}$$

$$I(v_1 \wedge v_2) = \max\{0, I(v_1) + I(v_2) - 1\}$$

max{0, 0.8+0.5−1} ≤ 0.5 d=0.0

max{0, 0.8+0.5−1} ≤ 0.2 d=0.1

# Distance from Satisfaction

similarAge(bob,ann) ∧ prefers(bob, ) → prefers(ann, )

0.8 0.9 0.2

$$d_r(I) = \max\{0, I(body) - I(head)\}$$

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  prefers(bob, ) $\geq$ prefers(bob, ) ?

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- MPE Inference
  prefers(bob, ) $\geq$ prefers(bob, ) ?
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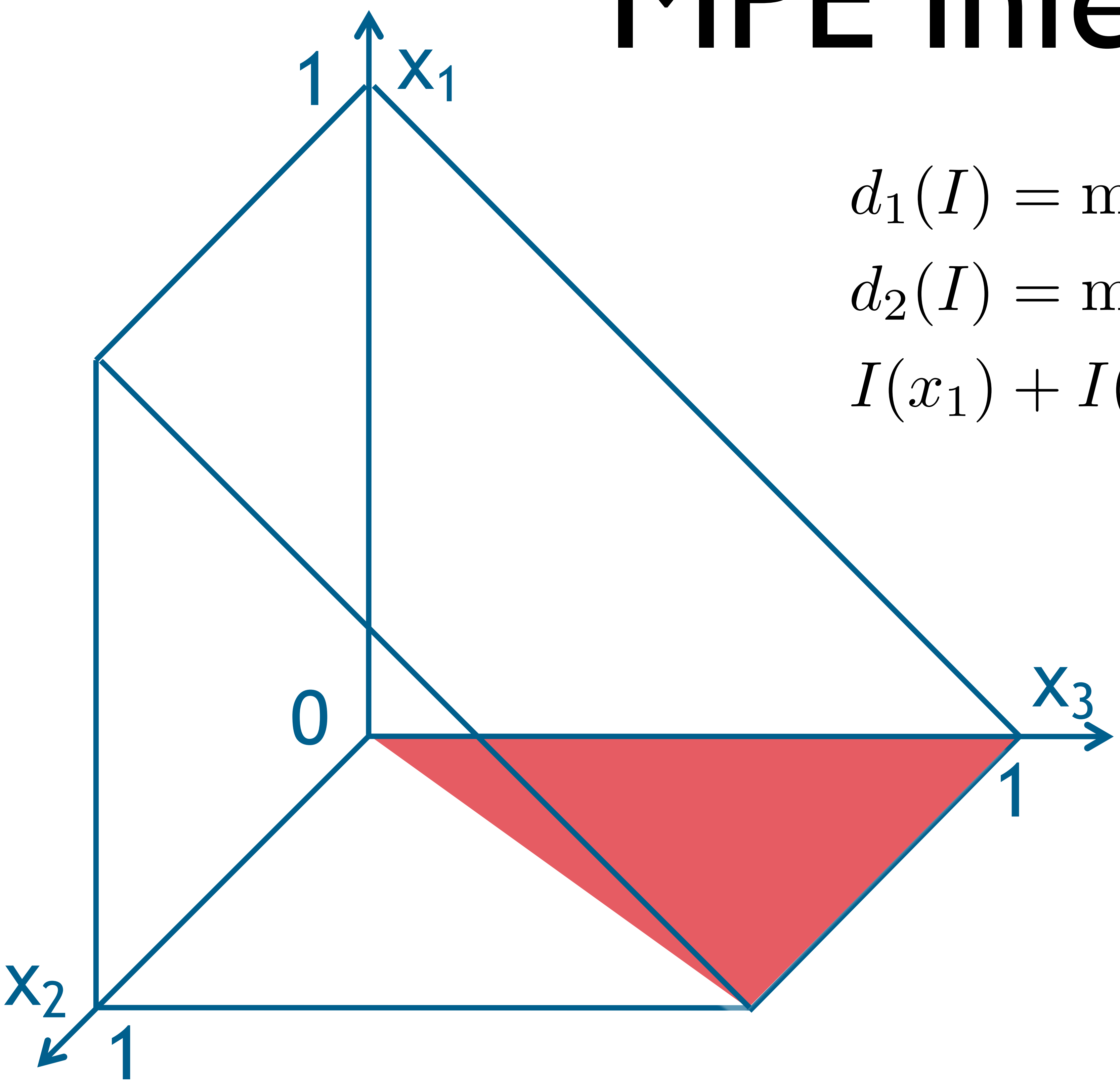
  P(prefers(bob, ) $\geq$ 0.8) ?
- Weight Learning
- Structure Learning

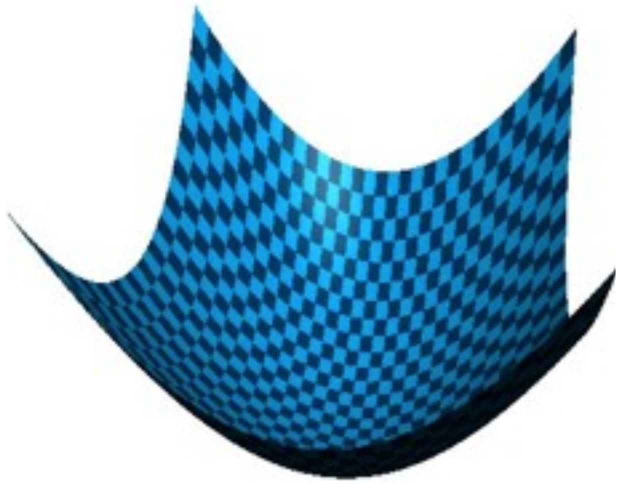
# Geometric Intuition: MPE Inference



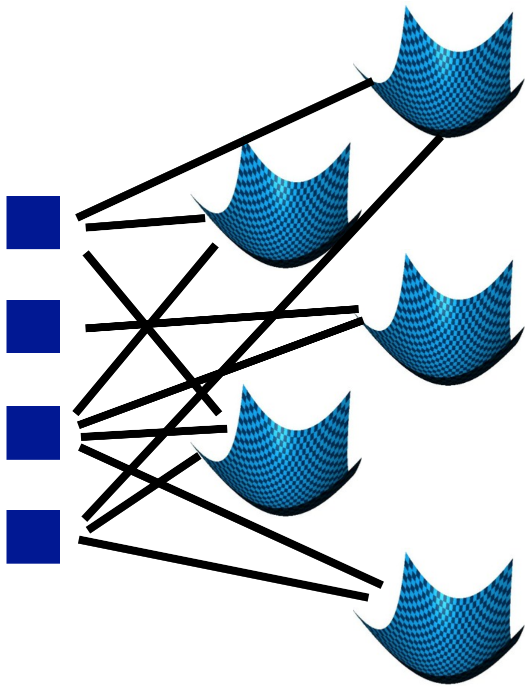

$$d_1(I) = \max\{0, I(x_1) - I(x_2)\}$$
$$d_2(I) = \max\{0, I(x_2) - I(x_3)\}$$
$$I(x_1) + I(x_3) \leq 1$$

most likely interpretations given $x_1$=0

# MPE Inference

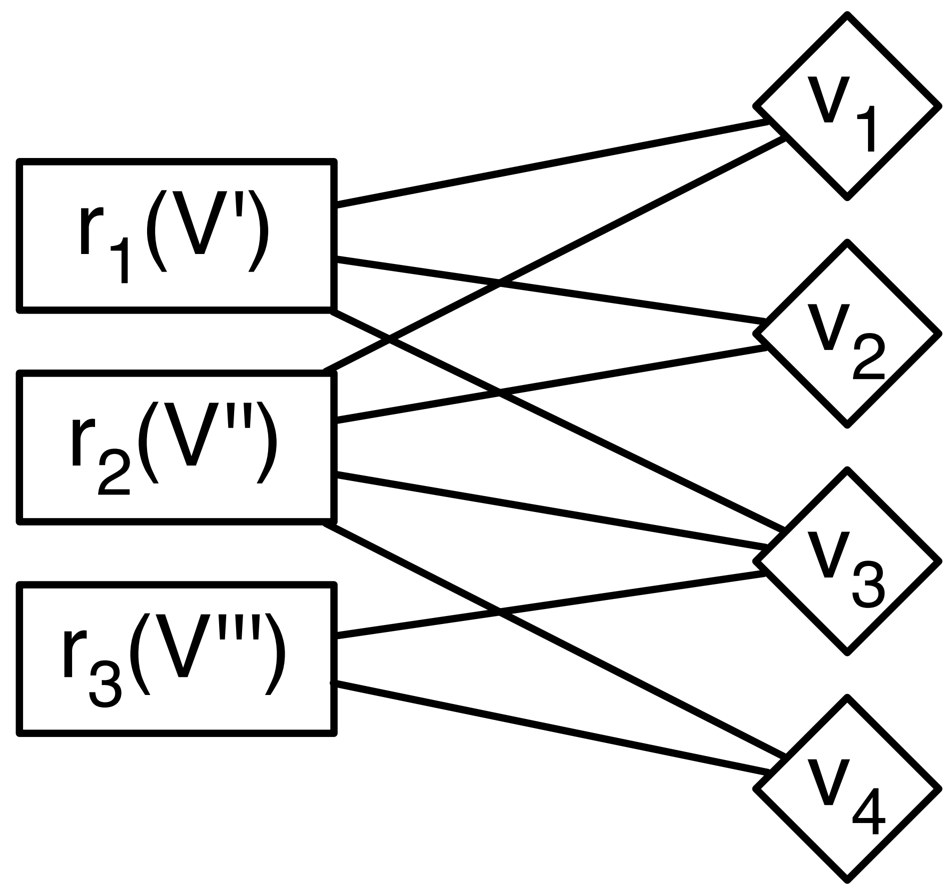


- Convex optimization problem
- New solver [Bach et al, NIPS 12]
  - Consensus optimization
  - Linear time in practice
  - Closed form solutions for subproblems



# Consensus Optimization

[Bach et al, NIPS 12]



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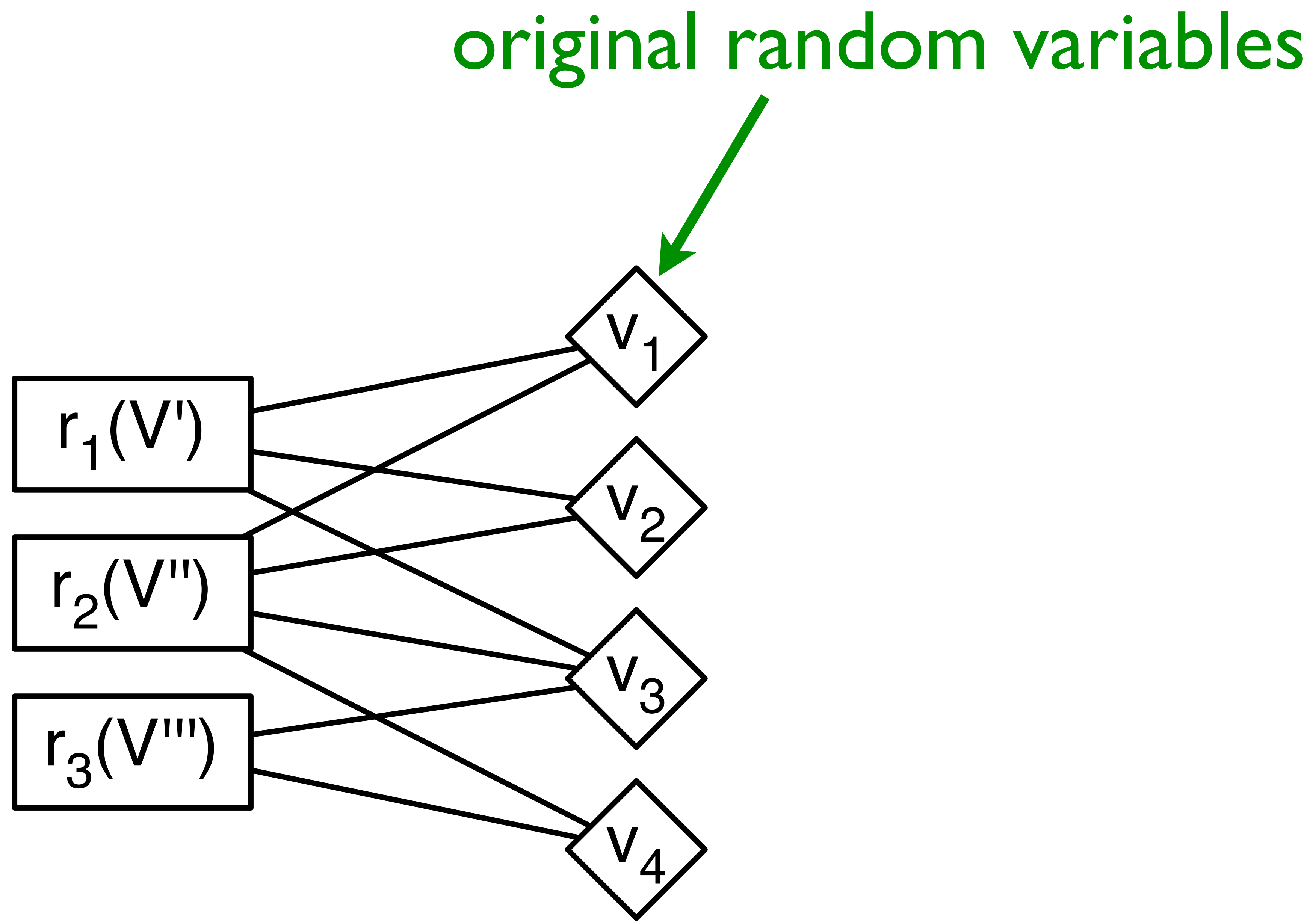

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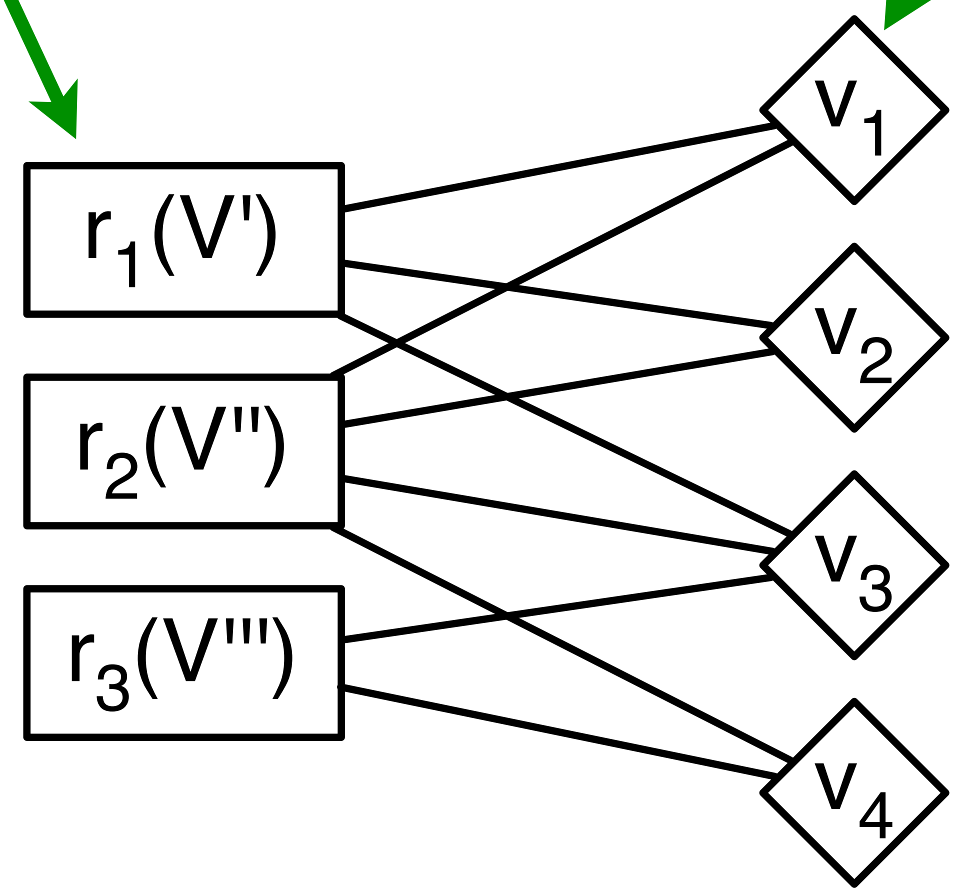
[Bach et al, NIPS 12]

rules with local copies of random variables

original random variables



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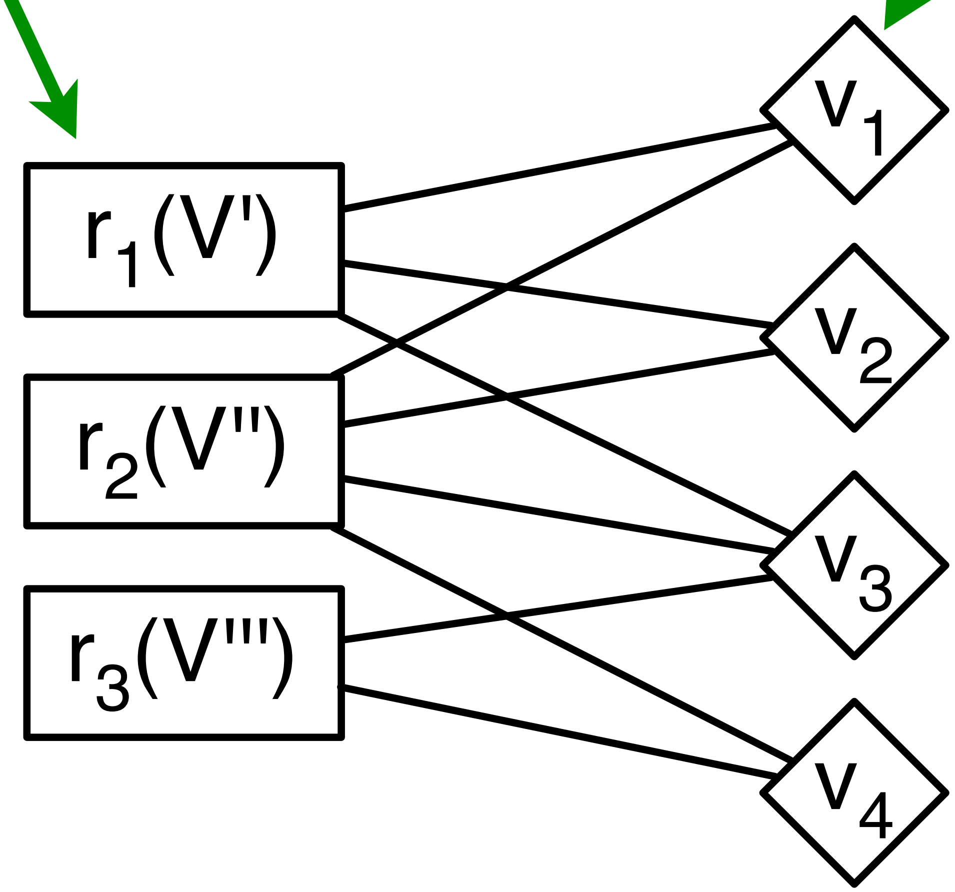

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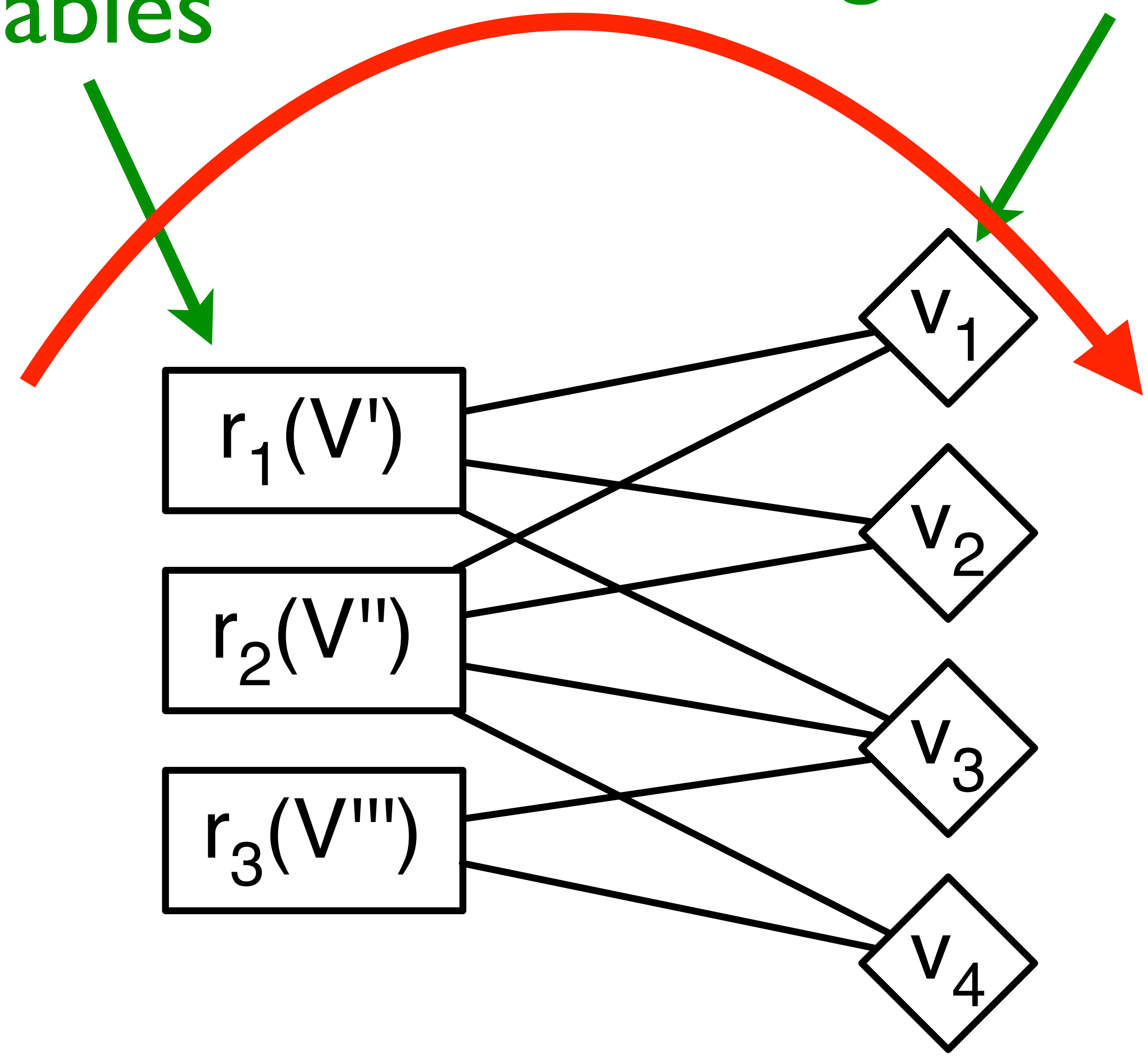


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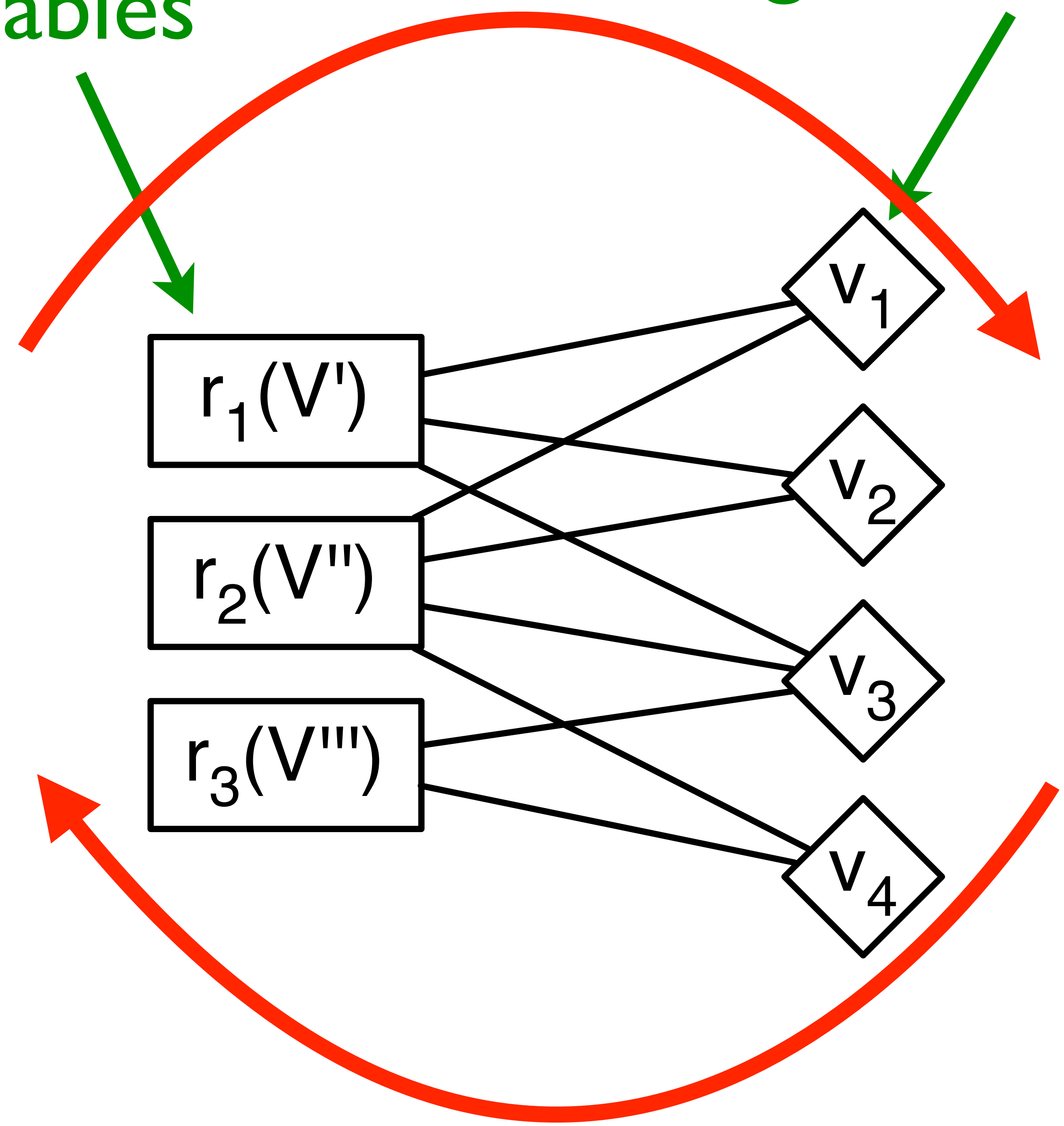


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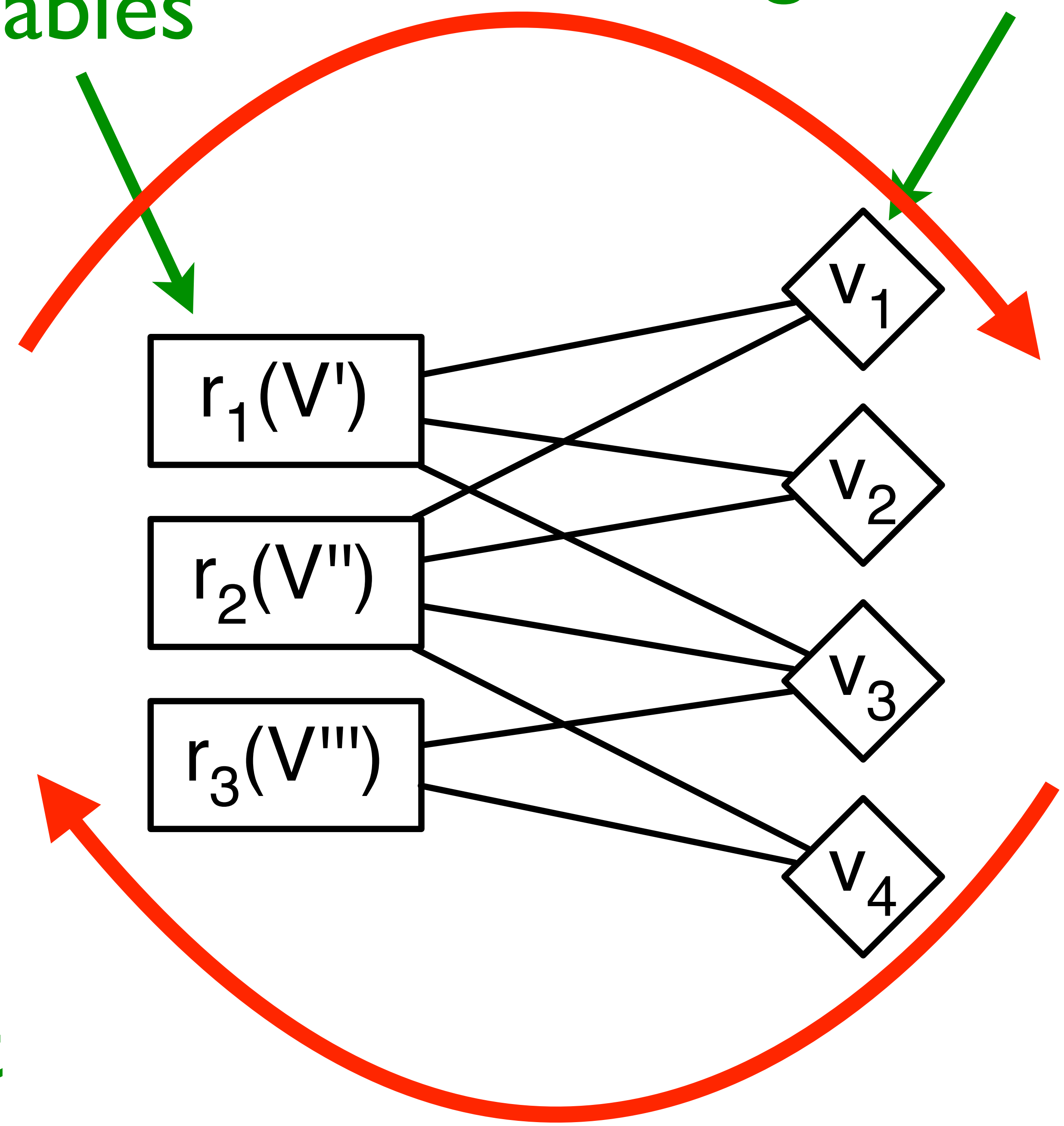
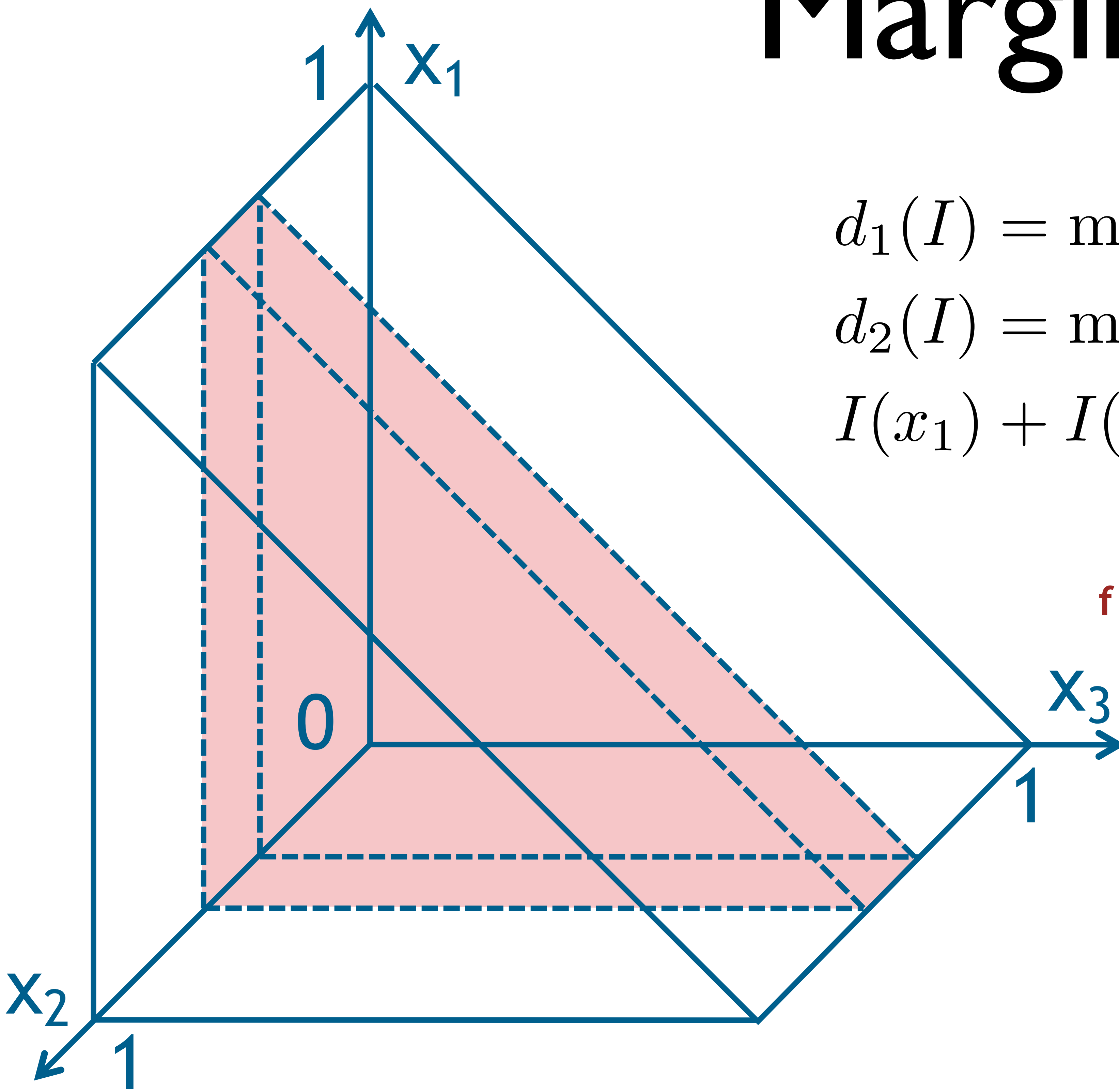



update variables to average of copies

new: fast solutions

# Geometric Intuition: Marginals

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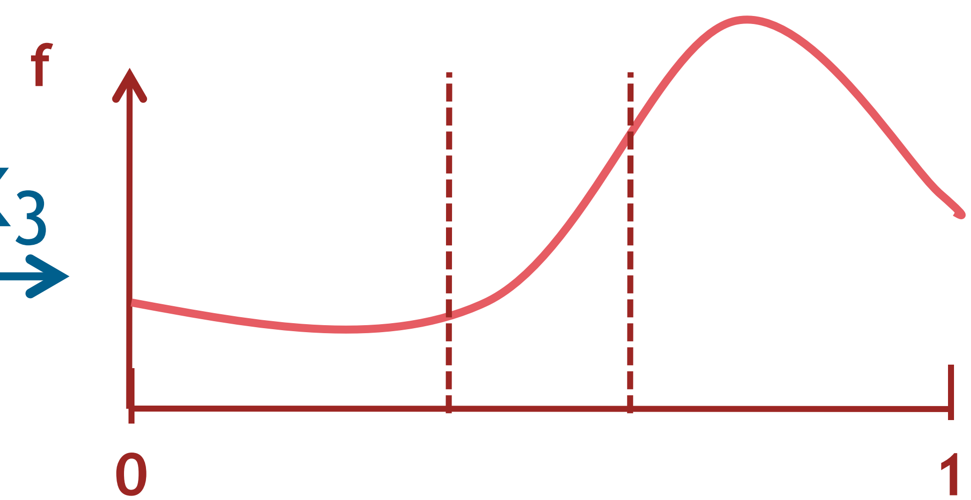
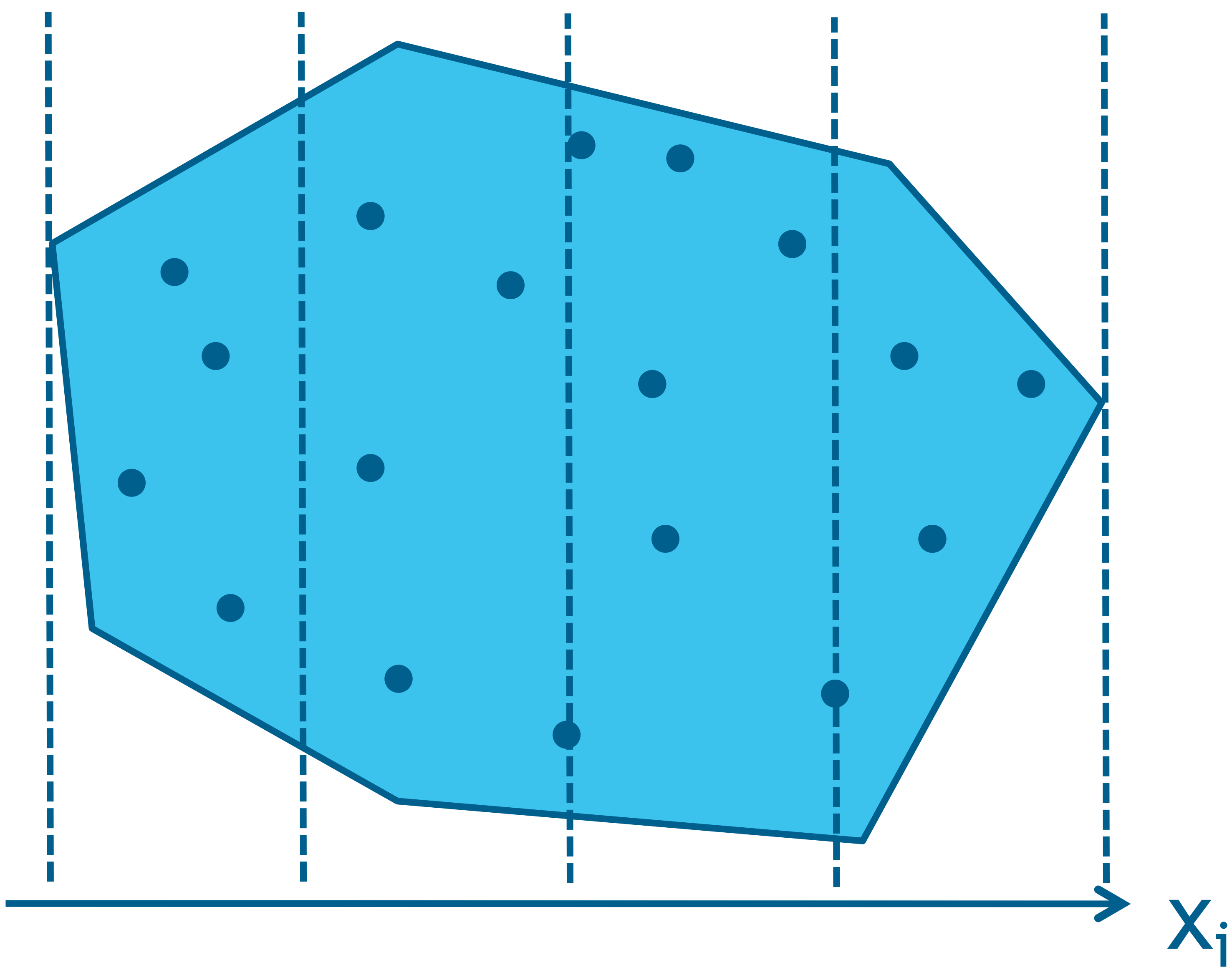



**$P(0.4 \leq x_2 \leq 0.6)$**

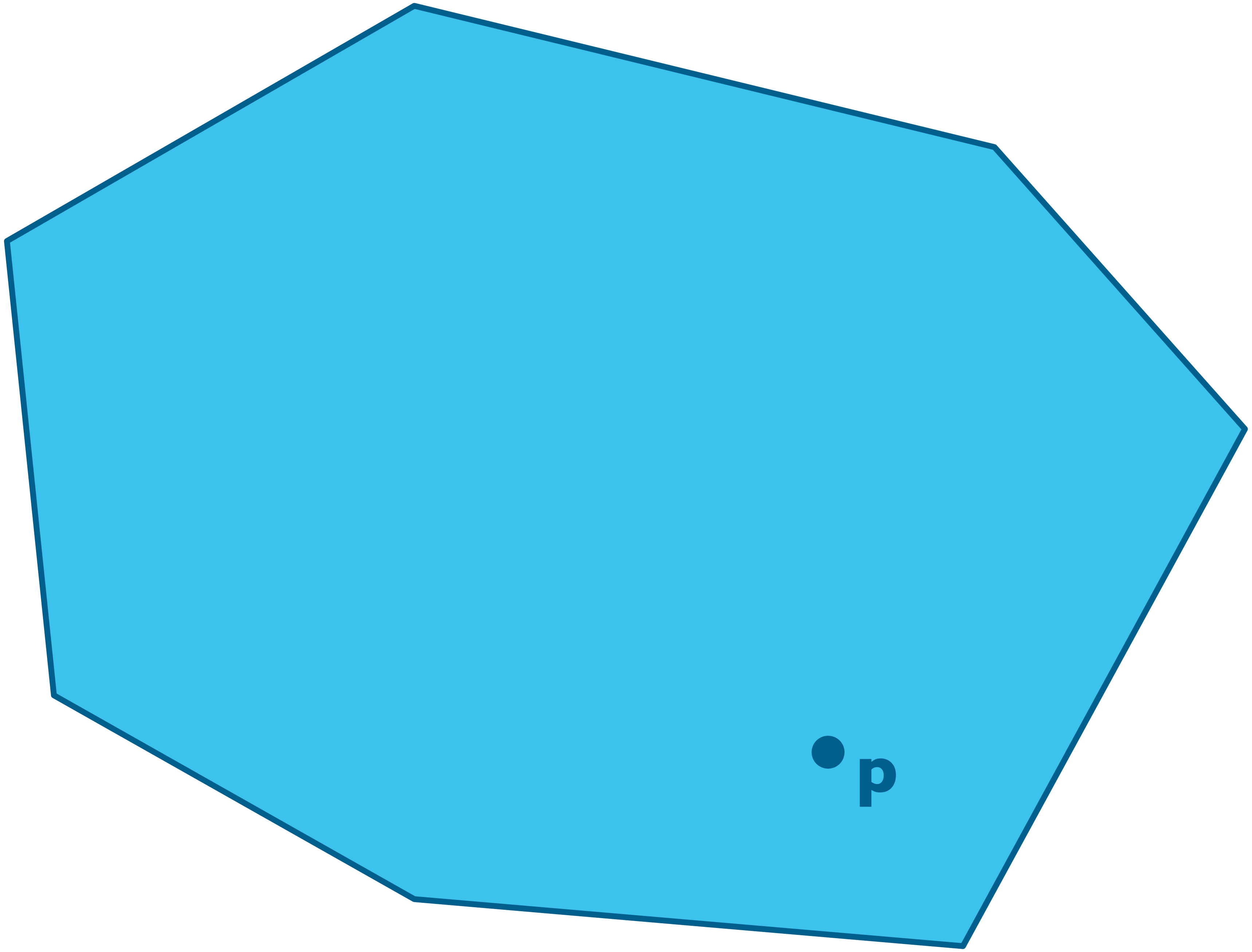
# Computing Marginals

[Broecheler and Getoor, NIPS 10]

Histogram sampling
using hit-and-run Monte Carlo scheme



# Hit-and-Run



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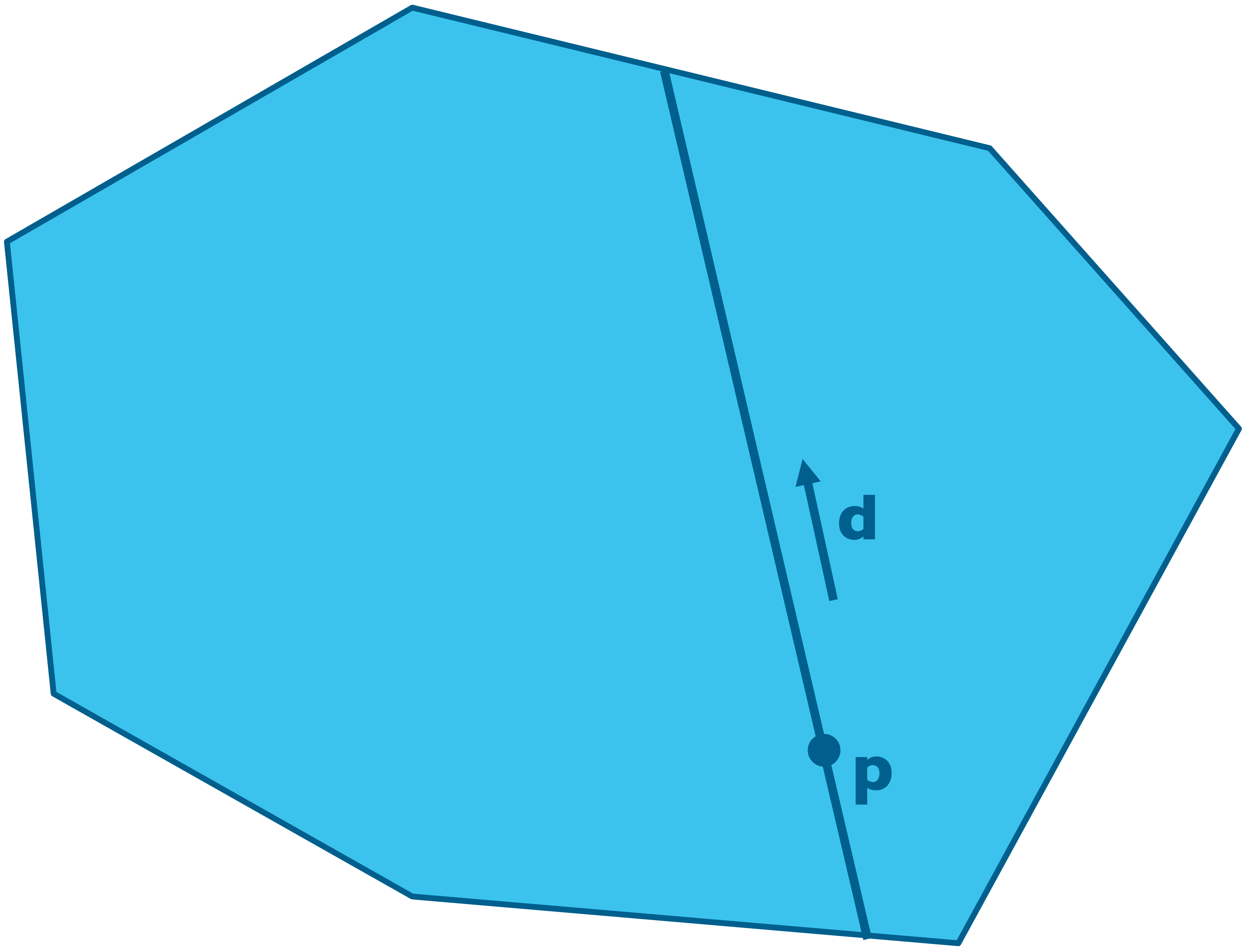

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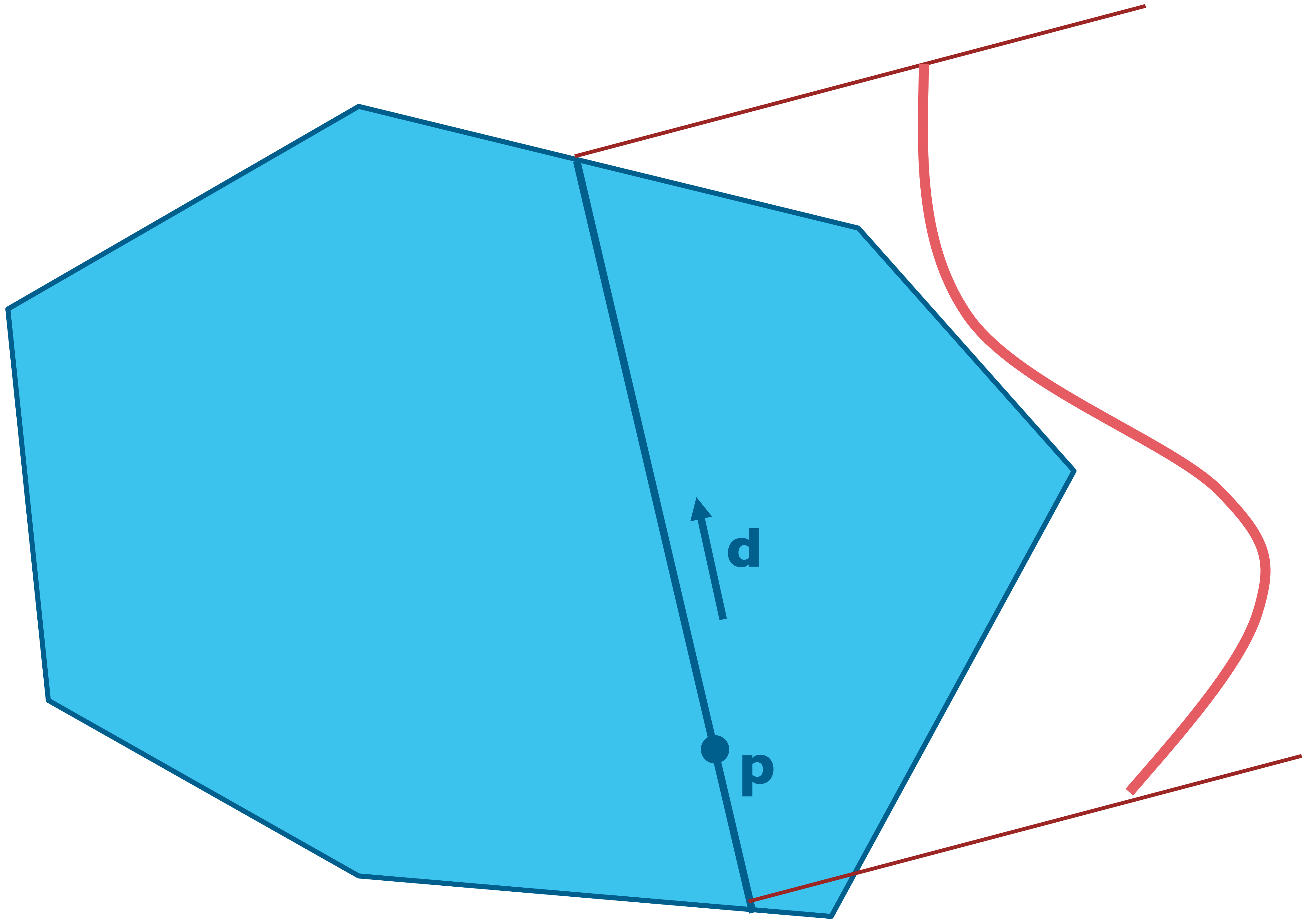

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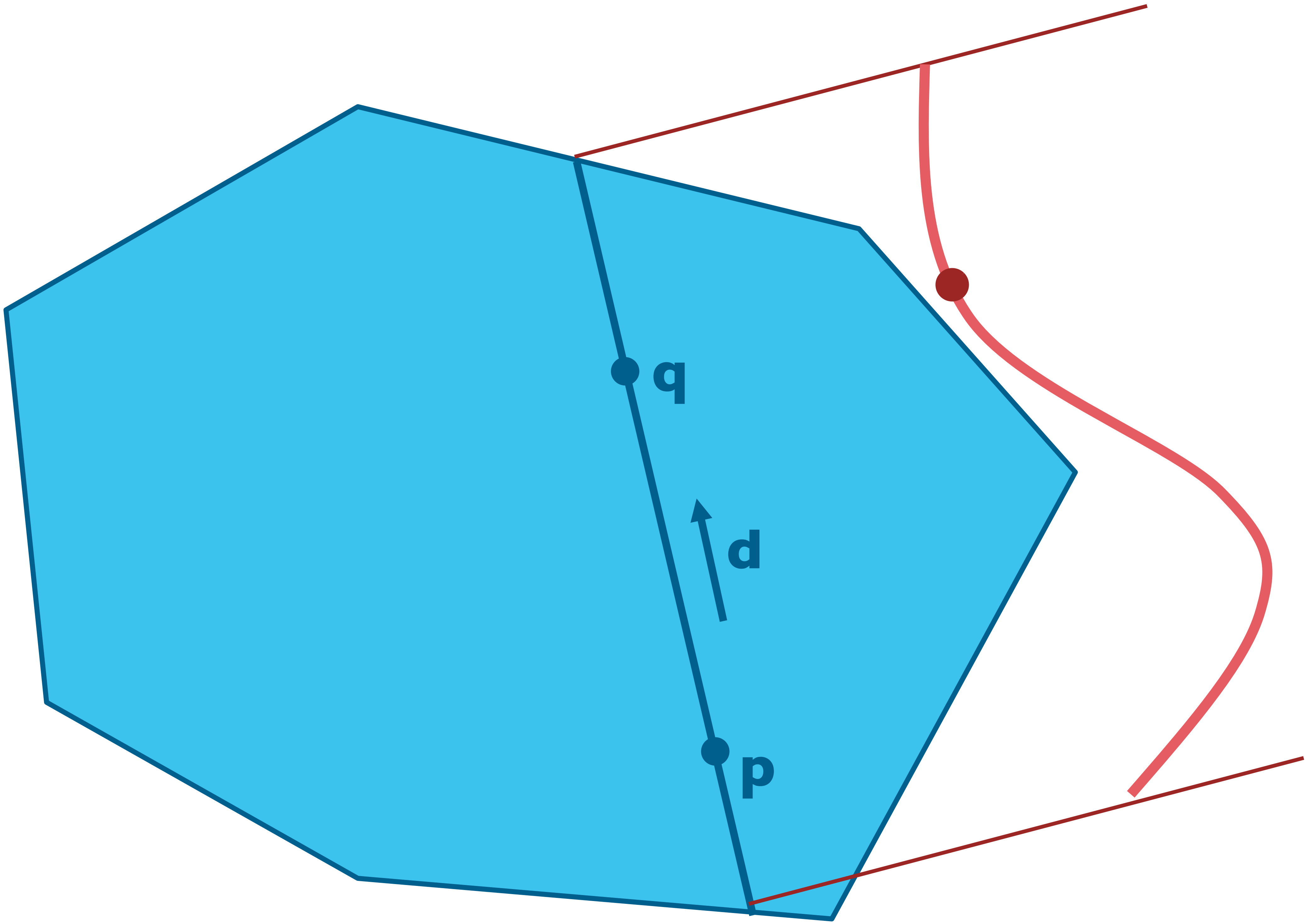

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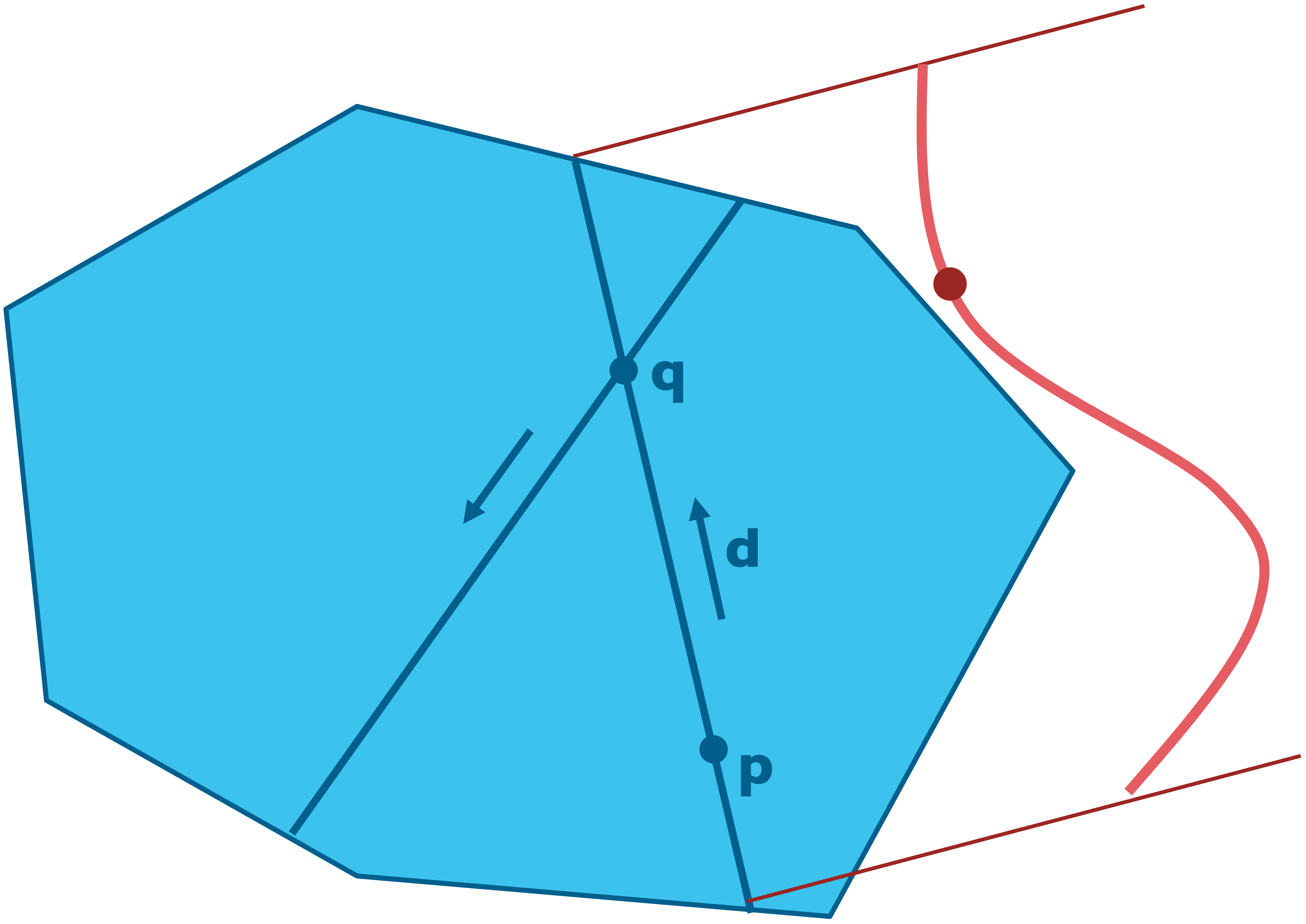

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- Dependencies as weighted first order rules
- Support for similarity functions and aggregation
- Linear (in)equality constraints

Inference

- MPE: consensus optimization
- Marginals: hit-and-run histogram sampling

# Thank you!

http://psl.umiacs.umd.edu